Nominal Rigidities and the Monetary Policy in Canada since 1981

Ali Dib*

May 1, 2002

Abstract

This paper develops and estimates a dynamic, stochastic, general-equilibrium model with price and wage stickiness to evaluate the monetary policy in Canada. The monetary policy is characterized as one that allows a central bank to systematically adjust the short-term nominal interest rate and the money growth in response to inflation and output deviations. The structural parameters of the model are estimated econometrically using a maximum likelihood procedure with a Kalman filter. The estimates reveal that either price or wage rigidities are key nominal frictions that generate real monetary effects. Furthermore, the simulation results show that, under such a policy, the Canadian monetary authority has, since 1981, increased the short-term nominal interest rate in response to exogenous positive demand-side disturbances, and used modest but persistent reductions to accommodate positive technology shocks.

JEL classification: E31, E32, E52.

Key words: Sticky prices; Sticky wages; Monetary rule; Endogenous persistence.

*Monetary and Financial Analysis Department, Bank of Canada. 234 Wellington St. Ottawa, ON. K1A 0G9. E-mail: adib@bank-banque-canada.ca, Tel.(613) 782-7851. I thank Robert Amano, Walter Engert, Scott Hendry, Jean-Paul Lam, Jinill Kim, Cessaire Meh, Kevin Moran, Louis Phaneuf, and seminar participants at the Bank of Canada for their discussion and useful comments. I also thank Veronika Dolar for her assistance. Of course, I am solely responsible for any remaining errors. The views expressed in this paper are mine. No responsibility for them should be attributed to the Bank of Canada.
1 Introduction

In recent years, there has been an extensive literature on the role of nominal rigidities in shaping key features of the business cycle and in evaluating short-run dynamic monetary policy effects on aggregate variables. Researchers have generally used dynamic, stochastic, general-equilibrium (DSGE) models in which price and/or nominal wages are sticky. These new sticky-price and/or sticky-wage models rest on the assumptions that private agents have rational expectations, and that their optimizing behaviour determines the time paths of nominal and real variables, such as output and inflation. Furthermore, in contrast to the previous generation models, these new models predict that the real effects of monetary policy shocks would differ sharply under sticky prices and sticky wages.\footnote{An example of this previous generation of models is Taylor (1980).} Chari, Kehoe and Mc Grattan (2000) establish that staggered price setting alone does not generate endogenous persistence in an economy of imperfectly competitive price-setters. Chrisiano, Eichenbaum and Evans (2001) find that a version of the DSGE model with only nominal wage rigidities does almost as well as the model with price and wage rigidities, while the version of their model with only nominal price rigidities performs very poorly. Andrew, Erceg, and Henderson (2000), Kim (2000), Huang and Liu (1998), and Huang, Phaneuf and Zheng (2000) find that combining staggered wage and imperfectly competitive households generates more output persistence in response to monetary policy shocks.

Since the early 1980s, the Bank of Canada has adopted a monetary policy that actively manages short-term nominal interest rates to control inflation. Figure 1 shows short-run dynamic relationships between short-term nominal interest rates, as measured by the three-month treasury-bill rate, inflation, as measured by quar-
terly changes in the GDP deflator, and M2 money growth for the Canadian economy; data are quarterly from 1980:1 to 2000:4. The inflation rate has significantly fallen from its peak in 1981 and has remained low and stable since then. Moreover, the longer-run inflation rate decline has been accompanied by a longer-run decline in the short-term nominal interest rate and money growth. However, several episodes of rising short-term nominal interest rate and money growth have interrupted this longer-run trend; in particular, the short-term nominal interest rate rose as inflation and money growth rates increased during 1987-1990 and 1993-1994.\textsuperscript{2} Nevertheless, during the periods 1983-1986 the money growth rate jumped higher and this movement was accompanied by a light increase in the inflation rate and by a significant decrease in the short-term nominal interest rate. Therefore, the figure could provide some evidence for the notion that monetary policy actions have contributed to the decline in inflation.\textsuperscript{3} Indeed, dynamic relationships between the nominal interest rate, money growth, and inflation may reflect both the way in which the monetary policy authority responds to economic disturbances and the way in which private agents respond to those same disturbances; in particular, monetary policy, money demand, technology, and preference shocks.

Following Christiano et al. (2001), Dib (2001), Dib and Phaneuf (2001), Ireland (1997, 2001a), Kim (2000), and Rotemberg and Woodford (1997), I develop and estimate an optimization-based model for the Canadian economy. The model features monopolistic competition between firms, and between households, nominal rigidities in the form of price- and wage-adjustment costs, and a real rigidity modelled as convex costs of adjusting capital. The developed model includes also four

\textsuperscript{2}The central bank may have increased the nominal interest rate to respond to financial market fluctuations, as in 1987, or to defend the Canadian dollar, as in 1994.

\textsuperscript{3}The interest rate has typically increased prior to inflation and money growth downturns.
sources of disturbances: monetary policy, money demand, technology, and preference shocks. Temporary rigidities in nominal prices and wages allow the monetary authority to affect the behaviour of real variables in the short term. Further, under these nominal rigidities, exogenous money demand shocks become a significant source of aggregate fluctuations. Empirical work shows that such shocks are large and highly persistent.4

This paper shall follow Taylor (1993), who describes Federal Reserve behaviour with monetary policy that adjusts the short-term nominal interest rate in response to output and inflation deviations, but I generalize the Taylor’s specification by allowing the central bank to respond to deviations of money growth as well. Such a policy implies a restrictedly endogenous money supply.5 An increase in inflation allows the interest rate to rise, reducing nominal asset demand and restraining money growth. Similarly, if money growth rose, reserve demand would rise, and the central bank would increase the nominal interest rate which should automatically reduce aggregate money demand.6

The presence of money growth in such a policy, can be considered as representing some omitted variables that monetary authority would respond to their changes, such as exchange rate and some financial variables. Alternatively, the central bank monetary policy can be characterized as a combination policy that adjusts a linear combination of the short-term nominal interest rate and the money growth in re-

4Examples of such work are, Dib (2001) for the Canadian economy, Dib and Phaneuf (2001) and Ireland (1997, 2000) for the U.S. economy.

5Under such a policy, the response of money growth to exogenous disturbances is restricted. However, under standard Taylor’s (1993), money supply is perfectly endogenous and freely responds to exogenous disturbances.

6If the money stock were growing faster than desired, the central bank would increase the nominal interest rate. This would in turn reduce money demand and tend to bring the money stock back to its starting point.
response to changes in output and inflation. Poole (1970) gives the classic analysis of the choice between employing an interest rate, a monetary aggregate, or any linear combination of the two as the principal central bank monetary policy instrument. He shows how the stochastic structure of the economy – the nature and relative importance of different types of disturbances – would determine the optimal instrument. Thus, if the central bank implements monetary policy by manipulating short-term nominal interest rates or any linear combination of nominal interest rate and money growth, the nominal stock of money is endogenous, but restricted. It is affected by policy actions as well as by other shocks hitting the economy.

To evaluate empirically this monetary policy under nominal rigidities, three versions of the DSGE model with quadratic price- and wage-adjustment costs are estimated using a maximum likelihood procedure with a Kalman filter applied to the state-space forms. Quarterly data on consumption, the three-month treasury-bill rate, inflation, and nominal monetary aggregate M2 are used. Since the Bank of Canada effectively abandoned M1 growth targeting by the middle of 1981, the data used cover the period 1981Q3 to 2000Q4. The estimates reveal that price- and wage-adjustment cost parameters are quite substantial and significant. Furthermore, the estimated values imply that, on average, prices remain unadjusted for more than two quarters in the standard sticky-price model and they are almost completely flexible when price and wage rigidities are combined together. However, the average estimated duration of unadjusted nominal wages is about five quarters in models including wage stickiness.\footnote{Since the introduction of the nominal price and wage rigidities using quadratic adjustment costs functions is equivalent to Calvo (1983) style nominal price and wage contracts, see Appendix C.} Moreover, the estimates of the capital-adjustment cost parameter indicate that it is costly to adjust capital and this form of real rigidity
helps to produce endogenously significant persistence in response of real variables to exogenous shocks.

The estimates of the monetary policy rule coefficients indicate that the central bank has positively responded to inflation, real output, and money growth deviations by increasing the nominal interest rate in order to control inflation. Thus, monetary policy during this period can be better described as following a modified Taylor (1993) rule that adjusts the short-term nominal interest rate in response to deviations of inflation, output, and money growth from their steady-state levels. A similar result is found by Ireland (2001a, 2001b) for the U.S. economy. Alternatively, monetary policy can be characterized as adjusting a linear combination of the nominal interest rate and the money growth rate to achieve a target for inflation.

