Financial Accelerator in a Small Open Economy Model with Heterogeneous Entrepreneurs

(Preliminary and Incomplete)

Dmitriy Kovtun

April 22, 2002

Department of Economics
University of Kentucky

e-mail: dkovt2@uky.edu

Abstract
This paper develops a theoretical model of how openness to international financial flows affects income volatility in developing countries. The underlying hypothesis is that the integration into world financial markets can have different implications for income volatility in developing and industrialized countries. This proposition is formed as an open-economy extension of the “financial accelerator” hypothesis based on the agency cost of external financing. The idea is that the agency costs of external financing can be responsible for persistent fluctuations in investment and income. A simple theoretical model is proposed to show how the distribution of the net worth of entrepreneurs can affect the dynamics of the economy after it opens to financial trade.
I. Introduction.

The main goal of this paper is to present a theoretical analysis of the effects of integration into international financial markets on income volatility in developing countries. Studying income volatility is important because volatility is a factor that directly affects welfare under an assumption that economic agents are risk-averse. There is a considerable amount of research on welfare costs of business cycles that finds sizeable welfare gains of eliminating business cycles (see, for example, Storesletten et al 2001). However, existing macroeconomic research on developing countries has focused primarily on long-term economic growth rather than on volatility.

The reason for concentrating on long-term growth is well-founded because even minor changes in the long-term growth rate affect future income levels dramatically. Lucas (1987) claims that the welfare costs of business fluctuations in industrialized countries are relatively minor compared to the costs of experiencing lower long-term economic growth. However, the volatility in the developing world is much higher and therefore plays a more significant role in welfare (Table A1 in the appendix shows that the standard deviation of the annual GDP growth rate is twice as big for the sample of developing countries than for a sample of OECD countries). Studying why income volatility is so much higher in developing than in industrialized countries is a very important task in itself, however, the main focus of this paper is to study implications of integration into the world financial markets for income volatility in developing countries.

This research is motivated in part by what seems to be a disagreement between standard neoclassical macro models and a casual observation of economic performance of developing countries. The macro models suggest that opening a country’s financial markets is generally welfare improving, because it provides more possibilities for intertemporal allocation of resources than under autarky. If financial markets are functioning perfectly, than an additional option to allocate resources across time cannot possibly be welfare-decreasing.

On the other hand, many economists believe that numerous financial crises that occurred in the developing world during the last two decades could have been exacerbated by the actions of international investors. Thus, “coordination problem”, “herd behavior”, “asset bubbles” and “contagion” became keywords in models in which the role played by financial markets is far from smoothing economic volatility. Historically, large increases of financial flows to the emerging economies were followed by drastic declines during times of financial crises. However, it is not entirely clear whether the crises occur due to inefficiency of the international financial markets or due to reasons that are specific to developing countries themselves, or maybe due to a combination of
both. It is difficult to give clear-cut predictions about the effects of liberalization of a country’s financial account unless the processes that drive foreign investments are well understood. Unfortunately, there are so many potentially important conditions that can affect the investment flows that it is virtually impossible to make any reasonably accurate predictions for individual countries. However, it is possible to point to the factors that probably have the greatest importance for the behavior of international capital flows and study them separately. This paper attempts to extend the financial accelerator hypothesis (Bernanke et al, 1997) to the open-economy environment in order to develop a theoretical model that would be suitable for analyzing the effects of financial market imperfections on the outcomes of the financial account liberalization in less developed countries.

The hypothesis proposes that the quality of the financial system has sizeable impacts on the level of real economic activity. Most neoclassical models consider the financial system as a perfect conduit of savings by households into productive investment by entrepreneurs, which does not affect the decision making of the agents in the economy. Along the same lines, the Modigliani-Miller theorem states that the choice of a firm to finance its investment by means of external or internal financing does not matter.

However, nothing could be further from the truth than a statement that the financial system is perfect, especially in the developing world. The asymmetry of information between borrowers and lenders, which simply means that borrowers are more knowledgeable about outcomes of their projects than the lenders, distorts economic incentives. That is, an entrepreneur who invests little of his own funds in a project has an incentive either to take on an excessive risk or to simply misrepresent the project’s outcome. In this case, the necessity to monitor an entrepreneur creates the agency cost of investment that varies negatively with the net worth of entrepreneurs.

It has generally been noted that during economic downturns, small firms, which do not have access to the equity market and therefore must rely on external financing, experience the most severe financial distress. Thus, tightening of the credit constraint can suppress the economic activity among small firms quite significantly. Therefore, exogenous negative economic shocks can reduce access to credit, which might result in a lower level of real economic activity.

Bernanke and Gertler (1989) developed a model that is considered to be one of the starting points of research in this area. In their model, the supply of credit is affected by the level of entrepreneurial net worth, which in turn creates a positive feedback effect on the output in the economy. Later attempts to build this idea into a standard real business cycle framework are presented by
Fuerst(1995) and Carlstrom and Fuerst (1997, 1998). However, there has been little research on the possible effects of the financial accelerator in an open economy environment.

