Noise Traders and the Exchange Rate Disconnect Puzzle

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Abstract

This paper integrated two lines of research on exchange rate: the “noise trader” approach and the new open macroeconomic approach. Two different types of foreign exchange traders and a foreign trader firm/industry are introduced into the dynamic general equilibrium framework of the new open economy macroeconomic literature. One type is the so called “rational/informed trader” and the other type is called the “noise trader”. Results from the model show that the introduction of the noise trader into new open economy macroeconomic model will help to explain the “exchange rate disconnect puzzle”. It also provides a well-defined context for exchange rate policy evaluation and welfare analysis for future research.

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1 Introduction

A central puzzle in international macroeconomics over the last twenty years is that real exchange rates are volatile and persistent. Furthermore, as Flood and Rose (1995) have elegantly documented, the exchange rate seems to “have a life of its own”, disconnected from other macroeconomic variables. For example, Meese and Rogoff (1983) show that standard macroeconomic exchange rate models, even with the aid of ex post data on the fundamentals, cannot do better in forecasting exchange rate at short to medium horizons than a naive random walk. Mussa (1986), Baxter and Stockman (1989) and Flood and Rose (1995) all find that transitions from fixed to floating exchange rate regimes lead to gigantic increases in nominal and real exchange rate volatility, while there are no such changes in the distributions of macroeconomic fundamentals. The exchange rate volatility also varies over time substantially. Obstfeld and Rogoff (2000) state this kind of “exceedingly weak relationship between the exchange rate and virtually any macroeconomic aggregates” as the “exchange rate disconnect puzzle”.

This regularity (which disappears in high-inflation conditions) casts some doubts on the macroeconomic model of exchange rates, especially those that embody the property of “nominal exchange regime neutrality”.1 These models cannot explain the difference in real exchange rate volatility across regimes. Mussa (1986) shows that sticky price theories can help to explain the observed regularities of real exchange behavior across nominal exchange rate regimes. However, even with sticky prices, most macroeconomic models of exchange rate still show that the exchange rate volatility will be transferred to other economic variables and thus cannot explain the “disconnect puzzle”.

This brings up another question: if exchange rate volatility can be reduced without incurring the cost of other macroeconomic volatility, then floating exchange rates may be too volatile and costly from a welfare point of view. However, it is impossible to make any policy recommendation in the absence of a fully articulated model that can explain exchange rate volatility and its relationship with macroeconomic fundamentals.

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1For instance, models that assume relative purchasing power parity (PPP) have this property.
Recently, developments in new open economy macroeconomic theory offer an explanation for exchange rate behavior and also provide a rigorous framework for policy analysis. The unifying characteristic of this literature is the introduction of nominal rigidities and market imperfections into a dynamic general equilibrium model with well-specified microfoundations. The interaction of monetary shocks, sticky prices and pricing to market is the most popular story provided by this line of research to explain this puzzle.

This new wave of research was initiated by Obstfeld and Rogoff (1995)’s Redux model. They introduced two key ingredients - imperfect competition and sticky prices - into new open economy macroeconomic models. Monopoly power permits the explicit analysis of the pricing decisions of firms. It also implies that equilibrium production will fall below the social optimum, a distortion that can potentially be corrected by monetary policy intervention. As mentioned above, sticky price models can explain the observed empirical regularities of the real exchange rate. It also alters the transmission mechanism for shocks and provides a more potent role for monetary policy. Therefore, by addressing issues of concern to policy makers, and presenting explicit utility and profit maximization problem, this model allows for the possibility of welfare analysis and policy evaluation.

But the original Redux model cannot solve the “disconnect puzzle” either, because it assumes that purchasing power parity (PPP) holds. With PPP, the “expenditure-switching” effect of changes in exchange rate will lead to substitution between domestically-produced goods and foreign goods. It implies that the exchange rate volatility will affect macroeconomic fundamentals. However, empirical evidence \(^2\) indicates that nominal exchange rate changes are not fully passed through to goods prices. In fact, it appears that consumer prices are very unresponsive to nominal exchange rate changes. In other words, the “expenditure-switching” effect might be very small. If this is true, then it might take large changes in exchange rates to achieve equilibrium after some shocks to fundamentals. That is, low pass-through of exchange rates might imply high exchange rate volatility in equilibrium. That intuition was explored by Betts and Devereux (1996, 2000) by introducing local currency pricing into the baseline model. In short, they assume that firms can

charge different prices for the same good in home and foreign markets and that the prices are sticky in each country in terms of the local currency. This allows the real exchange rate to fluctuate, and delinks home and foreign price levels.