The simulation results show that, using the short-term nominal interest rule, the Bank of Canada has successfully reduced the effects of negative money-demand shocks on aggregate output by modestly increasing the short-term nominal interest rate. More important, the monetary authority has also reduced the short-term nominal rate to accommodate technology shocks, despite the fact that this policy appears to focus on the behaviour of inflation and not on independent developments in the real economy.

The remainder of the paper is organized as follows. Section 2 presents the monopolistic competition model with price, wage, and capital rigidities. Section 3 describes the data, the calibration procedure and the econometric method used in the estimation of the models. Section 4 reports and discusses the empirical results. Finally, section 5 concludes.
2 The Model

The basic structure of the model is inspired essentially by Dib (2001), Ireland (1997, 2001a), Kim (2000), and Christiano et al. (2001). It is assumed the economy is populated by a continuum of households, a representative final-good-producing firm, a continuum of intermediate-goods-producing firms, and a monetary authority. Each household offers a distinct labour service on a monopolistically competitive market. Households also pay two distinct costs for adjusting nominal wages and the capital stock. The final-good-producing firm produces a final good, which sells on a perfectly competitive market. However, each intermediate-goods-producing firm produces a distinct, perishable intermediate good, which is sold on a monopolistically competitive market. Intermediate-goods-producing firms also pay a finite cost for changing their nominal prices.

2.1 The household

Household $i$ derives utility from consumption, $c_{it}$, real money balances, $M_{it}/p_t$, and leisure, $(1-h_{it})$. The household’s preferences are described by the expected utility function,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u \left( c_{it}, \frac{M_{it}}{p_t}, h_{it} \right),$$

where $\beta \in (0, 1)$ is the discount factor and the single-period utility function specified as:

$$u(\cdot) = \frac{\gamma z_t}{\gamma - 1} \log \left[ \frac{z_t^{\gamma - 1}}{c_{it}^{\gamma}} + b_t^{1/\gamma} \left( \frac{M_{it}}{p_t} \right)^{\frac{2-1}{\gamma}} \right] + \eta \log (1 - h_{it}),$$

where $\gamma$ and $\eta$ are positive structural parameters; $z_t$ and $b_t$ are both serially correlated shocks. As in Ireland (2001b), the preference shock $z_t$ enters into the Euler equation linking the household’s consumption growth to the real interest rate and
evolves according to
\[
\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt},
\]
where $\rho_z \in (-1, 1)$, and $\varepsilon_{zt}$ is a serially uncorrelated shock normally distributed with zero mean and standard deviation $\sigma_z$. As shown by McCallum and Nelson (1999), this type of disturbance resembles, in equilibrium, a shock to the IS curve in more traditional Keynesian analyses. However, shock $b_t$ is interpreted as a shock to money demand, and it follows the first-order autoregressive process:
\[
\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt},
\]
where $\rho_b \in (-1, 1)$, and the serially uncorrelated shock $\varepsilon_{bt}$ is normally distributed with zero mean and standard deviation $\sigma_b$.

As in Kim (2000), Eroeg et al. (2000), and Christiano et al. (2001), it is assumed that household $i$ is a monopoly supplier of a differentiated labour service, $h_{it}$. It sells this service to a representative, competitive firm which transforms it into an aggregate labour input, $h_t$, using the following technology
\[
h_t \leq \left( \int_0^1 \frac{\theta_h^{-1}}{h_{it}^{\theta_h}} \, di \right)^{\frac{\theta_h}{\theta_h - 1}}, \quad \theta_h > 1,
\]
where $\theta_h$ is the constant elasticity of substitution in the labour market. The demand curve for $h_{it}$ is given by
\[
h_{it} = \left( \frac{W_{it}}{W_t} \right)^{-\theta_h} h_t,
\]
where $W_{it}$ is the nominal wage of household $i$, $W_t$ is the wage index, i.e., the aggregate wage rate, which satisfies
\[
W_t = \left( \int_0^1 W_{it}^{1-\theta_h} \, di \right)^{-\frac{1}{1-\theta_h}}.
\]
The household takes $h_t$ and $W_t$ as given and beyond its control.
Household $i$ enters period $t$ with $k_{it}$ units of capital, $M_{it-1}$ units of money, and $B_{it-1}$ units of treasury bonds. During period $t$, it supplies labour and capital to firms and receives total factor payment $R_{kt}k_{it} + W_ith$, where $R_{kt}$ is the nominal rental rate for capital and $W_i$ is the individual nominal wage. Furthermore, household $i$ receives a lump-sum nominal transfer from the central bank, $T_{it}$, and dividend payments from intermediate-goods-producing firms, $D_{it} = \int_0^1 s_i D_{it} dj$, where $s_i$ is the share of household $i$ of the dividend payment by firm $j$. Household $i$ uses some of its funds to purchase the final good at the nominal price $p_t$, which it then divides between consumption and investment. Moreover, it is assumed that it is costly to intertemporally adjust the capital stock since there are adjustment costs specified as:

$$CAC_{it} = \frac{\phi_k}{2} \left( \frac{k_{it+1}}{k_{it}} - 1 \right)^2 k_{it}, \quad (8)$$

where $\phi_k > 0$ is the capital-adjustment cost parameter.

Wage stickiness is introduced through the cost of adjusting nominal wages. The functional form of these costs is assumed to be quadratic with a zero steady-state value. The real total wage-adjustment cost for household $i$ is given by

$$WAC_{it} = \frac{\phi_w}{2} \left( \frac{W_{it}}{W_{it-1} - \pi} \right)^2 \frac{W_{it}}{p_t}, \quad (9)$$

where $\phi_w \geq 0$ is the wage-adjustment cost scale parameter, $\pi$ is the steady-state value of the inflation rate, and $p_t$ is the final-good price index.

The budget constraint of household $i$ is given by

$$c_{it} + k_{it+1} - (1 - \delta)k_{it} + CAC_{it} + WAC_{it} + \frac{M_{it} + B_{it}/R_t}{p_t} \leq \frac{R_{kt}k_{it}}{p_t} + \frac{W_{it}}{p_t}h_{it} + \frac{M_{it-1} + B_{it-1} + T_{it} + D_{it}}{p_t}, \quad (10)$$

where $\delta \in (0, 1)$ and $R_t$ denote the constant capital depreciation rate and the gross nominal interest rate between $t$ and $t+1$, respectively.
Household $i$ chooses \{$c_{it}, M_{it}, h_{it}, W_{it}, k_{it+1}, B_{it}\}$ to maximize the expectation of the discounted sum of its utility flows subject to the labour demand that it faces, equation (6), and the budget constraint, equation (10). The first-order conditions for this problem are

\[
\frac{z_t c_{it}^{-\frac{1}{\gamma}}}{c_{it}^{-\frac{1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_{it}/p_t)^{\frac{1}{\gamma}}} = \lambda_t; \tag{11}
\]

\[
\frac{z_t b_t^{\frac{1}{\gamma}} (M_{it}/p_t)^{\frac{1}{\gamma}}}{c_{it}^{-\frac{1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_{it}/p_t)^{\frac{1}{\gamma}}} = \lambda_t - \beta E_t \left( \frac{p_t \lambda_{t+1}}{p_{t+1}} \right); \tag{12}
\]

\[
\frac{\eta}{1 - h_{it}} = \lambda_t \frac{W_{it}}{p_t} - \psi_t; \tag{13}
\]

\[
\theta_h \frac{\psi_t}{\lambda_t} \left( \frac{W_{it}}{W_t} \right)^{-\theta_h - 1} p_t h_{it} = 1 - \frac{\phi_w}{h_{it}} \left( \frac{W_{it}}{W_{it-1}} - \pi \right) \frac{W_{it}}{W_{it-1}} + \beta \frac{\phi_w}{h_{it}} E_t \left[ \left( \frac{W_{it+1}}{W_{it}} - \pi \right) \left( \frac{W_{it+1}}{W_{it}} \right)^2 \frac{p_t}{p_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right]; \tag{14}
\]

\[
\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{R_{kt+1}}{p_{t+1}} + 1 - \delta + \phi_k \left( \frac{k_{it+2}}{k_{it+1}} - 1 \right) \frac{k_{it+2}}{k_{it+1}} \right) \right] = 1 + \phi_k \left( \frac{k_{it+1}}{k_{it}} - 1 \right); \tag{15}
\]

\[
\frac{1}{R_t} = \beta E_t \left[ \frac{p_t \lambda_{t+1}}{p_{t+1} \lambda_t} \right]; \tag{16}
\]

where $\lambda_t$ is the Lagrangian multiplier of the budget constraint.