During the last two decades, there has been tremendous growth in the financial flows to emerging economies (IMF, 2001). Assuming that the reasoning behind the financial accelerator hypothesis described above is valid, the following story might be true for a country that relies extensively on foreign borrowing. In the time of economic expansion, the agency cost of investment is relatively low and a large number of entrepreneurs can obtain access to credit, which is expanded by the inflows of capital from abroad. During periods of rapid growth, credit will be given to even those entrepreneurs whose net worth is relatively low and who otherwise would not be able to gain access to the credits. This increased activity among the entrepreneurs with relatively low net worth contributes to the overall economic expansion. However, when the economy is subjected to a negative, exogenous shock, the financial system revises the “borderline” between the entrepreneurs who are allowed the credits and those who are not. In effect, the negative exogenous shock causes the financial system deny credit to those entrepreneurs who would still be given a credit in a perfectly functioning neoclassical economy. Thus, the negative shock could be exacerbated by the destruction of a certain fraction of businesses due to the financial constraint imposed on them.

An interesting question then is what would be the results of liberalizing the financial account in a country that has a large proportion of relatively poor entrepreneurs (this situation is quite typical of developing countries). If additional credits increase the proportion of “active” entrepreneurs with relatively low net worth, the country could experience an increase in “financial fragility” in a Bernanke/Gertler (1990) sense. Therefore, the increase in investment comes with a cost: if external financial conditions worsen, or the country experiences a negative exogenous shock, a relatively larger proportion of entrepreneurs will be subject to the risk of not being able to continue their activity due to facing a much tighter financial constraint. In other words, the “financial accelerator” effects could be much stronger in an economy where small changes in the credit standards affect a large number of entrepreneurs.

If, on the other hand, the majority of entrepreneurs in the economy is relatively rich so that the borderline between those who obtain credits and those who do not is located in the “less populated” part of the distribution, the financial accelerator effects could be less pronounced. This paper attempts to formalize this reasoning by offering a relatively simple theoretical framework that explicitly models the difference in the distribution of entrepreneurs over their net worth. The explicit treatment of heterogeneity among borrowers in the model is a significant innovation of this paper.
The paper is organized as follows. Section II explains the environment of the theoretical model. Section III develops the dynamic equation for the capital stock and the steady-state solutions for two autarkic economies with different distributions of entrepreneurial net worth. Section IV provides the dynamic equation for the capital stock and the steady-state solution for the small-open economy setting for the same economies. Section V explains how opening the financial markets changes the dynamics of certain aggregate variables in the two economies. Finally, Section VI summarizes the findings and suggests avenues for future research in this area.

II. Theoretical Model

The model developed in this paper is essentially an extension of the model by Huybens and Smith (JET, 1998) with an important additional feature of modeling the distribution of the entrepreneurs over their net worth. In other words, instead of using a conventional “representative agent” assumption, the model incorporates a continuum of entrepreneurs distributed over the level of their net worth. It is assumed that the shape of this distribution is one of the critical differences between developed and developing countries. As was stated above, the assumption of this paper is that developing countries tend to have a larger share of entrepreneurs with low net worth than do developed countries.

The economy is populated with a continuum of households and entrepreneurs, organized in two-period overlapping generations. There are two types of goods: a consumption good and capital. The economy contains perfectly competitive firms producing the consumption good and a continuum of capital mutual funds that channel household savings into capital-producing projects conducted by entrepreneurs. The following sections present more detailed discussions of each agent’s role in this economy.

2.1 Households

The total mass of households in the economy is \((1-\alpha)\). Households derive utility from consuming the consumption good in both periods of their lives, which could be represented by the following utility function:

\[
U^{HH} = \ln(c_Y) + \beta \ln(c_O),
\]

(2.1)

where \(\beta\) is a discount factor, \(c_Y\) and \(c_O\) are consumption during the first and the second period of the life of a household, respectively.
Every young household is endowed with one unit of labor, which is supplied to the labor market inelastically. The firms pay the perfectly competitive wage \( w \) that is equal to the marginal product of labor. Buying capital is the only means of savings available to households, so that households must acquire capital when young in order to consume during the second period of their lives.

