The Quiet Life of a Monopolist:
The Efficiency Losses of Monopoly Reconsidered

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Abstract

In this paper we reconsider the efficiency losses of monopoly by studying a monopolist's incentives to control production costs. To do so, we construct a model where a firm's total cost of production depends on output and the manager's effort. Total cost increases with output but decreases with effort. We analyse two separate cases with regard to ownership and control: (1) the owner of the firm manages the firm; and (2) the owner hires a manager to operate the firm. We demonstrate that even in the case where the owner is the manager of the firm, the effort level exerted by the owner-manager of a monopoly is generally not first-best. The direction of the bias depends on the relationship between output and effort in the cost function. The monopolist exerts too little effort if output and effort are complements in the cost function. The reverse is true if output and effort are substitutes in the cost function. The separation of ownership and control exacerbates monopoly slackness when output and effort are complements in the cost function. If output and effort are substitutes, on the other hand, the separation of ownership and control offsets the over-supply of effort by the owner-manager of the monopoly.

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"The best of all monopoly profits is a quite life" Hicks (1935)

1. Introduction

The study of the efficiency losses of monopoly has a long tradition in economics. It is well known that monopoly pricing leads to output distortion, with the resulting deadweight loss "triangle". However, the empirical significance of such output distortion was called into question by Harberger (1954), who reports that his estimated "triangle" deadweight welfare loss does not exceed 0.1 percent of the gross national product. In response economists have identified two other likely sources of efficiency losses, namely, rent seeking activities and the loss of productive efficiency. On the latter it has been suggested that monopoly may slack off and may exert too little effort in reducing costs. However, despite its intuitive appeal, this idea has rarely been rigorously analyzed. To the contrary, many economists react to such an idea with distress and puzzlement (Farrell 1983). Why would a monopolist spare efforts to reduce costs when it stands to reap all the incremental profits arising from the cost reduction (i.e. it does not have to worry about the incremental profits being competed away)?

The two exceptional papers that responded to this challenge were by Farrell (1983) and Nalebuff and Stiglitz (1983b). In both papers productive inefficiency was the result of managerial slack arising from the separation of ownership and control. Furthermore, Nalebuff and Stiglitz (1983b page 281) argue that monopoly does not cause productive efficiency loss in an owner-managed firm. Both analyses, however, are very brief and highly stylized.[2]

The objective of this paper is to conduct a more detailed analysis of the loss of productive efficiency caused by monopolist using a more general framework than those of Farrell (1983) and Nalebuff and Stigliz (1983b). To be more specific, we study a model where a firm's total cost of production depends on output and the manager's effort. Total cost increases with output but decreases with effort. In contrast to Nalebuff and Stiglitz (1983b), marginal cost of production in our model depends on effort as well. We analyze two separate cases with respect to ownership and control: (1) the owner of the firm manages the firm; and (2) the owner hires a manager to operate the firm.

We demonstrate that even in the case where the owner manages his firm, the level of effort exerted by an owner-manager monopolist generally cannot attain the first-best. The direction of the bias depends on the relationship between output and effort in the cost function. The

[2] A separate but related strand of literature studies the effects of competition from entrepreneurial firms on the managerial slackness in the managerial firms (Hart 1983 and Scharfstein 1988).

monopolist exerts too little effort if output and effort are complements in the cost function. The reverse is true if output and effort are substitutes in the cost function. As a result, the monopolist does not necessarily enjoy a "quite life". Furthermore, the effort distortion leads to higher marginal cost of production, which confirms Tirole's (1988, p.75-76) conjecture in this regard.

In the case of separation of ownership and control, we show that managerial slack exacerbates the monopolist's effort distortion if output and effort are complements in the cost function. On the other hand, if output and effort are substitutes, separation of ownership and control offsets the over-supply of effort by the owner-manager of the monopoly. Using numerical examples, we demonstrate that the agency costs in a monopoly firm are not necessarily higher than in a competitive firm. Taking this into consideration, the total welfare losses caused by a managerial monopolist are not necessarily larger than those by an entrepreneurial monopolist, even in the case where cost and effort are complements.

This paper is organized as follows. In section 2 we present two general models: an owner-manager monopolist's profit maximization model (case 1) and a principal-agent model (case 2), and we compare the monopoly equilibrium with competitive equilibrium in each of these two cases. In section 3 we parameterize the two general models and study a number of numerical examples. In section 4 we conclude.

2. The Model

Consider a product market characterized by the inverse demand function $p(q)$. To study the efficiency losses of monopoly, we will compare two situations, the one where the market is controlled by a monopolist and the one where it is supplied by a representative competitive firm. Each firm has access to a single-output technology associated with a stochastic cost function of production $C(q,e,\varepsilon)$, where q is firm's output quantity, e is cost-reducing effort and it is assumed to be one-dimensional, and ε is an idiosyncratic, random variable reflecting exogenous shocks to technology and giving rise to the stochastic fluctuations in the production cost. It is assumed to be a nondiversifiable risk that is cannot be controlled by firms and contracting parties. The set of possible realizations of ε is denoted $\Omega = [\underline{\varepsilon}, \bar{\varepsilon}]$. The cumulative distribution function of $\varepsilon \in \Omega$ and the associated probability density function are denoted respectively $F(\varepsilon)$ and $f(\varepsilon)$ with its mean $E[\varepsilon] = 0$, $E[\varepsilon_i \varepsilon_j] = 0$ and variance $\sigma_\varepsilon^2 \neq 0$ on its domain $\Omega = [\underline{\varepsilon}, \bar{\varepsilon}]$, and commonly known to all the firms. Some other basic assumptions are as follows.

***Assumption 1*:** The firm's cost function has the following properties: (a) $C(q,e,\varepsilon)$ is continuous and twice differentiable at all $q \geq 0$ and $e \geq 0$. $C(0,e,\varepsilon)=0$ and $C(q,0,\varepsilon)>0$ for all $q>0$. (b) $C(\cdot)$ is a *strictly increasing and convex* function of *q*, i.e. $C_q(q,e,\varepsilon)>0$, the cost is *increasing* for all $q \geq 0$; and $C_{qq}(q,e,\varepsilon) \geq 0$, the marginal cost of output is *nondecreasing* for all $q \geq 0$. (c) $C(\cdot)$ is a *strictly decreasing and convex* function of *e*, i.e. $C_e(q,e,\varepsilon)<0$ at all $e>0$, the high effort reduces the firm's cost. $C_{ee}(q,e,\varepsilon) \geq 0$ at all $e>0$, the marginal cost reduction of the firm's effort is *nonincreasing*. (d) $C_{qe}(q,e,\varepsilon) \neq 0$, which implies that the marginal cost of production depends on effort. To be more specific, we will consider both the case where $C_{qe}(q,e,\varepsilon)<0$ and the case where $C_{qe}(q,e,\varepsilon)>0$. In the former case, an increase in effort level reduces the marginal cost, implying *cost complementarities* between output and effort. In the latter case, an increase in effort level increases the marginal cost, implying *cost substitutability* between output and effort. (e) $C(q,e,\varepsilon)$ is continuous at $\varepsilon \in \Omega$ and a *strictly decreasing* function of ε, i.e. $C_\varepsilon(q,e,\varepsilon)<0$, the cost is *strictly decreasing* for all $\varepsilon \in \Omega$, this means that a positive shock to technology will reduce the production cost.

***Assumption 2*:** The owner or manager of a firm will incur a disutility from effort (the cost of effort) when he exerts his cost-reducing effort in the production process. The disutility function of effort (or the cost function of effort) expressed in monetary terms is denoted $\psi(e)$. It is twice continuously differentiable, strictly increasing and strictly convex. $\psi'(e)>0$, the cost-reducing effort is *costly*. $\psi''(e)>0$, the cost of effort is *convex*. $\psi'(0)=0$, and $\psi'(\infty)=\infty$.

***Assumption 3*:** The inverse market demand function $p(q)$ is twice continuously differentiable and strictly decreasing with $p'(q)<0$ and $p''(q) \leq 0$ at all $q \geq 0$, and $p'(0)>C_q(0,e,\varepsilon)$.

***Assumption 4*:** The owner is *a risk-neutral expected return maximizer*, and the manager is *a risk-averse expected utility maximizer*. The owner and the manager are identical in their firm management ability and they will choose optimal levels of output and effort quantity to maximize their objectives.

We will consider two separate cases with regard to ownership and control of the firm. In case 1 the owner of the firm also manages the firm. In case 2, there is separation of ownership and control and the owner delegates the management of the firm to an agent.

2.1 Case 1: The Non-Separation of Ownership and Control

In this case the owner does not hire the manager and he runs the firm by himself.

The firm's realized profit is

$$\pi(q,e,\varepsilon) = pq - C(q,e,\varepsilon).$$

The firm's expected cost of production is

$$E[C(q,e,\varepsilon)] = \int C(q,e,\varepsilon) f(\varepsilon) d\varepsilon.$$[3]

The owner's expected return is the expected profit minus the cost of effort,

$$E[R(q,e,\varepsilon)] = E[\pi(q,e,\varepsilon)] - \psi(e).$$

Monopoly Equilibrium

A monopoly firm has market power and takes the product market demand as given. It chooses the monopoly output and level of effort to maximize its expected return:

$$\max_{q\geq 0, e\geq 0} E[R(q,e,\varepsilon)] = p(q)q - \int C(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi(e)$$

The first-order conditions with respect to output and effort are

$$\frac{\partial E[R(q,e,\varepsilon)]}{\partial q} = p'(q)q + p(q) - \int C_q(q,e,\varepsilon) f(\varepsilon) d\varepsilon \leq 0, \text{ with equality if } q > 0.$$

$$\frac{\partial E[R(q,e,\varepsilon)]}{\partial e} = -\int C_e(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi'(e) \leq 0, \qquad \text{with equality if } e > 0.$$

The monopolist's optimal levels of output q_{M}^* and effort e_{M}^* satisfy the following two first-order conditions:[4]

[3] In what follows, given a density function $f(x)$ of *x*, the expected value of the function $g(x)$ over the possible realizations of the random variable *x* is given by $E[g(x)] = \int g(x) f(x) dx$. For notational simplicity, we do not explicitly write the limits of integration when the integral is over the entire range of possible realization values of $x \in X = [\underline{x}, \overline{x}]$.

$$p'(q_{\mathrm{M}}^*)q_{\mathrm{M}}^* + p(q_{\mathrm{M}}^*) = \int C_q(q_{\mathrm{M}}^*, e_{\mathrm{M}}^*, \varepsilon) f(\varepsilon) d\varepsilon,$$

i.e., the marginal revenue is equal to the firm's expected marginal cost, and

$$-\int C_e(q_{\mathrm{M}}^*, e_{\mathrm{M}}^*, \varepsilon) f(\varepsilon) d\varepsilon = \psi'(e_{\mathrm{M}}^*),$$

i.e., the expected marginal production cost saving from effort is equal to the marginal cost of effort.

Let $H^{\mathrm{M}} \equiv \begin{bmatrix} H_{11}^{\mathrm{M}} & H_{12}^{\mathrm{M}} \\ H_{21}^{\mathrm{M}} & H_{22}^{\mathrm{M}} \end{bmatrix}$ be the Hessian matrix of the second partial derivatives of the monopoly firm's expected return maximization problem, where

$$H_{11}^{\mathrm{M}} = p''(q)q + 2p'(q) - \int C_{qq}(q, e, \varepsilon) f(\varepsilon) d\varepsilon < 0,$$

$$H_{12}^{\mathrm{M}} = -\int C_{qe}(q, e, \varepsilon) f(\varepsilon) d\varepsilon \begin{matrix} > 0 \text{ if cost complementarities between output and effort} \\ < 0 \quad \text{if cost substiutions between output and effort} \end{matrix},$$

$$H_{21}^{\mathrm{M}} = -\int C_{eq}(q, e, \varepsilon) f(\varepsilon) d\varepsilon \begin{matrix} > 0 \\ < 0 \end{matrix},$$

$$H_{22}^{\mathrm{M}} = -\int C_{ee}(q, e, \varepsilon) f(\varepsilon) d\varepsilon - \psi''(e) < 0.$$

A sufficient second condition for the monopoly firm's expected return maximization problem is that the Hessian matrix is *negative semidefinite* at the optimal q_{M}^* and e_{M}^*, and that the determinant of H^{M} is *positive,*

$$\det(H^{\mathrm{M}}) = H_{11}^{\mathrm{M}} H_{22}^{\mathrm{M}} - H_{12}^{\mathrm{M}} H_{21}^{\mathrm{M}} > 0.$$

In what follows, the sufficient second condition is assumed to be satisfied.

