Overcoming the (near) Fetishization of a Small Group of 2×2 Games

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Abstract

Strictly speaking, there are 726 distinct 2×2 games. Despite this large number the majority of interest is given to only a small subset, the seventy-eight 2×2 games where indifference across outcomes does not exist for either player. Within this subset attention is further confined to the twelve games exhibiting symmetry across the outcomes for both players. This paper argues that in terms of the games that humans actually experience the near fetishization of this small number of games is justifiable only if two criteria are met. First, people must exert their preferences on a scale with at least twenty gradations (hopefully many more) and have a strong aversion to collapsing similar assessments together. Second, the distribution of the payoffs across the outcomes, as determined by the interaction of both the environment and the preferences of the agents over these outcomes, must not only not be random, but nonrandom in an appropriate way. The paper suggests that while it is likely the case that these conditions are satisfied for many situations, there are at least as many where they are not. The consequence of this is that many of the twelve 2×2 games receiving the most attention are experienced so infrequently (less than two thirds of one percent of the time) that the attention given to them is not justifiable. The paper concludes by noting interesting games from the mostly unexplored set of 2×2 games with indifference, most notably a game called "Red Dress".

A game is any situation where two or more players operate under a situation of strategic interdependence. The simplest models of agent interaction that can provide non-trivial multi-agent decision making scenarios are known as 2×2 games.[1] Simpler games result in at least one player not really being a player at all, but more of an observer. While there are certainly cases where people observe the decisions of others, it is rarely the case that such observers are without choices. It is simply that these choices do not matter from the perspective of generating outcomes that are meaningfully different and so simpler 1×1 and 1×2 games can be modelled by 2×2 games. More complicated games and scenarios can also be modelled by 2×2 games although such modelling can result in a loss of information through this simplification. Nevertheless there is reason to believe that human beings often perform similar simplifications when facing choice scenarios, reducing decisions into binary choice scenarios. In this way we present ourselves with simple choices such as "Should I stay or should I go?" rather than the more complicated choice scenario that this question may

[1] This name follows the standard method of classifying games which makes explicit the number of players and the total number of options available to each player through numbers separated by multiplication signs. This information is presented in the form A×B×C×..., where each letter represents the total number of options available to a specific player and the total number of players is indiciated by a count of the values separated by the character '×'. The '×' is read as "by" and may be interpreted as a multiplication symbol. The product of all options available to each player is the total number of possible outcomes for the game. For example, a 4×4×4 game has three players, each of whom faces a choice between four options which can be combined into a total of $4^3 = 64$ different outcomes. So, a 2×2 game has two players each of whom can choose between two options for a total of four possible outcomes.

simplify such as, "Should I stay and do 'w' or stay and do 'x' or stay and do... or go and do 'y' or go and do 'z' or...?". In this way 2×2 games are reasonable models for simplified real-world choices faced by real-world beings.

This simplification, in conjunction with a corresponding rigorous formality, has led to the widespread use of 2×2 to assess and analyze a wide range of human interactions, from nuclear deterrence to business partnerships to romantic relationships. Unfortunately, while there is a heavily explored core set of 2×2 games there is a larger penumbra of these games that have yet to be given much attention. With the notable exceptions of the Rapoport and Guyer (1966) taxonomy and a recent attempt at a topology by Robinson and Goforth (2005), an intense focus has been given to only a handful of 2×2 games—in particular The Prisoner's Dilemma, Stag Hunt, Battle of the Sexes, and Chicken. This narrow range of attention is problematic insofar as it suggests and promotes the idea that all meaningful human interaction can be captured with only a handful of games, 2×2 or otherwise.

This paper has one aim, to convince the reader that such a narrow focus is inappropriate given both the total number of 2×2 games that exist and also because the sorts of games that we play on a regular basis *and find challenging* go well beyond the tiny subset given so much attention. To this end the paper begins by arguing for a definition of "game" that supports the conclusion that there are exactly 726 2×2 games. With the size of the playing field set out it is argued that given some uncontroversial assumptions regarding human psychology there are many games deserving attention that are typically ignored. In conclusion some of these typically ignored games are shared, with particular attention being given to a game called "Red Dress".