### 2.2 Entrepreneurs

The total mass of entrepreneurs in the economy is \( \alpha \). The entrepreneurs are heterogeneous: each entrepreneur is endowed with a different amount of labor ranging from \( n_{\text{min}} \) to \( n_{\text{max}} \) and distributed according to a certain probability density function \( f(n) \) with the expected value \( \bar{n} \). Every entrepreneur supplies all of his labor inelastically to the labor market and receives the wage income proportional to the value of supplied labor. The wage income plays the role of entrepreneurial net worth, which is crucial for the dynamics in the general equilibrium. The entrepreneurs are the only agents in this economy possessing the capital-producing technology (“projects”). Each entrepreneur is capable of conducting a single project in the first period of his life, and launching the project requires \( q \) units of the consumption good. As soon as \( q \) consumption goods are invested, any additional investment does not alter the outcome of the project. A project outcome is stochastic: by the end of the period, the project yields \( z q \) units of capital, where \( z \) is a random variable with a support from \( z_{\text{min}} \) to \( z_{\text{max}} \), the expected value \( \bar{z} \), probability density function \( g(z) \) and cumulative probability function \( G(z) \). The crucial assumption of the model is that the project outcomes are private information known only to the entrepreneurs. If any other party wishes to observe the outcome of a specific project, it must pay monitoring cost \( \gamma \). Another important assumption of the model is that entrepreneurs do not have enough of their own funds to start a project, therefore, the necessary funds \( q - n \cdot w \), must be borrowed from the households via capital mutual funds, which play a role of the financial system in the model.

### 2.3. Capital Mutual Funds and Firms

Lending of consumption goods from households to entrepreneurs is conducted through a continuum of perfectly competitive capital mutual funds (CMFs). CMFs provide the means of perfect risk diversification for households. A typical CMF accepts savings from households in exchange for a promise to deliver a certain quantity of the capital good per each unit of the consumption good deposited. In other words, the households and the mutual funds sign contracts that promise to deliver capital at the market-clearing price \( p_t \) at the end of the period \( t \). Even though an outcome of an
individual project is random, there is no aggregate uncertainty about the quantity of capital produced by the end of the period, therefore the contracts between households and CMFs are fulfilled with certainty. Since the information about individual project outcomes is not available to CMFs, lending to entrepreneurs is organized by means of financial contracts that guarantee that entrepreneurs do not have incentives to lie about realizations of their projects.

Production of the consumption good is done via a continuum of perfectly competitive identical firms that utilize Cobb-Douglas technology as follows:

\[ Y_t = \theta_t K_t^{1/2} L_t^{1/2}, \]

where \( K_t \) and \( L_t \) are the amounts of capital and labor employed, respectively, and \( \theta_t \) is the total factor productivity (TFP) term, which is a random variable with the expected value \( \hat{\theta} \).

The inclusion of the random TFP term is motivated by the necessity to be able to “shock” the economy in order to analyze its dynamic behavior. As often assumed in the real business literature, the TFP term follows an autocorrelated process as follows:

\[ \theta_{t+1} = (1 - \phi) \hat{\theta} + \phi \theta_t + \varepsilon_t \]  \hspace{1cm} (2.2')

where \( \theta_t \) and \( \theta_{t+1} \) are the TFP realizations in periods \( t \) and \( t+1 \), respectively, \( \hat{\theta} \) is the long-term expected value of the TFP, \( \phi \) is the autocorrelation coefficient for the TFP, and \( \varepsilon_t \) is an independently and identically distributed random disturbance with the expected value equal to zero.

The firms rent capital from the households and entrepreneurs immediately before the beginning of the production cycle at period \( t \), and the marginal product of capital \( \rho_t \) is paid after the production cycle is completed. In order to simplify the model, an assumption is made that the capital depreciates fully in the process of production. As was stated above, the firms pay the marginal productivity of labor as wage to young households and entrepreneurs.

### 2.4 Financial Contracts

The properties of the optimal financial contract between CMFs and entrepreneurs are well-known (Huybens and Smith (1997) provide a good summary of the literature). The contract specifies the amount of lending and the contractual interest rate\(^1\). If an entrepreneur announces that the project outcome is insufficient to cover the interest payment, the project is declared bankrupt, and the entrepreneur is immediately monitored by the CMF, in which case all the capital from the project is

---

\(^1\) because the contract is intra-period, the term “contractual interest” is used simply for convenience. Because the contract is resolved by the end of the same period, this is just the price that borrowers agree to pay in order to get the loan.
seized by the CMF. If the entrepreneur pays contractually specified interest, no monitoring occurs and the entrepreneur keeps the excess capital. In order to make the model algebraically tractable, it is assumed that the entrepreneurs have absolute bargaining power. That is, entrepreneurs are maximizing the following expected yield of capital from a project:

$$\int_{\tilde{z}}^{\infty} q(z - \tilde{z}) g(z) dz, \quad (2.3)$$

where $\tilde{z}$ is a critical realization of capital productivity below which the project is monitored, and $g(z)$ is the probability density function of the random variable $z$.

In order to obtain the simplest algebraic expressions for the general equilibrium part of the model, it is assumed that the project outcome have the exponential distribution with the mean and variance equal to unity. Even though the exponentially distributed random variable has a distribution skewed to the right, it might not be far from reality in developing countries, where a lot of projects might have quite low outcomes while relatively low number of projects might be extremely profitable. Under the exponential distribution of $z$, the borrower’s yield becomes:

$$z e^{-z} \gamma = (2.3^{'})$$

The expected CMF’s capital yield from a project can be represented as follows:

$$\int_{\tilde{z}}^{\infty} q(z - \gamma) g(z) dz - r(q - n \cdot w_i)(1 - G(\tilde{z})) \quad (2.4)$$

where, $r$ is the interest payment, $n \cdot w_i$ is entrepreneurial own funds (“net worth”), $\gamma$ is the fraction of realized capital that is lost if the project is monitored, and $\tilde{z}$ is the critical value of the capital productivity realization below which the project is monitored.