The above two FOC equations characterize one output equation and one effort-exerting equation of the monopoly firm:

$$q = q_{\mathrm{M}}(e) \text{ and } e = e_{\mathrm{M}}(q).$$

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[4] For the reason of comparison convenience, in what follows, we use special notations of variables to indicate their optimal values. Superscripts with one asterisk denote the optimal values of variables in the case of non-separation of ownership. Superscripts with two asterisks denote the optimal values of variables in the case of separation of ownership under asymmetric information. Subscripts with capital letters C, M and a lowercase letter n are used to label respectively the representative competitive firm, the monopoly firm and the n competitive firms. Here, we summarize the optimal values of output and effort that we will obtain in the model as follows:

	Competitive Firm	**Monopoly Firm**
Case 1: The Non-Separation of Firm Ownership and Control	$(q_{\mathrm{C}}^*, e_{\mathrm{C}}^*)$	$(q_{\mathrm{M}}^*, e_{\mathrm{M}}^*)$
Case 2: The Separation of Firm Ownership and Control	$(q_{\mathrm{C}}^{**}, e_{\mathrm{C}}^{**})$	$(q_{\mathrm{M}}^{**}, e_{\mathrm{M}}^{**})$

Using the comparative static calculation, we have:

LEMMA 2.1: *By the properties of the Hessian matrix,* H^{M},

(i) slope of $q_{\mathrm{M}}(e) \equiv \frac{dq}{de} = -\frac{H_{12}^{\mathrm{M}}}{H_{11}^{\mathrm{M}}} \begin{matrix} > 0 \ (\textit{It is upward sloping if there is cost complementarities}) \\ < 0 \quad (\textit{It is downward sloping if there is cost substitutability}) \end{matrix}$

and slope of $e_{\mathrm{M}}(q) \equiv \frac{dq}{de} = -\frac{H_{22}^{\mathrm{M}}}{H_{21}^{\mathrm{M}}} \begin{matrix} > 0 \ (\textit{It is upward sloping if there is cost complementarities}) \\ < 0 \quad (\textit{It is downward sloping if there is cost substitutability}) \end{matrix}$.

(ii) slope of $e_{\mathrm{M}}(q) -$ slope of $q_{\mathrm{M}}(e) = \frac{\det(H^{\mathrm{M}})}{-H_{11}^{\mathrm{M}} H_{21}^{\mathrm{M}}} \begin{matrix} > \\ < \end{matrix} 0$ if $\begin{matrix} C_{qe}(q,e,\varepsilon) < 0 \\ C_{qe}(q,e,\varepsilon) > 0 \end{matrix}$, i.e. *the curve* $e_{\mathrm{M}}(q)$ *is steeper than the curve* $q_{\mathrm{M}}(e)$. *When two curves just have a single crossing point in* $\{q, e\}$ *space, and the unique stable equilibrium* $(q_{\mathrm{M}}^*, e_{\mathrm{M}}^*)$ *exists.* □

Competitive Equilibrium

Consider now the perfect competitive product market in the short run with two cases: a representative competitive firm and *n* identical competitive firms: each firm has the same cost function of production and disutility function of effort as the monopoly firm.

(1) In the competitive product market, the representative competitive firm takes the market price as given to choose the optimal quantity and effort to maximize its expected return:

$$\max_{q \geq 0, e \geq 0} E[R(q,e,\varepsilon)] = pq - \int C(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi(e)$$

The first-order conditions with respect to output and effort are

$$\frac{\partial E[R(q,e,\varepsilon)]}{\partial q} = p - \int C_q(q,e,\varepsilon) f(\varepsilon) d\varepsilon \leq 0, \qquad \text{with equality if } q > 0.$$

$$\frac{\partial E[R(q,e,\varepsilon)]}{\partial e} = -\int C_e(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi'(e) \leq 0, \quad \text{with equality if } e > 0.$$

At the market equilibrium the representative competitive firm's supply is equal to the market demand,

$$q = Q(p),$$

where $Q(p)$ is the market demand function, $Q'(p) < 0$ and $Q''(p) \leq 0$.

The representative competitive firm's optimal level of output q_{C}^* and level of effort e_{C}^* are characterized by the following two first-order conditions:

$$p(q_{\mathrm{C}}^*) = \int C_q(q_{\mathrm{C}}^*, e_{\mathrm{C}}^*, \varepsilon) f(\varepsilon) d\varepsilon,$$

the market price is equal to the firm's expected marginal cost,

$$-\int C_e\left(q_C^*, e_C^*, \varepsilon\right) f(\varepsilon) d\varepsilon = \psi'\left(e_C^*\right),$$

the expected marginal production cost saving from effort is equal to the marginal disutility of effort.

Let $H^C \equiv \begin{bmatrix} H_{11}^C & H_{12}^C \\ H_{21}^C & H_{22}^C \end{bmatrix}$ be the Hessian matrix of the second partial derivatives of the representative competitive firm's expected return maximization problem, where

$$H_{11}^C = p'(q) - \int C_{qq}(q,e,\varepsilon) f(\varepsilon) d\varepsilon < 0,$$

$$H_{12}^C = -\int C_{qe}(q,e,\varepsilon) f(\varepsilon) d\varepsilon \begin{matrix} > 0 \text{ if there is cost complementarities} \\ < 0 \quad \text{if there is cost substitutability} \end{matrix},$$

$$H_{21}^C = -\int C_{eq}(q,e,\varepsilon) f(\varepsilon) d\varepsilon \begin{matrix} > 0 \\ < 0 \end{matrix},$$

$$H_{22}^C = -\int C_{ee}(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi''(e) < 0.$$

A sufficient second condition for the competitive firm's expected return maximization problem is that the Hessian matrix is *negative semidefinite* at the optimal q_C^* and e_C^*, and the determinant of H^C is *positive*,

$$\det\left(H^C\right) = H_{11}^C H_{22}^C - H_{12}^C H_{21}^C > 0.$$

The above two FOC equations characterize one output equation and one effort-exerting equation of the representative competitive firm:

$$q = q_C(e) \text{ and } e = e_C(q).$$

Using the comparative static calculation, we have:

LEMMA 2.2: *By the properties of the Hessian matrix,* H^C,

(i) slope of $q_C(e) \equiv \dfrac{dq}{de} = -\dfrac{H_{12}^C}{H_{11}^C} \begin{matrix} > 0 \text{ (It is upward sloping if there is cost complementarities)} \\ < 0 \text{ (It is downward sloping if there is cost substitutability)} \end{matrix}$

and slope of $e_C(q) \equiv \dfrac{dq}{de} = -\dfrac{H_{22}^C}{H_{21}^C} \begin{matrix} > 0 \text{ (It is upward sloping if there is cost complementarities)} \\ < 0 \text{ (It is downward sloping if there is cost substitutability)} \end{matrix}.$

(ii) slope of $e_{\mathrm{C}}(q)$ − slope of $q_{\mathrm{C}}(e) = \dfrac{\det(H^{\mathrm{C}})}{-H_{11}^{\mathrm{C}}H_{21}^{\mathrm{C}}} \begin{smallmatrix}>\\<\end{smallmatrix} 0$ *if* $\begin{matrix} C_{qe}(q,e,\varepsilon) < 0 \\ C_{qe}(q,e,\varepsilon) > 0 \end{matrix}$, *i.e.* the curve $e_{\mathrm{C}}(q)$ is *steeper* than the curve $q_{\mathrm{C}}(e)$. When two curves just have a single crossing point in $\{q,e\}$ space, and the unique stable equilibrium $(q_{\mathrm{C}}^*, e_{\mathrm{C}}^*)$ exists. □

From Lemma 2.1 and 2.2 we have Lemma 2.3.

LEMMA 2.3: *(i)* slope of $q_{\mathrm{M}}(e) \begin{smallmatrix}<\\>\end{smallmatrix}$ slope of $q_{\mathrm{C}}(e)$ *if* $C_{qe}(q,e,\varepsilon) \begin{smallmatrix}<\\>\end{smallmatrix} 0$, *i.e. the output curve* $q_{\mathrm{M}}(e)$ *of the monopoly firm is flatter than* $q_{\mathrm{C}}(e)$ *of the competitive firm in* $\{q,e\}$ *space.*

(ii) slope of $e_{\mathrm{M}}(q)$ = slope of $e_{\mathrm{C}}(q)$, *i.e. the effort-exerting curve* $e_{\mathrm{M}}(q)$ *of the monopoly firm has the same slope as that of the competitive firm, so two curves,* $e_{\mathrm{C}}(q)$ *and* $e_{\mathrm{M}}(q)$, *coincide in* $\{q,e\}$ *space.*

(iii) At $e = 0$, $q_{\mathrm{M}}(0) < q_{\mathrm{C}}(0)$. $q_{\mathrm{M}}(e)$ *and* $q_{\mathrm{C}}(e)$ *are strictly increasing or decreasing and have no crossing point in* $\{q,e\}$, *so the curve* $q_{\mathrm{M}}(e)$ *is below the curve* $q_{\mathrm{C}}(e)$. □

Comparing the first-order conditions of the monopoly firm with those of the representative competitive firm, we directly have the following proposition.

PROPOSITION 2.1: *Under the assumptions made in our model, (1) the monopoly firm expends lower (respectively higher) effort than the representative competitive firm if there are cost complementarities (respectively cost substitutability) between output and the effort.*

(2) The monopoly firm always produces less output than the representative competitive firm.

PROOF: The proof is based on the first-order conditions of the expected return maximization problem for the monopoly firm and of the representative competitive firm. Suppose that the monopoly firm expends the same level effort, e_{C}^*, as the competitive firm does. By assumption 3, $p'(q) < 0$ for all $q \geq 0$, so it follows from the first-order condition with respect to output for the monopoly firm, $p'(q)q + p(q) = \int C_q(q, e_{\mathrm{C}}^*, \varepsilon) f(\varepsilon) d\varepsilon$, that

$$p(q) > \int C_q(q, e_{\mathrm{C}}^*, \varepsilon) f(\varepsilon) d\varepsilon ,$$

the price set by the monopoly firm is *higher* than the expected marginal cost. Let $q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right)$ denote the optimal level of output produced by the monopoly at e_{C}^{*}. Comparing with the pricing rule of the representative competitive firm under perfect competition,

$$p\left(q_{\mathrm{C}}^{*}\right)=\int C_{q}\left(q_{\mathrm{C}}^{*},e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon ,$$

since $p'(q)<0$ and $C_{qq}(q,e,\varepsilon)>0$ for all $q\geq 0$, we know that

$$q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right)<q_{\mathrm{C}}^{*},$$

the output produced by the monopoly firm is *less* than that by the competitive firm, implying that the monopoly leads to an *output distortion*. Thus, we know that

$$-\int C_{e}\left(q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right),e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon \underset{>}{<} -\int C_{e}\left(q_{\mathrm{C}}^{*},e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon \ \text{ if } \begin{array}{l} C_{qe}(q,e,\varepsilon)<0 \\ C_{qe}(q,e,\varepsilon)>0 \end{array},$$

it follows from the first-order condition with respect to effort for the representative competitive firm,

$$-\int C_{e}\left(q_{\mathrm{C}}^{*},e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon=\psi'\left(e_{\mathrm{C}}^{*}\right),$$

that

$$-\int C_{e}\left(q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right),e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon-\psi'\left(e_{\mathrm{C}}^{*}\right)\underset{>}{<}0 \ \text{ if } \begin{array}{l} C_{qe}(q,e,\varepsilon)<0 \\ C_{qe}(q,e,\varepsilon)>0 \end{array}.$$

By assumption, the Hessian matrix has a negative trace and a positive determinant, so the sufficient condition for *stability* of the equilibrium output and effort is satisfied. Hence, it can *decrease* or *increase* the levels of effort and correspondently *decrease* output to adjust to the expected return maximum.

To see it, differentiating the expression, $-\int C_{e}\left(q_{\mathrm{M}}(e),e,\varepsilon\right)f(\varepsilon)d\varepsilon-\psi'(e)$, with respect to effort, we have

$$-\left\{\int C_{eq}\left(q_{\mathrm{M}}(e),e,\varepsilon\right)f(\varepsilon)d\varepsilon\right\}q_{\mathrm{M}}'(e)-\int C_{ee}\left(q_{\mathrm{M}}(e),e,\varepsilon\right)f(\varepsilon)d\varepsilon-\psi''(e)$$

$$=H_{12}^{\mathrm{M}}\left[-\frac{H_{12}^{\mathrm{M}}}{H_{11}^{\mathrm{M}}}\right]+H_{22}^{\mathrm{M}}=\frac{\det\left(H^{\mathrm{M}}\right)}{H_{11}^{\mathrm{M}}}<0.$$

So this expression is strictly decreasing in effort.

When $-\int C_{e}\left(q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right),e_{\mathrm{C}}^{*},\varepsilon\right)f(\varepsilon)d\varepsilon-\psi'\left(e_{\mathrm{C}}^{*}\right)<0$ (if $C_{qe}(q,e,\varepsilon)<0$), then the monopoly firm can *decrease* the levels of effort e_{C}^{*} to e_{M}^{*} and correspondently *decrease* the level of output $q_{\mathrm{M}}\left(e_{\mathrm{C}}^{*}\right)$ to

$q_{\mathrm{M}}^* \equiv q_{\mathrm{M}}(e_{\mathrm{M}}^*)$ (since $q_{\mathrm{M}}'(e) > 0$ by Lemma 2.1) such that the first-order condition with respect to effort for the monopoly firm,

$$-\int C_e\left(q_{\mathrm{M}}(e_{\mathrm{M}}^*), e_{\mathrm{M}}^*, \varepsilon\right) f(\varepsilon) d\varepsilon - \psi'(e_{\mathrm{M}}^*) = 0$$

is satisfied.