The condition (14) implies that, under the symmetric hypothesis, the labour demand elasticity, $\epsilon_{ht}$, augmented with the wage-adjustment costs is\footnote{In fact,}

\[
\epsilon_{ht} = \theta_h \left\{ 1 - \frac{\phi_w}{h_t} \left( \frac{W_{it}}{W_{it-1}} - \pi \right) \frac{W_{it}}{W_{it-1}} + \beta \frac{\phi_w}{h_t} E_t \left[ \left( \frac{W_{it+1}}{W_{it}} - \pi \right) \left( \frac{W_{it+1}}{W_{it}} \right)^2 \frac{p_t}{p_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right] \right\}^{-1}.
\]
Thus, the wage-, \( q_{wt} \), which is the ratio of the real wage to the marginal rate of substitution of consumption for leisure (\( MRS_t \)), is derived from condition (13) as\(^9\)

\[
q_{wt} = \left(1 - \frac{1}{\epsilon_{ht}}\right)^{-1}.
\]

With finite elasticity of substitution, i.e., \( \epsilon_{ht} < \infty \), the wage-markup measures the household’s power in the labour market. If the nominal wages are perfectly flexible, i.e., \( \phi_w = 0 \), the wage-markup is constant at \( \theta_h/(\theta_h - 1) \). However, in the presence of wage-adjustment costs, i.e., \( \phi_w > 0 \), the exogenous disturbances directly affect the wage-markup which then affects the real variables. When log-linearized, conditions (13) and (14) imply that the wage-markup measures the discrepancies between actual nominal wages \( W_t \) and the nominal wages that would prevail in the absence of wage-adjustment costs \( W^*_t \) (see Appendix C).

As shown in Ireland (1997) and Dib (2001), combining conditions (11) and (12) yields the following standard money demand equation:

\[
\log \left( \frac{M_t}{p_t} \right) \approx \log(c_{it}) - \gamma \log(r_t) + \log(b_t),
\]

where \( r_t = R_t - 1 \) denotes the net nominal interest rate between \( t \) and \( t + 1 \), and \(-\gamma\) is the interest elasticity of money demand, while \( b_t \) is a serially correlated money demand shock.

### 2.2 The final-good-producing firm

The final good, \( y_t \), is produced by a perfectly competitive firm which uses a continuum of intermediate goods, indexed by \( j \in (0, 1) \). Its technology is

\[
y_t \leq \left( \int_0^1 y_j^\theta_j y^\theta_j \, dj \right)^{\theta_y - 1}, \quad \theta_y > 1,
\]

\(^9\text{With } MRS_t = \frac{w_y}{w_x}, q_{wt} = \frac{w_y}{MRS_t}.\)
where $y_{jt}$ denotes the time $t$ input of intermediate good $j$, and $\theta_y$ is the constant elasticity of substitution of intermediate goods.

Given the final-good price, $p_t$, and the intermediate-good price, $p_{jt}$, the final-good-producing firm chooses the quantity of intermediate-good $y_{jt}$ that maximizes its profits. Profit maximization implies the following demand function

$$y_{jt} = \left( \frac{p_{jt}}{p_t} \right)^{-\theta_y} y_t,$$

which expresses the demand for good $j$ as a function of its relative price and final output. The final-good price index satisfies

$$p_t = \left( \int_0^1 p_{jt}^{1-\theta_y} \, dj \right)^{-\frac{1}{1-\theta_y}}.$$

2.3 The intermediate-goods-producing firm

The intermediate-goods-producing firm $j$ hires $k_{jt}$ units of capital and $h_{jt}$ units of labour to produce output according to the following constant-returns-to-scale technology:

$$y_{jt} \leq k_{jt}^\alpha (A_t h_{jt})^{1-\alpha}, \quad \alpha \in (0, 1),$$

where $A_t$ is a technology shock which is common to all intermediate-goods-producing firms. The technology shock $A_t$ is assumed to follow the autoregressive process

$$\log A_t = (1 - \rho_A) \log (A) + \rho_A \log (A_{t-1}) + \varepsilon_{At},$$

where $\rho_A \in (-1, 1)$, and $\varepsilon_{At}$ is a serially uncorrelated shock that is normally distributed with mean zero and standard deviation $\sigma_A$.

Intermediate goods are imperfectly substitutable for one another in producing the final good, so the intermediate-goods-producing firm $j$ can set the price $p_{jt}$ that
maximizes its profits flows. Furthermore, as in Rotemberg (1982), firm $j$ faces a quadratic cost of adjusting its nominal price across periods. The price-adjustment costs are measured in terms of the final good and given by

$$PAC_{jt} = \frac{\phi_p}{2} \left( \frac{p_{jt}}{\pi p_{jt-1}} - 1 \right)^2 y_t,$$

where $\phi_p \geq 0$ is the price-adjustment cost parameter and $\pi$ is the steady-state value of the inflation rate. In the presence of such price-adjustment costs, the price-markup becomes endogenous and the intermediate-goods-producing firm’s problem is dynamic.

The intermediate-goods-producing firm $j$ chooses contingency plans for $h_{jt}$, $k_{jt}$, and $p_{jt}$ for all $t \geq 0$, that maximize its expected total profit flows

$$\max_{\{k_{jt}, h_{jt}, p_{jt}\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \lambda_t D_{jt}/p_t \right],$$

subject to the demand curve it faces, equation (19), and to the production technology, (21); where $\beta^t \lambda_t$ is the firm’s discount factor, and the instantaneous profit function is

$$D_{jt} = p_{jt} y_{jt} - R_{kt} k_{jt} - W_t h_{jt} - p_t PAC_{jt}.$$  

The first-order conditions for this optimization problem are

\begin{align*}
\alpha \frac{y_{jt}}{k_{jt}} \frac{\xi_t}{\lambda_t} &= \frac{R_{kt}}{p_t}; \\
(1 - \alpha) \frac{y_{jt}}{h_{jt}} \frac{\xi_t}{\lambda_t} &= \frac{W_t}{p_t}; \\
\frac{\xi_t}{\lambda_t} &= \frac{\theta_y - 1}{\theta_y} + \frac{\phi_p}{\theta_y} \left( \frac{p_{jt}}{\pi p_{jt-1}} - 1 \right) \frac{p_{jt}}{\pi p_{jt-1}} \frac{y_t}{y_{jt}} \\
&\quad \quad - \frac{\beta \phi_p}{\theta_y} \mathbb{E}_t \left[ \left( \frac{p_{jt+1}}{\pi p_{jt}} - 1 \right) \frac{p_{jt+1}}{\pi p_{jt}} \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_{jt}} \right]; \\
\left( \frac{p_{jt}}{p_t} \right)^{-\theta_y} y_t &= k_{jt}^\alpha (A_t h_{jt})^{1-\alpha};
\end{align*}
where $\xi_t > 0$ is the Lagrangian multiplier associated to the technology function.

As in Ireland (1997) and Dib (2001), conditions (26) and (27) imply that the price-markup, $q_{pt}$, which measures the ratio of price to marginal cost, is equal to $\lambda_t/\xi_t$. Moreover, condition (28) indicates that this price-markup responds endogenously to exogenous shocks in the presence of price-adjustment costs. However, $q_{pt}$ is constant at $\theta_y/(\theta_y - 1)$ if the prices are perfectly flexible.

Under the symmetry hypothesis, where all intermediate-goods-producing firms are identical, log-linearizing condition (28) yields

$$\log(p_t) - \log(p_{t-1}) - \log(\pi) = \left(\frac{\theta_y - 1}{\phi_p}\right) E_t \sum_{t=0}^{\infty} \beta^t \left[\log(p^*_{t+s}) - \log(p_{t+s})\right], \quad (30)$$

where $p^*_{t+s}$ is the nominal price that would prevail in the absence of price-adjustment costs.\(^{10}\) The discounted present value of current and future discrepancies between the desired price $p^*_{t+s}$ and the actual prices $p_{t+s}$ is given by the sum on the right-hand side of (30). As in Calvo (1983), the ratio $(\theta_y - 1)/\phi_p$ represents the fraction of intermediate firms that can change their prices in the current period. Price adjustment becomes more rapid when this fraction increases, so the price-adjustment cost becomes smaller as $\phi_p$ decreases.\(^{11}\)

2.4 The monetary authority

Following Ireland (2001a) and (2001b), I assume that the central bank conducts monetary policy by managing a linear combination of the short-term nominal in-

\(^{10}\)When log-linearized, condition (28) implies that the price-markup $q_{pt}$ measures the discrepancies between $p_{t+s}$ and $p^*_{t+s}$; equation (30) is derived from this relation.