Since $r(q - nw_i) = \tilde{z} q$, the expression above can be re-written as follows:

$$\int_{\tilde{z}}^{\infty} q(z - \gamma) g(z) dz + \tilde{z} q(1 - G(\tilde{z})) \quad (2.4^{'})$$

which, under exponentially distributed $z$, gives the following expression for the lender’s yield (LY)of capital from a project:

$$LY = e^{-\tilde{z}} (-1 + e^{\tilde{z}}) q(1 - \gamma) \quad (2.4^{''})$$

Because the expected lender’s yield must guarantee the purchase price of capital $p$ to households, the participation constraint for a CMF is as follows:

$$p_i \cdot LY \geq q \cdot nw_i \quad (2.5)$$
where \( p \) is the economy-wide price of capital. Because \( p \) is set by the general equilibrium forces, individual agents regard this price as given. Since entrepreneurs choose the terms of the contract, they choose such a \( z \) that the participation constraint above holds with equality:

\[
\bar{z} = \ln \left( \frac{p, q(1 - \gamma)}{p, q(1 - \gamma) + nw_t - q} \right)
\]

Substituting (2.6) into the entrepreneurial yield of capital (2.1’) gives the entrepreneurial expected yield as a function of \( q, n, w_t, p \) and \( \gamma \):

\[
BY = q \left( 1 - \frac{1}{p, (1 - \gamma)} \right) + \frac{nw_t}{p, (1 - \gamma)}
\]

Since each entrepreneur always has an option to become a lender rather than to deal with starting a project, there could be a case that due to low values of labor endowments, some proportion of entrepreneurs lend their wage income to CMFs (further referred as “discouraged entrepreneurs”). The critical level of labor endowment that separates “discouraged” entrepreneurs from active ones can be determined from the equality between entrepreneurial yield and the capital that would be obtained from investing into a CMF:

\[
q \left( 1 - \frac{1}{p, (1 - \gamma)} \right) + \frac{nw_t}{p, (1 - \gamma)} = \frac{nw_t}{p, (1 - \gamma)}
\]

From (2.8), the critical labor endowment is determined as:

\[
nw_{crit} = \frac{q[1 - p, (1 - \gamma)]}{w_t, \gamma}
\]

### III. General Equilibrium: Autarky

The central hypothesis in this model is that the distribution of the entrepreneurial net worth matters for the general equilibrium outcomes. Anecdotal evidence generally suggests that in many developing countries the distribution of entrepreneurs is skewed to the right -- a larger fraction has rather limited own funds, while a few have substantial possessions of their own net worth. Whether or not it is true remains an object for further empirical investigation, but in this paper it will be assumed that the left tail of the distribution of entrepreneurs according to their net worth in the developing countries is thicker than that of industrialized countries. In order to make necessary comparisons, the uniform distribution of net worth is chosen to represent the “industrialized” economies while the exponential distribution is chosen to represent the “developing” economies. Those distributions were chosen because of the following reasons. First, they successfully model the
main assumption in the paper that the proportion of poor entrepreneurs is larger in the developing economies. Secondly, these distributions make general equilibrium analysis more tractable and in some cases even lead to analytical solutions.

3.1. A Model of Autarkic Economy with Uniform Distribution of Net Worth.

The equilibrium in the economy is defined by the following conditions:

a. Households make consumption choices on the basis of maximizing their expected utility defined by equation (2.1).

b. Entrepreneurs maximize their expected capital yield from their project or from their deposits into CMFs.

c. Markets for capital and consumption goods clear.

It is more convenient to think about the market for capital good in terms of the market for loanable funds. The supply of loanable funds consists of savings by households and wage income of “discouraged” entrepreneurs. The household’s maximization problem is defined as follows:

\[
\text{Max } U_{HH} = \ln(c_Y) + \beta \ln(c_O),
\]

\[
\{c_Y, c_O\}
\]

subject to \( c_Y + \frac{p_Y c_O}{\rho_{y,i}} = w_i \)

From this maximization problem, the optimal amount of savings (deposits) in period \( t \) is found as follows:

\[
dt = \frac{\beta w_i}{1 + \beta}
\]