When $-\int C_e\left(q_{\mathrm{M}}(e_{\mathrm{C}}^*), e_{\mathrm{C}}^*, \varepsilon\right) f(\varepsilon) d\varepsilon - \psi'(e_{\mathrm{C}}^*) > 0$ (if $C_{qe}(q,e,\varepsilon) > 0$), then it can *increase* the levels of effort e_{C}^* to e_{M}^* and correspondently *decrease* the level of output $q_{\mathrm{M}}(e_{\mathrm{C}}^*)$ to $q_{\mathrm{M}}(e_{\mathrm{M}}^*)$ (since $q_{\mathrm{M}}'(e) < 0$ by Lemma 2.1). Hence, we obtain

$$e_{\mathrm{M}}^* \underset{>}{<} e_{\mathrm{C}}^*,\ q_{\mathrm{M}}^* < q_{\mathrm{C}}^* \text{ and } q_{\mathrm{C}}^* - q_{\mathrm{M}}^* > q_{\mathrm{C}}^* - q_{\mathrm{M}}(e_{\mathrm{C}}^*) \text{ if } \begin{matrix} C_{qe}(q,e,\varepsilon) < 0 \\ C_{qe}(q,e,\varepsilon) > 0 \end{matrix}. \qquad \square$$

Nalebuff and Stiglitz (1983b) in their simple model showed that managerial slack is the only source of the productive inefficiency of monopoly. They showed that the owner-manager monopolist expends the *same* first-best effort as competitive firms in the case of the *non-separation* of ownership and control. Because of more *managerial slack* arising in the moral hazard problem, monopoly firm expends *lower* cost-reducing effort than competitive firms in the case of the *separation* of ownership and control.

However, PROPOSITION 2.1 states that even though managerial slack is absent in the case of non-separation of ownership and control, the equilibrium effort expended by the monopoly firm cannot achieve the first-best. Furthermore, the equilibrium effort level can be either higher or lower than the first-best. In other word, the life of a monopolist may be “quiet” (expending lower effort) or “hard” (expending higher effort). Monopoly causes not only the output distortion, but also the effort distortion, so the monopoly firm is less efficient than the competitive firm in both the allocative efficiency and the productive efficiency. Furthermore, the monopoly output distortion leads to the effort distortion. This proposition shows that managerial slack is *not* the only source of the productive inefficiency of monopoly.

Proposition 2.1 implies that when there is cost substitutability between output and effort, the monopolist’s total cost of production, for any given output level, will be lower due to higher effort level. However, the same cannot be said about the marginal cost of production.

COROLLARY 2.1: *When the monopoly firm expends the inefficient level of cost-reducing effort, it always produces a given output at a higher marginal cost than the competitive firm.*

PROOF: From PROPOSITION 2.1, we know that if $\begin{matrix} C_{qe}(q,e,\varepsilon) < 0 \\ C_{qe}(q,e,\varepsilon) > 0 \end{matrix}$, then $e_{\mathrm{M}}^{*} \begin{matrix} < \\ > \end{matrix} e_{\mathrm{C}}^{*}$. So for given $q > 0$, $C_q\left(q, e_{\mathrm{M}}^{*}, \varepsilon\right) > C_q\left(q, e_{\mathrm{C}}^{*}, \varepsilon\right)$. □

2.2 Case 2: The Separation of Ownership and Control

In this case for simplicity we assume that in each firm one owner hires one manager to run the firm. The firm's performance such as the realized profit, $\pi(q,e,\varepsilon)$, not only depends on the manager's decisions on output quantity and his effort level, but also depends on the realizations of the stochastic shocks to the production cost. Assume that both the owner and the manager have no *ex ante* information about the exogenous random shocks to the production cost except for its probability density function and distribution function. Furthermore, assume that the owner even cannot observe its *ex post* realization.

The only asymmetric information between the owner and the manager is that the manager's actual effort level is *unobservable* to the owner. Although the owner can observe the realized output, cost and profit, he cannot infer the manager's effort level from the *stochastic* production cost function $C(q,e,\varepsilon)$ because the actual *ex post* realization of shocks is unobservable to him and only the realization of the outcome such output, costs and profit jointly determined by output, effort and random shocks is observable. Therefore, the contract cannot be specified based on the manager's actual effort level.

Assume that it is very costly for the owner to monitor the manager, so that it is impossible for him to do that. Since the outcome of unobservable effort — the realized profit is assumed to be observable, explicit incentive contracts are possible and feasible. To induce the manager to make desirable decisions, the owner will offer the manager an incentive compensation scheme based on the observable realized profit $\pi(q,e,\varepsilon)$. For simplicity, following the linear incentive scheme approach suggested Holmstrom and Milgrom (1987), we assume that the incentive scheme is linear, $s(\pi(q,e,\varepsilon)) = \alpha\pi(q,e,\varepsilon) + \beta$, where α is the incentive coefficient and $0 < \alpha < 1$, and β is the fixed wage.

By assumption 4, the owner is a risk-neutral expected return maximizer, and the manager is a risk-averse expected utility maximizer, his utility function is continuously differentiable, strictly increasing and concave, i.e.

$$u\left[s\left(\pi(q,e,\varepsilon)\right)-\psi(e)\right] \text{ with } u' > 0 \text{ and } u'' \leq 0\,,$$

where $\psi(e)$ is the cost of the manager's effort expressed in monetary terms.

The market demand, the production cost function, the owner's and the manager's preferences, and the cumulative distribution function $F(\varepsilon)$ of the random shocks ε and the associated probability density function $f(\varepsilon)$ are assumed to be common knowledge to both the owner and the manager in the sense of Aumann (1976).

In order to make a consistent comparison between the results obtained in case 2 and those in case 1, we adopt the state-space formulation approach suggested by Wilson (1969), Spence and Zeckhauser (1971) and Ross (1973) to express the principal-agent problem under asymmetric information.

The Manager's Problem

Given the compensation scheme $s\left(\pi(q,e,\varepsilon)\right)$, the manager chooses the output q and effort e to maximize his expected utility,

$$\max_{q\geq 0, e\geq 0} E(u) = \int u\left[\alpha\pi(q,e,\varepsilon)+\beta-\psi(e)\right]f(\varepsilon)d\varepsilon\,.$$

The first-order conditions with respect to output and effort are

$$\frac{\partial E(u)}{\partial q} = \int u'\left[\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right]f(\varepsilon)d\varepsilon \leq 0\,, \qquad \text{with equality if } q > 0\,.$$

$$\frac{\partial E(u)}{\partial e} = \int u'\left[\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right]f(\varepsilon)d\varepsilon \leq 0\,, \qquad \text{with equality if } e > 0\,.$$

The manager's optimal choices q^{**} and e^{**} are characterized by the following two first-order *necessary* conditions with equality for the manager's expected utility maximization:

$$\int u'\left[\alpha\frac{\partial\pi\left(q^{**},e^{**},\varepsilon\right)}{\partial q}\right]f(\varepsilon)d\varepsilon = 0\,,$$

$$\int u'\left[\alpha\frac{\partial\pi\left(q^{**},e^{**},\varepsilon\right)}{\partial e}-\psi'\left(e^{**}\right)\right]f(\varepsilon)d\varepsilon = 0\,.$$

The Owner's Problem

Assume that the manager market is perfectly competitive, and all the managers in this market are identical, so the owner knows the managers' reservation wages and their reservation utilities. The participation constraint is that the manager gets the expected utility at least as great as his reservation utility $\bar{u}$,

$$\int u[s(\pi(q,e,\varepsilon))-\psi(e)]f(\varepsilon)d\varepsilon \geq \bar{u} .$$

The incentive compatibility constraint is that the manager chooses output and effort, given $s(\pi(q,e,\varepsilon))$, to maximize his expected utility,

$$q = \arg\max \int u[s(\pi(q,e,\varepsilon))-\psi(e)]f(\varepsilon)d\varepsilon$$

and

$$e = \arg\max \int u[s(\pi(q,e,\varepsilon))-\psi(e)]f(\varepsilon)d\varepsilon .$$

Following the first-order approach suggested by Mirrlees (1974) and Holmstrom (1979), we can replace the IC constraint by the two first-order conditions of the manager's optimization problem.

The owner's problem is to design an optimal incentive scheme $s(\pi(q,e,\varepsilon))$ to induce the manager to choose output and effort that the owner desires, taking into account of the manager's reaction to this incentive scheme,

$$\max_{s(\pi)\geq 0,\, q\geq 0,\, e\geq 0} \int [\pi(q,e,\varepsilon)-s(\pi(q,e,\varepsilon))]f(\varepsilon)d\varepsilon ,$$

subject to

$$\int u[s(\pi(q,e,\varepsilon))-\psi(e)]f(\varepsilon)d\varepsilon \geq \bar{u} ,$$

$$(\text{IC}_1) \qquad \int u'\left[\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right] f(\varepsilon)d\varepsilon = 0 ,$$

$$(\text{IC}_2) \qquad \int u'\left[\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial e} - \psi'(e)\right] f(\varepsilon)d\varepsilon = 0 .$$

The Optimal Output and Effort under Asymmetric Information

The first-order conditions of the manager's problem can be rewritten in the expectation operator E as,

$$E\left[u' \cdot \left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right)\right] = 0 \text{ and } E\left[u' \cdot \left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial e} - \psi'(e)\right)\right] = 0 .$$

So we have

$$E\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)\right]=Eu'\cdot E\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)+Cov\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)\right]=0$$

and

$$E\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right)\right]=Eu'\cdot E\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right)+Cov\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right)\right]=0$$

LEMMA 2.4: *(i) If* $Cov\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)\right]=0$, then $E\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)=0$.

(ii) If $Cov\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right)\right]=0$*, then* $E\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial e}-\psi'(e)\right)=0$.

PROOF: (i) Profit $\pi(q,e,\varepsilon)$ is a concave function of q by the assumption of the convexity of $C(q,e,\varepsilon)$ in q. Suppose that $\frac{\partial\pi(q,e,\varepsilon)}{\partial q}>0$. As output q increases, $\frac{\partial\pi(q,e,\varepsilon)}{\partial q}$ *decreases*, profit $\pi(q,e,\varepsilon)$ increases, so the compensation scheme $s(\pi(q,e,\varepsilon))$ increases according to $0<\alpha<1$ and $u'[s(\pi(q,e,\varepsilon))-\psi(e)]$ *decreases* by the assumption of risk aversion of the manager, i.e., $u'>0$ and $u''\leq 0$. The sign of the covariance indicates the direction of covariation of u' and $\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}$. Hence, we obtain

$$Cov\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)\right]>0.$$

$Eu'>0$ and $E\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)>0$ imply

$$E\left[u'\cdot\left(\alpha\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right)\right]>0,$$

this contradicts the first FOC equation.

Suppose that $\frac{\partial\pi(q,e,\varepsilon)}{\partial q}<0$.

As output q decreases, $\frac{\partial\pi(q,e,\varepsilon)}{\partial q}$ *increases*, profit $\pi(q,e,\varepsilon)$ increases, so the compensation scheme $s(\pi(q,e,\varepsilon))$ increases according to $0<\alpha<1$ and $u'[s(\pi(q,e,\varepsilon))-\psi(e)]$ *decreases* by the assumption of risk aversion of the manager, i.e., $u'>0$ and $u''\leq 0$. Hence, we obtain

$$Cov\left[u' \cdot \left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right)\right] < 0 .$$

$Eu' > 0$ and $E\left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right) < 0$ imply

$$E\left[u' \cdot \left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right)\right] < 0 ,$$

this also contradicts the first FOC equation.

We conclude that

if $Cov\left[u' \cdot \left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right)\right] = 0$, then $E\left(\alpha \frac{\partial \pi(q,e,\varepsilon)}{\partial q}\right) = 0$, the first FOC equation is hence satisfied.

