Capital Tax Competition with Heterogeneous Firms

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1 A Regional Economy with Fixed Tax Rates

The economy consists of two regions, denoted A and B. There are H_i workers in region i (where i is either A or B). The workers are immobile, and each worker supplies one unit of labour at every wage rate. The government of region i taxes each firm's profits at the rate t_i. The only agents with decisions to make are the mobile and immobile firms:

- A **mobile firm** is able to locate in either region. The productivity of each mobile firm depends upon the region in which it locates, and within each region, productivity varies across firms. Each mobile firm knows its productivity in each region, the tax rate in each region, and the competitive wage rate in each region. Given this information, it chooses its location and employment of labour so as to maximize its after-tax profits.
- An **immobile firm** is unable to relocate. The immobile firms in each region, knowing the competitive wage rate, choose their employment of labour so as to maximize their after-tax profits.

The productivity of a mobile firm in each region is determined in part by its **productivity factors** (θ_A, θ_B). The distribution of productivity factors across mobile firms is described by the density function $g(\theta_A, \theta_B)$. The density function is restricted in several ways.

ASSUMPTION A1: *Let S be the set of pairs (θ_A, θ_B) such that $g(\theta_A, \theta_B)$ is positive. Assume that there exists a positive number $\underline{\theta}$ such that $(\underline{\theta}, \underline{\theta})$ is a lower bound of S, and that there exists a positive finite number $\overline{\theta}$ such that $(\overline{\theta}, \overline{\theta})$ is an upper bound of S. Assume also that $g(\theta', \theta'')$ is equal to $g(\theta'', \theta')$ for every pair (θ', θ'') contained in S. Finally, for simplicity, assume that:*

$$\int_S \theta_A g(\theta_A, \theta_B) = \int_S \theta_B g(\theta_A, \theta_B) = 1$$

Another consideration that influences a firm's output in a particular region is agglomeration effects—that is, each firm benefits from the presence of other firms in the region. It might be that a greater concentration of firms makes each firm more productive because the transportation costs are lower or the opportunities to trade more plentiful. Alternatively, it might be easier to learn about state-of-the-art production technologies when firms cluster together. The exact nature of the agglomeration effect is left unspecified; instead, it is simply assumed that each firm's output is greater when economic activity in its region is greater.

Let y_i be a mobile firm's output in region i, and let h be the quantity of labour employed by the firm. Let Y_i be regional output. Each mobile firm's production function is:

$$y_i = (1/\beta)(\theta_i)^{1-\beta}(Y_i)^{\alpha}h^{\beta}$$

where the parameters α and β are positive and smaller than one. This production function incorporates the basic assumptions: the firm's output rises with its own productivity factor, and both regional output and employment have positive but diminishing marginal products. A Cobb-Douglas function is used for tractability (the relative sizes of α and β play a key role in the analysis).

The immobile firms in each region are treated as an aggregate. The productivity factor of the immobile firms in region i is η_i, and these firms hire $\overline{h}_i$ units of labour. Their aggregate output is $\overline{Y}_i$, where:

$$\overline{Y}_i = (1/\beta)(\eta_i)^{1-\beta}(Y_i)^{\alpha}(\overline{h}_i)^{\beta}$$

An equilibrium in this economy is formally described as follows.

DEFINITION: *An* **equilibrium with fixed tax rates** *has these properties:*
(a) Each mobile firm, knowing the regional outputs, the regional tax rates and the regional wage rates, chooses a location (either region A or region B) and a level of employment to maximize its after-tax profits.
(b) The immobile firms in each region, knowing the wage rate and the output in the region, choose a level of employment to maximize their aggregate after-tax profits.
(c) The wage rate w_i clears the labour market in region i.
(d) Regional output is the aggregate of the outputs of the individual firms.

1.1 The Employment and Location Decisions

A mobile firm which has decided to locate in region i can hire any quantity of labour at the market wage w_i. It will choose the quantity of labour h that maximizes its after-tax profits:

$$\pi_i = (1 - t_i)\left\{(1/\beta)(\theta_i)^{1-\beta}(Y_i)^{\alpha}h^{\beta} - w_i h\right\}$$

This quantity of labour is:

$$h = \theta_i (Y_i)^{\frac{\alpha}{1-\beta}} (w_i)^{-\frac{1}{1-\beta}} \tag{1}$$

Evaluating the firm's output and profits at this employment level gives:

$$y_i = (1/\beta)\theta_i (Y_i)^{\frac{\alpha}{1-\beta}} (w_i)^{-\frac{\beta}{1-\beta}} \tag{2}$$

$$\pi_i = (1 - \beta)(1 - t_i)y_i \tag{3}$$

A mobile firm locates in the region in which its after-tax profits are greater. It will locate in region A if:

$$\pi_A \geq \pi_B$$
$$(1 - t_A)y_A \geq (1 - t_B)y_B$$

A firm's output in each region depends on its productivity factor in that region *and* on the regional output and wage rate. Consequently, a mobile firm will locate in region A if θ_A/θ_B is greater than or equal to the critical value k, where:

$$k = \left(\frac{1 - t_B}{1 - t_A}\right)\left(\frac{Y_B}{Y_A}\right)^{\frac{\alpha}{1-\beta}}\left(\frac{w_A}{w_B}\right)^{\frac{\beta}{1-\beta}} \tag{4}$$

and it will locate in region B otherwise. This decision rule is the one required by part (a) of the definition of equilibrium. The employment and output rules required by the same part are (1) and (2).

The immobile firms employ the quantity of labour that maximizes their after-tax profits. Their employment of labour, output and after tax profits are:

$$\overline{h}_i = \eta_i (Y_i)^{\frac{\alpha}{1-\beta}} (w_i)^{-\frac{1}{1-\beta}} \tag{5}$$

$$\overline{Y}_i = (1/\beta)\eta_i (Y_i)^{\frac{\alpha}{1-\beta}} (w_i)^{-\frac{\beta}{1-\beta}} \tag{6}$$

These are the employment and output rules required by part (b) of the definition of equilibrium. The after-tax profits of the immobile firms are:

$$\overline{\pi}_i = (1-\beta)(1-t_i)\overline{Y}_i \tag{7}$$

1.2 The Regional Labour Markets

Define the function:

$$\widehat{h}(x, Y_i, w_i) \equiv x(Y_i)^{\frac{\alpha}{1-\beta}}(w_i)^{-\frac{1}{1-\beta}}$$

