Optimal Airport Charges and Capacity Expansion: Effect of Concession Revenues

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Abstract: This paper is concerned with the price decision of an unregulated profit-maximizing airport with profitable concession activities. We show that a privatized airport will levy a lower airport charge if it has profitable concession operations than if the airport does not have concession operations. As a special case, the pricing policy may even attain social optimum. In general, however, the profit-maximizing price deviates from social optimum provided that the demand is growing over time and the per-flight concession profit is independent of the total number of flights. We further show that, without concession operations, a privatized airport tends to add capacity too late from a social point of view. However, when the airport has profitable concession operations, its timing of capacity expansion will be earlier and thus might be closer to social optimum.

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1. Introduction

An airport is a multi-product firm. It has two obvious sides to its business: aeronautical and concession activities. Aeronautical revenues include aircraft landing fees, airport parking and hangar fees, passenger terminal fees and air traffic control charges (if the service is provided by the airport authority), with landing and passenger terminal charges being most important. Concession revenues are those generated from non-aircraft related commercial activities in the terminals and on airport land. Concession operations include running or leasing out shopping concessions of various kinds, car parking and rental, banking and catering, with terminal concessions and car parking and rental being most important.

Many airports generate a much higher proportion of their income from concession activities than from aeronautical operations. Doganis (1992, pp. 56-58) reported that in medium to large U.S. airports, concession operations had contributed between 75-80 per cent of total airport revenue. Indeed, in 1990, more than 90 per cent of total revenue at Los Angeles airport was from commercial operations. Furthermore, concession revenue has grown faster than aeronautical revenue. For example, the Hong Kong International Airport generated the same amount of revenue from its aeronautical and commercial operations, while in late 1980s and 1990s concession revenue accounted for 66-70 percent of total revenue (Zhang and Zhang, 1997). More importantly, concession operations tend to be profitable. Jones, Viehoff and Marks (1993) showed that, in 1990-1991, concession activities at BAA’s three airports around London (Heathrow, Gatwick, and Stansted) were far more profitable to BAA than the activities financed through aeronautical charges. The operating margin from aeronautical charges was –7 per cent for the three airports as a group, while the operating margin from concession revenue was 64 per cent.

Concession operations at a large, busy airport can achieve high returns for the landlord (the airport). The better the location, the greater will be the locational rents. Large, busy airports clearly have location-based market power, with both their aeronautical and concession services. Many airports in the world today are privatized, pursuing profit-maximization, rather than social-welfare-maximization as a public-owned airport should do. Could we expect that rent-seeking behaviour from the airports lead to social welfare maximization, or would there be a conflict between maximizing social welfare and maximizing profits?

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1 In a separate study, Jones, Viehoff and Marks (1993) showed that, in 1990-1991, approximately 60 per cent of revenue at Heathrow, Gatwick, and Stansted airports came from concession activities carried on by BAA and its agents.
2 In effect, at Hong Kong airport concession income had been rising more rapidly than its traffic. The average annual growth rate was 17.3 per cent for concession revenue as against 10.3 per cent for passenger traffic during the period of 1979-1995.
The perception that airports are local monopolies and need to be regulated to prevent them from abusing their market power, has figured into the regulation of airport charges. This includes, predominantly, charges for aeronautical activities such as landing and passenger terminal services. The form that this price regulation has taken varies from country to country, with rate of return, price-cap or cost-based price regulations being the norm. Recent studies on country-specific options and experience of airport regulation include Forsyth (1997), Beesley (1999), Starkie and Yarrow (2000), Starkie (2001), Tretheway (2001), and Gillen and Morrison (2001). Tretheway (2001) pointed out that the rate of return regulation tends to be complex, unresponsive and expensive to administer. Beesley (1999) was critical of the approach used in the UK to regulate its airports. In particular, he argued that the price-cap regulation was inappropriate in the case of Heathrow. Starkie (2001) further concluded that regulation for airports might be unnecessary because the airports are unlikely to abuse their monopoly power due to the existence of a complementarity between demand for aviation (runways) and demand for concession (retailing) activities. Essentially, since concession activities can gain superior locational rents, increases in traffic volumes at an airport will often produce significant increases in their profitability. Therefore, for a profit-maximizing airport company with market power, the effect of the demand complementarity, linked to locational rents, is to attenuate the normal, downward pressure on profits that would arise when increased air traffic volumes have to be bought at the expense of lower prices. This then means that, as long as an airport combines both runway and retailing activities, the incentive will be to set charges lower than if runways were a stand-alone facility (Starkie, 2001).