The proof of Part (ii) is analogous to the proof of (i), so it is omitted. □

Now we can compare the two first-order conditions for the manager's expected utility maximization in case 2

$$\int \left[\alpha \frac{\partial \pi(q^{**},e^{**},\varepsilon)}{\partial q}\right] f(\varepsilon) d\varepsilon = 0 \text{ and } \int \left[\alpha \frac{\partial \pi(q^{**},e^{**},\varepsilon)}{\partial e} - \psi'(e^{**})\right] f(\varepsilon) d\varepsilon = 0$$

with those in case 1

$$\int \left[\frac{\partial \pi(q^{*},e^{*},\varepsilon)}{\partial q}\right] f(\varepsilon) d\varepsilon = 0 \text{ and } \int \left[\frac{\partial \pi(q^{*},e^{*},\varepsilon)}{\partial e} - \psi'(e^{*})\right] f(\varepsilon) d\varepsilon = 0 .$$

We find that the pricing rule is *not* affected by asymmetric information and it is the same as that in case 1, but the effort-exerting rule is affected by asymmetric information. Hence, we obtain the dichotomy property of the pricing rule and incentive under asymmetric information. Notice, although the pricing rule is independent of informational problem, this does not imply that the manager's production decision does not depend on his effort-exerting decision.

Before turning to the efficiency losses of monopoly under separation of ownership and control, we compare the effort and output levels in this case with those in the case where the owner also manages the firm.

PROPOSITION 2.2: *The effort exerted by the manager of the monopoly firm under separation of ownership and control is less than that by a manager who also owns the firm. The equilibrium* *output produced by the monopoly firm under separation of ownership and control is smaller (higher) if there are cost complementarities (cost substitutability) between output and effort.*

PROOF: Since $0<\alpha<1$, we have

$$\int\left[\frac{\partial\pi\left(q^{**},e^{**},\varepsilon\right)}{\partial e}\right]f(\varepsilon)d\varepsilon>\psi'\left(e^{**}\right).$$

The assumption $\psi'(e)>0$ and $\psi''(e)>0$ implies that the manager's effort e^{**} in case 2 is *less* than e^{*} in case 1, $\psi'\left(e^{**}\right)<\psi'\left(e^{*}\right)$, so we have

$$e^{**}<e^{*}.$$

By the assumption of the *cost complementarities* or *cost substitutions* in output and effort, $\int\left[\alpha\frac{\partial\pi\left(q^{**},e^{**},\varepsilon\right)}{\partial q}\right]f(\varepsilon)d\varepsilon=0$ and $\int\left[\frac{\partial\pi\left(q^{*},e^{*},\varepsilon\right)}{\partial q}\right]f(\varepsilon)d\varepsilon=0$ hold if the output q^{**} in case 2 is *less* or greater than q^{*} in case 1, i.e.,

$$q^{**}\begin{matrix}<\\>\end{matrix}q^{*}\ \ if\ \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0. \quad \square$$

To analyze the efficiency losses of monopoly in this case, consider now the optimal solutions for the monopoly firm and the competitive firm under asymmetric information. By Lemma 2.4, for the interior solutions, two simplified first-order conditions for the manager's expected utility maximization under asymmetric information can be rewritten as

$$\int\left[\frac{\partial\pi(q,e,\varepsilon)}{\partial q}\right]f(\varepsilon)d\varepsilon=0,$$

$$\alpha\int\left[\frac{\partial\pi(q,e,\varepsilon)}{\partial e}\right]f(\varepsilon)d\varepsilon-\psi'(e)=0,$$

where $\pi(q,e,\varepsilon)=p(q)q-C(q,e,\varepsilon)$.

(1) Monopoly equilibrium with moral hazard

At the equilibrium of the monopoly product market, the manager's optimal level of output q_{M}^{**} and level of effort e_{M}^{**} are characterized by the following two simplified first-order *necessary* conditions for the manager's expected utility maximization:

$$p'\left(q_{\mathrm{M}}^{**}\right)q_{\mathrm{M}}^{**} + p\left(q_{\mathrm{M}}^{**}\right) - \int C_q\left(q_{\mathrm{M}}^{**}, e_{\mathrm{M}}^{**}, \varepsilon\right) f(\varepsilon) d\varepsilon = 0,$$

$$-\alpha_{\mathrm{M}} \int C_e\left(q_{\mathrm{M}}^{**}, e_{\mathrm{M}}^{**}, \varepsilon\right) f(\varepsilon) d\varepsilon - \psi'\left(e_{\mathrm{M}}^{**}\right) = 0.$$

It follows from the above result that

$$e_{\mathrm{M}}^{**} < e_{\mathrm{M}}^{*} \text{ and } q_{\mathbf{M}}^{**} \begin{smallmatrix} < \\ > \end{smallmatrix} q_{\mathbf{M}}^{*} \text{ if } \begin{matrix} C_{qe}(q,e,\varepsilon) < \\ C_{qe}(q,e,\varepsilon) > \end{matrix} 0.$$

Note that the manager of the monopoly firm exerts less effort on cost reduction in case 2 than that in case 1. Moreover, the decrease in his level of effort also causes the decrease or increase in the level of output and the cost distortions. Although the manager of the representative competitive firm has to bear some risk of his income, there is managerial slack due to the moral hazard problem. Comparing with the first-best solution of the competitive firm, the managerial monopoly firm engages in two types of slack: monopoly slack and managerial slack.

Let $\tilde{H}^{\mathrm{M}} \equiv \begin{bmatrix} \tilde{H}_{11}^{\mathrm{M}} & \tilde{H}_{12}^{\mathrm{M}} \\ \tilde{H}_{21}^{\mathrm{M}} & \tilde{H}_{22}^{\mathrm{M}} \end{bmatrix}$ be the Hessian matrix of the second partial derivatives of two simplified FOC equations of the manager's expected utility maximization problem, where by the assumptions we have

$$\tilde{H}_{11}^{\mathrm{M}} = p''(q)q + 2p'(q) - \int C_{qq}(q,e,\varepsilon) f(\varepsilon) d\varepsilon < 0,$$

$$\tilde{H}_{12}^{\mathrm{M}} = -\int C_{qe}(q,e,\varepsilon) f(\varepsilon) d\varepsilon \begin{matrix} > 0 \text{ if there is cost complementarities} \\ < 0 \quad \text{ if there is cost substiutability} \end{matrix},$$

$$\tilde{H}_{21}^{\mathrm{M}} = -\alpha \int C_{eq}(q,e,\varepsilon) f(\varepsilon) d\varepsilon > 0,$$

$$\tilde{H}_{22}^{\mathrm{M}} = -\alpha \int C_{ee}(q,e,\varepsilon) f(\varepsilon) d\varepsilon - \psi''(e) < 0.$$

A sufficient second condition for the competitive firm's expected return maximization problem is that the Hessian matrix is *negative semidefinite* at the optimal q_{M}^{**} and e_{M}^{**}, and the determinant of $\tilde{H}^{\mathrm{M}}$ is *positive,*

$$\det\left(\tilde{H}^{\mathrm{M}}\right) = \tilde{H}_{11}^{\mathrm{M}} \tilde{H}_{22}^{\mathrm{M}} - \tilde{H}_{12}^{\mathrm{M}} \tilde{H}_{21}^{\mathrm{M}} > 0.$$

The above two FOC equations characterize one output equation and one effort-exerting equation of the monopoly firm:

$$q = \tilde{q}_{\mathrm{M}}(e;\alpha) \text{ and } e = \tilde{e}_{\mathrm{M}}(q;\alpha).$$

Using the comparative static calculation, we have:

Lemma 2.5: *For any incentive coefficient,* $0<\alpha<1$*, by the properties of the Hessian matrix,* $\tilde{H}^{\mathrm{M}}$*, (i) Output increases or decreases with the incentive coefficient if there are cost complementarities or cost substitutability in output and effort, i.e.*

$$\frac{d\tilde{q}_{\mathrm{M}}(\alpha)}{d\alpha}=\frac{\tilde{H}_{12}^{\mathrm{M}}}{\det\left(\tilde{H}^{\mathrm{M}}\right)}\left[-\int C_e(q,e,\varepsilon)f(\varepsilon)d\varepsilon\right]\begin{matrix}>\\<\end{matrix}0 \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0.$$

(ii) Effort always increases with the incentive coefficient, i.e.

$$\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha}=\frac{-\tilde{H}_{11}^{\mathrm{M}}}{\det\left(\tilde{H}^{\mathrm{M}}\right)}\left[-\int C_e(q,e,\varepsilon)f(\varepsilon)d\varepsilon\right]>0.$$ □

Lemma 2.6: *Given incentive coefficient,* $0<\alpha<1$*, by the properties of the Hessian matrix,* $\tilde{H}^{\mathrm{M}}$*,*

(i)

$$\text{slope of } \tilde{q}_{\mathrm{M}}(e;\alpha)\equiv\frac{dq}{de}=-\frac{\tilde{H}_{12}^{\mathrm{M}}}{\tilde{H}_{11}^{\mathrm{M}}}\begin{matrix}>0 \; (\textit{It is upward sloping if there is cost complementarities})\\<0 \quad (\textit{It is downward sloping if there is cost substitutability})\end{matrix}$$

and

$$\text{slope of } \tilde{e}_{\mathrm{M}}(q;\alpha)\equiv\frac{dq}{de}=-\frac{\tilde{H}_{22}^{\mathrm{M}}}{\tilde{H}_{21}^{\mathrm{M}}}\begin{matrix}>0 \; (\textit{It is upward sloping if there is cost complementarities})\\<0 \quad (\textit{It is downward sloping if there is cost substitutability})\end{matrix}.$$

(ii) $\text{slope of } \tilde{e}_{\mathrm{M}}(q;\alpha)-\text{slope of } \tilde{q}_{\mathrm{M}}(e;\alpha)=\dfrac{\det\left(\tilde{H}^{\mathrm{M}}\right)}{-\tilde{H}_{11}^{\mathrm{M}}\tilde{H}_{21}^{\mathrm{M}}}\begin{matrix}>\\<\end{matrix}0 \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0$*, i.e. the curve* $\tilde{e}_{\mathrm{M}}(q;\alpha)$ *of the monopoly firm is steeper than the curve* $\tilde{q}_{\mathrm{M}}(e;\alpha)$*. When two curves just have a single crossing point in* $\{q,e\}$ *space, and the unique stable equilibrium* $\left(q_{\mathrm{M}}^{**},e_{\mathrm{M}}^{**}\right)$ *exists.*

(iii) $\text{slope of } \tilde{q}_{\mathrm{M}}(e;\alpha)=\text{slope of } q_{\mathrm{M}}(e)$*, i.e. the output curve* $\tilde{q}_{\mathrm{M}}(e;\alpha)$ *of the monopoly firm in case 2 has the same slope as that in case 1, so two curves* $\tilde{q}_{\mathrm{M}}(e;\alpha)$ *and* $q_{\mathrm{M}}(e)$ *coincide in* $\{q,e\}$ *space.*

(iv) $\text{slope of } \tilde{e}_{\mathrm{M}}(q;\alpha)\begin{matrix}>\\<\end{matrix}\text{slope of } e_{\mathrm{M}}(q) \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0$*, i.e. the effort-exerting curve* $\tilde{e}_{\mathrm{M}}(q;\alpha)$ *of the monopoly firm in* $\{q,e\}$ *space in case 2 is steeper than* $e_{\mathrm{M}}(q)$ *in case 1, so the effort-exerting curve of the monopoly firm rotates leftward.* □

(2) Competitive equilibrium with moral hazard

At the equilibrium of the competitive product market, the manager's optimal level of output q_{C}^{**} and level of effort e_{C}^{**} are characterized by the following two simplified first-order *necessary* conditions for the manager's expected utility maximization:

$$p\left(q_{\mathrm{C}}^{**}\right)-\int C_q\left(q_{\mathrm{C}}^{**},e_{\mathrm{C}}^{**},\varepsilon\right)f(\varepsilon)d\varepsilon=0\,,$$

$$-\alpha_{\mathrm{C}}\int C_e\left(q_{\mathrm{C}}^{**},e_{\mathrm{C}}^{**},\varepsilon\right)f(\varepsilon)d\varepsilon-\psi'\left(e_{\mathrm{C}}^{**}\right)=0\,.$$

It follows from the above result that

$$e_{\mathrm{C}}^{**}<e_{\mathrm{C}}^{*} \text{ and } q_{\mathrm{C}}^{**} \begin{matrix}<\\>\end{matrix} q_{\mathrm{C}}^{*} \text{ if } \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0\,.$$

Note that under asymmetric information the manager of the representative competitive firm does not exert the first-best effort. Moreover, the decrease in his level of effort causes the inefficient level of output. Here, the effort distortion causes the quantity distortion and the cost distortions. These consequences are referred to as managerial slack. However, the manager of the representative competitive firm has to bear some risk of his income.