Then the quantities of labour employed by a mobile firm with productivity factor θ_i in region i, and by the immobile firms in the same region, are $\widehat{h}(\theta_i, Y_i, w_i)$ and $\widehat{h}(\eta_i, Y_i, w_i)$. The total demand for labour in region A is:

$$H_A^D = \widehat{h}(\eta_A, Y_A, w_A) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{k\theta_B}^{\overline{\theta}} \widehat{h}(\theta_A, Y_A, w_A) g(\theta_A, \theta_B) d\theta_A d\theta_B$$

and the total demand for labour in region B is:

$$H_B^D = \widehat{h}(\eta_B, Y_B, w_B) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{k\theta_B} \widehat{h}(\theta_B, Y_B, w_B) g(\theta_A, \theta_B) d\theta_A d\theta_B$$

Alternatively,

$$H_i^D = (Y_i)^{\frac{\alpha}{1-\beta}}(w_i)^{-\frac{1}{1-\beta}}\{\eta_i + z_i(k)\}$$

where $z_i(k)$ aggregates the productivity factors of the mobile firms that locate in region i under a particular value of k:

$$z_A(k) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \int_{k\theta_B}^{\overline{\theta}} \theta_A g(\theta_A, \theta_B) d\theta_A d\theta_B$$

$$z_B(k) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{k\theta_B} \theta_B g(\theta_A, \theta_B) d\theta_A d\theta_B$$

Note that:

$$z_A'(k) = -kz_B'(k) \leq 0 \tag{8}$$

An increase in the critical value k causes some firms that had previously located in A to shift to B, causing z_A to fall and z_B to rise. Each of the firms that switches from one region to the other is a marginal firm at the time of the switch, implying that θ_A is just equal to

$k\theta_B$. The loss in region A's aggregate productivity is therefore k times as large as region B's gain.[1]

Part (c) of the definition of equilibrium requires labour demand to be equal to labour supply in each region. This requirement is satisfied when:

$$(Y_i)^{\frac{\alpha}{1-\beta}} (w_i)^{-\frac{1}{1-\beta}} \{\eta_i + z_i(k)\} = H_i \qquad i = A, B \tag{9}$$

1.3 Regional Output

Part (d) of the definition of equilibrium requires that the output of a region be equal to the sum of the outputs of the individual firms; but there is a simpler way of finding regional output. Since each firm's profits are the fraction $1 - \beta$ of its output, each firm's wage bill is the fraction β of its output. The regional outputs therefore satisfy the conditions:

$$Y_i = (1/\beta) w_i H_i \qquad i = A, B \tag{10}$$

1.4 Equilibrium

An equilibrium with fixed tax rates consists of the following information:

- The location, output and employment of each mobile firm.
- The output and employment of the immobile firms in each region.
- The market-clearing wage in each region.
- The output of each region.

Note, however, that if the values taken by five of the variables are known (k, Y_A, Y_B, w_A, w_B), all of the remaining information can be deduced from them. The location of every mobile firm is determined by k. Once the location of a mobile firm is known, its employment of labour and its output is given by (1) and (2). The employment and output of the immobile firms in each region are determined by (5) and (6). The only arguments in these equations are Y_A, Y_B, w_A and w_B.

[1] The inequality in (8) is weak because $z_i'(k)$ is equal to zero at any k for which there are no (θ_A, θ_B) in S such that θ_A is equal to $\theta_B k$ (so that a small change in k does not induce any firm to switch regions).

The five key variables are themselves determined by the five equation system consisting of (4), the two equations in (9), and the two equations in (10). This system can be considerably simplified. Use the last four equations (in pairs) to obtain each region's output and wage rate in terms of k:

$$Y_i = \left[(1/\beta)\,(H_i)^{\beta}\,(\eta_i + z_i(k))^{1-\beta}\right]^{\frac{1}{1-\alpha}} \tag{11}$$

$$w_i = \left[(1/\beta)^{\alpha}\,(H_i)^{\alpha+\beta-1}\,(\eta_i + z_i(k))^{1-\beta}\right]^{\frac{1}{1-\alpha}} \tag{12}$$

Output rises as the supply of labour rises and as the aggregate productivity of the firms in the region rises. The wage rises as the aggregate productivity of the firms rises, but might either rise or fall as the supply of labour rises. The latter result follows from the presence of agglomeration. Suppose that the supply of labour rises and that the wage adjusts to absorb all of the additional labour. The marginal product of labour at each firm falls because more labour is being used; but it rises because every firm is using more labour and producing more output, causing regional output to rise. The wage rate falls if the former effect dominates, and rises if the latter effect dominates.

Now consider the determination of k. Substituting (12) into (4) gives:

$$k = \left(\frac{1-t_B}{1-t_A}\right)\left(\frac{H_B}{H_A}\right)^{\frac{\beta}{1-\beta}}\left(\frac{Y_B}{Y_A}\right)^{\frac{\alpha-\beta}{1-\beta}} \tag{13}$$

An increase in the value of k means that fewer firms will choose to locate in region A. The influence of agglomeration can also be seen in this decision rule. An increase in region B's output could cause k to either rise or fall. The marginal product of labour (and therefore the wage rate) in region B rises, pushing some of the firms into region A in pursuit of lower wages. However, a greater regional output raises the output of every firm that chooses to locate in the region, so that firms are drawn into the region. The former effect dominates (*i.e.*, k rises) if β is greater than α, and the latter effect dominates if β is smaller than α.

Now substitute (11) into (13) to obtain:

$$k = \left(\frac{1-t_B}{1-t_A}\right)\left(\frac{H_B}{H_A}\right)^{\frac{\beta}{1-\alpha}}\left(\frac{\eta_B + z_B(k)}{\eta_A + z_A(k)}\right)^{\frac{\alpha-\beta}{1-\alpha}}$$

or:

$$k = TL\mu(k) \tag{14}$$

where:

$$T \equiv \frac{1-t_B}{1-t_A}$$

$$L \equiv \left(\frac{H_B}{H_A}\right)^{\frac{\beta}{1-\alpha}}$$

$$\mu(k;\eta_A,\eta_B) \equiv \left(\frac{\eta_B + z_B(k)}{\eta_A + z_A(k)}\right)^{\frac{\alpha-\beta}{1-\alpha}}$$

To interpret (14), imagine that the mobile firms initially allocate themselves to the two regions according to the rule:

Locate in A if θ_A is greater than or equal to $k\theta_A$; otherwise, locate in A.