1 726 2×2 Games

While it seems as though how many 2×2 games there are should be a settled issue a browse through the literature on the subject reveals otherwise: the original taxonomy by Rapoport and Guyer (1966) counts 78 2×2 games, a paper byWang and Yang (2003) claims that there are 6,561 2×2 games in total, and a recent book dedicated to producing a topology of 2×2 games by Robinson and Goforth (2005) claims that there are 144. These differences are not simply a matter of restricting discussion to some particularly interesting or important subset of 2×2 games, although this motivation certainly accounts for part of the difference in these numbers. Rather, the main source of these wildly different values is an unspoken disagreement among these authors (and others) regarding *exactly what counts as a game.*

The core of the disagreement about just how many 2×2 games there are arises from a decision on the part of some authors to count what amount to different representations of the same game as different games, leading to anywhere from double to octuple counting. For example, each of the games in Table 1 is a Prisoner's Dilemma.[2]

(a)

2, 2	0, 3
3, 0	1, 1

(b)

3, 0	1, 1
2, 2	0, 3

(c)

0, 3	2, 2
1, 1	3, 0

(d)

1, 1	3, 0
0, 3	2, 2

Table 1: Four Ways to Represent the Prisoner's Dilemma in Strategic Form

Prisoner's Dilemma (a) is perhaps the most common way to represent the game, (b) represents a swapping of the rows from the representation shown in (a), (c) represents a

[2]There is some inconsistency in the literature regarding what counts as a prisoner's dilemma. The structure given here is the one used by Rapoport and Guyer Rapoport and Guyer (1966) and has become the standard way to understand this 2×2 game. However, Luce and Raiffa (1989) use a different payoff structure and hence present an entirely different game as a PD showing that even this now iconic game is not without its small definitional controversies.

swapping of the columns from the representation shown in (a), and (d) represents a swapping of both rows and columns from the representation shown in (a). Despite the change of appearance the game being played remains the same for all ideally rational agents. Wang and Yang (2003) count each of these presentations as a different game while both Rapoport and Guyer (1966) and Robinson and Goforth (2005) agree that each representation is just that, a *representation* of the same game. This agreement between the earlier work of Rapoport and Guyer and the more recent work of Robinson and Goforth is not perfect, extending only to games that, like the Prisoner's Dilemma, have symmetry across the payoffs available to the players. Such player-symmetric games have the exact same payoff matrix when subject to R&G reflections.[3]

Table 2 provides eight ways of representing a 2×2 game that does not have player-symmetry and so appears different under an R&G reflection. It is this lack of symmetry across the payoffs available to the players that allows for the full spectrum of ideas about what constitutes the game to be examined.

For Rapoport and Guyer each of the normal form game matrices in this table are simply alternative ways of representing the same game. Allowing matrix (a) to act as the base case, matrices (b) through (c) can be created through row or column swaps (e.g. (b) can be attained from (a) simply by swapping the second column of outcomes with the first). The matrices (e) through (h) are the R&G reflections of the game immediately above (e.g. (e) is the R&G reflection of (a)). Robinson and Goforth are of a different opinion regarding what is captured in Table 2: rather than eight representations of one game they see two games, each with four representations: one captured by the four representations in the top line and one captured by the four representations in the bottom line. Why this discrepancy?

3, 2	1, 1
2, 0	0, 3

(a)

1, 1	3, 2
0, 3	2, 0

(b)

2, 0	0, 3
3, 2	1, 1

(c)

0, 3	2, 0
1, 1	3, 2

(d)

2, 3	0, 2
1, 0	3, 0

(e)

0, 2	2, 3
3, 0	1, 0

(f)

1, 0	3, 0
2, 3	0, 2

(g)

3, 0	1, 0
0, 2	2, 3

(h)

Table 2: Eight Ways to Represent a Game Lacking Symmetric Payoffs

The discrepancy arises in part because Robinson and Goforth are not so interested in counting games accurately as they are interested in building a *topology of games* that will provide a tool for game theorists with a value similar to what the periodic table of the elements provides chemists. By explicitly ignoring R&G reflections as alternative representations of the same game they are able to create a topological description of 2×2 games that is both beautiful and useful in its illustration the relationships between each game. The problem is not their admirable topology but their insistence in referring to the products of R&G reflections, what are often colloquially referred to as "player swaps", as *different* games. The reasoning behind this insistence is their interpreting R&G reflections as involving the reassigning of roles, which:

> ...is not conventional geometric or group-theoretic reflection. "Rapoport and Guyer reflections" (R&G reflections) are behaviourally equivalent if players fac-

[3]R&G reflections are "Rapoport and Guyer reflections" and are the equivalent of swapping the row player's payoffs with the column player's payoffs. If the same payoff arrangement is returned following the swap it is an indication that an ideally rational player whose von Neumann and Morgenstern utilities are fully captured by the current utility assignments for both players, would have no preference in choosing to be either the row or the column player. The name "R&G reflection" is used by Robinson and Goforth (2005) and is a reference to Rapoport and Guyer (1966) who made use of the reflection to eliminate the number of payoff matrix combinations that they needed to count when building their original taxonomy.