In this paper, we examine the price decision of an unregulated profit-maximizing airport with profitable concession activities. Our model incorporates two other important features of the airport business. First, the demand for aviation (runway) services grows over time. This is because the demand for aviation services changes with general economic growth (and has over the past four decades generally grown at twice the GDP growth rate). Second, airports exhibit significant indivisibilities. Particularly with large, busy airports, capacity is increased in large indivisible lumps, such as when a new runway is built. We find that when the privatized airport has profitable concession operations, its pricing policy will move closer to social optimum than if the airport had no concession activities or if runway and concession activities were treated separately. In effect, the pricing policy may even attain the social optimum, making pricing regulations unnecessary as suggested by Starkie (2001). However, such coincidence occurs only at a particular point of time. At most times, the profit-maximizing price deviates from social optimum provided that the per-flight concession profit is unrelated to the total number of flights. Essentially, the two airport features imply that both (optimal) prices and delays vary over time. When there is a major investment in an airport, there will be a very large increase in capacity that may not be fully utilized for some time. When capacity is in excess, the airport’s optimal prices may be below first-best prices so as to stimulate total concession revenues. As demand presses against capacity, airport congestion sets in and optimal prices will need to rise to offset the negative impact of delays. In these cases, the optimal prices are higher than first-best prices.
Our second objective is to examine the differences, if any, in the timing of capacity expansion between a privatized airport and a public-owned airport. We find that given lumpy capacity, a privatized airport tends to make capacity expansion later than a comparable public-owned airport would do. In other words, decisions on capacity expansion by a privatized airport are sub-optimal from a social point of view. Further, we show that when the privatized airport has profitable concession operations in the airport, its timing of capacity expansion may be adjusted towards social optimum.

There is a large body of literature on airport pricing and capacity investment. Useful references include Levine (1969), Carlin and Park (1970), Walters (1973), Morrison (1983, 1987), Gillen, Oum and Treheway (1987, 1989), Oum and Zhang (1990), Oum, Zhang and Zhang (1996), and Zhang and Zhang (1997, 2001). Oum and Zhang (1990) first studied capacity expansion of airports when capacity is lumpy and the airport is public owned. Zhang and Zhang (1997) considered the effect of concession operations by airports in a setting where capacity is divisible, while Zhang and Zhang (2001) investigated the effect of financial constraints on a public-owned airport with lumpy capacity. The present paper contributes to the literature in that we look at decisions on airport charges and timing of capacity expansion by airports of different ownership, recognizing that airport capacity is largely indivisible. We first show the difference in pricing policies and capacity decisions between airports pursuing social-welfare maximization and those pursuing profit maximization. Then we proceed to uncover the interesting linkage effect that concession operations bring between the two types of airports.

The paper is organized as follows. Section 2 investigates the pricing policy based on two alternative airport objectives: one is to maximize social welfare and the other is to maximize profits. Section 3 studies the timing of capacity expansion under the alternative airport objectives. Section 4 concludes the paper.

2. The model of airport pricing

An airport supplies aviation services to airlines and passengers. These services include runway facilities for aircraft landing and take-offs, terminal buildings for passenger traffic, technical services to the aircraft such as fueling and maintenance, and navigational services, etc. Although the demand is growing over time, the supply of airport services remains largely fixed for an airport during its economic life span. This is because the capacity of an airport is constrained by its runways and terminal buildings. Once put in place, the runways and terminal buildings can serve for many years and cannot be expanded quickly or in fraction. Of course, given fixed number of runways, the capacity can still expand by improving the airport's navigation and traffic control systems, but these marginal expansions do have limits. In this paper, we assume that the capacity can only be added in lumpy units at discrete times.