For any given value of k, a firm maximizes its after-tax profits by locating in A if θ_A is greater than or equal to $TL\mu(k)\theta_B$ and by locating in B it is not. The equilibrium value of k is the one under which no firm regrets its location because the profit-maximizing critical value, $TL\mu(k)$, is equal to the current critical value, k.[2]

If $TL\mu(k)$ is greater than k, some of the firms that are initially located in A will move to B, causing k to rise; and if $TL\mu(k)$ is smaller than k, some of the firms that are initially located in B will move to A, causing k to fall. This observation motivates the following definition:

DEFINITION: *Let k^* be the critical value in an equilibrium with fixed tax rates. The equilibrium is* **stable** *if $TL\mu(k)$ is greater than k when k is less than k^*, and less than k when k is greater than k^*. It is* **unstable** *if $TL\mu(k)$ is less than k when k is less than k^*, and greater than k when k is greater than k^*.*

It will be shown below that economies with strong agglomeration effects are not always

[2] Actually, (14) only describes an *interior* equilibrium. Suppose that all of the mobile firms locate in A when k is equal to k', and that all of the mobile firms locate in B when k is equal to k''. Then k' is an equilibrium if $TL\mu(k')$ is less than k', and k'' is an equilibrium if $TL\mu(k'')$ is greater than k''. This kind of equilibrium only arises when the interior equilibrium is unstable, and situations that give rise to unstable interior equilibria are largely ignored in the discussion that follows.

stable.

1.5 Total Output

Equation (11) shows that in any equilibrium with fixed tax rates, each region's output is determined by k. Since the total output of the economy is the sum of the regional outputs, it is also determined by k. Differentiating $Y_A + Y_B$ with respect to k and simplifying the resulting expression shows that:

$$\text{sign}\left[\frac{d(Y_A + Y_B)}{dk}\right] = \text{sign}\left[\, L\mu(k) - k\right]$$

Let k^{**} be a stationary point of $Y_A + Y_B$. The stationary point is a local maximum if $L\mu(k)$ is great than k when k is less than k^{**}, and less than k when k is greater than k^{**}. It is a local minimum if $L\mu(k)$ is less than k when k is less than k^{**}, and greater than k when k is greater than k^{**}.

Comparing these results with the above definition leads immediately to the following observation:

> LEMMA: *Let k^* be the unique critical value in an equilibrium in which the tax rates are fixed and equal (so that T is equal to one). Then k^* maximizes total output if and only if the economy is stable, and it minimizes total output if and only if the economy is unstable.*

If the agglomeration effects are very strong, the economy is unstable and total output is maximized when all of the mobile firms are located in the same region. Nevertheless, weaker agglomeration effects could lead to a stability equilibrium in which output is maximized when the mobile firms are divided between the two regions.

2 Equilibrium with Symmetric Regions

The firms' location decision is governed by two opposing factors: profits are higher where economic activity is high, and profits are higher where labour is cheap. These factors are highlighted in this section by imagining that the two regions are identical.

If the tax rates, the labour supplies and the productivities of the immobile firms are the same in both regions, the equilibrium value of k is characterized by:

$$k = \mu(k)$$

where:

$$\mu(k) = \left(\frac{\eta + z_B(k)}{\eta + z_A(k)}\right)^{\frac{\alpha-\beta}{1-\alpha}}$$

The assumption that the frequency distribution is symmetric implies that $z_A(1)$ is equal to $z_B(1)$, so the equilibrium value of k is 1.

2.1 Cheap Labour Dominates

If β is greater than α, $\mu(k)$ is non-increasing in k (specifically, it is negatively sloped if the z_i' are non-zero and flat if they are equal to zero). This case is shown in Figure 1. The graphs of $\mu(k)$ and k are shown for all values of k between $\underline{k}$ and $\overline{k}$, where:

$$\underline{k} \equiv \underline{\theta}/\overline{\theta} < 1$$

$$\overline{k} \equiv \overline{\theta}/\underline{\theta} > 1$$

When k is equal to $\underline{k}$, every mobile firm goes to A, implying:

$$\mu(\underline{k}) = \left(\frac{\eta}{\eta + 1}\right)^{\frac{\alpha-\beta}{1-\alpha}} > 1$$

Similarly, when k is equal to $\overline{k}$, every mobile firm goes to B, implying:

$$\mu(\overline{k}) = \left(\frac{\eta + 1}{\eta}\right)^{\frac{\alpha-\beta}{1-\alpha}} < 1$$

Since $\mu(k)$ is continuous and non-increasing, the graphs of $\mu(k)$ and k intersect exactly once between $\underline{k}$ and $\overline{k}$. As noted above, that intersection occurs when k is equal to 1.

The equilibrium is unique and stable. The equilibrium also maximizes the economy's total output. Since the equilibrium value of k is 1, every mobile firm locates in the region in which it has the higher productivity factor θ_i.

2.2 Agglomeration Dominates

If α is greater than β, $\mu(k)$ is non-decreasing in k. Multiple equilibria are possible, but the remainder of the paper will assume that there is only one equilibrium. There are nevertheless two possible cases, both of which are illustrated in Figure 2.

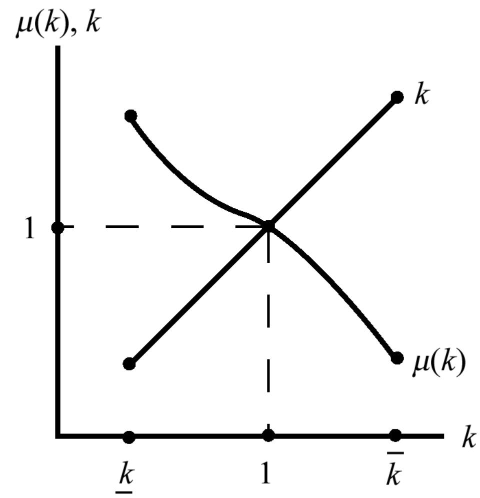

Figure 1: The equilibrium value of k when β is greater than α.

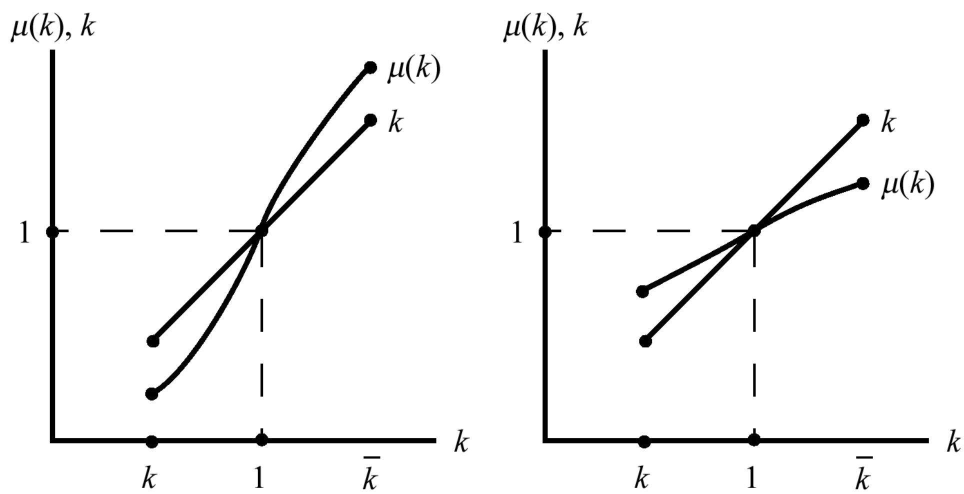

Figure 2: Equilibrium when α is greater than β. These figures assume that the equilibrium is unique. The left-hand figure corresponds to a small value of η, and the right-hand figure corresponds to a large value of η.