> ing the same payoff structure always behave the same way. In other words, reflections are indistinguishable if players are indistinguishable. (Robinson and Goforth, 2005, p. 17)

The strongly implied premise in their reasoning is that many possible players, especially human players, are clearly *not* indistinguishable.[4] Consequently, it seems that we should consider R&G reflections as different games; otherwise we will be failing to account for all the 2×2 games that non-indistinguishable beings might play.

Another reason for counting R&G reflections as different games is that, under Robinson and Goforth's approach to producing the topology, including these reflections produces a stack of games consisting of four layers of games, six games wide by six games deep. This block of games and the specific ordering they provide allows for numerous relationships between the 2×2 games to be clearly illustrated just as the periodic table of the elements allows elements with similar properties to be grouped and more easily understood.

Despite the pragmatic and aesthetic usefulness of accounting for R&G reflections when investigating the structure of games counting such reflections as entirely new games is a mistake. The source of the mistake rests in a confusion concerning what constitutes a game and how this is importantly separate from the way in which the game is represented.

The founders of game theory, John Von Neumann and Oskar Morgenstern, define games succinctly, "[A] *game* is simply the totality of rules which describe it" (von Neumann and Morgenstern, 1967, p.49).[5] More recent authoritative works include Luce and Raiffa (1989) and Fudenberg and Tirole (1991) and the definitions they provide can be found in table 3.[6]

Fudenberg and Tirole (1991)	**Luce and Raiffa (1989)**
A game in strategic (or normal) form has three elements: 1. the set of players $i \in I$, which we take to be the finite set $\{1, 2, \ldots, I\}$, 2. the *pure-strategy space* S_i for each player i, and 3. *payoff functions* u_i that give player i's von Neumann-Morgenstern utility $u_i(s)$ for each profile $s = (s_1, \ldots, s_I)$ of strategies.	...a *game in normal form* consists of: 1. The set of n players. 2. n sets of pure strategies S_i, one for each player. 3. n linear payoff functions M_i, one for each player, whose values depend upon the strategy choices of all the players.

Table 3: Comparing Game Definitions

None of these definitions say anything at all about the method of representation that is to be used when presenting any game. The reason for this silence is that the representation is irrelevant to the constitution of a game. Robinson and Goforth go wrong when they claim, " 'Rapoport and Guyer reflections'... are behaviourally equivalent if players facing

[4] As an example of how players fail to be indistinguishable consider that it is well known that human beings are not purely rational and so changes that should make no difference whatsoever to how the game is played, such as rearranging rows and columns or changing the names used to refer to various actions can influence human behaviour, and thus the play of games, significantly. See framing effects in Kahneman and Tversky (2000), Gilovich et al. (2002), and Dawson, Gilovich, and Regan (Dawson et al.).

[5] Of course, after putting it succinctly they go to great lengths to spell out an axiomatic definition of games spanning roughly twenty pages and ending with a complete list of all axioms. See section 10 of von Neumann and Morgenstern (1967) for this list.

[6] The strategic/normal form definitions are used simply because they are simpler to write down. It should be recalled by the reader that well-formed normal form representations are completely translatable into an extensive form representation of the same game and vice-versa.

the same payoff structure always behave the same way [and they do not]". This position seems plausible because it is certainly true that when faced with the same choices and consequences as another person some people will not necessarily chose the same way as the original person, but this is not the sort of swap that R&G reflections create. R&G reflections are not about changing the options or outcomes available to a player but about *changing the way that the current options and outcomes for the given players are represented.*

To help see this, imagine that matrix (a) in table 2 correctly captures the interaction of Todd and Linda, with Todd being the row player and Linda being the column player. What then are the other seven matrices in the table? Seven different representations of the same game. Choosing to represent the game by (e) instead of (a) does not mean that a different game is being played, only that in (e) Todd's preference across the outcomes are now being represented by the column player. It is in principle no different from having a group of people sitting at a table playing cards all move one seat to the right *and taking their current hands and all other relevant assignments of powers with them.* Consequently, the R&G reflection is no different in terms of changing the representation than is either a row or a column swap and Robinson and Goforth's claim that there are 144 2×2 games is thus mistaken because they double count every 2×2 game with a payoff structure that is non-symmetric across players.

Wang and Yang (2003) make the same mistake, but on a much larger scale, arguing that there are $81^2 = 6,561$ different 2×2 games. They begin their proof by showing that for each player there are four ways that a preference ordering may be applied across the four cells in a normal form representation of a 2×2 game: no outcomes are equally preferred; two and only two cells are equally preferred; two pairs of equally preferred cells or a single set of three equally preferred cells; or all cells being valued equally. From this they determine that there are 24, 36, 20, and 1 (respectively) possible ways of arranging the preferences for a player over the cells. The error is made when they reason that because there are 81 possible ways of distributing the preferences of one player over the four cells in a normal form distribution that each application of a payoff ordering from one player will produce a meaningfully different arrangement when combined with each payoff ordering of the other player. Thus, they reason that there are $81 \times 81 = 81^2 = 6,561$ different 2×2 games. In making this jump they implicitly assume that different arrangements of preferences within the cells of a normal form distribution are meaningfully distinct under the definitions of a game given earlier, but this is not the case.