\(^{11}\)In Calvo's model, $(\theta_y - 1)/\phi_p$ is interpreted at the individual firm level as the probability that the firm will change its price, and at the aggregate level, is the fraction of firms that can adjust their price in any given period. Using a version of Calvo's model of price adjustment, King and Watson (1996) assume that each firm has a probability 0.10 adjusting its price each quarter. This probability is independent over time for each firm.
Interest rate, $R_t$ and the money growth rate, $\mu_t = M_t/M_{t-1}$, in response to changes in output, $y_t$, and inflation, $\pi_t = p_t/p_{t-1}$. Thus, the monetary policy rule evolves according to:

$$\log(R_t/R) - \rho_\mu \log(\mu_t/\mu) = \rho_y \log(y_t/y) + \rho_\pi \log(\pi_t/\pi) + \varepsilon_{Rt}, \tag{31}$$

where $R$, $\mu$, $y$, and $\pi$ are the steady-state values of $R_t$, $\mu_t$, $y_t$, and $\pi_t$, and where $\varepsilon_{Rt}$ is a zero-mean, serially uncorrelated monetary policy shock with standard deviation $\sigma_R$.

The policy coefficients $\rho_\mu$, $\rho_y$, and $\rho_\pi$ are chosen by the central bank. When $\rho_\mu = 0$, $\rho_y > 0$, and $\rho_\pi > 0$, monetary policy follows a Taylor (1993) rule in which the central bank increases the nominal interest rate in response to deviations of output and inflation from their steady-state values.\footnote{Under the standard Taylor’s (1993) rule, money supply freely responds to exogenous disturbances.} In this case, a unique equilibrium exists only if $\rho_\pi$ is greater than 1.\footnote{If $\rho_\pi > 1$, an increase in the inflation rate of 1 per cent generates an increase in the nominal interest rate of more than 1 per cent which, in turn, increases the real interest rate.}

Nevertheless, if $\rho_\mu$ is different of zero, two interpretations are possible. First, the monetary policy can be described as following a modified Taylor (1993) rule that adjusts the short-term nominal interest rate in response to money growth rate as well as output and inflation. Herein, the money growth rate may be interpreted as a proxy of some omitted variables that monetary policy should respond to their changes, such as exchange rate and financial variables. Therefore, money supply becomes endogenous and responds systematically, but restrictedly, to exogenous disturbances.\footnote{In contrast, an exogenous monetary policy would keep the money supply growing at a constant rate so that $\mu_t = \mu$ for all $t \geq 0$.} Alternatively, as Ireland (2001b) points out, the central bank monetary policy can be characterized as a combination policy, as shown in Poole (1970),
that adjusts a linear combination of the interest rate and the money growth rate to achieve a target for inflation.

2.5 Symmetric equilibrium

In a symmetric equilibrium, all households and intermediate-goods-producing firms make identical decisions, so that

\[ c_{it} = c_t, \quad M_{it} = M_t, \quad h_{it} = h_t, \quad k_{it} = k_t, \quad W_{it} = W_t, \quad B_{it} = B_t, \quad T_{it} = T_t, \]

and

\[ p_{jt} = p_t, \quad y_{jt} = y_t, \quad k_{jt} = k_t, \quad h_{jt} = h_t, \quad D_{jt} = D_t \]

for all \( i, j \in (0, 1) \) during each period \( t \geq 0 \). Further, the market-clearing conditions \( M_t = M_{t-1} + T_t \) and \( B_t = 0 \) must hold for all \( t \geq 0 \). Let \( r_{kt} = R_{kt}/p_t, \quad w_t = W_t/p_t, \) and \( m_t = M_t/p_t \) denote the real rental rate on capital services, the real wage rate, and real balances, respectively. Thus, a non-linear symmetric equilibrium system consists of an allocation \( \{y_t, c_t, m_t, h_t, k_t\}_{t=0}^{\infty} \) and a sequence of prices and co-state variables \( \{w_t, r_{kt}, R_t, \pi_t, \lambda_t, q_{pt}, q_{wt}\}_{t=0}^{\infty} \) that satisfy the household’s first-order conditions \((11)-(16)\), the intermediate-goods-producing firm’s first-order conditions \((26)-(29)\), the aggregate resource constraint, the money supply rule, and the stochastic processes of preference, money demand, technology, and monetary policy shocks, equations \((3), (4), (22), \) and \((31)\) (see Appendix A).

Taking a log-linear approximation of the equilibrium system around steady-state values and using the method of Blanchard and Kahn \((1980)\), yields a state-space
solution of the form:\footnote{For any stationary variable $x_t$, I define $\hat{x}_t = \log(x_t/x)$ as the deviation of $x_t$ from its steady-state value, $x$ (see Appendix B for the steady-state ratios).}

$$\tilde{s}_{t+1} = \Phi_1 \tilde{s}_t + \Phi_2 \varepsilon_{t+1}, \quad (32)$$
$$\hat{d}_t = \Phi_3 \tilde{s}_t, \quad (33)$$

where $\tilde{s}_t$ is a vector of state variables which includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector $\varepsilon_{t+1}$ contains technology, money demand, monetary policy, and preference shocks.\footnote{This solution is a restricted vector autoregression (VAR) in the sense that the coefficient matrices, $\Phi_1$, $\Phi_2$, and $\Phi_3$, have elements that depend on the structural parameters of the model describing the household’s preferences, technologies, and central bank’s monetary policy rule.} This solution is estimated as 12.37 in Kim (2000) for the U.S. economy. Moreover, Christiano

\section{Calibration, Data, and Estimation}

As in Dib (2001), five structural parameters of the model are set prior to estimation, since the data used contain little information about them. The parameter $\eta$, denoting the weight on leisure in the utility function, is set equal to 1.35, so that households spend roughly 32 per cent of their time in market activities. The share of capital in production, $\alpha$, and the depreciation rate, $\delta$, are assigned values of 0.33 and 0.025, respectively; these values are commonly used in the literature.\footnote{Estimating a standard RBC model for the Canadian economy, Dolar and Moran (2001) find that $\alpha$ is about 0.3. Using this value does not affect the estimates in the present model.} The parameter $\theta_h$, which measures the degree of monopoly power in the labour market,
et al. (2001) set the wage-markup equal to 1.05. Since wages are more rigid in Canada than in the U.S. economy, \( \theta_h \) is set equal to 15, implying a gross steady-state wage-markup of 1.07.\(^{18}\) Finally, the parameter measuring monopoly power in intermediate goods markets, \( \theta_y \) is set equal to 9, implying a steady-state markup of price over marginal cost equal to 12.5 per cent, which matches the values usually used in similar studies.\(^{19}\) The calibration of these parameters is summarized in Table 1.

**Table 1:** Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \eta )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \theta_y )</th>
<th>( \theta_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>1.35</td>
<td>0.025</td>
<td>0.33</td>
<td>9.00</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Except for the wage- and price-adjustment cost parameters (\( \phi_w \) and \( \phi_p \)), the choices of \( \theta_h \) and \( \theta_y \) do not affect the estimated values of the model’s remaining parameters. Moreover, the degrees of nominal rigidity associated with wage and price stickiness are not affected as long as the ratios \( (\theta_h - 1)/\phi_w \) and \( (\theta_y - 1)/\phi_p \) remain constant.\(^{20}\)

The non-calibrated parameters are estimated using the method of Hansen and Sargent (1998). This procedure consists of applying a Kalman filter to a model’s state-space form to generate series of innovations, which are then used to evaluate the likelihood function for the sample. Since the solution (32)-(33) is a state-space econometric model, driven by four innovations in \( \varepsilon_t \), the underlying structural pa-\(^{18}\)The presence of unions is more important in Canada than in the U.S.\(^{19}\)However, following Rotemberg and Woodford (1995), Ireland (1997) and Dib (2001) set \( \theta_y \) equal to 6. The choice of \( \theta_y \) affects only proportionally the estimated value of \( \phi_p \).\(^{20}\)I also estimate the model’s structural parameters with \( \theta_h = 21 \) and \( \theta_y = 6 \); I find that the estimated values of \( \phi_w \) and \( \phi_p \) change proportionally with the choices of \( \theta_h \) and \( \theta_y \), so the ratios \( (\theta_h - 1)/\phi_w \) and \( (\theta_y - 1)/\phi_p \) remain constant. However, the estimates of the other parameters are not affected by the choice of \( \theta_h \) and \( \theta_y \).
rameters embedded in $\Phi_1$, $\Phi_2$, and $\Phi_3$, can be estimated by a maximum-likelihood procedure using data for four series, in particular $c_t$, $\pi_t$, $R_t$, and $m_t$ (see also Hamilton 1994, chapter 13).\footnote{The vector of structural parameters to estimate is}

\[
(\phi_p, \phi_w, \gamma, \beta, b, \rho_b, \sigma_b, A, \rho_A, \sigma_A, \pi, \rho_y, \rho_p, \sigma_R, \rho_z, \sigma_z)^T.
\]

Using quarterly Canadian data that run from 1981Q3 through 2000Q4, I estimate three versions of the model. The first is a sticky-price model (SP model) where nominal wages are flexible (model with $\phi_w = 0$). The second is a sticky-wage model (SW model) with flexible prices (i.e. $\phi_p = 0$). Finally, the third version has both sticky prices and sticky wages (SPSW model), in which both $\phi_p$ and $\phi_w$ are greater than zero.