Let $\widetilde{H}^{\mathrm{C}}\equiv\begin{bmatrix}\widetilde{H}_{11}^{\mathrm{C}} & \widetilde{H}_{12}^{\mathrm{C}}\\ \widetilde{H}_{21}^{\mathrm{C}} & \widetilde{H}_{22}^{\mathrm{C}}\end{bmatrix}$ be the Hessian matrix of the second partial derivatives of two simplified FOC equations of the manager's expected utility maximization problem, where by the assumptions we have

$$\widetilde{H}_{11}^{\mathrm{C}}=p'(q)-\int C_{qq}(q,e,\varepsilon)f(\varepsilon)d\varepsilon<0\,,$$

$$\widetilde{H}_{12}^{\mathrm{C}}=-\int C_{qe}(q,e,\varepsilon)f(\varepsilon)d\varepsilon \begin{matrix}>0 \text{ if there is cost complementarities}\\ <0 \quad\quad \text{ if there is cost substiutability}\end{matrix}\,,$$

$$\widetilde{H}_{21}^{\mathrm{C}}=-\alpha\int C_{eq}(q,e,\varepsilon)f(\varepsilon)d\varepsilon>0\,,$$

$$\widetilde{H}_{22}^{\mathrm{C}}=-\alpha\int C_{ee}(q,e,\varepsilon)f(\varepsilon)d\varepsilon-\psi''(e)<0\,.$$

A sufficient second condition for the competitive firm's expected return maximization problem is that the Hessian matrix is *negative semidefinite* at the optimal q_{C}^{**} and e_{C}^{**}, and the determinant of $\widetilde{H}^{\mathrm{C}}$ is *positive*,

$$\det\left(\widetilde{H}^{\mathrm{C}}\right)=\widetilde{H}_{11}^{\mathrm{C}}\widetilde{H}_{22}^{\mathrm{C}}-\widetilde{H}_{12}^{\mathrm{C}}\widetilde{H}_{21}^{\mathrm{C}}>0\,.$$

The above two FOC equations characterize one output equation and one effort-exerting equation of the monopoly firm:

$$q=\widetilde{q}_{\mathrm{C}}(e;\alpha) \text{ and } e=\widetilde{e}_{\mathrm{C}}(q;\alpha)\,.$$

Using the comparative static calculation, we have:

LEMMA 2.7: *For any incentive coefficient,* $0<\alpha<1$, *by the properties of the Hessian matrix,* $\tilde{H}^{\mathrm{C}}$, *(i) Output increases or decreases with the incentive coefficient if there are cost complementarities or cost substitutability in output and effort, i.e.*

$$\frac{d\tilde{q}_{\mathrm{C}}(\alpha)}{d\alpha}=\frac{\tilde{H}_{12}^{\mathrm{C}}}{\det\left(\tilde{H}^{\mathrm{C}}\right)}\left[-\int C_e(q,e,\varepsilon)f(\varepsilon)d\varepsilon\right]\begin{matrix}>\\<\end{matrix}0 \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0.$$

(ii) Effort always increases with the incentive coefficient, i.e.

$$\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}=\frac{-\tilde{H}_{11}^{\mathrm{C}}}{\det\left(\tilde{H}^{\mathrm{C}}\right)}\left[-\int C_e(q,e,\varepsilon)f(\varepsilon)d\varepsilon\right]>0.\ \square$$

From Lemma 2.5 and 2.7, we have Lemma 2.8.

LEMMA 2.8: *For any incentive coefficient,* $0<\alpha<1$,

(i) $\frac{d\tilde{q}_{\mathrm{M}}(\alpha)}{d\alpha}\begin{matrix}<\\>\end{matrix}\frac{d\tilde{q}_{\mathrm{C}}(\alpha)}{d\alpha}\begin{matrix}>\\<\end{matrix}0 \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0.$

(ii) $0<\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha}<\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}.$ $\square$

LEMMA 2.9: *Given incentive coefficient,* $0<\alpha<1$, *by the properties of the Hessian matrix,* $\tilde{H}^{\mathrm{C}}$,

(i)

$$\text{slope of } \tilde{q}_{\mathrm{C}}(e;\alpha)\equiv\frac{dq}{de}=-\frac{\tilde{H}_{12}^{\mathrm{C}}}{\tilde{H}_{11}^{\mathrm{C}}}\begin{matrix}>0\ (\textit{It is upward sloping if there is cost complementarities})\\<0\ \ (\textit{It is downward sloping if there is cost substitutability})\end{matrix}$$

and

$$\text{slope of } \tilde{e}_{\mathrm{C}}(q;\alpha)\equiv\frac{dq}{de}=-\frac{\tilde{H}_{22}^{\mathrm{C}}}{\tilde{H}_{21}^{\mathrm{C}}}\begin{matrix}>0\ (\textit{It is upward sloping if there is cost complementarities})\\<0\ \ (\textit{It is downward sloping if there is cost substitutability})\end{matrix}.$$

(ii) $\text{slope of } \tilde{e}_{\mathrm{C}}(q;\alpha)-\text{slope of } \tilde{q}_{\mathrm{C}}(e;\alpha)=\frac{\det\left(\tilde{H}^{\mathrm{C}}\right)}{-\tilde{H}_{11}^{\mathrm{C}}\tilde{H}_{21}^{\mathrm{C}}}\begin{matrix}>\\<\end{matrix}0 \;\; if \;\; \begin{matrix}C_{qe}(q,e,\varepsilon)<\\C_{qe}(q,e,\varepsilon)>\end{matrix}0$, *i.e. the curve* $\tilde{e}_{\mathrm{C}}(q;\alpha)$ *of the representative competitive firm is steeper than the curve* $\tilde{q}_{\mathrm{C}}(e;\alpha)$. *When two*

curves just have a single crossing point in $\{q,e\}$ *space, and the unique stable equilibrium* $\left(q_{\mathrm{C}}^{**},e_{\mathrm{C}}^{**}\right)$ *exists.*

(iii) slope of $\tilde{q}_{\mathrm{C}}(e;\alpha)=$ slope of $q_{\mathrm{C}}(e)$, *i.e. the output curve* $\tilde{q}_{\mathrm{C}}(e;\alpha)$ *of the representative competitive firm in case 2 has the same slope as that in case 1, so two curves* $\tilde{q}_{\mathrm{C}}(e;\alpha)$ *and* $q_{\mathrm{C}}(e)$ *coincide in* $\{q,e\}$ *space.*

(iv) slope of $\tilde{e}_{\mathrm{C}}(q;\alpha)\,{>\atop<}\,$ slope of $e_{\mathrm{C}}(q)$ *if* ${C_{qe}(q,e,\varepsilon)<\atop C_{qe}(q,e,\varepsilon)>}0$, *i.e. the effort-exerting curve* $\tilde{e}_{\mathrm{C}}(q;\alpha)$ *of the representative competitive firm in* $\{q,e\}$ *space in case 2 is steeper than* $e_{\mathrm{C}}(q)$ *in case 1, so the effort-exerting curve of the representative competitive firm rotates leftward.*

□

From Lemma 2.6 and 2.9 we have Lemma 2.10.

LEMMA 2.10: *Given incentive coefficient,* $0<\alpha<1$, *(i)* slope of $\tilde{q}_{\mathrm{M}}(e;\alpha)\,{<\atop>}\,$ slope of $\tilde{q}_{\mathrm{C}}(e;\alpha)$ *if* $C_{qe}(q,e,\varepsilon)\,{<\atop>}\,0$, *i.e. the output curve* $\tilde{q}_{\mathrm{M}}(e;\alpha)$ *of the monopoly firm is flatter than* $\tilde{q}_{\mathrm{C}}(e;\alpha)$ *of the competitive firm in* $\{q,e\}$ *space in case 2.*

(ii) slope of $\tilde{e}_{\mathrm{M}}(q;\alpha)=$ slope of $\tilde{e}_{\mathrm{C}}(q;\alpha)$, *i.e. the effort-exerting curve* $\tilde{e}_{\mathrm{M}}(q;\alpha)$ *of the monopoly firm has the same slope as* $\tilde{e}_{\mathrm{C}}(q;\alpha)$ *of the competitive firm, so two curves* $\tilde{e}_{\mathrm{M}}(q;\alpha)$*, and* $\tilde{e}_{\mathrm{C}}(q;\alpha)$*, also coincide in* $\{q,e\}$ *space.*

(iii) At $e=0$, $\tilde{q}_{\mathrm{M}}(0;\alpha)<\tilde{q}_{\mathrm{C}}(0;\alpha)$. $\tilde{q}_{\mathrm{M}}(e;\alpha)$ *and* $\tilde{q}_{\mathrm{C}}(e;\alpha)$ *are strictly increasing or decreasing and have no crossing point in* $\{q,e\}$, *so the curve* $\tilde{q}_{\mathrm{M}}(e;\alpha)$ *is below the curve* $\tilde{q}_{\mathrm{C}}(e;\alpha)$. □

Given the incentive coefficient α of the optimal linear incentive scheme, $s(\pi(q,e,\varepsilon))=\alpha\pi(q,e,\varepsilon)+\beta$, let $\tilde{q}(\alpha)$ and $\tilde{e}(\alpha)$ are the optimal output and effort of the manager's expected utility maximization.

PROPOSITION 2.3: *Given any incentive coefficient,* $0<\alpha<1$, if ${C_{qe}(q,e,\varepsilon)<\atop C_{qe}(q,e,\varepsilon)>}0$, *then the following relationships hold for the equilibrium efforts and outputs:*

$$\tilde{e}_{\mathrm{M}}(\alpha) \lessgtr \tilde{e}_{\mathrm{C}}(\alpha),\ \tilde{q}_{\mathrm{M}}(\alpha) < \tilde{q}_{\mathrm{C}}(\alpha) \text{ and } \tilde{q}_{\mathrm{C}}(\alpha) - \tilde{q}_{\mathrm{M}}(\tilde{e}_{\mathrm{M}}(\alpha)) > \tilde{q}_{\mathrm{C}}(\alpha) - \tilde{q}_{\mathrm{M}}(\tilde{e}_{\mathrm{C}}(\alpha)).$$

PROOF: Given incentive coefficient, the two simplified first-order conditions of manager's expected utility maximization are given by

$$\int \left[\frac{\partial \pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon)}{\partial q} \right] f(\varepsilon) d\varepsilon = 0,$$

$$\alpha \int \left[\frac{\partial \pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon)}{\partial e} \right] f(\varepsilon) d\varepsilon - \psi'(\tilde{e}(\alpha)) = 0,$$

where $\pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) = p(\tilde{q}(\alpha))\tilde{q}(\alpha) - C(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon)$.

The rest of the proof of this proposition is analogous to the proof of PROPOSITION 2.1 by using Lemmas 2.6, 2.9 and 2.10, and is omitted. □

The owner's expected return is $E(R(\alpha)) = (1-\alpha)\int \pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) f(\varepsilon) d\varepsilon - \beta$. The owner's problem is to choose the coefficients α and β, to maximize his expected return,

$$\max_{\alpha, \beta} (1-\alpha)\int \pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) f(\varepsilon) d\varepsilon - \beta,$$

subject to

$$\int u[\alpha\pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) + \beta - \psi(\tilde{e}(\alpha))] f(\varepsilon) d\varepsilon \geq \bar{u}.$$

When the manager's participation constraint is binding, by inverting the function, $\int u[\alpha\pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) + \beta - \psi(\tilde{e}(\alpha))] f(\varepsilon) d\varepsilon = \bar{u}$, we can express β as a function of α and $\bar{u}$,

$$\beta = \beta(\alpha; \bar{u}).$$

So $\dfrac{d\beta(\alpha;\bar{u})}{d\alpha} = -\dfrac{\partial Eu/\partial \alpha}{\partial Eu/\partial \alpha} = -\dfrac{E(u' \cdot \pi)}{E(u')} < 0$.

PROPOSITION 2.4: *Assume that the variance of profit is constant. The incentive coefficient,* α, *set by the owner of the monopoly firm is less than that by the owner of the competitive firm if the technology exhibits cost complementarities between output and the effort. The relative magnitudes of* α *in these two cases cannot be determined if the technology exhibits cost substitutability.*

PROOF: By the envelope theorem, the optimal incentive coefficient, α, satisfies the first-order condition of the owner's expected return maximization,

$$E(R'(\alpha)) = (1-\alpha)\frac{\partial E\pi}{\partial e}\frac{d\tilde{e}(\alpha)}{d\alpha} - E\pi - \frac{d\beta(\alpha;\bar{u})}{d\alpha} = 0.$$

Using Taylor first-order series expansion, we approximate the functions, $E(u' \cdot \pi)$ and $E(u')$, based at the manager's mean net income, $\bar{w} \equiv \alpha E\pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) + \beta - \psi(\tilde{e}(\alpha))$,

$$E(u' \cdot \pi) = u'(\bar{w})E\pi + u''(\bar{w})\alpha E(\pi(\pi - E\pi)) = u'(\bar{w})E\pi + u''(\bar{w})\alpha Var(\pi),$$

$$E(u') = u'(\bar{w}) + u''(\bar{w})E(\alpha(\pi - E\pi)) = u'(\bar{w}).$$

Substituting these two expressions in the first-order condition, it is approximated by

$$E(R'(\alpha)) = (1-\alpha)[-EC_e(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon)]\frac{d\tilde{e}(\alpha)}{d\alpha} - r(\bar{w})\alpha Var(\pi) = 0,$$

where $r(\bar{w}) \equiv -u''(\bar{w})/u'(\bar{w})$ is the Arrow-Pratt coefficient of absolute risk aversion at $\bar{w}$.