To see that this is a mistake consider again the eight representations of the unnamed game given in table 2. According to Wang and Yang each of these representations is a *different* game because each shows a different arrangement of preferences over the cells within each matrix. While it is true that each is a different arrangement of preferences across cells they fail to recognize that these different arrangements of *preferences over cells* can produce the same arrangement of *preferences over outcomes*. Again, changing the representation does not and *should not* prima facie produce a new game.

So, if neither Wang and Yang nor Robinson and Goforth assert the correct number of 2×2 games, then just how many are there? Rapoport and Guyer's original 1966 taxonomy was restricted to only include those games that could be constructed when:

1. payoffs are taken into account on an ordinal scale,
2. transformations resulting in equivalent games are ignored, and
3. the payoff scale is strict (i.e. no items on the scale may be judged as equivalent).

Given these restrictions they correctly identified 78 meaningfully different 2×2 games. However, their restriction to strict payoff scales, while useful in constraining the number of games to be considered to an easily manageable number within their taxonomy, leaves out a significant number of 2×2 games; namely those within which at least one of the players is indifferent across at least one pair of outcomes. Removing the strict payoff scale restriction results in a significant expansion of the number of 2×2 games from 78 to 726.

Two methods of proof for this claim are offered, neither of which is reproduced here. The first breaks the set of possible 2×2 games into fifteen cases depending on the number of equivalent payoffs that are offered to each player of the players and then counts the number of possible combinations in each. The second uses a computer program to produce every possible combination of ordinal values and testing each combination to see if it is simply a new representation of a previously identified game and then throwing away the duplicates. Both these proofs and the complete taxonomy produced by the computer program are available for inspection in the appendices of Simpson (2010).

2 2×2 Game Frequency

So, given that there are 726 2×2 games, which ones matter? Or, put another way, which ones do we expect to come up most frequently in our day-to-day existence so that we can focus future analysis as is appropriate? While the exact answer to this question will vary from context to context, there are a few key factors that will contribute to any such analysis, as follows:

1. The structure of human preferences. In particular, the number of meaningfully different gradations on the human preference scale.
2. The relative importance of small changes in the value of outcomes as determined by the environment.
3. The distribution of payoffs across outcomes as determined by the interaction of the environment and an individual's preferences.

The first of these is important simply because the number of gradations on the scale will change both the number and sorts of games that people play. To see this in the most trivial sense note that a group of people who evaluate all outcomes as being equally valuable only play one game. Increasing the number of gradations on the personal scales for one or more players increases the number of possible 2×2 games that can be played until four distinct gradations are available to each player, at which point all 726 games become available. Once the number of gradations is increased beyond four the total number of 2×2 games becomes fixed, but the frequency that each game may be encountered in the world is altered.

The environment stands to play an important role in just what games are important in a number of ways. The first is that an environment where resources are plentiful and preferences are easily satisfied will stand to reduce the number of conflicts that are likely to arise. Not only will there be less reason for conflict in the first place (typically it is less efficient to pick a fight over food when simply picking up food is an option), but conflicts that do arise will have a very different character than they would otherwise, as a result of the interaction between preferences and the environment (the interacting players are likely to have different preferences than they would in a harsher environment that produced "do or die" scenarios). The reverse stands to be true in environments where survival becomes increasingly difficult.

So it can be seen how the intersection of environmental conditions and innate human persnicketiness might combine to alter the gaming landscape available for humans, but can any more be said here and with any more precision? Put crudely, let the question being asked be, "How would the sorts of games that people are presented with change if the world presented payoffs randomly and the ability/inclination that people had to differentiate across these presentations was varied?" In answering this question two small assumptions are entertained. The first is simply that the there is no reason why the environment, broadly construed, should provide one payoff over another across the range of possible circumstances that players, human or otherwise, might find themselves in. Taken in full force this assumption allows us to model the environment as presenting players with payoffs across outcomes that are completely random. The second assumption is that human psychology is constructed such that, for the most part, the preferences that people have correspond to the survival value of the outcomes that they are associated with. This allows it to be the case that the players will behave in the standard ways that we expect surviving agents in an evolutionary environment to behave, they will, for the most part, have behaviours and attributes that support the survival of both themselves and their kin. With these two assumptions in place it becomes possible to calculate the frequency with which each of the 726 2×2 games might be encountered for a given range of preference gradations and to make some inferences based on this result.