Therefore, the total supply of loanable funds in period \( t \) in the economy is:

\[
S^{LF} = \int_{n_{\text{min}}}^{n_{\text{crit}}} n \cdot w_i \cdot f(n)dn + \frac{\beta w_i}{1 + \beta},
\]

where \( f(n) \) is the probability density function for entrepreneurial labor endowment. After substituting the expressions for the density function and the critical value of entrepreneurial labor endowment \( n_{\text{crit}} \), the supply of loanable funds is found as follows:

\[
S^{LF} = \frac{w_i (1 - \alpha) \beta}{1 + \beta} + \frac{\alpha(1 - p_i (1 - \gamma))^2}{w_i \gamma^2}
\]

The demand for loanable funds is created by those entrepreneurs whose net worth exceeds \( n_{\text{crit}} w_i \):
After substituting expressions for the density function and the critical value of entrepreneurial labor endowment \( n_{\text{crit}} \), the demand for loanable funds is as follows:

\[
D^{\text{LF}} = \alpha \int_{n_{\text{crit}}}^{n_{\max}} (q - n \cdot w_i) \cdot f_{\text{EXP}}(n) dn \\
(3.5)
\]

The equilibrium price that clears the loanable funds market \(( S^{\text{LF}} = D^{\text{LF}} )\) is found as follows:

\[
p_t = \frac{2\alpha(1 + \beta) + w_i \gamma [w_i \beta + \alpha(w_i - 2(1 + \beta))]}{2\alpha(1 + \beta)(1 - \gamma)} \\
(3.7)
\]

The next step in finding the evolution paths of the aggregate variables in the economy is to find the dynamic equation for capital that shows the amount of capital in period \( t+1 \) as a function of capital and random technological disturbances in period \( t \).

The expected capital contribution by an individual entrepreneur with net worth \( (n - w_i) \) is found by integrating over all possible project realizations over states with monitoring and without monitoring:

\[
k^{\text{EXP}}(n) = \int_{z_{\text{min}}}^{z_{\text{max}}} \int g(z - \gamma) g(z) dz + \int g(z) g(z) dz \\
(3.8)
\]

which, given the exponential distribution of \( z \), is found as

\[
k_{n, t}^{\text{EXP}}(n) = \frac{2p_t(1 - \gamma) - 2\gamma - nw_i \gamma}{p_t(1 - \gamma)} \\
(3.9)
\]

Then, the economy-wide aggregate capital stock is found by integrating the individual expected contributions over the entrepreneurial endowments of labor above \( n_{\text{crit}} \):

\[
\alpha \int_{n_{\text{crit}}}^{n_{\max}} k_{n, t}^{\text{EXP}}(n) f(n) dn, \\
(3.10)
\]

where \( f(n) \) stands for the probability density function for entrepreneurial labor. Given the uniform distribution of the labor endowments with support \([0, 2]\) and expected value of unity, the aggregate capital stock is found as follows:

\[
k_{t+1} = \frac{\alpha(1 + p_i - (2 + p_i - w_i) \gamma)(1 - p_i(1 - \gamma) - w_i \gamma)}{p_i w_i (1 - \gamma) \gamma} \\
(3.11)
\]

Substituting the value of the market-clearing price for \( p \) into (3.11), one gets:

\[
k_{t+1} = \frac{w_i(\alpha + \beta)(w_i \gamma^2 (\alpha + \beta) + 4\alpha(1 + \beta)(1 - \gamma))}{2(1 + \beta)(w_i \gamma^2 (\alpha + \beta) + 2\alpha(1 + \beta)(1 - 1w_i \gamma))} \\
(3.12)
\]
Finally, substituting the marginal product of labor into (3.12), a dynamic equation is obtained as follows:

\[
k_{t+1} = \frac{\sqrt{k_t \theta_t (\alpha + \beta) \left( \frac{1}{4} (\alpha + \beta) \cdot k_t \theta_t^2 + 4 \alpha (1 + \beta) (1 - \gamma) \right)}}{4 (1 + \beta) \left( \frac{1}{4} (\alpha + \beta) \cdot k_t \theta_t^2 + \alpha (1 + \beta) (2 - \gamma \sqrt{k_t \theta_t}) \right)}
\]

(3.13)

Although it is difficult to obtain the explicit algebraic solution for the steady-state value of capital, it is possible to analyze the steady state using a graphical approach. Figure 2.1 plots \( k_{t+1} \) as a function of \( k_t \), given a certain vector of the exogenous parameter values. The figure indicates that there exists only one steady state where the plot of \( k_{t+1}(k_t) \) intersects the 45-degree line.

![Figure 3.1](Illustrating the Steady State in Autarky)

(The following values of the exogenous parameters were chosen for plotting the graph: \( \alpha=0.25, \beta=0.75, \gamma=0.4, \theta=2 \)).