In airports with basically fixed capacity in short run, congestion will build up as traffic increases over time. Flights are often delayed due to congestion. The costs of the
delay are part of the total travel costs of the passengers and also constitute part of the operating costs of airlines due to increased flight time either in the air or on the ground. The delay might be anticipated and internalized by the airlines, since the airlines will over time learn where and when they need to extend or even pad the schedule. In any case, this delay time, being anticipated or not, increases with the level of congestion at the airport and imposes costs on passengers and airlines. Consequently, passengers will view their total costs (the full price) of taking a flight as the sum of ticket price and costs of delay.

Let

\[ \rho = P + D(Q,K) \]

be the full price perceived by the passengers which is reflected in the carriers' demand for airport facilities,

\[ P = \text{airport charges for a flight, including both landing fees and passenger services fees,} \]

\[ Q(\rho,t) = \text{the demand (number of flights) per period for landing which depends on the perceived full price, } \rho, \text{ and time, } t, \text{ where } \partial Q/\partial t > 0, \text{ reflecting the trend of growing demand,} \]

\[ K = \text{capacity of the airport,} \]

\[ D(Q,K) = \text{flight delay costs experienced by each aircraft (D for "delay"), which depends on total traffic level and airport capacity,} \]

\[ C(Q) = \text{operating costs of the airport per period, and} \]

\[ r = \text{cost of capital per period including interest and depreciation.} \]

We assume that the capacity of the airport, K, is fixed until the time that the airport takes next capacity expansion.

We first look at a public-owned airport whose objective is to maximize social welfare. Social welfare is defined, as is common in the literature, as the sum of consumer surplus and producer surplus. Let T denote the time of the next capacity expansion, the objective of the airport can be written as:

\[ \max \sum_{t=0}^{T} (P - \rho(t)) \]

where

\[ \rho(t) = P + D(Q(t),K(t)) \]

is the perceived price at time t, and

\[ Q(t) \]

is the demand at time t.

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3 In an interesting study, Brueckner (2001) assumes non-atomistic carriers at an airport and shows that a monopoly airline fully internalizes congestion, so there is no role for congestion pricing. Under Cournot oligopoly, carriers only internalize the congestion they impose on themselves. Allocation can be improved by a toll that captures the un-internalized congestion. Airport congestion pricing has been widely discussed in policy circles as well as in the literature. Other recent academic papers include Daniel (1985, 2001), and Daniel and Pahwa (2000).
\[
\max_P \int_0^\tau \left( P Q - \rho Q + P Q - C(Q) - Kr \right) e^{-rt} dt
\]

(1)

Here, the future revenues and costs are discounted using the cost of capital as the discount rate. The Euler equation for the optimum leads to (Kamien and Schwartz, 1991):

\[
P_w = C' + D'Q, \quad \forall t
\]

(2)

This is the familiar social-marginal-cost pricing. Note that, since capacity is fixed, over time traffic volume will increase and so will marginal costs and the airport charge.

For a privatized (and unregulated) airport with no concession operations (or concession and aeronautical operations being treated independently), the objective is to maximize profits as follows:

\[
\max_P \int_0^\tau \left( PQ - C(Q) - Kr \right) e^{-rt} dt
\]

(3)

The first-order condition for profit maximization is given by:

\[
P_w = C' + D'Q + \frac{P}{\varepsilon}, \quad \forall t
\]

(4)

where \( \varepsilon \) is the (positive) elasticity of demand with respect to full price, \(- (\partial Q / \partial \rho) (\rho / Q)\). Comparing (4) with (2), we find that the profit-maximizing pricing is the social-marginal cost plus a markup, which is inversely related to the demand elasticity. As can be seen from Figure 1, owing to the markup over the social-marginal cost, the pricing policy of a privatized airport is sub-optimal from a social point of view.