This calculation was initially performed for a total number of possible preference gradations ranging from 4 to 25 via a computer program written specifically for the task. This program attempted every payoff combination possible with the number of values allowed at each stage of the calculation. Each combination was checked to see which of the 726 possible 2×2 games it is a representation of and then a counter associated with that game is incremented. When all the combinations have been tried for a given number of possible values a payoff can take then the number of times each game was encountered was divided by the total number of combinations possible. The resulting probabilities were tracked over the entire range of values being explored.[7] Charts of the results are presented in Figures 1 and 2.

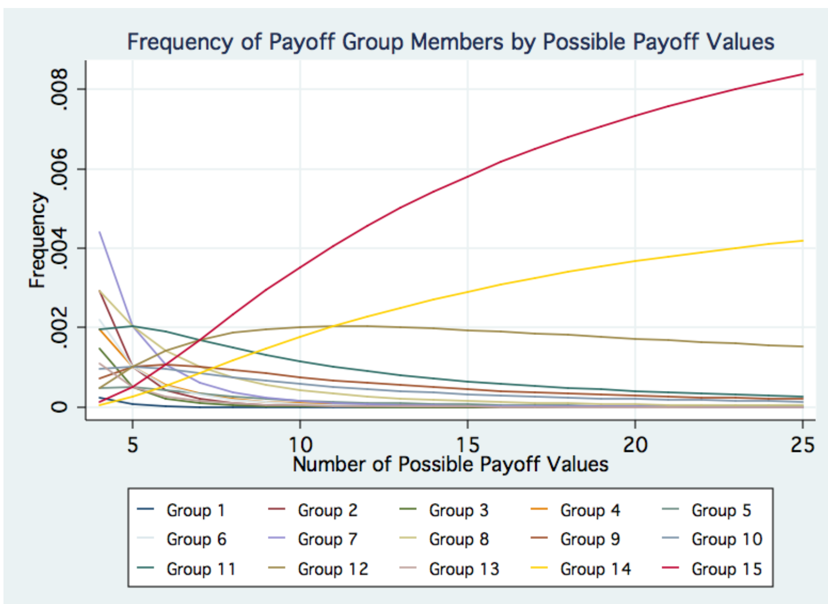

Figure 1: Frequency of Single Members with the 2×2 Probability Groups

Before looking at these figures specifically it is important to note an important general outcome: there are clearly 15 lines represented, indicating that there are 15 importantly different sorts of games regarding the sorts of frequencies that we are interested in. The first important insight from looking at the results of this process was that there were clearly 15 different distributions of frequency across the 726 2×2 games. While it was initially suspected that these 15 probability distribution groups would overlap perfectly with the 15 groups used to count the games in the first place (See Simpson (2010)) this did not turn

[7]This brute force approach was necessary to gain some initial insight into this problem, but its limitations became apparent quickly—at 25 possible values the number of combinations to be checked and tracked was $25^8 \approx 1.5x10^11$, over 150 billion possibilities. With this insight into the frequency of games under an increasing number of possible payoff values it became possible to construct a formula to achieve this result more efficiently. This formula will be shared shortly.

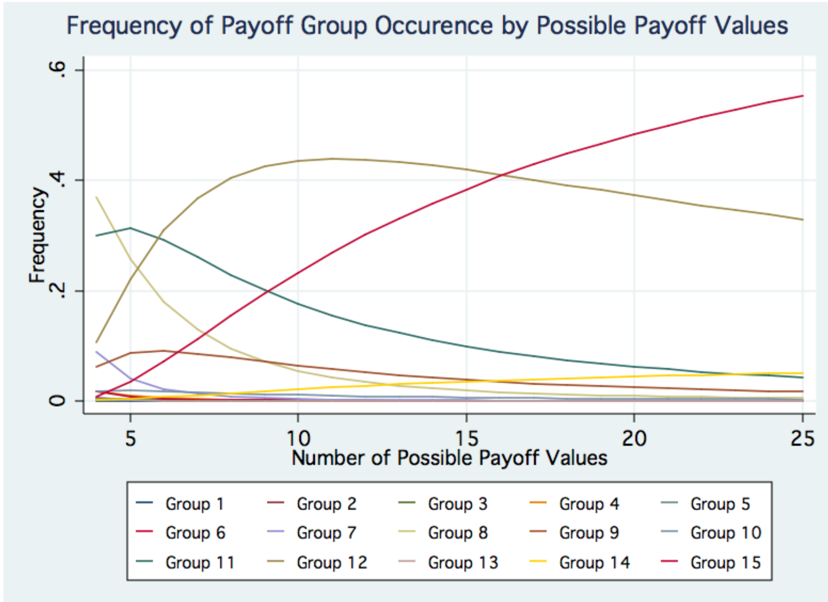

Figure 2: Frequency of 2×2 Probability Groups

out to be the case.[8] Instead the games within the groups produced for counting are split over the groups produced when investigating raw game frequency in various ways. Exactly where these two grouping methods overlap in terms of number of games is shown in figure 3.