If η is sufficiently small,

$$\mu(\underline{k}) < \underline{k} < 1 < \overline{k} < \mu(\overline{k})$$

so that the graph of $\mu(k)$ cuts the graph of k from below, as shown in the left-hand panel of Figure 2. The equilibrium is unstable, and total output is maximized when all of the firms locate in one region.[3]

Somewhat larger values of η lead to the configuration:[4]

$$\underline{k} < \mu(\underline{k}) < 1 < \mu(\overline{k}) < \overline{k}$$

The graph of $\mu(k)$ then cuts the graph of k from below, as shown in the right-hand panel of Figure 2. The equilibrium is stable. Total output attains its maximum value, with each mobile firm locating in the region where its productivity factor is highest.

3 Comparative Statics

The comparative statics of an equilibrium with exogenous tax rates are derived in the appendix and summarized here. Changes in the tax rates have very simple effects. An increase in T causes k to increase, so that fewer firms locate in A and more firms locate in B. This migration of firms reduces region A's output and increases region B's output.

$$\frac{\partial k}{\partial T} > 0 \qquad \frac{\partial Y_A}{\partial T} < 0 \qquad \frac{\partial Y_B}{\partial T} > 0$$

[3] If all of the mobile firms locate in one region, some of them are choosing to locate in that region even though their productivity factors in that region is lower—possibly substantially lower—than their productivity factors in the other region. They do so because the agglomeration effect caused by concentrating all of the firms in one region more than compensates them for choosing the region in which their productivity factor is lower.

[4] Every mobile firm finds it more profitable to locate in the same region as every other mobile firm only if η is very small, so that a defecting firm (whose productivity factors cause it to strongly favour the other region) would find its profits lower in the other region. Since there is a diminishing marginal product to economic activity, an increase in η causes each firm's profits to go up more in the low-activity (immobile firms only) region than in the high-activity region—that is, it would decrease the advantage of remaining in the high-activity region. A sufficiently large increase in η would raise the profits of some firms to switch regions, leading to an interior solution.

Changes in the supply of labour also have relatively simple effects. An increase in a region's supply of labour raises the output of that region, and therefore raises the output of every firm that chooses to locate in that region. Some firms will be induced to move into that region, generating a further increase in that region's output and a decrease in the other region's output.

$$\frac{\partial k}{\partial L} > 0 \qquad \frac{\partial Y_A}{\partial L} < 0 \qquad \frac{\partial Y_B}{\partial L} > 0$$

The effects of changes in the productivity of immobile firms depend upon the importance of agglomeration,—specifically, on the relative sizes of α and β. Suppose that η_B increases. The direct effect of this change will be to increase the output of the immobile firm's in region B, but this increase induces two further effects.

- Every firm is more productive when economic activity is higher, so every mobile firm that chooses to locate in region B would have higher output now than previously. This effect induces some mobile firms to locate in B rather than A.
- The higher level of output in region B means that the marginal product of labour, and hence the wage rate is higher. This effect reduces the profits of firms in region B, and will induce some mobile firms to leave the region in search of lower wages.

The former effect dominates if α is greater than β, so k rises. The latter effect dominates if α is less than β, causing k to fall. An increase in k reinforces the initial increase in region B's output and causes region A's output to fall. A decrease in k partially—but not entirely—offsets the initial increase in region B's output and causes region A's output to rise.

$$\text{sign}\left[\frac{\partial k}{\partial \eta_B}\right] = \text{sign}\left[\alpha - \beta\right] \qquad \text{sign}\left[\frac{\partial Y_A}{\partial \eta_B}\right] = \text{sign}\left[\beta - \alpha\right] \qquad \frac{\partial Y_B}{\partial \eta_B} > 0$$

Similar arguments apply when η_A rises, implying:

$$\text{sign}\left[\frac{\partial k}{\partial \eta_A}\right] = \text{sign}\left[\beta - \alpha\right] \qquad \frac{\partial Y_A}{\partial \eta_A} > 0 \qquad \text{sign}\left[\frac{\partial Y_B}{\partial \eta_A}\right] = \text{sign}\left[\beta - \alpha\right]$$

4 Tax Competition

Assume that the workers in region i own the fraction γ_i of each firm, whether mobile or immobile, and that:

$$\gamma_A + \gamma_B = 1$$

Let the utility of a worker in region i be:

$$U_i = u(G_i, c_i)$$

where G_i is the quantity of public good provided by the government, and c_i is the worker's consumption of private goods. The government can impose a tax on the workers' incomes, and since labour is immobile, this tax will be a lump sum tax. The revenue from this tax can be combined with the revenue from the tax on corporate profits to finance the public good, or the revenue from the wage tax could be split between the provision of the public good and the subsidization of the firms. (This subsidization would take the form of a negative tax on corporate profits.) In either case, the choice of G_i and c_i is constrained by the government's budget constraint. The budget constraint for region i is:

$$G_i = R_i - c_i H_i$$

where R_i is the resources available to region i. Each worker in region i is allowed to retain a part of his wage earnings c_i as his private consumption, and the remainder of the region's resources are allocated to the public good. The resources of the two regions are:

$$R_A = Y_A + (1-\beta)[\gamma_A(1-t_B)Y_B - \gamma_B(1-t_A)Y_A]$$

$$R_B = Y_B - (1-\beta)[\gamma_A(1-t_B)Y_B - \gamma_B(1-t_A)Y_A]$$

Profits constitute the fraction $1-\beta$ of each region's output. The resources available to region A include all of its own output except for region B's share of region A's after-tax profits, plus region A's share of region B's after-tax profits. The resources of region B are calculated in a similar fashion.

Assume that the government chooses c_i and t_i to maximize the worker's utility. If it chooses c_i optimally, the worker's utility under any tax t_i is:

$$V(R_i) \equiv \max_{c_i} u(R_i - c_i H_i, c_i)$$

Clearly, the government maximizes the worker's utility by choosing t_i to maximize its resources R.

The government of each region takes the other region's tax rate as given when it chooses its own tax rate. Each government will recognize that the effects of any tax rate that they choose depends upon the tax rate chosen by the other government. The two governments are involved in a game.