With all the three key ingredients: sticky prices, imperfect competition and local currency pricing, many authors have developed fully calibrated and very complicated new open economy macroeconomic models of exchange rates. These models can generate exchange rate movements but typically predict a strong relationship between exchange rates and ratios of business cycles. A monetary shock simultaneously raises domestic real GDP (by more than it raises foreign GDP) and creates a (temporary) depreciation of home currency. Consequently, these models almost generically predict a strong positive correlation between depreciation and (relative) business cycle booms. The problem is that we do not see such co-movements of exchange rates and fundamentals in the data. Another failure of these models is their incapability in explaining why the exchange rate volatility is so time-varying. Chari, Kehoe and McGrattan (2000), for example, find with high risk aversion, separable preferences in leisure and long price-stickiness, their model could produce volatile and persistent real exchange rates. However, it also predicts a strongly counterfactual relation between exchange rates and international ratios of GDP.

One explanation for this discrepancy might lie in the fact that the new open economy macroeconomic framework ignores the old views that nominal exchange rate is also an asset price which is decided on an asset market. What do changes in exchange rates mean to practitioners in the financial markets? Do they believe that the swings of exchange rates are justified by macroeconomic fundamentals? From the large body of evidence documenting deviations from rational expectations in foreign exchange markets (Frankel and Froot (1990), Taylor and Allen (1992) and Cheung and Wong (1998)), the answer to this question is probably no. The first paragraph in Richard Lyon (1998)’s book is a good example:

As I sat there, my friend traded furiously, all day long, racking up over 1 billion in trades each day(USD)····· Despite my belief that exchange rates depend on macroeconomics, only rarely was news of this type his primary concern.
Most of the time my friend was reading tea leaves that were, at least to me, not so clear.

As an asset price, the exchange rate will be inevitably affected by any imperfections of financial markets. These imperfections may include incomplete financial markets, deviations from rational expectations about future economic conditions and the presence of herd behavior and noise traders. Working together with sticky prices, these are all important reasons why the real exchange rate persistently deviates from the level predicted by the fundamentals-based models. Although the financial economist cares about the high frequency data while international macroeconomists focus more on low frequency data, it is still surprising how little the financial practitioners’ concern about exchange rate in the real world has been considered in the macroeconomic theory of exchange rates. The presence of the exchange rate disconnect puzzle also suggests that exchange rate models based only on macroeconomic fundamentals are unlikely to be very successful. As Flood and Rose (1995) pointed out at the end of their paper, more microeconomic detail should be taken into consideration. I believe a fully articulated model that can explain exchange rate behavior and international business cycles should incorporate microeconomic imperfections of financial markets.

Although there are many “microstructure” models of exchange rates and behavior finance models, what we need is something that can be embedded into the general equilibrium framework. In this sense, the influential noise trading literature on asset price volatility initiated by De Long et al. (1990) is a good candidate. De Long et al. (1990) present a model of asset markets in which irrational noise traders with erroneous stochastic beliefs affect the prices and may earn higher expected returns. The unpredictability of noise traders’ beliefs helps them to “creates their own space” - creates a risk that deters the rational traders from betting against them and makes it possible that they can survive on the market. As a result, asset prices can diverge significantly from fundamental values. Jeanne and Rose (2002) develop a model of noise traders to explain exchange rate behavior. They show that based on endogenous noise trading, their model can generate multiple equilibria: for the same level of volatility of fundamentals, it is possible to have different
levels of noise trading and thus different levels of exchange rate volatility. Their findings have important policy implications: exchange rate policy can also work by affecting the composition of foreign exchange market.

However, the macroeconomic part of Jeanne and Rose’s (2002) model is a very simple monetary model of exchange rates with PPP. Neither nominal rigidities nor pricing to market is considered there. Moreover, intertemporal optimizing agents and profit maximizing firms are not present in this model, which shuts off most channels through which the exchange rates affect macroeconomic fundamentals. Another problem is that it is a partial equilibrium model without explicit welfare specifications for households, so rigorous policy analysis and welfare evaluation are impossible in this model. The new open economy macroeconomic framework has natural advantages in these aspects.

Therefore, my paper intends to propose a new approach to study exchange rates, which combines the macroeconomic models of exchange rates and the microstructure approach of foreign exchange markets. This approach is implemented within a specific model: a model in which noise traders are introduced into the new open economy macroeconomic framework. The combination of these two approaches will help us to understand the behavior of exchange rates and their relationship with the macroeconomic fundamentals better. It also gives more rigorous microeconomic foundations to the “noise trader” approach and enriches the new open economy macroeconomic framework with a more realistic setting of the microstructure of foreign exchange market. Moreover, it will provide a well-defined framework for welfare analysis and policy evaluations, especially for policies that are designed to control non-fundamental volatilities.