The reason that games that belong to one counting group do not necessarily be long to the same probability group is the existence of symmetries within certain games that are not shared or acknowledged between the two groups. The existence of symmetries reduces the likelihood of a game occurring because it is no longer simply a matter of a certain collection of payoffs being on the board but a matter of their specific arrangement. These symmetries amount to the presentation of the game remaining unchanged through any of the various transformations that are available with a normal form game (row swap, column swap, and player swap).

Returning to the charts produced, Figure 1 shows the behaviour of *single members* within each of the 15 probability groups. This graph shows that as the number of possible values that a payoff can take on increases there are are only two probability groups that have members who occur with increasing frequency, groups 14 and 15. These two probability groups belong to count group 15 which is made up of those 2×2 games where neither player

[8]The groups used to count games result from seeing that there are five ways that payoffs across outcomes might be seen as equivalent for a player, as follows:

1. All the outcomes are seen as equivalent.
2. Three outcomes are judged as being equivalent.
3. Two pairs of outcomes each have the payoffs within them judged as being equivalent, but no member of a pair is seen as being equivalent to either member of the other pair.
4. One pair and only one pair of outcomes is judged as being equivalent.
5. No outcomes are judged as being equivalent.

Since there are five ways that that equivalencies might be distributed by a single player across the outcomes the next step is to figure out how many ways that these might be distributed across two players. This is a matter of determining how many pairs can be generated from five items when repetition is allowed. The number of combinations of k objects that can be generated from a total of n objects when allowing repetition is $\binom{n+k-1}{k}$. This means that there are $\binom{5+2-1}{2} = \binom{6}{2} = 6! \div 2! \times 4! = 15$ total cases to be considered.

Probability Group

Counting Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
1	1															*1*
2		2														*2*
3			3													*3*
4				9												*9*
5					6											*6*
6						4	6									*10*
7							12									*12*
8								72								*72*
9									48							*48*
10						4	2						2			*8*
11								54								*54*
12									36							*36*
13										18	153					*171*
14												216				*216*
15														12	66	*78*
Total	*1*	*2*	*3*	*9*	*6*	*8*	*20*	*126*	*84*	*18*	*153*	*216*	*2*	*12*	*66*	***726***

Figure 3: Count of Games Shared Between Count and Probability Groups

is indifferent between any combination of outcomes. Probability group 15 captures the games which have no symmetry and group 14 captures those with symmetry. Given that as the number of possible values a payoff can take increases the likelihood of two payoffs taking on the same value over a random distribution decreases, this result should not be surprising. What is surprising is that when the members of each probability grouping are added together and the results of this aggregation plotted, as in figure 2, groups 14 and 15 become much less significant when the number of values that a payoff may take are relatively small. To see this more clearly note that the *individual members* of probability group 15 become dominant when as few as 6 possible values for a payoff to take are allowed, but that the *group as a whole* does not become dominant until 16 possible values. The situation is even more striking for probability group 14. Individually, its members become the second most dominant by 12 possible payoff values, but as a group it is barely able to make third place by 25 possible values. Most surprising of all, probability group 14 is where the the majority of the most famous 2×2 games can be found: The Prisoners' Dilemma, Stag Hunt, Chicken, Battle of the Sexes, etc.

The probability group that manages to dominate this grouping of famous games for second place, at least initially, is group 12. Looking at figure 3 it can be seen that this probability group is composed of all the games from count groups 9 and 12. Count group 9 is made up of games where one player values three of the four outcomes equally and the fourth either more or less than these while the second player values every outcome differently. Count group 12 is made up of games where one player's preferences across outcomes may be grouped into two pairs where the members within each pair are valued equally while the second player values every outcome differently. Combined, these two count groups have $48 + 36 = 92$ members, dwarfing the 12 members of probability group 14. This, in combination with simple probability, accounts for the dominance of probability group 12 over group 14 despite any intuitions that may be initially had that it might be otherwise.