Consumption is measured by real personal spending on non-durable goods and services. The inflation rate is measured by changes in the GDP deflator, while the short-term nominal interest rate is measured by the rate on three-month Treasury bills. Real balances are measured by dividing the M2 money stock by the GDP implicit price deflator. The series for consumption and real balances are expressed in per capita terms; they are divided by the civilian population aged 15 and over. The model implies that all variables are stationary and fluctuate around a constant mean. Thus, before estimating the model, the data are rendered stationary by regressing the logarithm of each variable on a constant and a time trend.\footnote{In the data, $m_t$ growth rate is much larger in the pre-1992 period than the post-1992 period, so I have used 1991Q4 as a break point of its linear trend.}
4 Empirical Results

This section discusses the estimated parameter values, displays the impulse-response functions to exogenous shocks, and reports the forecast-error variance decomposition of detrended output and inflation implied by the three estimated models. Table 2 displays the maximum-likelihood estimates of the SP, SW and SPSW models’ structural parameters with their standard errors.\footnote{I have tested the SP and SW models against the SPSW model. At the 5 per cent of significance level, the likelihood-ratio test, which has a chi-squared distribution with one degree of freedom, rejects the hypothesis that \( \phi_w = 0 \) in the SP model, but it does not reject the hypothesis that \( \phi_p = 0 \) in the SW model.} First, the estimates of parameters describing the price- and wage-adjustment costs, \( \phi_p \) and \( \phi_w \), are reported and their implications are discussed. The estimated value of \( \phi_p \) is 16.86 in the sticky-price model where \( \phi_w \) is set equal to 0. These values imply that, in any given period, 47.45 per cent of the intermediate-goods-producing firms can adjust their price in the SP model.\footnote{When \( \theta_p = 6 \), the estimate of \( \phi_p \) is equal to 10.52 in the SP model, and it is equal to 2.53 in the SPSW model. Thus, the ratio \( (\theta_p - 1)/\phi_p \) remains constant in the two models. Furthermore, with \( \theta_p = 6 \), Dib (2001) estimates a value for \( \phi_p \) equal to 14.36 in the model with price- and capital-adjustment costs and with money growth used as monetary policy instrument.} Thus, on average, the prices remain unadjusted for 2.10 quarters in the SP model. However, the estimate of \( \phi_p \) is only 4.01 in the SPSW model that combine price- and wage-adjustment costs. This finding indicates that prices are almost flexible in the presence of wage stickiness. Christiano et al. (2001) find a similar result. The estimates of \( \phi_w \) are 22.60 and 24.88 in the SW and SPSW models, respectively. These estimates are slightly imprecise, with high standard errors. The estimated values imply that, in any period, 18.75 per cent of nominal wages can be adjusted in the SW model; whereas, the percentage is about 17 per cent in the SPSW model.\footnote{When \( \theta_h = 21 \), the estimates of \( \phi_w \) are 32.21 in the SW model and 34.64 in the SPSW model. Therefore, the ratio \( (\theta_h - 1)/\phi_w \) remains almost constant compared to the standard estimations.} This result means that the average duration of wage stickiness

19
in the SW and SPSW models is roughly 5.33 and 5.67 quarters, respectively.\footnote{In the estimated version of the Christiano \textit{et al.} (2001) model with staggered prices and wages, the average duration of price and wage contracts is roughly 2 and 3 quarters, respectively.}

The estimates of the capital-adjustment cost parameter, $\phi_b$, are 8.52, 7.18, and 5.76 in the SP, SW, and SPSW models, respectively. These values produce an average cost of adjusting capital of about 1.5 per cent of quarterly investment.\footnote{The estimated value for $\phi_b$ in Ireland (2001b) is 32.13 and 17.41 in his sticky-price and flexible-price models for the post-1979 period for the U.S. economy.}

The discount parameter $\beta$ is estimated at 0.98 in the three models; while, the estimate of the gross quarterly steady-state inflation rate, $\pi$, is 1.028. The parameter $b$, determining the steady-state ratio of real balances to consumption, is estimated at 0.24. The estimate of $\gamma$, the constant elasticity of substitution between real consumption and real balances, is 0.40 and 0.43 in the SP and SW models, respectively; however, it is about 0.47 in the SPSW model. The estimates of $\rho_A$ are relatively small, but have high standard errors; they are 0.63 and 0.76 in the SP and SW models, respectively. However, the estimates of $\rho_b$ are close to 0.99, revealing that money demand shocks are highly persistent; whereas, the estimates of $\rho_z$ are around 0.93.\footnote{I have also estimated the models with $\rho_b$ fixed at 0.93. The only parameters affected by this constraint are $\gamma$ and $b$, which are estimated in the constrained models to be about 0.24 and 0.42, respectively. The remaining parameters are only marginally affected.}

On the other hand, the estimates of $\sigma_A$, $\sigma_b$, and $\sigma_z$ indicate that these exogenous shocks are quite volatile.

Finally, the estimates of the monetary policy parameters are reported. They are all statistically different from zero. The estimates of $\rho_\pi$, the coefficient that measures the response of monetary policy to the inflation deviations, are significant. The estimated values are 0.67, 0.62, and 0.60 in the SP, SW, and SPSW models, respectively. In contrast, the estimates for $\rho_y$, the coefficient measuring the response of monetary policy to output deviations are close to 0.08 in the SP and SPSW
models, while it is estimated at 0.085 in the SPSW model. However, the coefficient measuring the response of monetary policy to the deviations of money growth from its steady-state level, $\rho_\mu$, is substantial and statistically significant; it is estimated at 0.37, 0.42, and 0.44 in the SP, SW and SPSW models, respectively. The estimates of $\sigma_R$, which are close to 0.006, indicate that monetary policy shocks are quite volatile. Thus, this finding supports the hypothesis that, since 1981, the central bank has actively managed the short-term nominal interest rates, in response to the deviations of inflation, output, and money growth. Similarly, the central bank has used a linear combination of short-term nominal interest rate and money growth as its principal policy instrument to achieve its objectives.

Figures 2-5 display the impulse responses of detrended output, the short-term nominal interest rate, inflation, and money growth to a one per cent shock to monetary policy, money demand, technology, and preferences using the estimated SP, SW, and SPSW models. These impulse responses are generated from the state-space forms in (32)-(33), and each response is expressed as the percentage deviation of a variable from its steady-state level.

Figure 2 plots the different impulse responses to a 1 per cent positive monetary policy shock, i.e., $\varepsilon_R = 0.01$. This shock represents an exogenous tightening of monetary policy. Detrended output, inflation, and money growth fall sharply on impact, and the first two variables remain below their steady-state levels for several quarters after the shock. This merely reflects the slow adjustment of prices and nominal wages to their steady-state levels in response to the shock. Further, since the estimated degree of nominal rigidity is higher in the SW and SPSW models, nominal wages decrease by a lesser amount in response to the policy shock. Thus, the return of detrended output to its steady-state level in these models is slower and
more persistent compared to the SP model.\textsuperscript{29}

Nevertheless, the nominal short-term interest rate responds sharply but positively to this shock, and its positive response persists for at least five quarters, above its steady-state level, after the shock. As shown by Ireland (2001a), persistent movements in detrended output, inflation, and money growth, the variables on the right-hand side of the rule (31), generate persistence in the nominal interest rate response, even without the smoothing terms that Clarida, Gali, and Gelter (2000) and Ireland (2000) include in their specifications. More importantly, endogenous money helps to create a liquidity effect in the estimated models: an instantaneous increase in short-term nominal interest rate is accompanied with a decrease in money growth. Therefore, a recession, like one of 1990-1991, could be a result of a tightening monetary policy that increases the short-term nominal interest rate and decreases the money stock, as what happened in 1989, see Figure 1.