For *any* incentive coefficient, $0 < \alpha < 1$, the owner's expected *marginal return* of the monopoly firm and the owner's expected marginal return of the representative competitive firm are respectively give by

$$E(R'_{\mathrm{M}}(\alpha)) = (1-\alpha)[-EC_e(\tilde{q}_{\mathrm{M}}(\alpha), \tilde{e}_{\mathrm{M}}(\alpha), \varepsilon)]\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha} - r(\bar{w}_{\mathrm{M}})\alpha Var(\pi_{\mathrm{M}}),$$

$$E(R'_{\mathrm{C}}(\alpha)) = (1-\alpha)[-EC_e(\tilde{q}_{\mathrm{C}}(\alpha), \tilde{e}_{\mathrm{C}}(\alpha), \varepsilon)]\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha} - r(\bar{w}_{\mathrm{C}})\alpha Var(\pi_{\mathrm{C}}).$$

Comparing each term in these two expressions of the owner's expected *marginal return*, we want to figure out which one is *greater*.

(1) Since $\tilde{q}_{\mathrm{M}}(\alpha)$, $\tilde{e}_{\mathrm{M}}(\alpha)$, $\tilde{q}_{\mathrm{C}}(\alpha)$ and $\tilde{e}_{\mathrm{C}}(\alpha)$ are the optimal output and effort of the manager's expected utility maximization given the coefficients α, by PROPOSITION 2.4,

$$\tilde{e}_{\mathrm{M}}(\alpha) \substack{< \\ >} \tilde{e}_{\mathrm{C}}(\alpha) \text{ if } \begin{matrix} C_{qe}(q,e,\varepsilon) < \\ C_{qe}(q,e,\varepsilon) > \end{matrix} 0.$$

In addition, $\tilde{q}(\alpha)$ and $\tilde{e}(\alpha)$ satisfy the first-order conditions with respect to effort of the manager's expected utility maximization, $-EC_e(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) = \psi'(\tilde{e}(\alpha))$, the convexity of $\psi(\cdot)$ implies that

$$\psi'(\tilde{e}_{\mathrm{M}}(\alpha)) \substack{< \\ >} \psi'(\tilde{e}_{\mathrm{C}}(\alpha)) \text{ if } \begin{matrix} C_{qe}(q,e,\varepsilon) < \\ C_{qe}(q,e,\varepsilon) > \end{matrix} 0,$$

Hence, $-EC_e(\tilde{q}_{\mathrm{M}}(\alpha), \tilde{e}_{\mathrm{M}}(\alpha), \varepsilon) \substack{< \\ >} -EC_e(\tilde{q}_{\mathrm{C}}(\alpha), \tilde{e}_{\mathrm{C}}(\alpha), \varepsilon)$ if $\begin{matrix} C_{qe}(q,e,\varepsilon) < \\ C_{qe}(q,e,\varepsilon) > \end{matrix} 0$.

(2) By LEMMA 2.8, for any incentive coefficient, $0 < \alpha < 1$, $0 < \dfrac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha} < \dfrac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}$.

(3) For *any* incentive coefficient, $0 < \alpha < 1$, the managers' mean net incomes minus the fixed wage satisfies $\overline{w}_{\mathrm{M}} - \beta_{\mathrm{M}} > \overline{w}_{\mathrm{C}} - \beta_{\mathrm{C}}$ by COROLLARY 2.4, but the manager's participation constraint is binding, $\int u[\alpha\pi(\tilde{q}(\alpha),\tilde{e}(\alpha),\varepsilon) + \beta - \psi(\tilde{e}(\alpha))]f(\varepsilon)d\varepsilon = \overline{u}$. So $\overline{w}_{\mathrm{M}} = \overline{w}_{\mathrm{C}}$ and $\beta_{\mathrm{M}} < \beta_{\mathrm{C}}$. We have

$$r(\overline{w}_{\mathrm{M}}) = r(\overline{w}_{\mathrm{C}}).$$

(4) By the assumption of constant variance of the profit, $Var(\pi_{\mathrm{M}}) = Var(\pi_{\mathrm{C}})$.

On the basis of the above four results, we obtain

$$E(R'_{\mathrm{M}}(\alpha)) < E(R'_{\mathrm{C}}(\alpha)) \text{ if } C_{qe}(q,e,\varepsilon) < 0.$$

$$E(R'_{\mathrm{M}}(\alpha)) \substack{<\\>} E(R'_{\mathrm{C}}(\alpha)) \text{ if } C_{qe}(q,e,\varepsilon) > 0.$$

The optimal incentive coefficients, α_{M}, chosen by the owner of the monopoly firm must satisfy this first-order condition,

$$E(R'_{\mathrm{C}}(\alpha_{\mathrm{M}})) > E(R'_{\mathrm{M}}(\alpha_{\mathrm{M}})) = 0,$$

the concavity of $E(R(\alpha))$ implies that the owner can increases the incentive coefficient from α_{M} to α_{C} such that his expected return attains a maximum, $E(R'_{\mathrm{C}}(\alpha_{\mathrm{C}})) = 0$ at the optimal α_{C}. Hence, we obtain

$$\alpha_{\mathrm{M}} < \alpha_{\mathrm{C}} \text{ if } C_{qe}(q,e,\varepsilon) < 0.$$

By similar reasoning,

$$\alpha_{\mathrm{M}} \substack{<\\>} \alpha_{\mathrm{C}} \text{ if } C_{qe}(q,e,\varepsilon) > 0. \qquad \square$$

PROPOSITION 2.5: *Assume that the variance of profit is constant.*

(i) If the technology exhibits cost complementarities between output and the effort, the equilibrium effort and output of the monopoly firm are less than those of the competitive firm.

(ii) If the technology exhibits cost substitutability between output and the effort and $\alpha_{\mathrm{M}} < \alpha_{\mathrm{C}}$*, it is ambiguous as to whether the equilibrium effort and output of the monopoly firm is higher or lower than those of the competitive firm.*

(iii) If the technology exhibits cost substitutability between output and the effort and $\alpha_{\mathbf{M}} > \alpha_{\mathbf{C}}$*, then the equilibrium effort exerted by the manager of the monopoly firm is greater than that by the manager of the competitive firm but the equilibrium output produced by the monopoly firm is less than that by the competitive firm.*

PROOF: (i) If $C_{qe}(q,e,\varepsilon)<0$, then $\tilde{e}_{\mathrm{M}}(\alpha)<\tilde{e}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.3. Now $\alpha_{\mathrm{M}}<\alpha_{\mathrm{C}}$ by PROPOSITION 2.4 and $0<\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha}<\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}$ by Lemma 2.8, implying that

$$e_{\mathrm{M}}^{**}\equiv\tilde{e}_{\mathrm{M}}(\alpha_{\mathrm{M}})<\tilde{e}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv e_{\mathrm{C}}^{**}.$$

By similar reasoning, $\tilde{q}_{\mathrm{M}}(\alpha)<\tilde{q}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.4 and $\frac{d\tilde{q}_{\mathrm{C}}(\alpha)}{d\alpha}>\frac{d\tilde{q}_{\mathrm{M}}(\alpha)}{d\alpha}>0$ by Lemma 2.8, implying that

$$q_{\mathrm{M}}^{**}\equiv\tilde{q}_{\mathrm{M}}(\alpha_{\mathrm{M}})<\tilde{q}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv q_{\mathrm{C}}^{**}.$$

(ii) If $C_{qe}(q,e,\varepsilon)>0$, then $\tilde{e}_{\mathrm{M}}(\alpha)>\tilde{e}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.3. Now $\alpha_{\mathrm{M}}<\alpha_{\mathrm{C}}$ by PROPOSITION 2.4 and $0<\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha}<\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}$ by Lemma 2.8, implying that

$$e_{\mathrm{M}}^{**}\equiv\tilde{e}_{\mathrm{M}}(\alpha_{\mathrm{M}})\lessgtr\tilde{e}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv e_{\mathrm{C}}^{**}.$$

By similar reasoning, $\tilde{q}_{\mathrm{M}}(\alpha)<\tilde{q}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.4 and $\frac{d\tilde{q}_{\mathrm{C}}(\alpha)}{d\alpha}<\frac{d\tilde{q}_{\mathrm{M}}(\alpha)}{d\alpha}<0$ by Lemma 2.8, implying that

$$q_{\mathrm{M}}^{**}\equiv\tilde{q}_{\mathrm{M}}(\alpha_{\mathrm{M}})\lessgtr\tilde{q}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv q_{\mathrm{C}}^{**}.$$

(iii) If $C_{qe}(q,e,\varepsilon)>0$, then $\tilde{e}_{\mathrm{M}}(\alpha)>\tilde{e}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.3. Now $\alpha_{\mathrm{M}}>\alpha_{\mathrm{C}}$ by PROPOSITION 2.5 and $0<\frac{d\tilde{e}_{\mathrm{M}}(\alpha)}{d\alpha}<\frac{d\tilde{e}_{\mathrm{C}}(\alpha)}{d\alpha}$ by Lemma 2.8, implying that

$$e_{\mathrm{M}}^{**}\equiv\tilde{e}_{\mathrm{M}}(\alpha_{\mathrm{M}})>\tilde{e}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv e_{\mathrm{C}}^{**}.$$

By similar reasoning, $\tilde{q}_{\mathrm{M}}(\alpha)<\tilde{q}_{\mathrm{C}}(\alpha)$ by PROPOSITION 2.3 and $\frac{d\tilde{q}_{\mathrm{C}}(\alpha)}{d\alpha}<\frac{d\tilde{q}_{\mathrm{M}}(\alpha)}{d\alpha}<0$ by Lemma 2.8, implying that

$$q_{\mathrm{M}}^{**}\equiv\tilde{q}_{\mathrm{M}}(\alpha_{\mathrm{M}})<\tilde{q}_{\mathrm{C}}(\alpha_{\mathrm{C}})\equiv q_{\mathrm{C}}^{**}. \quad \square$$

A comparison of Proposition 2.1 with Proposition 2.5 suggests that under certain conditions the effects of monopoly under separation of ownership and control are qualitatively similar to those in the case where the owner manages the firm himself. That is, monopoly leads to lower efforts if there are cost complementarities between output and effort; but the opposite is true if there is cost substitutability.

Another way of considering the effects of monopoly under separation of ownership and control is to use the competitive entrepreneurial firm (i.e. the first best equilibrium) as the benchmark. Combining the insights from Propositions 2.1 and 2.2 we see that separation of ownership and control exacerbates the problem of monopoly if there are cost complementarities between quantity and effort. On the other hand, if there is cost substitutability, separation of ownership and control mitigates the problem of monopoly by offsetting the entrepreneurial monopolist's tendency to increase effort level and reduce output.

3. A Parameterized Model and Numerical Examples

To push further with our analysis, we consider a parameterized model in which demand function is linear,

$$p = a - bq \text{, with } a, b > 0 \text{,}$$

and the cost function is strictly convex in quantity,[5]

$$C(q, e, \varepsilon) = q^3 + (k - e)q - \varepsilon \text{, with } a > k > 0 \text{.}$$

Note that this cost function has the property that $C_{eq} < 0$, implying cost complementarities between output and effort. Parameter ε reflects an exogenous random shock to the costs of production, and is assumed to have a normal distribution with mean zero and variance σ_ε^2.[6] The firm or any contracting party has no information the *ex post* realizations of the random variable except for its distribution.

Thus, the firm's profit is

$$\pi(q, e, \varepsilon) = pq - C(q, e, \varepsilon) = pq - q^3 - (k - e)q + \varepsilon \text{.}$$

The disutility function of effort expressed in monetary terms is assumed to be quadratic,[7]

$$\psi(e) = (R/2)e^2 \text{, with } R > 0 \text{,}$$

where R is the parameter of disutility of effort.

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[5] This is in contrast to the parameterized model of Nalebuff and Stiglitz (1983b), in which production technology exhibits constant returns to scale.

[6] The restriction on the normal distribution of the random variable is to pair it with exponential utility when we consider the case of the separation of ownership and control. Holmstrom and Milgrom (1987) suggested such a pair is convenient to solve some examples of the principal-agent model. For the reason of comparison, we take the same restriction on the distribution of the random variable.

[7] This specification of the disutility function of effort is inspired from the example of Nalebuff and Stiglitz (1983a), the example of Holmstrom and Milgrom (1987) and Tirole's (1988) example that is taken from Parsons (1984).

Note that all the assumptions made in our general model are satisfied in the specifications of these functions. As in section 2, we will consider both the case of an entrepreneurial firm and the case of a managerial firm (i.e. separation of ownership and control).