Eventually, probability groups 15 and 14 win out over all other probability groups. This is most easily seen by noting that as the number of possible values that the payoffs can take increases the likelihood of the same payoff values being randomly assigned to two autcomes drops to zero. More formally, the probability of any 2×2 game being generated is given by the formula[9]:

$$P(x) = \frac{r\binom{x}{p}\binom{x}{q}}{x^8}$$

where

- x is the number of integer values that each payoff might take on from $4 \rightarrow \infty$
- r is a measure of the symmetry within the game in question. There are four possible values that r might take on. A value of 1 means that the game is symmetric through all possible changes in presentation. Only the game where all the payoffs to each player are equivalent receives a value of 1. A value of 2 means that there is strong symmetry that does not manage to be perfect. Such games maintain their presentation in both the case of an R&G reflection (player swap) and in the case of a row swap followed by a column swap. A value of 4 means that there is only one form of symmetry and the presentation cannot be maintained through more than one kind of change. A value of 8 means that there is no symmetry at all.
- p is the number of different values that the row player is allowed to have appear as payoffs across the outcomes. p must be an integer between 0 and 4.
- q is the number of different values that the row player is allowed to have appear as payoffs across the outcomes. q must be an integer between 0 and 4.

The binomial nature of this formula in conjunction with the fact that it only takes integer values removes the possibility of simply taking the limit to find out which games survive at the highest values, it is possible to put the formula into a high-end math program that is able to perform the necessary calculation at increasingly larger values of x. At $x = 1,000,000$ Maple 13 approximates the probability of generating a game from probability group 15 ($r = 8$,$p = q = 4$) at 0.92 and the probability of generating a game from probability group 14 ($r = 4$,$p = q = 4$) at 0.08. All other groups have the probability of their members being generated go to zero. This happens because games outside groups 14 and 15 require player indifference across a set of outcomes and this becomes increasingly unlikely as the range of values randomly assigned to each outcome expands.

3 Directing Attention to Other Games

Given the attention that has been bestowed upon the games of probability group 14—and only a small portion of these 12 games at that—it should be surprising that they compose only 8 percent of the possible games that might be produced as the number of possible preferences approaches the limit. Granted, they are interesting in their own right and it may well be the case that they are more frequently encountered in the real world due to anything from a psychological disposition towards forcing symmetries to environmental structures that are unaccounted for in this simplistic treatment, but if such reasons exist then surely it is the responsibility of those studying such games to carry the burden of proof regarding their existence. It would appear that such proofs do not currently exist, not because they are not available, but because it is a generally accepted assumption that these are the only games that matter, an assumption that we are now in the position to challenge.

Given the results above it is the case that the disproportionate focus on games in probability groups 14 is justified only to the extent that we either expect the world (as a combination of the environment and human psychological predispositions) to actually present us

[9] I believe that this formula is generalizable such that it will provide the probability of *any* game of any class of games being randomly generated when the corresponding information regarding the relevant structural components is provided. It is my hope that this will be an important step towards being able to count the games of any class without making use of the clumsy method shared in the appendices. It may also aid in developing a non-arbitrary system for naming games, but realizing this is likely a long way off.

with symmetrical outcomes across players or that we find it useful to suppose that it does for teaching and research purposes. Even if we have grounds for supposing either of these possibilities we surely cannot imagine that group 14 is a fair representation of the the sorts of games that we play, or even only the games that we play that are interesting. There are clearly asymmetries in the world and the raw probabilities of encountering these shared above, in conjunction with our own thoughtful reflection on their actual frequency make it the case that the focus on games in group 14 is somehow misplaced and disingenuous, we should at least be directing attention towards the games in probability group 15 as well.

Even more important than arguments for considering both probability groups 14 and 15 are the arguments available for considering games from the other thirteen groups. Since our experiences in the world make it clear that people are either regularly indifferent or behave in ways that are functionally equivalent to indifference these groups must be considered as well in any comprehensive treatment of 2×2 games.

Some of the inattention given to games involving indifference games seems justified as there are many that can uncontroversially be considered trivial as the following examples show:

0, 0	0, 0
0, 0	0, 0

1, 1	0, 0
0, 0	0, 0

1, 1	1, 0
0, 1	0, 0

In the first game what either player does makes no difference to the payoff that either player can receive. Such a state of affairs is ripe for being considered trivial because neither player cares how they or the other player behave. In the second and third cases playing for the top-left outcome is “obvious” to both players, so obvious that such games might also be labeled as trivial. Still, just because something is trivial does not mean that it is irrelevant and so such games should not be dismissed from consideration without further cause. Consider that it is quite possible that despite their triviality these games are important because we play them so frequently, but because they cause us so little problem they are mostly invisible to us. They may also be important as substitutes for more complicated games that we would be forced to play if we did not reframe situations in various important ways, such as taking a situation that might otherwise be a prisoner’s dilemma and either ignoring it entirely (the first trivial game listed) or recasting it through a lens that places a heavy emphasis on cooperation (the second and third trivial games listed).