**3.2. A Model of Autarkic Economy with the Exponential Distribution of Entrepreneurial Net Worth.**

The derivation of the model with the exponential distribution follows the same steps outlined in the previous section. The market-clearing price is as follows:

\[
p_t = \frac{4 + w \gamma \ln \left( \frac{w_t (\alpha + 2 \beta - \alpha \cdot \beta)}{4 \alpha (1 + \beta)} \right)}{4 (1 - \gamma)}
\]

(3.14)

The expression for the aggregate stock of capital, which corresponds to (3.10) is the following:
\[ k_{t+1} = \frac{\alpha(4-(4-w_t)\gamma)}{2\rho_t(1-\gamma)} e^{\frac{4(p_t(1-\gamma)-1)}{w_t\gamma}}, \]  

which, after substituting the expression of the market-clearing price of capital becomes:

\[ k_{t+1} = \frac{w_t(\alpha + 2\beta - \alpha \cdot \beta)(4-(w_t-4)\gamma)}{2(1+\beta)\left(4+\gamma \cdot w_t \ln \left(\frac{w_t(\alpha + 2\beta - \alpha \cdot \beta)}{4\alpha(1+\beta)}\right)\right)}, \]  

so that the final dynamic equation of capital stock is as follows:

\[ k_{t+1} = \frac{\sqrt{k_t \cdot \theta} \cdot (\alpha + 2\beta - \alpha \cdot \beta) \left(4 - \gamma \left(\frac{\sqrt{k_t \cdot \theta}}{2} - 4\right)\right)}{2(1+\beta) \left(4 + \frac{1}{2} \gamma \cdot \sqrt{k_t \cdot \theta} \ln \left(\frac{\sqrt{k_t \cdot \theta} \cdot (\alpha + 2\beta - \alpha \cdot \beta)}{8\alpha(1+\beta)}\right)\right)}. \]  

Since the equation is transcendental, it does not have algebraic solutions, so that the same graphical method of analyzing the steady states of the economy must be used. Figure 3.2 shows the plot of \( k_{t+1} \) as a function of \( k_t \) given the same values of the exogenous parameters:

As in the case of the model with the uniform distribution of labor endowments, there is one steady state of capital in the economy. All the aggregate variables are functions of the capital stock in the economy.

### IV. Equilibrium in a Small Open Economy

#### 4.1. A Model of a Small Open Economy with the Uniform Distribution of Entrepreneurial Net Worth.

It is relatively easy to modify the model of an autarkic economy to allow for the presence of foreign lenders willing to buy domestic capital. The main difference is that the price of capital will be
set not by the domestic supply and demand for loanable funds but by the world interest rate and the expected marginal productivity of domestic capital during the next period:

\[ p^F = \rho_{t+1}/ r^w \]  

(4.1)

After substituting (4.1) into the expression for the aggregate capital stock (3.11), one gets the following expression for aggregate capital stock in the open economy with the uniform distribution of entrepreneurial net worth:

\[ k_{t+1} = \frac{\alpha(\gamma \cdot r^w (1 - \gamma(w_t - 2)) + \rho_{t+1}^r (1 - \gamma) r^w (\gamma(w_t - 1)) + \rho^t (1 - \gamma))}{\gamma p_{t+1}^r \cdot r^w (1 - \gamma)} \]  

(4.2)

After substituting \( w_t \) and \( \rho_{t+1} \) for the values of the marginal productivity of labor during period \( t \) and the expected marginal productivity of capital during period \( t+1 \), the dynamic equation for capital is obtained as follows:

\[ k_{t+1} = \frac{4\alpha \sqrt{k_{t+1}} \left[ r^w \left( \frac{\gamma\sqrt{k_{t+1}}}{2} - 1 \right) + \frac{(1 - \gamma)(\theta + (1 - \phi)\theta)}{2\sqrt{k_{t+1}}} \right] \left[ r^w \left( 1 + \gamma \left( -2 + \frac{\sqrt{k_{t+1}}}{2} \right) \right) + \frac{(1 - \gamma)(\theta + (1 - \phi)\theta)}{2\sqrt{k_{t+1}}} \right]}{\sqrt{k_{t+1} r^w (1 - \gamma)\theta(\theta + (1 - \phi)\theta)}}, \]  

(4.3)

where \( r^w \) is the world interest rate, \( \phi \) is the autocorrelation coefficient of the total factor productivity shock and \( \phi(1 - \phi) \) is the expected value of the total factor productivity shock in period \( t+1 \).

Because the equation is nonlinear, it is rather difficult to obtain the analytical solutions for steady-state aggregate capital stock. However, a numerical simulation can be used. Using the following vector of exogenous parameters, \( (\alpha, \beta, \gamma, r^w, \theta, \phi) = (0.25, 0.98, 0.4, 1.075, 2, 0.8) \), equation (4.3) yields one real solution which is the steady state aggregate capital stock in the economy:

\[ k^{SS} = 0.3254. \]

4.2. A Model of a Small Open Economy with the Exponential Distribution of Entrepreneurial Net Worth.

The dynamic equation for capital stock in the economy with the exponential distribution of entrepreneurial labor endowments is found in a similar way. Substituting the price of capital defined by (4.1) into the expression for the aggregate value of capital (3.15), the following dynamic equation is obtained:
Since it is generally impossible to find analytical solutions for equations in which the unknown enters both multiplicatively and also as a part of an exponential expression, the same numerical method as in the previous section is used. Using the following vector of exogenous parameters \((\alpha, \beta, \gamma, \rho, \theta, \phi) = (0.25, 0.98, 0.4, 1.075, 2, 0.8)\), equation (4.4) yields one real solution which is the steady state aggregate capital stock in the economy: \(k = 0.312\).