However, in the case that the airport also has concession operations, as is the fashion nowadays, the integration of concession revenues may bring the pricing policy of the airport closer to the social optimum than what a privatized airport without concession revenues would do.

To see this, let \( R \) denote the (positive) profits per flight from the airport’s concession operations, with \( R \) being independent of the total number of flights. Then the objective of the airport with combined aeronautical and concession operations becomes:

\[
\max_P \int_0^\tau \left( PQ - C(Q) - Kr + RQ \right) e^{-rt} dt
\]

(5)
Notice the complementarity between aeronautical and concession operations: an increase in air traffic volumes $Q$ will increase the profits from concession activities. The optimal pricing of the airport in this case can be derived as:

$$P_{R} = C + D'Q + \frac{P}{\varepsilon} - R, \quad \forall t$$

(6)

It is obvious that the existence of concession profits gives the airport an incentive to lower airport charges. What is more, the reduction in aeronautical charges per flight is exactly the expected concession profits per flight. The situation is depicted in Figure 1.

Equation (6) reveals that the markup on the social-marginal cost by the privatized airport can be offset by a markdown arising from concession operations. Specifically, equations (2) and (6) show that, in static case, the privatized airport would still charge the social-marginal cost if $R = \rho/\varepsilon$ happens to hold. The static case is of course unrealistic because while traffic has a growing trend, the concession profits per flight, $R$, may not vary. When capacity is in excess, the airport’s optimal prices may be below social optimal prices so as to stimulate total concession revenues. As demand presses against capacity, airport congestion sets in and optimal prices will need to rise to offset the negative impact of delays. In these cases, the optimal prices are higher than first-best prices. Nevertheless, allowing the privatized airport to earn concession profits might be welfare improving if $R$ and $\rho/\varepsilon$ are not too apart on average.

3. The model of capacity expansion

Now we study the timing of capacity expansion. Given that airport capacity cannot be adjusted continuously with infinitesimal increases, we assume that the capacity can only expand by one unit at a time. Suppose that up to time $T$, the airport has $K$ units
of capacity and that at time T, one unit of additional capacity is acquired by the airport. We first derive the optimality condition for the choice of T when the airport has no concession operations (or concession and aeronautical activities are treated as stand-alone operations).

3.1. The case of no concession operations

For a public-owned airport, the objective is to maximize social welfare given the nature of capacity constraints. This can be written as follows:

\[
\max_{T} \int_{0}^{T} [\rho dQ - \rho_Q + PQ - C(Q) - Kr]e^{-rt} \, dt + \int_{T}^{\infty} [\rho dQ - \rho_Q + PQ - C(Q) - (K+1)r]e^{-rt} \, dt \quad (7)
\]

We can, without loss of generality, assume the upper limit of the second integration in equation (7) to be infinity. For airports with possibility of multiple capacity expansions in the future, the second integration can be decomposed into sum of integrations with upper limits setting at times when additional capacity is added. However, as we will see below, the optimality condition for T is completely independent to additional capacity expansions. Therefore, conditions based on (7) will also be true for the airports with multiple capacity expansions in the future.

Let

\[
W = \int_{0}^{Q} [\rho dQ - \rho_Q + PQ - C(Q), \quad \pi = PQ - C(Q)
\]

where W represents social welfare and \(\pi\) represents the airport’s operating profit from aeronautical activities (both W and \(\pi\) exclude concession profits and capital costs). Then (7) can be expressed as:

\[
\max_{T} \int_{0}^{T} [W(t, K) - Kr]e^{-rt} \, dt + \int_{T}^{\infty} [W(t, K+1) - (K+1)r]e^{-rt} \, dt \quad (8)
\]

The optimality condition with respect to T can be derived as:

\[
W(T_w, K+1) - W(T_w, K) = r \quad (9)
\]

or, in short notation,

\[
\Delta W(T_w, K) = r \quad (10)
\]
This condition has clear interpretation: the capacity should be expanded at the moment, \( T_w \), when the gain in social welfare per period due to an additional unit of capacity just reaches the additional cost of capital per period.