Figure 3 shows the impulse responses to a positive 1 per cent money demand shock, i.e. $\varepsilon_{bt} = 0.01$. This shock exogenously increases households’ money demand. In the three estimated models, the impact of this shock is highly persistent since the estimates of money demand shock autocorrelation coefficient, $\rho$, is close to one. Nevertheless, the impact of this shock is systematically very small. In fact, output decreases sharply but modestly before slowly returning to its steady-state level. On the other hand, inflation modestly responds to this shock, but it jumps slightly above its steady-state level two quarters after and is highly persistent thereafter.

On the other hand, the nominal short-term interest rate and money growth respond positively to the money demand shock. These responses are highly persistent particularly in the SP and SW models. As expected, the positive money demand

\textsuperscript{29}Real wages respond countercyclically in the SW and SPSW models where nominal wages are sticky, while they respond procyclically in the SP model where nominal wages are perfectly flexible.
shock increases real balances held by households, so that, with endogenous money, the money supply adjusts to equate money demand. More important, the central bank increases, modestly but persistently, the short-term nominal interest rate to respond positively to money growth, as suggested by the monetary rule (31). By doing so, real balances held by households eventually decrease and inflation returns back to its level before the shock. This result matches Poole’s (1970) classic analysis in which monetary authority should change the short-term nominal interest rate to react to exogenous demand-side disturbances.

Figure 4 displays the effects of a one per cent positive technology shock, $\varepsilon_{At} = 0.01$, in the three estimated models. As the estimates of technology autocorrelation coefficient, $\rho_A$, are relatively small, the impact of this shock on the real and nominal variables is not persistent. So, in response to a positive technology shock, detrended output jumps up instantaneously before returning gradually to its steady-state level. On the other hand, the nominal interest rate and inflation fall below their steady-state levels. Money growth responds positively to the shock before falling below its steady-state level after two quarters. The negative nominal interest rate response is very persistent and associated with the decrease in the money growth.

Responding to the technology shock, inflation falls sharply before returning to its steady-state level. However, the deflationary pressure, brought about by the positive technology shock, calls for a transitory increase in money growth and for a modest but sustained easing of monetary policy. This mechanism helps to accelerate and magnify the increase in output, which peaks above its steady-state level several quarters after the shock in the SP and SPSW models. Therefore, this central bank’s response helps the economy to adjust to supply-side disturbances as it would in the absence of nominal rigidities.
Figure 5 shows the impulse responses to a one per cent positive preference shock, \( \varepsilon_{zt} = 0.01 \); this is an exogenous shock to the household’s marginal utility of consumption and real balances. In equilibrium, this shock acts like a disturbance that looks like the IS shock in traditional Keynesian analyses, as shown by McCallum and Nelson (1999). Hence, in response to this shock, detrended output, nominal interest rate, inflation and money growth jump immediately above their steady-state levels before returning gradually. Since the estimate of the preference autocorrelation coefficient is higher in the SP model, the computed impulse responses show more persistence. To control the impact of preference shocks on the output and inflation, the central bank increases modestly but persistently the short-term nominal interest rate.

By actively managing the short-run nominal interest rate in response to changes in inflation, output, and money growth, monetary policy allows the economy to respond more efficiently to exogenous demand- and supply-side disturbances. A similar result was found by Ireland (2000) for the U.S. economy.

For the three estimated models, the decomposition of the forecast-error variance for detrended output and inflation into components due to policy, technology, money demand, and preference shocks is reported in Tables 3 and 4 at various horizons. Table 3 decomposes the forecast-error variance for detrended output. As shown in Panel A, the SP model implies that policy and money demand shocks explain a significant fraction of the output fluctuations in the short term, but preference shocks largely explain the observed variations in output even at the short horizon. Further, Panels B and C show that the SW and SPSW models predict that policy and money demand shocks are important sources of output fluctuations in the short term, and that preference shocks contribute very substantially to the variance of
output in the long term. At the one-quarter-ahead horizon, policy shocks account for at least 21 to 26 per cent of the forecast-error variance for detrended output in the SW and SPSW models. Nevertheless, up to the one-year-ahead horizon, money demand shocks explain at least 7 to 11 per cent of the output forecast-error variance.

Table 4 reports the forecast-error variance decomposition for inflation. Panel A and B, which present the SP and SW models’ results, show that policy shocks contribute to most of the observed variation in the inflation rate, even in the medium term. The fraction of the total variance explained by these shocks is about 50 per cent at the one-quarter-ahead horizon. This fraction decreases to 34 per cent at the four-quarter-ahead horizon in both models, even though money demand shocks contribute little to the inflation variations in the short term; however the fraction attributed to these shocks significantly increases in the long-term horizon. Technology shocks still explain a substantial fraction of inflation variation in the short and long horizons. On the other hand, Panel C shows that the model combining price and wage stickiness (the SPSW model) predicts that both policy and technology shocks are the important factors determining the movements in the inflation rate in the short and long term, even though money demand shocks account for over 80 per cent of the inflation variation in the long term and preference shocks modestly contribute to the inflation variations in the short and long terms.\footnote{When I set $\rho_A$, the autocorrelation coefficient of technology shock, equal to 0.95, technology and policy shocks explain, at one-quarter-ahead horizon, 87 and 11 per cent of the inflation variance, respectively; however, these fractions decrease as long as the horizon increases.}

To compare the performance of the economy under the estimated policy to its hypothetical performance under an alternative policy specified by the standard Taylor’s (1993) rule, I present the impulse response of output, nominal interest rate, inflation, and money growth to a one per cent positive monetary policy and money
demand shocks.\textsuperscript{31} In the alternative policy, the central bank makes no attempt to respond to money growth deviations. Therefore, the money supply freely responds to the state of the economy, but the central bank responds aggressively to output and inflation changes. As shown in Figures 6, under the alternative policy, the policy shock produces large nominal effects: nominal interest rate and money growth significantly jump above and under their steady-state levels, compared to their responses under the estimated policy. However, the negative inflation response lasts only one quarter after the shock before becoming positive in the second period. Thus, by responding or manipulating the money growth deviations, as in the estimated policy, the central bank has succeeded in reducing the variation of nominal variables.

Nevertheless, Figure 7, which displays the impulse response to one per cent positive money demand shock, shows that the central bank can significantly reduce the negative effects of money demand shocks on output, nominal interest rate and inflation by allowing to money supply to adjust freely. As the money supply response is restricted under the estimated policy, money demand shocks affect substantially the aggregate output and nominal interest rate; however, this is not the case when money supply freely responds. As expected by Poole's (1970) analysis, to completely isolate the economy from the negative effects of money demand shocks, the central bank should only and unrestrictedly adjust the money supply.\textsuperscript{32}

\textsuperscript{31}The coefficient $\rho_\mu$ is fixed equal to 0 and $\rho_x$ and $\rho_\eta$ are set equal to 1.5 and 0.5, as in Taylor (1993). The remaining parameters are set equal to their values calibrated or estimated in the SPSW model.

\textsuperscript{32}Nevertheless, under an exogenous monetary policy that keep the money supply growing at a constant rate, $\mu_t = \mu$, the effects of money demand shocks on output, nominal interest rate and inflation are much higher than those under the estimated policy.
5 Conclusion

Since the early 1980s, the Bank of Canada has followed a monetary policy that actively manages short-term nominal interest rates to control inflation. Under a such policy, the money supply is endogenous. Instead of Taylor’s (1993) rule, the monetary authority is assumed to adjust short-term nominal interest rates to respond not only to the deviations of output and inflation from their steady-state levels, but also to those of money growth. To evaluate this policy, a dynamic, stochastic, general-equilibrium model with nominal and real rigidities is developed and estimated. To compare the implications of price and wage stickiness, three versions of the DSGE model are estimated and simulated: sticky-price, sticky-wage, and a combined sticky-price and sticky-wage models. The structural parameters are estimated using a maximum-likelihood procedure with a Kalman filter applied to the state-space forms. The estimates reveal that either price or wage rigidities are key nominal friction to account for real monetary policy effects. Furthermore, the estimates of the monetary rule coefficients indicate that, since 1981, Canadian policy has actively responded to changes in inflation and money growth, but modestly to output deviations. Similarly, one can consider that the central bank has used a linear combination of the short-term nominal interest rate and the money growth to achieve their objectives.