DEFINITION: *The* **tax competition game** *has these characteristics:*
(a) The players are the two governments, A and B.
(b) Government i (where i is either A or B) chooses the tax rate t_i. The tax rate must be smaller than 1, but it can be either positive or negative.
(c) The governments recognize that, for any pair of tax rates they choose, the economy will reach an equilibrium with fixed tax rates. Hence, each government recognizes that the output of its own region depends upon both tax rates.
(d) Each government wishes to maximize R_i.

A **Nash equilibrium** in the tax competition game is a pair (t_A^*, t_B^*) such that neither government can increase its own resources by unilaterally deviating from the equilibrium.

Region A's best tax rate is the solution to the problem:

$$\max_{t_A} R_A = \widetilde{Y}_A(T) + (1-\beta)[\gamma_A(1-t_B)\widetilde{Y}_B(T) - \gamma_B(1-t_A)\widetilde{Y}_A(T)]$$

where $\widetilde{Y}_i(T)$ is region i's output under an equilibrium with fixed tax rates. The first-order condition for a maximum is:

$$\gamma_B(1-\beta)Y_A + \left\{[1-\gamma_B(1-\beta)(1-t_A)]\frac{\partial \widetilde{Y}_A}{\partial T} + \gamma_A(1-\beta)(1-t_B)\frac{\partial \widetilde{Y}_B}{\partial T}\right\}\frac{\partial T}{\partial t_A} = 0 \tag{15}$$

Likewise, the first-order condition for region B's best tax rate is:

$$\gamma_A(1-\beta)Y_B + \left\{[1-\gamma_A(1-\beta)(1-t_B)]\frac{\partial \widetilde{Y}_B}{\partial T} + \gamma_B(1-\beta)(1-t_A)\frac{\partial \widetilde{Y}_A}{\partial T}\right\}\frac{\partial T}{\partial t_B} = 0 \tag{16}$$

Since T only affects Y_i through its effect on k,

$$\frac{\partial \widetilde{Y}_i}{\partial T} = \frac{\partial Y_i}{\partial k}\frac{\partial k}{\partial T}$$

Using (11),

$$\frac{\partial Y_i}{\partial k} = Y_i \left(\frac{1-\beta}{1-\alpha}\right)\frac{1}{\phi_i(k)}$$

where:

$$\phi_i(k) \equiv \frac{\eta_i + z_i(k)}{z_i'(k)}$$

Using the comparative statics results in the appendix,

$$\frac{\partial k}{\partial T} = \left(\frac{1}{T}\right)\left[\frac{1}{k} - \left(\frac{\alpha-\beta}{1-\alpha}\right)\left(\frac{1}{\phi_B(k)} - \frac{1}{\phi_A(k)}\right)\right]^{-1}$$

The pair (t_A^*, t_B^*) is a Nash equilibrium if the triplet (t_A^*, t_B^*, k) is a solution to (14), (15) and (16).

5 Tax Competition in a Symmetric Economy

Assume that the regions have equal labour supplies, own equal shares of the firms, and that their immobile firms are equally productive:

$$L = 1$$

$$\gamma_A = \gamma_B = 1/2$$

$$\eta_A = \eta_B = \eta$$

Since the density function has been assumed to be symmetrical, the regions are then identical and will choose the same tax rate. Every mobile firm will locate in the region in which its productivity factor is higher.

$$T = 1$$

$$k = 1$$

The last restriction implies that (14) can be removed from the three equation system determining t_A, t_B and k. Imposing all of the above restrictions, and recalling (8), shows that the common value of t_A^* and t_B^* is:

$$t^* = 1 - \left[(1-\alpha)\left(1 + \frac{\phi_B(1)}{2}\right)\right]^{-1} \tag{17}$$

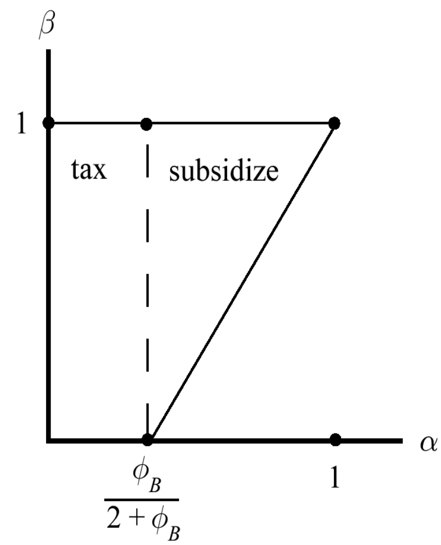

Figure 3: This figure shows the pairs (α, β) that lead to a stable equilibrium under fixed taxes. The region is divided into two sections, one in which the symmetric equilibrium involves taxes and one in which it involves subsidies. An increase in η or a decrease in $z'_B(1)$ shifts the dividing line to the right, and expands the set of stable pairs.

A critical issue in tax competition is whether regions will tax profits at all: the fear of losing firms to other regions might drive the tax rates ever downward, until each region is subsidizing profits because the other region is subsidizing profits. Equation (17) offers some insight into the conditions under which regions subsidize and the conditions under which they tax. The tax is positive if the expression in square brackets is greater than one, and it is negative—that is, it is a subsidy—if the expression is less than one. Equivalently,

$$\text{sign}\,[t^*] = \text{sign}\,[(1-\alpha)\phi_B(1) - 2\alpha]$$

Then Figure 3 shows the pairs (α, β) for which the equilibrium outcome involves taxes and the pairs for which it involves subsidies. (The pairs below the diagonal line are unstable, and hence have been excluded from consideration.) Note that:

- The more important is agglomeration (that is, the greater is α), the less likely the regions are to tax firms. Agglomeration increases the value of having a firm locate in a region (through its external effects on other firms), and hence increases the cost of driving firms out of the region through high taxes.

- The greater the aggregate productivity of immobile firms, the more likely the regions are to tax. The presence of the immobile firms makes a region's tax base less responsive to the migraton of firms.
- The less the firms' productivity factors vary across regions, the greater is $z'_B(1)$ and the less likely the regions are to tax. For example, if all of the firms are virtually identical, all of the firms will want to be in B if k exceeds 1 by an arbitrarily small amount, and all of the firms will want to be in A when k falls short of 1 by an arbitrarily small amount. If z_B is continuous, z'_B must approach infinity in the neighbourhood of 1. It is then almost certain that the regions will subsidize the firms.