Case 1: Non-Separation of Ownership and Control

The expected return or net profit of the owner-manager's is

$$E(R) = E(\pi(q, e, \varepsilon)) - \psi(e) = pq - q^3 - (k - e)q - (R/2)e^2 .$$

In the *monopoly equilibrium*, the first-order conditions of the expected profit maximization problem are given by

$$a - 2bq - 3q^2 - k + e = 0$$

and

$$q - eR = 0 .$$

The equilibrium output and effort are

$$q_{\mathrm{M}}^* = \frac{1 - 2bR + \sqrt{12(a-k)R^2 + (2bR-1)^2}}{6R} ,$$

$$e_{\mathrm{M}}^* = \frac{1 - 2bR + \sqrt{12(a-k)R^2 + (2bR-1)^2}}{6R^2} .$$

From the first of the two first-order conditions we obtain the output as a function of effort,

$$q_{\mathrm{M}}(e) = \frac{1}{3}\left(-b + \sqrt{3(a - k + e) + b^2}\right),$$

which is strictly increasing in effort.

For a representative competitive firm, the first-order conditions of the expected profit maximization problem are given by

$$a - bq - 3q^2 - k + e = 0$$

and

$$q - eR = 0 .$$

The equilibrium output and effort are

$$q_{\mathrm{C}}^* = \frac{1 - bR + \sqrt{12(a-k)R^2 + (bR-1)^2}}{6R} ,$$

$$e_{\mathrm{C}}^* = \frac{1 - bR + \sqrt{12(a-k)R^2 + (bR-1)^2}}{6R^2} .$$

We can make some interesting comparisons between our equilibrium outcome and the outcome of the traditional model. Here by traditional model we mean the model where the effort variable is not considered. In our framework we can obtain a version of the traditional model by setting $e = 0$. The monopoly equilibrium output and the competitive equilibrium output in this traditional model are respectively

$$q_{\mathrm{M}}^{0} = \frac{1}{3}\left(-b + \sqrt{3(a-k) + b^{2}}\right),$$

$$q_{\mathrm{C}}^{0} = \frac{1}{6}\left(-b + \sqrt{12(a-k) + b^{2}}\right),$$

Then $q_{\mathrm{M}}^{0} < q_{\mathrm{M}}^{*}$ and $q_{\mathrm{C}}^{0} < q_{\mathrm{C}}^{*}$. In addition, the effect of effort on output for the monopoly firm and competitive firm, evaluating at $e = 0$, are

$$\left.\frac{dq_{\mathrm{M}}(e)}{de}\right|_{e=0} = \frac{1}{\sqrt{12(a-k) + 4b^{2}}}$$

and

$$\left.\frac{dq_{\mathrm{C}}(e)}{de}\right|_{e=0} = \frac{1}{\sqrt{12(a-k) + b^{2}}},$$

implying

$$\left.\frac{dq_{\mathrm{M}}(e)}{de}\right|_{e=0} < \left.\frac{dq_{\mathrm{C}}(e)}{de}\right|_{e=0}.$$

In other words, staring from $e = 0$, as the level of effort increases, the increase in the level of output produced by the monopoly is less than that by the competitive firm. Comparing the equilibrium value of effort and output for the monopoly firm with e_{C}^{*} and q_{C}^{*}, we know that monopoly causes a reduction in effort as well as a reduction in output. The monopoly indeed engages in slack even in the absence of the moral hazard problem.

From the first of the competitive firm's first-order conditions we obtain the output as a function of effort,

$$q_{\mathrm{C}}(e) = \frac{1}{6}\left(-b + \sqrt{12(a-k+e) + b^{2}}\right),$$

which is also strictly increasing in effort. Then the output supposed to be produced by the competitive firm at the inefficient level of effort e_{M}^{*} is

$$q_{\mathrm{C}}\left(e_{\mathrm{M}}^{*}\right) = \frac{1}{6}\left[-b + \sqrt{12(a-k)R^{2} + b^{2}R^{2} + 2\left(1 - 2bR + \sqrt{12(a-k)R^{2} + (2bR-1)^{2}}\right)} \Big/ R\right],$$

which is less than $q_{\mathrm{C}}^{*} \equiv q_{\mathrm{C}}\left(e_{\mathrm{C}}^{*}\right)$ since $e_{\mathrm{M}}^{*} < e_{\mathrm{C}}^{*}$.

Case 2: Separation of Ownership and Control

Let $s(\pi(q,e,\varepsilon))$ denote the linear incentive contract offered to the manager. Then

$$s(\pi(q,e,\varepsilon)) = \alpha\pi(q,e,\varepsilon) + \beta ,$$

where α is a share of profit and β is a fixed income. The manager's net income is

$$w = s(\pi(q,e,\varepsilon)) - \psi(e) .$$

Suppose that the manager has a constant absolute risk averse utility function,[8]

$$u(w) = -e^{-rw} ,$$

where r is the constant coefficient of absolute risk aversion, and w is the manager's net monetary income. Then his expected utility is given by

$$Eu(w) = -e^{-r[E(w)-(r/2)Var(w)]} = u[E(w) - (r/2)Var(w)] .$$

It follows that the manager's certainty equivalent given $s(\pi)$ is given by

$$E(w) - (r/2)Var(w) = Es(\pi(q,e,\varepsilon)) - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 ,$$

where $(r/2)\alpha^2\sigma_\varepsilon^2$ is the manager's risk cost. His reservation wage is $\overline{w}$, so the participation constraint is

$$Es(\pi(q,e,\varepsilon)) - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 \geq \overline{w} .$$

Since the expected utility is increasing in his certainty equivalent, the maximization of his expected utility is equivalent to the maximization of his certainty equivalent.

Note that the owner is risk neutral, his expected return is

$$E[\pi(q,e,\varepsilon) - s(\pi(q,e,\varepsilon))] .$$

Given the incentive contract, the manager chooses q and e to maximize his certainty equivalent,

$$\max_{q,e} \ \beta + \alpha E(\pi(q,e,\varepsilon)) - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 ,$$

The first-order conditions of this problem are

$$\frac{\partial E(\pi(q,e,\varepsilon))}{\partial q} = 0 \tag{1}$$

and

$$\alpha \frac{\partial E(\pi(q,e,\varepsilon))}{\partial e} - \psi'(e) = 0 \,. \tag{2}$$

Let $q(\alpha)$ and $e(\alpha)$ denote the equilibrium output and effort given the incentive parameter, α.

Totally differentiating two first-order conditions with respect to q, e and α, we have

$$H\begin{bmatrix} dq \\ de \end{bmatrix} = \begin{bmatrix} 0 \\ -\dfrac{\partial E\pi}{\partial e} d\alpha \end{bmatrix} = 0\,,$$

where $H = \begin{bmatrix} \dfrac{\partial^2 E\pi}{\partial q^2} & \dfrac{\partial^2 E\pi}{\partial e \partial q} \\ \alpha \dfrac{\partial^2 E\pi}{\partial q \partial e} & \alpha \dfrac{\partial^2 E\pi}{\partial e^2} - \psi''(e) \end{bmatrix}$ is the Hessian matrix of the second derivatives of the first-order conditions of the manager's certainty equivalent maximization.

So the effects of the incentive parameter on output and effort are

$$\begin{bmatrix} dq/d\alpha \\ de/d\alpha \end{bmatrix} = H^{-1}\begin{bmatrix} 0 \\ -\dfrac{\partial E\pi}{\partial e} d\alpha \end{bmatrix} = \frac{1}{\det(H)}\begin{bmatrix} \dfrac{\partial^2 E\pi}{\partial e \partial q}\dfrac{\partial E\pi}{\partial e} \\ -\dfrac{\partial^2 E\pi}{\partial q^2}\dfrac{\partial E\pi}{\partial e} \end{bmatrix}$$

$$= \frac{1}{\det(H)}\begin{bmatrix} EC_{eq}EC_e \\ \dfrac{\partial^2 E\pi}{\partial q^2} EC_e \end{bmatrix},$$

where $\det(H)$ is the determinant of H, which is positive.

Hence, we have

$$\frac{dq}{d\alpha} = \frac{EC_{eq}EC_e}{\det(H)} \begin{matrix} > 0 & \text{if} & C_{eq} < 0 \\ = 0 & \text{if} & C_{eq} = 0 \\ < 0 & \text{if} & C_{eq} < 0 \end{matrix}\,,$$

$$\frac{de}{d\alpha} = \frac{1}{\det(H)}\left[\frac{\partial^2 E\pi}{\partial q^2}\right] EC_e < 0\,.$$

The owner's problem is to choose the parameters of the incentive contract, α and β, to maximize his expected return, subject to the participation constraint and to two incentive constraints (1) and (2). When the participation constraint and two incentive constraints are binding, the owner's problem can be rewritten as

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[8] The constant absolute risk averse utility function is a special case of mean-variance utility functions and the very usual specification in the principal-agent literature. It is taken from Lazear and Rosen (1981), and Holmstrom and Milgrom (1987).

$$\max_{q,e,\alpha} \; E(\pi(q,e,\varepsilon)) - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 - \overline{w}$$

$$\text{s.t.} \qquad \frac{\partial E(\pi(q,e,\varepsilon))}{q} = 0 ,$$

$$\alpha \frac{\partial E(\pi(q,e,\varepsilon))}{e} - \psi'(e) = 0 .$$

The Lagrangian for the owner's problem is

$$L = E(\pi(q,e,\varepsilon)) - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 - \overline{w} + \mu_1\left(\frac{\partial E(\pi(q,e,\varepsilon))}{q}\right) + \mu_2\left(\alpha \frac{\partial E(\pi(q,e,\varepsilon))}{e} - \psi'(e)\right).$$

In the *monopoly equilibrium with moral hazard*, the first-order conditions for the manager's certainty equivalent maximization are given by

$$a - 2bq_{\mathrm{M}} - 3q_{\mathrm{M}}^2 - k + e_{\mathrm{M}} = 0$$

and

$$\alpha_{\mathrm{M}} q_{\mathrm{M}} - e_{\mathrm{M}} R = 0 .$$

Let $q_{\mathrm{M}}(\alpha_{\mathrm{M}})$ and $e_{\mathrm{M}}(\alpha_{\mathrm{M}})$ denote the equilibrium output and effort given the incentive parameter, α_{M},

$$q_{\mathrm{M}}(\alpha_{\mathrm{M}}) = \frac{\alpha_{\mathrm{M}} - 2bR + \sqrt{12(a-k)R^2 + (2bR - \alpha_{\mathrm{M}})^2}}{6R} ,$$

$$e_{\mathrm{M}}(\alpha_{\mathrm{M}}) = \frac{\alpha_{\mathrm{M}}\left(\alpha_{\mathrm{M}} - 2bR + \sqrt{12(a-k)R^2 + (2bR - \alpha_{\mathrm{M}})^2}\right)}{6R^2} .$$

The Hessian matrix is

$$H^{\mathrm{M}} = \begin{bmatrix} -2b - 6q_{\mathrm{M}} & 1 \\ \alpha_{\mathrm{M}} & -R \end{bmatrix},$$

$$\det(H^{\mathrm{M}}) = 2R(b + 3q_{\mathrm{M}}) - \alpha_{\mathrm{M}} > 0 ,$$

$$EC_e = -q_{\mathrm{M}} ,$$

$$EC_{eq} = -1,$$

$$\frac{\partial^2 E\pi}{\partial q^2} = -2(b + 3q_{\mathrm{M}}) < 0 .$$

We obtain the effects of the incentive parameter on output and effort,

$$\frac{dq_{\mathrm{M}}}{d\alpha_{\mathrm{M}}} = \frac{EC_{eq}EC_e}{\det(H^{\mathrm{M}})} = \frac{q_{\mathrm{M}}}{\det(H^{\mathrm{M}})} > 0 ,$$

$$\frac{de_{\mathrm{M}}}{d\alpha_{\mathrm{M}}} = \frac{1}{\det\left(H^{\mathrm{M}}\right)}\left[\frac{\partial^2 E\pi}{\partial q^2}\right] EC_e = \frac{2\left(b + 3q_{\mathrm{M}}\right)q_{\mathrm{M}}}{\det\left(H^{\mathrm{M}}\right)} > 0 .$$

It follows that the *elasticities* of output and effort with respect to the incentive parameter are given respectively by

$$\eta_{q_{\mathrm{M}},\alpha_{\mathrm{M}}} = \frac{dq_{\mathrm{M}}}{d\alpha_{\mathrm{M}}}\frac{\alpha_{\mathrm{M}}}{q_{\mathrm{M}}} = \frac{\alpha_{\mathrm{M}}}{\det\left(H^{\mathrm{M}}\right)} < 1$$

and

$$\eta_{e_{\mathrm{M}},\alpha_{\mathrm{M}}} = \frac{de_{\mathrm{M}}}{d\alpha_{\mathrm{M}}}\frac{\alpha_{\mathrm{M}}}{e_{\mathrm{M}}} = \frac{2\left(b + 3q_{\mathrm{M}}\right)q_{\mathrm{M}}}{\det\left(H^{\mathrm{M}}\right)}\frac{\alpha_{\mathrm{M}}}{e_{\mathrm{M}}} = \frac{2\left(b + 3q_{\mathrm{M}}\right)R}{\det\left(H^{\mathrm{M}}\right)} > 1 .$$