More importantly, the results suggest that, by a small increase in the short-term nominal interest rate and in the money growth, monetary policy has been able to reduce the negative effects of money demand shocks on the real economic activity. Furthermore, monetary policy has accommodated positive technology shocks by persistently reducing the short-term nominal interest rates and provisionally increasing the money growth. As a positive technology shock produces temporary
deflationary pressures, the central bank responds to these pressures with a modest but persistent reduction in short-term nominal interest rates. However, to reduce the inflationary pressures implied by positive demand-side (preferences) shocks, the central bank modestly, but persistently, increases the short-term nominal interest rate and transitively decreases the money growth.

This paper focuses on the historical monetary policy rule that the central bank has followed since 1981. It does not investigate the choice of an optimal monetary rule that should be followed. This question is left for the future work. Moreover, since Canada is a small open economy, the effects of foreign economic disturbances on its domestic economy should be considered. As such, future work consists of extending this framework to develop and estimate an optimizing DSGE model for a small open economy.
References


Table 2:
Maximum-likelihood estimates and standard errors: 1981Q3 to 2000Q4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SP model</th>
<th>SW model</th>
<th>SPSW model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>Std. er.</td>
<td>Est.</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>16.861</td>
<td>11.271</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>-</td>
<td>-</td>
<td>22.597</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>8.5824</td>
<td>10.648</td>
<td>7.1826</td>
</tr>
<tr>
<td>$A$</td>
<td>2446.7</td>
<td>29.514</td>
<td>2451.5</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.6284</td>
<td>0.1206</td>
<td>0.7637</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0071</td>
<td>0.0033</td>
<td>0.0043</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9805</td>
<td>0.0007</td>
<td>0.9804</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4048</td>
<td>0.1567</td>
<td>0.4346</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2585</td>
<td>0.1229</td>
<td>0.2359</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9984</td>
<td>0.0127</td>
<td>0.9950</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.0247</td>
<td>0.0069</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.0285</td>
<td>0.0024</td>
<td>1.0287</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.3686</td>
<td>0.0885</td>
<td>0.4165</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>0.0720</td>
<td>0.0248</td>
<td>0.0850</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>0.6717</td>
<td>0.0832</td>
<td>0.6242</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.0061</td>
<td>0.0007</td>
<td>0.0064</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.9392</td>
<td>0.0154</td>
<td>0.9375</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0153</td>
<td>0.0015</td>
<td>0.0160</td>
</tr>
<tr>
<td>$LL$</td>
<td>1470.6</td>
<td></td>
<td>1467.5</td>
</tr>
</tbody>
</table>
Table 3:
Forecast-error variance decomposition of detrended output

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Variance</th>
<th>Policy</th>
<th>Technology</th>
<th>Money demand</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A. The SP model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000057</td>
<td>17.36</td>
<td>15.496</td>
<td>9.226</td>
<td>57.93</td>
</tr>
<tr>
<td>2</td>
<td>0.000098</td>
<td>10.96</td>
<td>18.47</td>
<td>7.17</td>
<td>63.40</td>
</tr>
<tr>
<td>3</td>
<td>0.000130</td>
<td>8.34</td>
<td>17.98</td>
<td>6.29</td>
<td>67.38</td>
</tr>
<tr>
<td>4</td>
<td>0.000156</td>
<td>6.97</td>
<td>16.66</td>
<td>5.91</td>
<td>70.46</td>
</tr>
<tr>
<td>5</td>
<td>0.000177</td>
<td>6.13</td>
<td>15.33</td>
<td>5.75</td>
<td>72.78</td>
</tr>
<tr>
<td>10</td>
<td>0.000248</td>
<td>4.39</td>
<td>11.47</td>
<td>6.17</td>
<td>77.96</td>
</tr>
<tr>
<td>50</td>
<td>0.000355</td>
<td>3.09</td>
<td>8.16</td>
<td>17.54</td>
<td>71.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B. The SW model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000041</td>
<td>21.67</td>
<td>26.37</td>
<td>9.69</td>
<td>42.75</td>
</tr>
<tr>
<td>2</td>
<td>0.000073</td>
<td>16.09</td>
<td>23.30</td>
<td>8.52</td>
<td>52.08</td>
</tr>
<tr>
<td>3</td>
<td>0.000101</td>
<td>12.66</td>
<td>20.61</td>
<td>7.70</td>
<td>59.02</td>
</tr>
<tr>
<td>4</td>
<td>0.000126</td>
<td>10.49</td>
<td>18.44</td>
<td>7.17</td>
<td>63.88</td>
</tr>
<tr>
<td>5</td>
<td>0.000147</td>
<td>9.08</td>
<td>16.74</td>
<td>6.83</td>
<td>67.34</td>
</tr>
<tr>
<td>10</td>
<td>0.000218</td>
<td>6.18</td>
<td>12.35</td>
<td>6.62</td>
<td>74.84</td>
</tr>
<tr>
<td>50</td>
<td>0.000309</td>
<td>4.41</td>
<td>9.00</td>
<td>15.11</td>
<td>71.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C. The SPSW model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000065</td>
<td>26.38</td>
<td>28.38</td>
<td>16.15</td>
<td>29.07</td>
</tr>
<tr>
<td>2</td>
<td>0.000104</td>
<td>20.30</td>
<td>29.40</td>
<td>13.69</td>
<td>36.56</td>
</tr>
<tr>
<td>3</td>
<td>0.000132</td>
<td>16.96</td>
<td>27.018</td>
<td>12.37</td>
<td>43.64</td>
</tr>
<tr>
<td>4</td>
<td>0.000155</td>
<td>14.78</td>
<td>24.35</td>
<td>11.51</td>
<td>49.35</td>
</tr>
<tr>
<td>5</td>
<td>0.000175</td>
<td>13.24</td>
<td>22.11</td>
<td>10.90</td>
<td>53.74</td>
</tr>
<tr>
<td>10</td>
<td>0.00024</td>
<td>9.69</td>
<td>16.36</td>
<td>9.84</td>
<td>64.10</td>
</tr>
<tr>
<td>50</td>
<td>0.00033</td>
<td>7.12</td>
<td>12.06</td>
<td>17.24</td>
<td>63.57</td>
</tr>
</tbody>
</table>
Table 4:
Forecast-error variance decomposition of inflation

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Variance</th>
<th>Policy</th>
<th>Technology</th>
<th>Money demand</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. The SP model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000022</td>
<td>59.32</td>
<td>38.59</td>
<td>0.84</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>0.000026</td>
<td>53.56</td>
<td>33.77</td>
<td>10.42</td>
<td>2.23</td>
</tr>
<tr>
<td>3</td>
<td>0.000030</td>
<td>46.61</td>
<td>29.28</td>
<td>21.16</td>
<td>2.95</td>
</tr>
<tr>
<td>4</td>
<td>0.000035</td>
<td>40.80</td>
<td>25.63</td>
<td>30.14</td>
<td>3.42</td>
</tr>
<tr>
<td>5</td>
<td>0.000039</td>
<td>36.21</td>
<td>22.75</td>
<td>37.31</td>
<td>3.73</td>
</tr>
<tr>
<td>10</td>
<td>0.000061</td>
<td>23.17</td>
<td>14.57</td>
<td>58.00</td>
<td>4.25</td>
</tr>
<tr>
<td>50</td>
<td>0.000229</td>
<td>6.22</td>
<td>3.95</td>
<td>87.52</td>
<td>2.30</td>
</tr>
<tr>
<td>B. The SW model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000025</td>
<td>50.18</td>
<td>43.06</td>
<td>0.33</td>
<td>6.42</td>
</tr>
<tr>
<td>2</td>
<td>0.000029</td>
<td>43.92</td>
<td>37.28</td>
<td>13.13</td>
<td>5.66</td>
</tr>
<tr>
<td>3</td>
<td>0.000033</td>
<td>38.56</td>
<td>32.67</td>
<td>23.57</td>
<td>5.20</td>
</tr>
<tr>
<td>4</td>
<td>0.000038</td>
<td>34.17</td>
<td>28.96</td>
<td>31.94</td>
<td>4.92</td>
</tr>
<tr>
<td>5</td>
<td>0.000042</td>
<td>30.60</td>
<td>25.96</td>
<td>38.67</td>
<td>4.74</td>
</tr>
<tr>
<td>10</td>
<td>0.000064</td>
<td>20.12</td>
<td>17.14</td>
<td>58.47</td>
<td>4.25</td>
</tr>
<tr>
<td>50</td>
<td>0.000211</td>
<td>6.20</td>
<td>3.32</td>
<td>86.19</td>
<td>2.29</td>
</tr>
<tr>
<td>C. The SPSW model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000023</td>
<td>38.33</td>
<td>56.22</td>
<td>0.03</td>
<td>5.42</td>
</tr>
<tr>
<td>2</td>
<td>0.000026</td>
<td>36.20</td>
<td>50.19</td>
<td>8.28</td>
<td>5.32</td>
</tr>
<tr>
<td>3</td>
<td>0.000029</td>
<td>32.21</td>
<td>44.63</td>
<td>18.12</td>
<td>5.03</td>
</tr>
<tr>
<td>4</td>
<td>0.000033</td>
<td>28.69</td>
<td>39.80</td>
<td>26.69</td>
<td>4.81</td>
</tr>
<tr>
<td>5</td>
<td>0.000036</td>
<td>25.78</td>
<td>35.77</td>
<td>33.78</td>
<td>4.67</td>
</tr>
<tr>
<td>10</td>
<td>0.000055</td>
<td>17.01</td>
<td>23.60</td>
<td>55.15</td>
<td>4.22</td>
</tr>
<tr>
<td>50</td>
<td>0.000186</td>
<td>5.08</td>
<td>7.06</td>
<td>85.75</td>
<td>2.09</td>
</tr>
</tbody>
</table>
Figure 1:
Inflation, nominal interest, and money growth in Canada