Similarly, the objective of the privatized airport can be expressed as:

\[
\max_T \int_T^\infty \{\pi(t, K) - Kr\}e^{-rt} dt + \int_T^\infty \{\pi(t, K + 1) - (K + 1)r\}e^{-rt} dt
\]

The optimality condition with respect to \( T \) then becomes

\[
\Delta \pi(T, K) = r
\]  

Thus, for the privatized airport, the capacity should be expanded at the moment, \( T_p \), when the gain in its operating profit per period due to an additional unit of capacity just reaches the additional cost of capital per period.

Before comparing the optimal timing decisions by the public-owned airport and the privatized airport, we first make the following trivial assumption. We assume that the delay cost function \( D(Q, K) \) is differentiable with respect to \( K \), with

\[
\frac{\partial D}{\partial K} < 0, \quad \frac{\partial^2 D}{\partial Q \partial K} < 0
\]  

(12)

This assumption is quite general, only requiring that adding capacity will reduce congestion and that the effect is more pronounced when there is more congestion. Note that we require the function \( D(Q, K) \) to be differentiable only in formality. The variable \( K \) will still take only discrete values. The following functional form, for example, estimated from steady-state queuing theory (see Lave and DeSalvo, 1968; U.S. Federal Aviation Administration, 1969; Horonjeff, 1975; Morrison, 1987) satisfies all our requirements:

\[
D(Q, K) = \alpha \frac{Q}{K(K - Q)}
\]

Condition (12) will be used in the remainder of the paper.

Now we proceed to analyze the timing of capacity expansion by the airports.

**Lemma 1.** At any time, to add an additional unit of capacity, the perceived benefit is larger to a public-owned airport than to a privatized airport. In other words,

\[
\Delta W(t, K) > \Delta \pi(t, K), \quad \forall t
\]
Proof. See the appendix.

Note that the perceived benefit to a public-owned airport is the gain in social welfare excluding the cost of capital, while the perceived benefit to a privatized airport is the gain in the airport’s operating profits. The airport will undertake capacity expansion if the perceived benefits just outweigh the cost of capital.

**Lemma 2.** For an additional unit of capacity to either the public-owned airport or the privatized airport, the perceived benefits increase over time. In other words,

\[ \frac{\partial}{\partial t} \Delta W(t, K) > 0, \quad \frac{\partial}{\partial t} \Delta \pi(t, K) > 0 \]

Proof. See the appendix.

Now we are able to state our main results.

**Proposition 1.** In the case of no concession operations, the privatized airport is less inclined in capacity expansion than the public-owned airport. Therefore, timing of capacity expansion by the privatized airport is sub-optimal from a social point of view.

Proof. The public-owned airport will expand capacity at the time when \( \Delta W = r \) is satisfied. By Lemma 1, however, the privatized airport will find \( \Delta \pi < r \) at the same time and will not expand the capacity. Further, Lemma 2 implies that the time when \( \Delta \pi = r \) will come later than the time when \( \Delta W = r \) holds, so that the privatized airport will expand capacity at a later time than the public-owned airport. This shows that the privatized airport’s timing of capacity expansion is socially sub-optimal. \( QED. \)
Proposition 1 is depicted in Figure 2. The intuition of the above result may be explained as follows. The socially optimal timing for capacity expansion is the time when the benefit of additional capacity, which includes benefits to both passengers and the airport, just outweighs the cost of the capital. However, since the privatized airport does not care for the consumer surplus but has to bear the cost of capital, at the social optimum the airport will find the benefit to itself insufficient. The privatized airport will add capacity only when congestion is heavy enough such that the benefit to the airport alone would outweigh the cost of capital. Thus, the privatized airport tends to add capacity “too late” by the social point of view.