There are, however, games involving indifference that are not so trivial. Take for example the game called “Red Dress” displayed in Table 4.[10]

		Girl	
		Red Dress	*Jeans*
Boy	*Suit*	1, 1	0, 1
	Jeans	2, 0	0, 0

Table 4: Red Dress

In this game a boy and a girl have decided to visit friends for dinner. They have talked on the phone about what to wear and did not come to an agreement. The girl simply wanted the boy to wear a suit rather than jeans. The boy would really like to balance the comfort of showing off his girlfriend in her red dress with the comfort that jeans provide over his suit. Having his girlfriend wear jeans is the worst outcome from the boy’s perspective

[10] The name and explanation arose in response to presenting a class I taught with the game shown and asking the students to come up with a description that would fit the game. I regret that I no longer remember the name of the student who presented it.

since he is applying a double standard and is worried about how she will look in jeans. This combination makes choosing to wear jeans a weakly dominant strategy for the boy because if the girl wears her dress then he can improve the outcome by wearing his jeans and if she is going to wear jeans then he does no worse wearing jeans than if he had worn the suit. However the existence of this dominant strategy fails to provide a robust solution to the game. Why? The girl has similar motives for having the boy wear his suit and is in a position to make a credible threat in any single iteration of the game and is able to punish the boy in any iterative version. This threat exists because while the girl is indifferent to what she wears she is not indifferent to what the boy wears, giving her the leverage of threatening to wear jeans should the boy not agree in advance to wear his suit. However, should such an agreement be made the boy then has reason to wear his jeans as a weakly dominant strategy again and so begins a cycle not unlike that experienced in reasoning about the Prisoner's Dilemma. Yes, mixed strategies are often suggested as a way to solve such games, but it can take a long time for each player to discover the appropriate probabilities to assign each outcome and mixed strategies do not, at least in this case, take into account the credible threat available to the girl as a consequence of her indifference about what she wears nor the human predisposition for carrying out such a threat.

What makes this game interesting is the combined indifference across the players and it is interesting on two levels. Let the first be called the pragmatic level. This level is concerned with how the game should be solved both in general and from the perspective of both players. After sharing this game with many classes of students and colleagues it is not clear that there is a clear answer that is reasonable given the power that indifference gives to the girl. Despite some controversy on this front it is likely the case that on further investigation and consideration that the controversy surrounding what to do amounts to a simple misunderstanding of what is being represented and what a solution to a game involves.

More interesting is considering the game on the second level, what we might consider to be the meta-game level. Here we can sensibly ask, "Can the girl be said to be truly indifferent?" Yes, given the representation of the game shared here there is a sense in which she should be taken to be truly indifferent since it is common practice to analyze the game given and not the one we want to analyze, but in the real world is such a situation possible? Put another way, can games of this sort survive the application of reason? In this particular case these question amount to wondering whether or not the girl can remain indifferent about her own choices as the extent to which she is aware that she is aware that she may be able to leverage her own choice (or the presentation thereof) to influence the boy and his choice. Ultimately this is a question to be answered in the lab or through systematic and detailed real-world observations of actual humans in such situations.

It would be particularly useful if such a study was combined with or part of an investigation into the construction of human preferences and how indifference both creates and solves various coordination problems, both when introduced by the subjects themselves and the researchers. If there is any question that there are other games involving indifference that are frustrating and in need of (at least practical) solutions on a regular basis simply consider the following:

1, 0	0, 0
1, 0	0, 0

3, 2	0, 2
1, 0	2, 1

In the first the row player faces the frustrating situation of having a clear preference for one set of outcomes that they can do nothing to bring about as it is left entirely to the whim of the column player whether or not they will receive one of these outcomes. The column player is, of course, completely indifferent. In the second the column player faces a choice between attempting to coordinate on the top-left outcome (a clear focal point) or defecting from this via a weakly dominant strategy, a strategy the existence of which both puts them in a position to protect themselves from a similar defection by the row player while simultaneously prompting that defection in the first place.

Clearly this is not a comprehensive treatment of any of the games presented here and it in hoped that this can be forgiven since this was not the intent in sharing them. Rather, they were shared to simply provide evidence that there are games with interesting properties that not only fall outside the set of player-symmetric games but outside the set of games that ignore the possibility of player indifference, a possibility that seems to be a very real one. In identifying the existence of such games, calling attention to the large set of games that they belong to, and suggesting that how frequently we encounter them may have a great deal to do with how we see the world, it is hoped that further discussion can follow–despite their apparent simplicity 2×2 games still promise to be a rich source for prompting and testing further investigations about human psychology and behaviour.

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