V. The Dynamics

The original purpose of building four different models was to analyze how opening to financial trade affects the dynamics of capital stock and income in the economies with different distributions of the entrepreneurial net worth. The analysis is done in a usual way: certain values for the exogenous parameters are assumed, the steady states are found as outlined in the section IV, and the impulse-response graphs showing how an economy reacts to a small total factor productivity shock over several periods are plotted. The magnitude and the persistence of the impulse-response plots will indicate the degree of volatility when the economy is subjected to total factor productivity shocks every period.

Finding the impulse-response plots for the autarkic models is relatively straightforward because the value of the capital stock in period \(t + 1\) is an explicit function of capital stock in period \(t\). Setting the initial value of the capital to the steady-state value, it is possible to calculate the amount of aggregate capital stock for each following period iteratively.

However, analyzing the dynamics in the open economies is not a trivial exercise because the function \(k_{t+1}(k_t)\) is given by the dynamic equations implicitly. Instead of performing conventionally used in the literature method of linearization around the steady state, the following indirect numerical method was used. After specifying the values of exogenous parameters, setting the value of capital in the first period to its steady state value and feeding the value of the TFP term for period 1, it is possible to solve numerically for the capital stock next period \(k_{t+1}\). As a rule, there are multiple solutions. However, it is most likely that the economy’s capital stock will accept the value that is the closest to what it was during the last period. Therefore, it is possible to pick the most probable
realization of the capital stock for period $t+1$. Thus, picking most probable realization of capital for $t+1$, it is possible to use it as a starting point for the same procedure for period $t+3$, and so on.

Figures A.1 and A.2 (page 22 in the appendix) show the impulse-response plots for the capital stock per capita as deviations from the steady state values for the uniform and exponential distributions of entrepreneurial labor, respectively. The economies are subjected to a 1% total factor productivity shock in period 3. The black and blue solid lines on the graphs are chosen to draw a small open economy and an autarky, respectively. Figures A.3 and A.4 show similar impulse-response plots for output per capita, and figures A.5 and A.6 show the impulse-response plots for consumption per capita.

The plots of the capital stock of the economies under consideration indicate that the initial deviation from the steady state is the biggest for the economy with the exponential distribution of entrepreneurial net worth. The plots suggest that the capital stock reacts less to an exogenous productivity shock, and returns to its steady state value faster in a small open economy with the uniform distribution of net worth than in autarky. It must be noted, however, that the graph says nothing about the absolute deviations. Since the steady-state value of the capital stock in an open economy is higher, the same absolute deviations are represented by bigger percentage deviations from the steady-state values for an autarkic economy with lower steady-state capital.

The impulse-response plots of output per capita are qualitatively similar to the impulse-response plots for the capital stock. Per capita output in an open economy with the exponential distribution deviates more than per capita output in an open economy with the uniform distribution, but the difference is smaller. The impulse–response graphs for the consumption per capita do not indicate any particular difference in how the economies with different types of distributions of entrepreneurial net worth react to the financial account liberalization.

The next experiment is using the models to simulate the behavior of output and capital stock time series over a large number of periods that would allow calculating such conventional statistics of volatility as the standard deviation and the variability coefficient. In order to conduct the simulation, a sequence of 1000 normally distributed random shocks with zero mean and standard deviation 0.01 were used for calculating a series of autocorrelated TFP terms (the autocorrelation coefficient $\phi=0.8$ was used). The TFP terms were then used for obtaining the “time series” for capital stock and output for 1000 periods. The table below summarizes the mean values and the standard deviations of capital stock and output for the four economies under consideration.
The standard deviations for the capital stock appear to be consistent with prior expectations. Standard deviation of capital stock in the open economy with exponential distribution is bigger by 16% than in an open economy with the uniform distribution, and it is more than twice as large as in the autarky. The results for output are qualitatively similar but weaker as far as the comparison between open economies is concerned – the standard deviation of output in an economy with the exponential distribution appears to be only marginally bigger than that of the economy with the uniform distribution of entrepreneurial labor.