Inflation rate (solid line), 3-month T-bill rate (dash line), and M2 growth rate (dotted line).
Figure 2:
The effects of monetary policy shocks in the three estimated models

The impulse responses are computed for the SP model (dash line), the SW model (dotted line), and the SPSW model (solid line).
Figure 3:
The effects of money demand shocks in the three estimated models

The impulse responses are computed for the SP model (dashed line), the SW model (dotted line), and the SPSW model (solid line).
Figure 4:
The effects of technology shocks in the three estimated models

The impulse responses are computed for the SP model (dashed line), the SW model (dotted line), and the SPSW model (solid line).
Figure 5:
The effects of preference shocks in the three estimated models

The impulse responses are computed for the SP model (dashed line), the SW model (dotted line), and the SPSW model (solid line).
Figure 6:
The effects of policy shocks, in the SPSW model, under estimated and alternative policies

The impulse responses are computed for the SPSW model under alternative monetary policy (dashed line) and estimated monetary policy (solid line).
Figure 7:
The effects of money demand shocks, in the SPSW model, under estimated and alternative policies

The impulse responses are computed for the SPSW model under alternative monetary policy (dashed line) and estimated monetary policy (solid line).
Appendix A: The Symmetric Equilibrium

\[ \frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{1-\gamma} + b_t^{1/\gamma} m_t^{-\frac{\gamma}{2}}} = \lambda_t; \]  

\[ \frac{z_t b_t^{1/\gamma} m_t^{-\frac{\gamma}{2}}}{c_t^{1-\gamma} + b_t^{1/\gamma} m_t^{-\frac{\gamma}{2}}} = \lambda_t - \beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right); \]  

\[ \frac{\eta}{1 - h_t} = \frac{\lambda t w_t}{q_{wt}}; \]  

\[ q_{wt}^{-1} = \frac{\theta_h - 1}{\theta_h} + \frac{\phi_w}{\theta_h h_t} \left( \frac{\pi_t w_t}{w_{t-1}} - \pi \right) \frac{\pi_t \mu_t}{w_{t-1}} \]  

\[ - \beta \phi_w E_t \left[ \left( \frac{\pi_{t+1} w_{t+1}}{w_t} - \pi \right) \left( \frac{w_{t+1}}{w_t} \right)^2 \frac{\pi_{t+1} \lambda_{t+1}}{\lambda_t} \right]; \]  

\[ \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( r_{kt+1} + 1 - \delta + \phi_k \left( \frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}} \right) \right] \]  

\[ = 1 + \phi_k \left( \frac{k_{t+1}}{k_t} - 1 \right); \]  

\[ \frac{1}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{p_t}{p_{t+1}} \right]; \]  

\[ y_t = k_t^\alpha (A_t h_t^{1-\alpha}); \]  

\[ \frac{\alpha y_t}{k_t} = q_{pt} r_t; \]  

\[ \frac{(1 - \alpha) y_t}{h_t} = q_{pt} w_t; \]  

\[ q_{pt}^{-1} = \frac{\theta_y - 1}{\theta_y} - 1 + \frac{\phi_p}{\theta_y} \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \phi_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1} \lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \right]; \]  

\[ y_t = c_t + k_{t+1} - (1 - \delta) k_t + C A C_t + C A P_t; \]  

\[ \mu_t = \frac{m_t \pi_t}{m_{t-1}}; \]  

\[ \log(A_t) = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \]  

\[ \log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \]  

\[ \log \left( \frac{R_t}{R} \right) = \rho_y \log \left( \frac{y_t}{y} \right) + r h o_{\pi} \log \left( \frac{\pi_t}{\pi} \right) + \rho_{\mu} \log \left( \frac{\mu_t}{\mu} \right) + \varepsilon_{Rt}; \]  

\[ \log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}. \]
Appendix B: The Steady-State Equilibrium

\[ \mu = \pi; \]  
\[ R = \frac{\pi}{\beta}; \]  
\[ r_k = \frac{1}{\beta} - 1 + \delta; \]  
\[ q_{pt} = \frac{\theta_y}{\theta_y - 1}; \]  
\[ q_{wt} = \frac{\theta_h}{\theta_h - 1}; \]  
\[ \lambda_c = \left[ 1 + b^{1/\gamma} \left( \frac{b^{1/\gamma} \mu}{\mu - \beta} \right)^{\gamma - 1} \right]^{-1}; \]  
\[ \lambda_m = \lambda_c \left( \frac{b^{1/\gamma} \mu}{\mu - \beta} \right) \gamma; \]  
\[ \frac{k}{y} = \frac{\alpha}{r_k q_p}; \]  
\[ \frac{c}{y} = 1 - \delta \left( \frac{k}{y} \right); \]  
\[ wh\lambda = \frac{(1 - \alpha) (\lambda_c)}{q_p \left( c/y \right)}; \]  
\[ h = \frac{wh\lambda}{\eta q_w + wh\lambda}; \]  
\[ y = h A \left( \frac{k}{y} \right)^{\frac{\alpha}{\alpha}}. \]
Appendix C: Nominal Wage Discrepancies

By log-linearizing the equation (37), in Appendix A, around the steady-state values of the variables, one may get

$$
\beta(\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1}) - \hat{w}_t + \hat{w}_{t-1} - \hat{\pi}_t = \frac{\theta_h - 1}{\phi_w} \frac{h}{\pi^2} \hat{q}_{wt};
$$

which implies that

$$
(1 - \beta F)(\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t) = -\frac{\theta_h - 1}{\phi_w} \frac{h}{\pi^2} \hat{q}_{wt},
$$

where $F$ is a forward operator. Hence,

$$
\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t = -\frac{\theta_h - 1}{\phi_w} \frac{h}{\pi^2} E_{t} \sum_{s=0}^{\infty} \beta^s \hat{q}_{wt+s},
$$

Since the nominal wage is equal to the wage-markup times the labour-marginal cost, $lmc_t$, i.e. $W_t = q_{wt}lmc_t$ in the presence of wage rigidities. Similarly, $W_t^* = q_wlmc_t$, where $W_t^*$ is the nominal wages that would prevail in the absence of wage-adjustment costs and $q_w$ is a constant wage-markup rate. Therefore, the ratio of nominal wages that would prevail in the absence of wage-adjustment costs to nominal wages under wage-adjustment costs is

$$
\frac{W_t^*}{W_t} = \frac{q_w}{q_{wt}}.
$$

Thus, by taking the log of both sides

$$
\log(W_t^*) - \log(W_t) = -\log(q_{wt}/q_w) = -\hat{q}_{wt},
$$

Therefore, the equation (64) becomes

$$
\log(W_t) - \log(W_{t-1}) - \log(\pi) = \frac{\theta_h - 1}{\phi_w} \frac{h}{\pi^2} E_{t} \sum_{s=0}^{\infty} \beta^s \left[ \log(W_{t+s}^*) - \log(W_{t+s}) \right];
$$

44