The first-order conditions of the owner's problem of the *monopoly* firm are

$$-2\left(b + 3q_{\mathrm{M}}\right)\mu_{\mathrm{M1}} + \alpha_{\mathrm{M}}\mu_{\mathrm{M2}} = 0 ,$$

$$q_{\mathrm{M}} - e_{\mathrm{M}}R + \mu_{\mathrm{M1}} - \mu_{\mathrm{M2}}R = 0 ,$$

$$\mu_{\mathrm{M2}}q_{\mathrm{M}} - r\alpha_{\mathrm{M}}\sigma_{\varepsilon}^2 = 0 ,$$

$$a - 2bq_{\mathrm{M}} - 3q_{\mathrm{M}}^2 - k + e_{\mathrm{M}} = 0 ,$$

$$\alpha_{\mathrm{M}}q_{\mathrm{M}} - e_{\mathrm{M}}R = 0 .$$

In the *competitive equilibrium with moral hazard for the representative competitive firm*, the first-order conditions for the manager's certainty equivalent maximization are given by

$$a - bq_{\mathrm{C}} - 3q_{\mathrm{C}}^2 - k + e_{\mathrm{C}} = 0$$

and

$$\alpha_{\mathrm{C}}q_{\mathrm{C}} - e_{\mathrm{C}}R = 0 .$$

Let $q_{\mathrm{C}}\left(\alpha_{\mathrm{C}}\right)$ and $e_{\mathrm{C}}\left(\alpha_{\mathrm{C}}\right)$ denote the equilibrium output and effort given the incentive parameter, α_{M},

$$q_{\mathrm{C}}\left(\alpha_{\mathrm{C}}\right) = \frac{\alpha_{\mathrm{C}} - bR + \sqrt{12(a-k)R^2 + (bR - \alpha_{\mathrm{C}})^2}}{6R} ,$$

$$e_{\mathrm{C}}\left(\alpha_{\mathrm{C}}\right) = \frac{\alpha_{\mathrm{C}}\left(\alpha_{\mathrm{C}} - bR + \sqrt{12(a-k)R^2 + (bR - \alpha_{\mathrm{C}})^2}\right)}{6R^2} .$$

The Hessian matrix is

$$H^{\mathrm{C}} = \begin{bmatrix} -b - 6q_{\mathrm{C}} & 1 \\ \alpha_{\mathrm{C}} & -R \end{bmatrix},$$

$$\det\left(H^{\mathrm{C}}\right) = R\left(b + 6q_{\mathrm{C}}\right) - \alpha_{\mathrm{C}} > 0\,,$$

$$EC_e = -q_{\mathrm{C}}\,,$$

$$EC_{eq} = -1\,,$$

$$\frac{\partial^2 E\pi}{\partial q^2} = -\left(b + 6q_{\mathrm{C}}\right) < 0\,.$$

We obtain the effects of the incentive parameter on output and effort,

$$\frac{dq_{\mathrm{C}}}{d\alpha_{\mathrm{C}}} = \frac{EC_{eq}EC_e}{\det\left(H^{\mathrm{C}}\right)} = \frac{q_{\mathrm{C}}}{\det\left(H^{\mathrm{C}}\right)} > 0\,,$$

$$\frac{de_{\mathrm{C}}}{d\alpha_{\mathrm{C}}} = \frac{1}{\det\left(H^{\mathrm{C}}\right)}\left[\frac{\partial^2 E\pi}{\partial q^2}\right]EC_e = \frac{\left(b + 6q_{\mathrm{C}}\right)q_{\mathrm{C}}}{\det\left(H^{\mathrm{C}}\right)} > 0\,.$$

It follows that the *elasticities* of output and effort with respect to the incentive parameter are given respectively by

$$\eta_{q_{\mathrm{C}},\alpha_{\mathrm{C}}} = \frac{dq_{\mathrm{C}}}{d\alpha_{\mathrm{C}}}\frac{\alpha_{\mathrm{C}}}{q_{\mathrm{C}}} = \frac{\alpha_{\mathrm{C}}}{\det\left(H^{\mathrm{C}}\right)} < 1$$

and

$$\eta_{e_{\mathrm{C}},\alpha_{\mathrm{C}}} = \frac{de_{\mathrm{C}}}{d\alpha_{\mathrm{C}}}\frac{\alpha_{\mathrm{C}}}{e_{\mathrm{C}}} = \frac{\left(b + 6q_{\mathrm{C}}\right)q_{\mathrm{C}}}{\det\left(H^{\mathrm{C}}\right)}\frac{\alpha_{\mathrm{C}}}{e_{\mathrm{C}}} = \frac{\left(b + 6q_{\mathrm{C}}\right)R}{\det\left(H^{\mathrm{C}}\right)} > 1\,.$$

The first-order conditions of the owner’s problem of the *representative competitive* firm are

$$-\left(b + 6q_{\mathrm{C}}\right)\mu_{\mathrm{C}1} + \alpha_{\mathrm{C}}\mu_{\mathrm{C}2} = 0\,,$$

$$q_{\mathrm{C}} - e_{\mathrm{C}}R + \mu_{\mathrm{C}1} - \mu_{\mathrm{C}2}R = 0\,,$$

$$\mu_{\mathrm{C}2}q_{\mathrm{C}} - r\alpha_{\mathrm{C}}\sigma_{\varepsilon}^2 = 0\,,$$

$$a - bq_{\mathrm{C}} - 3q_{\mathrm{C}}^2 - k + e_{\mathrm{C}} = 0\,,$$

$$\alpha_{\mathrm{C}}q_{\mathrm{C}} - e_{\mathrm{C}}R = 0\,.$$

Note that the first-order conditions of the owner’s problems in the cases of the monopoly firm and the representative competitive firm are systems of nonlinear equations, more precisely, systems of multivariate polynomial equations. Because of their complexity, we are not able to obtain analytical solution for q, e and α. However, we can solve these systems of equations by numerical methods.

In the base scenario we use the following parameter values:

$$a = 60,\ b = 10,\ k = 10,\ R = 1,\ \sigma_\varepsilon^2 = 0.81,\ \overline{w} = 1,\ r\overline{w} = 9.$$ [9]

In addition, we also conducted sensitivity analysis by changing the parameter values one at a time.

In Tables 1 to 4 we present a sample of these numerical exercises. In Table 1, equilibrium values of quantity, effort, and, where applicable, the incentive parameter are presented. In Table 2 are various aspects of efficiency losses. The following are the mathematical definitions of the notations in Table 2:

$$DWL_q = \int_{q_{\mathrm{M}}(e)}^{q_{\mathrm{C}}(e)} \left[p(x) - EC_q(x, e, \varepsilon)\right] dx,$$

$$DWL_e = \int_0^{q(e_1)} \left[p(x) - EC_q(x, e_1, \varepsilon)\right] dx - \int_0^{q(e_2)} \left[p(x) - EC_q(x, e_2, \varepsilon)\right] dx,$$

$$\Delta\psi = \psi(e_1) - \psi(e_2),$$

$$\Delta RC = (r/2)\sigma_\varepsilon^2\left(\alpha_{\mathrm{C}}^2 - \alpha_{\mathrm{M}}^2\right),$$

$$\Delta E(NSW) = DWL_q + DWL_e - \Delta\psi - \Delta RC.$$

In Tables 3 and 4 we take a more detailed look at the agency costs. Note that the equilibriums in these four tables are associated with two sets of parameter values, the base scenario and sensitivity analysis with the value of R being raised to 3.

Three interesting observations can be made from these tables. First, as expected, the introduction of the effort variable increases the welfare losses of monopoly. As we can see from Table 2, compared with the traditional model the welfare losses of monopoly are higher both in the entrepreneurial firm (“case 1”) and in the case of separation of ownership and control (“case 2”).

Second, the moral hazard problem arising from the separation of ownership of control does not necessary exacerbate the efficiency losses caused by monopoly. From the last two columns of Table 2 (with the parameter value R = 3), we see that the efficiency losses arising from a monopoly run by a manager are smaller than those from an entrepreneurial monopolist.

[9] $r\overline{w}$ is average relative risk aversion and its value is taken from Caballero’s (1990) numerical examples.

Third and finally, it is not always the case that the agency costs are higher in a monopolistic industry than in a competitive industry. As we can see from Tables 3 and 4, the agency costs in the monopolistic industry can be higher (when $R = 1$) or lower (when $R = 3$) than those in the competitive industry. This in part explains the second observation that the separation of ownership and control does not necessarily make the efficiency losses of monopoly larger.

4. Concluding Remarks

This paper is a work in progress. There is obviously much room for improvement. For example, the numerical examples in section 3 are only for the case where quantity and effort are complements in the cost function. It would be useful to study also examples for the alternative case where the two variables are substitutes. In addition, there are a couple of avenues for future research that we will explore. First, we will examine whether the adverse selection problem would increase the monopoly inefficiency. By introducing heterogeneity among owners and managers in their firm management ability into a model we can compare the efficiency loss of entrepreneurial monopoly firm with that of managerial monopoly firm. Second, we will examine whether yardstick competition would alleviate managerial slack to increase the efficiency of competitive firms. Given that yardstick competition would not be possible for a monopolist firm, the welfare loss of monopoly would be higher if yardstick competition can be used to reduce the agency costs in a competitive industry.

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Table 1: The equilibrium values of output, effort and incentive parameter

Models / Firm	Traditional Model	Our Model									
		Case 1 ($R=1$)		**Case 2** ($R=1$)			**Case 1** ($R=3$)		**Case 2** ($R=3$)		
	q^0	q^*	e^*	q^{**}	e^{**}	α^{**}	q^*	e^*	q^{**}	e^{**}	α^{**}
Monopoly Firm	1.94	2.0	2.0	1.96	0.68	0.35	1.96	0.65	1.94	0.10	0.15
Competitive Firm	2.74	2.85	2.85	2.80	1.46	0.52	2.78	0.93	2.75	0.24	0.26

Table 2: The welfare losses of monopoly

Welfare Losses / **Models**	**Traditional Model**	**Our Model**			
		Case 1 ($R=1$)	**Case 2** ($R=1$)	**Case 1** ($R=3$)	**Case 2** ($R=3$)
The Deadweight Welfare Loss from *Quantity Distortion*, DWL_q	8.066	8.452	8.198	8.192	8.085
The Deadweight Welfare Loss from *Effort Distortion*, DWL_e	N/A	2.407	2.177	0.758	0.3888
Saving on the *Cost of Effort*, $-\Delta\psi$	N/A	-2.059	-0.838	-0.647	-0.070
The Saving on the *Risk Cost*, $-\Delta RC$	N/A	N/A	-0.556	N/A	-0.163
The Expected Net Social Welfare Loss, $\Delta E(NSW)$ (The monopoly firms versus the representative competitive firm)	8.066	8.800	8.980	8.303	8.239

Table 3: The agency cost of the monopoly firm and the representative firm of our model ($R=1$)

	Monopoly Firm	**Representative Competitive Firm**
The Deadweight Welfare Loss from the *Effort Distortion*, $DWL_e\left(q\left(e^*\right), q\left(e^{**}\right), e^*, e^{**}\right)$	3.432	3.916
The Saving on the Cost of Effort, $-\Delta\psi\left(e^*, e^{**}\right) = -\left[\psi\left(e^*\right) - \psi\left(e^{**}\right)\right]$	-1.769	-2.990
The Incentive Cost, $IC = DWL_e\left(q\left(e^*\right), q\left(e^{**}\right), e^*, e^{**}\right) - \Delta\psi\left(e^*, e^{**}\right)$	1.663	0.996
The Risk Cost, $RC = (r/2)\alpha^2\sigma_\varepsilon^2$	0.440	0.996
The Certainty Equivalent, $CE = \overline{w}$	1.0	1.0
Total Agency Cost, $AC\left(q^*, e^*\right) = IC + RC + CE$	3.103	2.922

Table 4: The agency cost of the monopoly firms and the representative firms of our model ($R=3$)

	Monopoly Firms	**Representative Competitive Firms**
The Deadweight Welfare Loss from the *Effort Distortion*, $DWL_e\left(q\left(e^*\right), q\left(e^{**}\right), e^*, e^{**}\right)$	1.429	1.906
The Saving on the Cost of Effort, $-\Delta\psi\left(e^*, e^{**}\right) = -\left[\psi\left(e^*\right) - \psi\left(e^{**}\right)\right]$	-0.625219	-1.202
The Incentive Cost, $IC = DWL_e\left(q\left(e^*\right), q\left(e^{**}\right), e^*, e^{**}\right) - \Delta\psi\left(e^*, e^{**}\right)$	0.804	0.704
The Risk Cost, $RC = (r/2)\alpha^2\sigma_\varepsilon^2$	0.079	0.242
The Certainty Equivalent, $CE = \overline{w}$	1.0	1.0
Total Agency Cost, $AC\left(q^*, e^*\right) = IC + RC + CE$	1.883	1.946