3.2. The case with concession operations

It is interesting to note that, when the privatized airport has profitable concession operations, its decision on the timing of capacity expansion may move towards the social optimum.

**Proposition 2.** If the privatized airport has profitable concession operations, its timing of capacity expansion will be earlier than the timing in the case of no concession operations.

**Proof.** When the privatized airport has profitable concession operations, the optimality condition for capacity expansion becomes

\[
\Delta \pi(T_{aR}, K) + R \Delta Q(T_{aR}, K) = r
\]

where \( \pi \) still denotes the operating profit from aeronautical operations and \( R \) represents per-flight concession profits, \( \Delta Q(T_{aR}, K) = Q(T_{aR}, K + 1) - Q(T_{aR}, K) \), and \( T_{aR} \) denotes the profit-maximizing timing of capacity expansion with concession operations. Since additional capacity reduces congestion and will stimulate demand, \( \Delta Q \) is positive. Thus, condition (13) brings the timing condition for the privatized airport, (11), closer to that for the public-owned airport, (10).

\[ QED. \]

Intuitively, the effect of concession profits would enhance the benefit to the privatized airport if the new capacity can stimulate demand. As a result, the timing of capacity expansion for the privatized airport with concession operations would tend to be earlier, and so might be closer to social optimum, than the timing in the case of no concession operations.

4. Conclusion

In recent years, concession operations have brought considerable amount of revenues and profits to the airports around the world. At the same time, airport deregulation and privatization have become a growing trend. There are strong public concerns that the airports may abuse their monopoly power in a captive market. Our analysis in this paper suggests that allowing profitable concession operations of the
airports may be welfare-improving than the alternative of depriving the airports of all profits from concession operations. Specifically, we have shown that, while pursing profit-maximization, a privatized airport will levy a lower airport charge if the airport has profitable concession operations than if the airport does not have concession operations. In effect, the pricing policy may even attain the social optimum, making pricing regulations unnecessary as argued by Starkie (2001). However, this coincidence between the profit-maximizing price and the social optimal price may occur only at a particular point of time. At most times, the profit-maximizing price deviates from the socially optimal price provided that the demand is growing over time and the per-flight concession profit is unrelated to the total number of flights going through the airport. This suggests that a careful examination of the nature of concession operations may be warranted when one approaches to questions concerning the need and forms of airport regulation (or deregulation).

We have further shown that, without concession operations, a privatized airport tends to add capacity too late from a social point of view. However, when the airport has profitable concession operations, its timing of capacity expansion will be earlier than otherwise, and thus might be closer to social optimum.
Appendix

**Proof of Lemma 1.** By definition,

\[ W = \int_0^Q \rho dQ - \rho Q + PQ - C(Q), \quad \pi = PQ - C(Q) \]

Differentiating \( W \) with respect to \( K \), we get

\[
\frac{\partial W}{\partial K} = -Q \frac{d\rho}{dK} + Q \frac{dP}{dK} + (P - C') \frac{\partial Q}{\partial \rho} \frac{d\rho}{dK} \quad (A1)
\]

Note that \( \rho = P+D(Q,K), \quad Q = Q(\rho,t) \). It follows that

\[
\frac{\partial \rho}{\partial t} = \frac{(\partial D / \partial Q)(\partial Q / \partial t)}{1 - (\partial D / \partial Q)(\partial Q / \partial \rho)}, \quad (A2)
\]

\[
\frac{\partial \rho}{\partial P} = \frac{1}{1 - (\partial D / \partial Q)(\partial Q / \partial \rho)}, \quad (A3)
\]

\[
\frac{\partial \rho}{\partial K} = \frac{\partial D / \partial K}{1 - (\partial D / \partial Q)(\partial Q / \partial \rho)}, \quad (A4)
\]

and so

\[
\frac{d\rho}{dK} = \frac{dP / dK + \partial D / \partial K}{1 - (\partial D / \partial Q)(\partial Q / \partial \rho)} \quad (A5)
\]