These results extend previous results in the literature. Burbidge, DePater, Myers and Sengupta [1997] show, in a model with no agglomeration and no heterogeneity, that regions that import capital will reduce the price of these imports by taxing capital, while regions that export capital will raise the price of the exported capital by subsidizing capital at home. In a symmetric equilibrium, of course, no region is a net exporter or a net importer of capital, so that the tax rate on capital is equal to zero. Burbidge and Cuff [2002] generalize these findings by imaging an economy with agglomeration. They show that, when capital has a positive external effect on the region's output, each region subsidizes capital in an attempt to gain the benefit of the externality. Their model is a special case of the current model. Capital is homogeneous when $z'_B(1)$ is infinite, or equivalently, then $\phi_B(1)$ is equal to zero. The diagonal line (in Figure 3) then shifts upward to pass through the origin, eliminating the region in which capital is taxed. If α is equal to zero, capital is neither taxed nor subsidized (as in Burbidge, Myers and Sengupta); and if α is greater than zero, capital is subsidized.

The novel aspect of the current model is the heterogeneity of firms. In any equilibrium, almost all of the firms are inframarginal: they are earning rents that they would not be earning if they relocated to the other region. The regions have an incentive to tax the rents of the firms. The greater is the degree of heterogeneity—the farther to the right is the diagonal line in Figure 3—the greater the region in which the incentive to tax inframarginal firms dominates the incentive to subsidize capital that has a positive externality. Also, given the degree of heterogeneity of the firms, the externality becomes larger as α becomes larger, and hence firms tax when α is small and subsidize when it is large.

6 Comparative Statics of the Tax Competition Game

Equation (11) determines the values of Y_A and Y_B for any given value of k. The economy's **production possibility frontier** consists of the pairs (Y_A, Y_B) associated with the values of k between $\underline{k}$ and $\overline{k}$. A restriction on the shape of this forntier is equivalent to placing additional restrictions on the set S and on the form of the function g. The following restriction is imposed for the remainder of this section:

> ASSUMPTION A2: *The production possibility frontier of the economy is concave.*

The economy then has two useful properties:

- The equilibrium with fixed tax rates is stable.
- Total output is maximized when the mobile firms divide themselves between the two regions. In a symmetric economy, total output is maximized when there is an equal number of firms in each region ($k = 1$).

The comparative statics results below imagine that the regions are initially symmetric in every way, and that a small change in one of the parameters is then made. The behaviour of the economy farther from a symmetric equilibrium are illustrated by simulations. In these simulations, it is assumed that all of the pairs (θ_A, θ_B) lie on a line segment which is centered on $(1, 1)$ and has slope -1.

$$S = \{\,(\theta_A, \theta_B)|\ \theta_A \in [1-h, 1+h], \theta_B = 2 - \theta_A\}$$

The distribution g is uniform on this set.

First, imagine that γ_A rises from 1/2 by a small amount, and that γ_B falls from 1/2 by an equal amount.

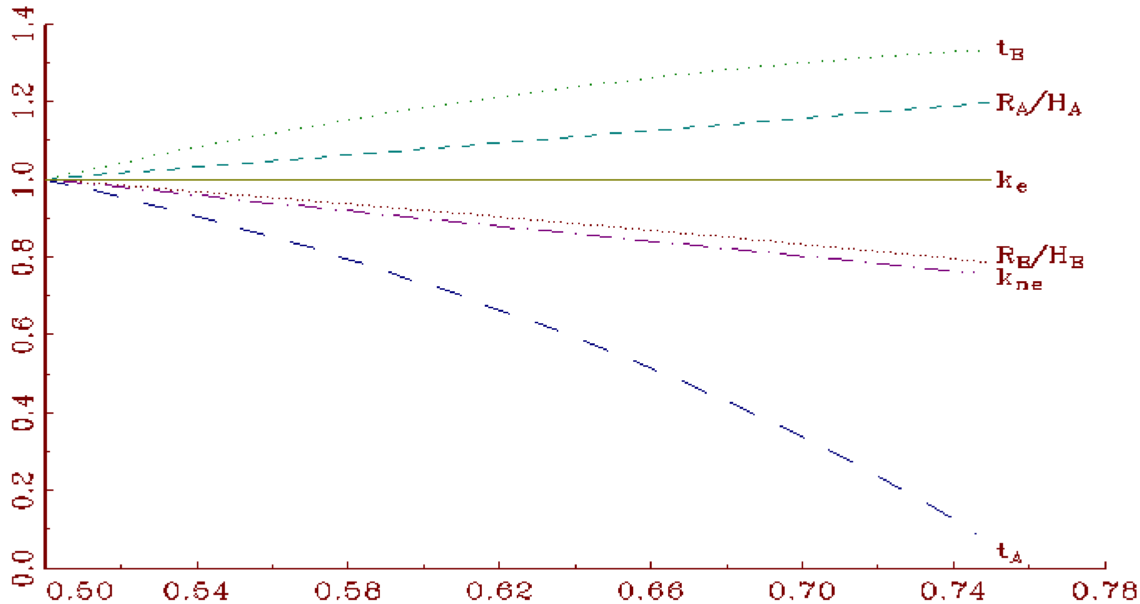

Figure 4: Here, α and β are 0.4 and 0.5, and h is 0.5. The horizontal axis shows the value of γ_A; the value of γ_B is $1 - \gamma_A$. The value of k that maximizes total output is k_e, while the Nash equilibrium value of k is k_{ne}. The initial values of all variables have been normalized at 1.

PROPOSITION 1: *Assume A1 and A2, and assume that the regions are initially symmetric. A small increase in γ_A, accompanied by an equal decrease in γ_B, causes region A to reduce its tax rate and region B to increase its tax rate.*

This result conforms to the earlier argument that capital-importing regions will tax capital and capital-importing regions will subsidize it. Figure 4 illustrates the effects of changing ownership. Clearly, t_A falls continuously and t_B rises continuously as γ_A rises from 1/2. The efficient (in the sense of maximizing total output) value of k is always equal to 1, but the Nash equilibrium value of k falls farther and farther below the efficient value as γ_A rises. (That is, more and more firms are induced to move from B to A.) Also shown in the figure are the per capita resources R_i/H_i of each region i. Not surprisingly, the per capita resources of region A rises with its claims to capital, while the per capita resources