Table 5.1

<table>
<thead>
<tr>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniform Distribution of n</strong></td>
<td></td>
</tr>
<tr>
<td>Autarky</td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.1911</td>
</tr>
<tr>
<td>output</td>
<td>.8743</td>
</tr>
<tr>
<td>consumption</td>
<td>.6028</td>
</tr>
<tr>
<td>SMOPEC</td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.3254</td>
</tr>
<tr>
<td>output</td>
<td>1.1401</td>
</tr>
<tr>
<td>consumption</td>
<td>.6808</td>
</tr>
<tr>
<td><strong>Exponential Distribution of n</strong></td>
<td></td>
</tr>
<tr>
<td>Autarky</td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.1171</td>
</tr>
<tr>
<td>output</td>
<td>.6845</td>
</tr>
<tr>
<td>consumption</td>
<td>.0508</td>
</tr>
<tr>
<td>SMOPEC</td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.3120</td>
</tr>
<tr>
<td>output</td>
<td>1.1172</td>
</tr>
<tr>
<td>consumption</td>
<td>.6137</td>
</tr>
</tbody>
</table>

Since the mean values of the capital stock and output differ between closed and open economies, the output shocks of the same absolute magnitude might be “felt” much stronger in an economy with a smaller expected value of output. Table 5.2 below shows the “index of variability”, which is calculated as a ratio of the standard deviation to the mean value of the time series under consideration. The index of variability appears to be bigger for autarkic economies. The result is due to the fact that in this model, liberalizing the financial account raises both capital stock and output faster than their volatility, which leads to lower variability indexes.

Overall, the numerical simulation shows that the open economy with an exponential distribution of entrepreneurial net worth has a bigger absolute volatility (as measured by the standard deviation) for both capital stock and output than other economies. However, the differences are quite small in the magnitude. The expected results do not appear if the “relative” measure of volatility is
used. It is possible that making the outcome of projects contingent on entrepreneurial efforts can improve the results. Thus, the future research might incorporate the endogenous effort by entrepreneurs to account for the possibility of the moral hazard problem.

**Table 5.2**

**Coefficient of Variability for Capital, Output and Consumption per Capita**

<table>
<thead>
<tr>
<th></th>
<th>Uniform Distribution of n</th>
<th>Exponential Distribution of n</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autarky</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.0337</td>
<td>.0286</td>
</tr>
<tr>
<td>output</td>
<td>.0298</td>
<td>.0276</td>
</tr>
<tr>
<td>consumption</td>
<td>.0298</td>
<td>0.276</td>
</tr>
<tr>
<td><strong>SMOPEC</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital stock</td>
<td>.0190</td>
<td>.0232</td>
</tr>
<tr>
<td>output</td>
<td>.0236</td>
<td>.0256</td>
</tr>
<tr>
<td>consumption</td>
<td>.0282</td>
<td>.0272</td>
</tr>
</tbody>
</table>

**VI. Conclusions**

This paper presented an attempt to model the heterogeneity of the entrepreneurial net worth in the dynamic equilibrium setting in order to study possible financial accelerator effects arising due to imperfections of the financial market. The main contribution of the paper is the explicit theoretical treatment of the difference between the distributions of net worth among entrepreneurs. As far as absolute volatility of output capital stock is concerned, the results shown by the mode agree with prior expectations – liberalizing the financial account makes standard deviation of capital stock and income bigger in the economy with relatively larger proportion of poor entrepreneurs.

However, the model did not present evidence of increasing relative volatility (as measured by the ratio of the standard deviation to the mean value). Also, the model predicted that liberalizing the financial account significantly raises the steady-state levels of capital stocks in economies with both types of distribution of entrepreneurial net worth. It is still too early to use this model for obtaining definitive conclusions about the welfare implications of liberalizing the financial account. However, one policy can always be suggested as welfare-improving: Improving the financial system, which means decreasing the costs of monitoring, leads to the smaller sensitivity of credit policies on entrepreneurial net worth and decreases potential volatility. Overall, this model should be considered
a first step in a bigger research program that incorporates heterogeneity of borrowers into a general equilibrium framework.

References


Table A.1

Means and Standard Deviations of Annual Growth Rates for Samples of OECD and Developing Countries

<table>
<thead>
<tr>
<th></th>
<th>Mean of Annual Growth Rate</th>
<th>Standard Deviation of the Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrialized (OECD) countries</td>
<td>3.61%</td>
<td>2.98%</td>
</tr>
<tr>
<td>Developing Countries</td>
<td>3.78%</td>
<td>6.75%</td>
</tr>
</tbody>
</table>

The sample covered period from 1975 to 1999.
Figure A.1

Impulse–Response Plots of Capital per Unit of Labor,
Uniform Distribution of Entrepreneurial Net Worth

Figure A.2

Impulse–Response Plots of Capital per Unit of Labor,
Exponential Distribution of Entrepreneurial Net Worth
Figure A.3

Impulse–Response Plots of Output per Unit of Labor, Uniform Distribution of Entrepreneurial Net Worth

Figure A.4

Impulse–Response Plots of Output per Unit of Labor, Exponential Distribution of Entrepreneurial Net Worth
Figure A.5

Impulse-Response Plots of Consumption per Unit of Labor, Uniform Distribution of Entrepreneurial Net Worth

Figure A.6

Impulse-Response Plots of Consumption per Unit of Labor, Exponential Distribution of Entrepreneurial Net Worth