Substituting (A5) into (A1), using the first-order condition (2) of the text and simplifying, we get

\[
\frac{\partial W}{\partial K} = -Q \frac{\partial D}{\partial K} \quad (A6)
\]

Next, differentiating \( \pi \) with respect to \( K \) gives

\[
\frac{\partial \pi}{\partial K} = Q \frac{dP}{dK} + (P - C') \frac{\partial Q}{\partial \rho} \frac{d\rho}{dK} \quad (A7)
\]

Substituting (A5) into (A7), using the first-order condition (4) of the text (noting \( \varepsilon = -(\partial Q / \partial \rho)(\rho/Q) \)) and simplifying, we have
\[
\frac{\partial \pi}{\partial K} = -Q \frac{\partial D}{\partial K} \quad \text{(A8)}
\]

Although \( \partial W / \partial K \) and \( \partial \pi / \partial K \) have the same functional form, they have different values because (A6) is evaluated with the airport charge, \( P_W \), determined by (2) whereas (A8) is evaluated with \( P_\pi \) determined by (4). Since with \( P_\pi > P_W \), the difference between \( \partial W / \partial K \) and \( \partial \pi / \partial K \) can be expressed as follows:

\[
\frac{\partial W}{\partial K} - \frac{\partial \pi}{\partial K} = -Q \frac{\partial D}{\partial K} \bigg|_{P_W} + Q \frac{\partial D}{\partial K} \bigg|_{P_\pi} = \int_{P_W}^{P_\pi} \frac{\partial}{\partial P} (Q \frac{\partial D}{\partial K}) dP \quad \text{(A9)}
\]

Since

\[
\frac{\partial}{\partial P} (Q \frac{\partial D}{\partial K}) = \frac{\partial Q}{\partial P} \frac{\partial D}{\partial K} + Q \frac{\partial^2 D}{\partial Q \partial K} \frac{\partial Q}{\partial P} \frac{\partial P}{\partial P}
\]

Using (A3) and assumption (12), we see that the integrand on the right hand side of (A9) is positive and so

\[
\frac{\partial W}{\partial K} > \frac{\partial \pi}{\partial K}
\]

Finally, by the identity

\[
\Delta W = \int_{K}^{K+1} \frac{\partial W}{\partial K} dK, \quad \Delta \pi = \int_{K}^{K+1} \frac{\partial \pi}{\partial K} dK
\]

where integration is taken only in formality since \( K \) in fact will only take discrete values, we conclude that \( \Delta W > \Delta \pi \). \quad QED.

\textit{Proof of Lemma 2.} Differentiating \( \partial W / \partial K \) with respect to \( t \), we have

\[
\frac{\partial^2 W}{\partial t \partial K} = - \frac{d}{dt} \left( Q \frac{\partial D}{\partial K} \right) = -\left[ \frac{\partial D}{\partial K} \frac{\partial Q}{\partial D} \frac{\partial Q}{\partial Q} + Q \frac{\partial^2 D}{\partial Q \partial K} \frac{\partial Q}{\partial D} \right] \frac{dQ}{dt}
\]

Note that

\[
\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial \rho} \frac{d\rho}{dt}
\]
The demand has a positive trend, so $\frac{\partial Q}{\partial t} > 0$. Then using (A2) implies $dQ/dt > 0$. By assumption (12), we have

$$\frac{\partial^2 W}{\partial t \partial K} > 0$$

By similar argument, we can also get

$$\frac{\partial^2 \pi}{\partial t \partial K} > 0$$

Finally, by the identity

$$\frac{\partial}{\partial t} \Delta W = \int_\kappa^{\kappa+1} \frac{\partial^2 W}{\partial t \partial K} dK, \quad \frac{\partial}{\partial t} \Delta \pi = \int_\kappa^{\kappa+1} \frac{\partial^2 \pi}{\partial t \partial K} dK$$

we conclude that $\frac{\partial \Delta W(t, K)}{\partial t} > 0$ and $\frac{\partial \Delta \pi(t, K)}{\partial t} > 0$. \textit{QED.}
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