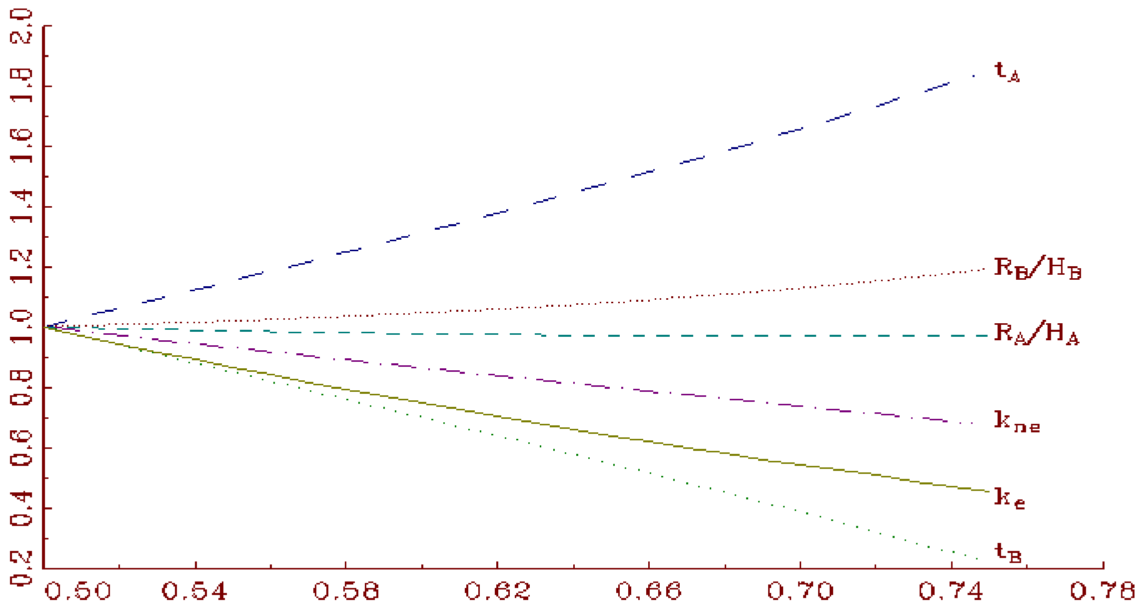

Figure 5: Here, α and β are 0.4 and 0.5, and h is 0.5. The horizontal axis shows the value of H_A; the value of H_B is $1 - H_A$. The value of k that maximizes total output is k_e, while the Nash equilibrium value of k is k_{ne}.

of region B fall.

Now consider the effects of a reallocation of labour.

PROPOSITION 2: *Assume A1 and A2, and assume that the regions are initially symmetric. A small shift of the labour force from region B to region A causes region A to raise its tax rate and region B to reduce its tax rate.*

Shifting labour from region B to region A pushes down the wage rate in region B and raises the wage rate in region A. This adjustment to labour costs causes some firms to migrate from region B to region A, so that B becomes a capital exporter and A becomes a capital importer. The tax rates adjust in the predicted fashion.

Figure 5 shows that these tax adjustments continue as the economy moves farther away from the symmetric equilibrium. As H_A rises, total output is maximized by moving more

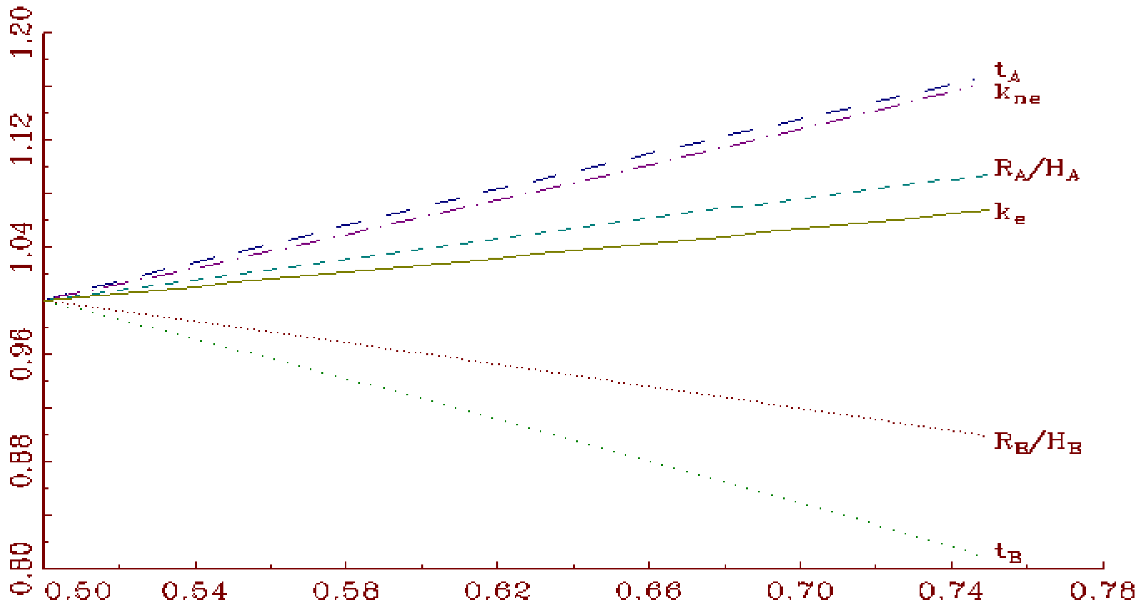

Figure 6: Here, α and β are 0.4 and 0.5, and h is 0.5. The horizontal axis shows the value of η_A; the value of η_B is $1 - \eta_A$. The value of k that maximizes total output is k_e, while the Nash equilibrium value of k is k_{ne}.

of the firms into region A, so that the efficient value of k falls. The Nash equilibrium value of k also falls, but too slowly, so that there are always too few firms in region A. The per capita resources of region A fall while those of region B rise. (Note, however, that the region with the bigger population is able to provide units of the public good more cheaply, so welfare in region A will nevertheless rise.)

Finally, consider the effects of assigning more immobile firms to one region than to the other.

PROPOSITION 3: *Assume A1 and A2, and assume that the regions are initially symmetric. A small increase in η_A, accompanied by an equal decrease in η_B, causes region A to raise its tax rate and region B to reduce its tax rate.*

This result is most easily understood by examining Figure 6. When immobile firms are

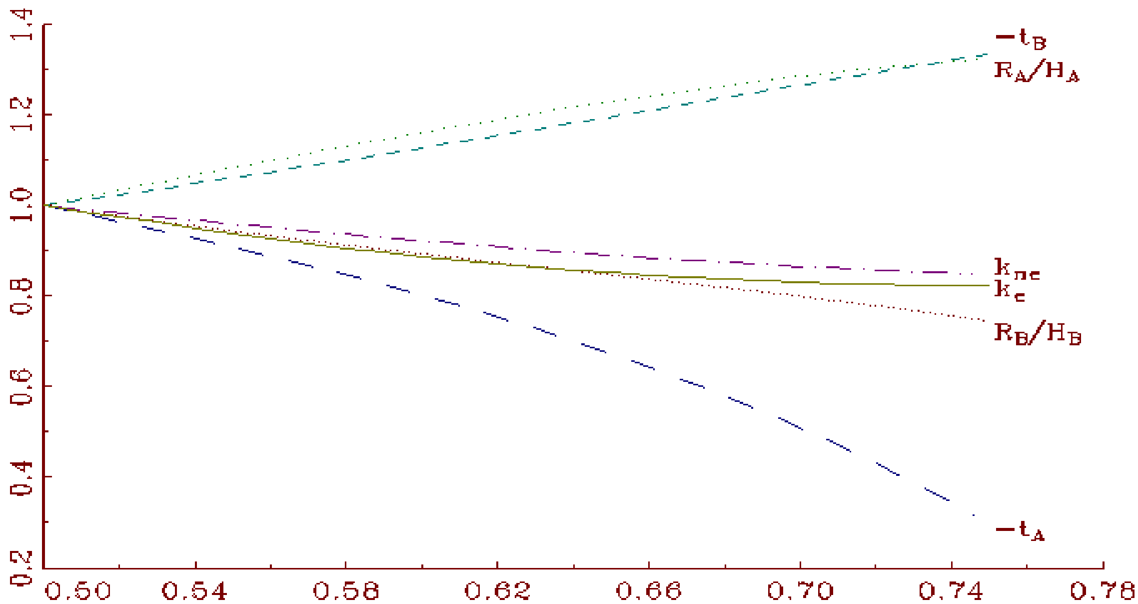

Figure 7: Here, α and β are 0.4 and 0.5, h is 0.1, and η_A and η_B are equal to zero. The horizontal axis shows the value of *both* γ_A and H_A; the values of γ_B and H_B are $1 - \eta_A$ and $1 - H_A$ respectively. The value of k that maximizes total output is k_e, while the Nash equilibrium value of k is k_{ne}.

shifted from region B to region A, region A has a larger fixed tax base, which it exploits by raising its tax rate. Region B's fixed tax base has contracted, so it reduces its tax rate. The adjustment to the tax rates causes some firms to move from region A to region B, so that the Nash equilibrium value of k rises. The *efficient* value of k also rises. Shifting immobile firms into region A makes labour relatively scarce in region A; this scarcity of labour is reversed by moving some of the mobile firms in the opposite direction. The Nash equilibrium value of k, however, is always greater than the efficient value of k—that is, there are always too few firms in region A.

Figure 7 shows the effects of making one region absolutely larger than the other: the ownership of capital and supply of labour in region A is greater than that in region B. Both regions are initially subsidizing capital, and as region A becomes larger, its subsidy falls while region B's subsidy becomes larger. Both the efficient and Nash equilibrium values of k fall, but there are always too few firms in A.

Appendix:

Comparative Statics of an Equilibrium with Fixed Tax Rates

Write (14) in logarithmic form:

$$\ln k = \ln T + \ln L + \left(\frac{\alpha-\beta}{1-\alpha}\right) M(k, \eta_A, \eta_B) \tag{A1}$$

where:

$$M(k, \eta_A, \eta_B) \equiv \ln\left(\eta_B + z_B(k)\right) - \ln\left(\eta_A + z_A(k)\right)$$

Note that:

$$\frac{\partial M}{\partial k} \geq 0, \frac{\partial M}{\partial \eta_A} < 0, \frac{\partial M}{\partial \eta_B} > 0$$

Let R be the ratio Y_B/Y_A. By (11),

$$\ln R = \ln L + \left(\frac{1-\beta}{1-\alpha}\right) M(k, \eta_A, \eta_B) \tag{A2}$$

Treat (A1) and (A2) as a system in which k and R are endogenous and T, L, η_A and η_B are exogenous. The objective is to find the impact of the exogenous variables on the endogeneous variables and on the output levels Y_A and Y_B, given that the equilibrium is stable.

Differentiating the equation system gives:

$$\begin{bmatrix} \frac{1}{k} - \left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{\partial M}{\partial k} & 0 \\ -\left(\frac{1-\beta}{1-\alpha}\right)\frac{\partial M}{\partial k} & \frac{1}{R} \end{bmatrix} \begin{bmatrix} dk \\ dR \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{L} & \frac{1}{T} & \left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{\partial M}{\partial \eta_B} & -\left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{\partial M}{\partial \eta_A} \\ \frac{1}{L} & 0 & \left(\frac{1-\beta}{1-\alpha}\right)\frac{\partial M}{\partial \eta_B} & -\left(\frac{1-\beta}{1-\alpha}\right)\frac{\partial M}{\partial \eta_A} \end{bmatrix} \begin{bmatrix} dL \\ dT \\ d\eta_B \\ d\eta_A \end{bmatrix}$$

The stability condition is:

$$\frac{1}{k} - \left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{\partial M}{\partial k} > 0$$

which is necessarily satisfied if β is greater than α but might not be satisfied if α is greater than β. Applying Cramer's rule gives the following results:

$$\frac{dk}{dL} > 0$$

$$\frac{dk}{dT} > 0$$

$$\text{sign}\left[\frac{dk}{d\eta_B}\right] = \text{sign}\left[\alpha - \beta\right] \qquad \frac{dR}{d\eta_B} > 0$$

If region B becomes innately more productive, and if agglomeration has the stronger impact on firms, the higher level of output in region B will draw mobile firms into that region and out of region A. However, if the drive for cheap labour is dominant, region B's increase in productivity will drive up the wage rate, driving firm's out of the region and into region A. The same kind of effects are observed when region A becomes more productive.

$$\text{sign}\left[\frac{dk}{d\eta_A}\right] = \text{sign}\left[\beta - \alpha\right] \qquad \frac{dR}{d\eta_A} < 0$$

Now consider the regional outputs, which are given by (11). A change in a tax rate only affects output through the change in k, so:

$$\frac{dY_A}{dT} < 0 \qquad \frac{dY_B}{dT} > 0$$

A change in H_A or H_B affects one of the outputs directly and both outputs through k. Their effects are unambiguous.

$$\frac{dY_A}{dL} < 0 \qquad \frac{dY_B}{dL} > 0$$

Now consider an increase in η_B. If α is greater than β, the increase in η_B and the ensuing increase in k are reinforcing, causing Y_B to rise. The out-migration of firms causes region A's output to fall. If α is less than β, the two effects are not reinforcing but region B's output will nevertheless rise. The in-migration of firms cause region A's output to rise, and it was shown above that Y_B/Y_A rises when η_B rises, Y_B must also rise. That is,

$$\frac{dY_B}{d\eta_B} > 0$$

$$\text{sign}\left[\frac{dY_A}{d\eta_B}\right] = \text{sign}\left[\beta - \alpha\right]$$

Similarly,

$$\frac{dY_A}{d\eta_A} > 0$$

$$\text{sign}\left[\frac{dY_B}{d\eta_A}\right] = \text{sign}\left[\beta - \alpha\right]$$