In this paper, two different types of foreign exchange traders and a foreign trader firm/industry are introduced into the general equilibrium framework of new open economy macroeconomic literature. One type is the so called “rational/informed trader” as they have rational expectations, while the other type could not forecast the future exchange rate correctly and is called the “noise trader”. The trader firm/industry acts as a “middleman” through which the home household and the foreign exchange traders are connected.
I firstly solve the model when the entry cost is zero, which means that all the existing noise traders will enter the foreign bond market. The results from this specification show that the volatility of exchange rate is increasing when the number of noise traders is increasing. Meanwhile, the volatilities of other macroeconomic fundamentals is invariant to the number of noise traders. This implies that it is possible to solve the “exchange rate disconnect puzzle” by the suggested approach. Then by introducing heterogenous entry cost of traders, the size of the noise component is endogenized: only noise traders whose entry costs are low enough will choose to enter the foreign exchange market. Our findings are: 1. The exchange rate disconnection still holds in this specification. 2. Given the number of informed trader: \( N_I \), increasing the entry cost \( \bar{c} \) will reduce the noise component on the foreign exchange market (the number of incumbent noise trader: \( n \)), and this helps to reduce the exchange rate volatility. The second finding shows that the exchange rate policies that aim at the non-fundamental risk can be justified theoretically.

This paper is organized as follows. In section 2, I construct a model which embeds noise traders into a new open economy macroeconomic framework. In the first specification, the number of incumbent noise traders is exogenously given. For the endogenous entry specification, the noise component of the foreign exchange market is endogenously determined. In Section 3 we will discuss features of the solution to the model. Section 4 gives the results of the model and shows that exchange rate disconnect puzzle could be explained in the model. The paper concludes with a brief summary and suggestions for subsequent research.

2 The model

The world economy consists of two countries, denominated home and foreign, each specialized in the production of a composite traded good. Variables in the foreign country will be denoted by an asterisk. In addition, a subscript \( h \) will denote a variable originating from the home country; a subscript \( f \) will denote a variable used in the foreign country.
As in Betts and Devereux (1996) and Chari, Kehoe and McGrattan (2000), I assume local currency pricing of the intermediate goods producer. Besides the infinitely lived household, a second type of representative agent is introduced into the model: the foreign exchange trader in the home country, who only lives two periods. In the first period, he chooses how many foreign bonds to purchase; while in the second period, he gets the excess return of his investment, consumes and dies. Therefore, these foreign exchange traders live in an ‘overlapping generation’ demographic structure. It is also assumed that there exists a foreign exchange trader industry/firm in the home country. This industry acts as a “middle-man” through which the home household and the foreign exchange traders are connected. Specifically, the domestic bond market functions as financial intermediary between home household and trader industry/firm; meanwhile, the industry/firm helps the foreign exchange traders to invest in the foreign bond market. In the foreign country, there is only one type of representative agent, the foreign household.

Moreover, I assume that there exist two types of foreign currency traders in the home market. One is the so called ‘informed trader’, the other is called the ‘noise trader’. The relationship between the exchange rate volatility and the noise component of the economy will be explored in this model.

2.1 Households

In each period every household in the home country is endowed with one unit of time, which he divides between leisure and work. His consumption is a continuum of differentiated goods. Home households can hold two types of nominal assets: non-interest bearing home money \( M \), and a one-period home currency denominated bond \( B \) which pays a riskless nominal interest rate \( r \).

Preferences The lifetime expected utility of the home representative household is:

\[
\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^H)^{1-\rho}}{1-\rho} + \frac{1}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} L_t^{1+\psi} \right] \right\} 
\]  

(2.1)
Subject to
\[ P_t C_t^H + B_{t+1} + M_t = W_t L_t + \Pi_t + \Pi_t^T + M_{t-1} + T_t + B_t(1 + r_t) \]  
(2.2)

where \( E_0 \) denotes the mathematical expectation conditional on information available at time \( t = 0 \) and \( \beta \in (0, 1) \) is the discount rate. The household’s instantaneous utility increases with \( C_t^H \), the composite consumption of the home household at time \( t \), and real money balances, \( M_t \), where \( M_t \) is the nominal balances held at the beginning of period \( t \) and \( P_t \) is a consumption based price index for period \( t \). It depends negatively on \( L_t \), the labor effort at time \( t \).

Households’ income is derived from labor income \( W_t L_t \), profits from domestic intermediate goods producers (which we assumed to be owned by domestic households) \( \Pi_t \) and domestic foreign currency trader industry \( \Pi_t^T \), and from the interest received on bond \( B_t(1 + r_t) \) and lump-sum government transfers \( T_t \).

Solving the household’s problem, the optimal money demand schedule can be written as:
\[
\left( \frac{M_t}{P_t} \right)^\epsilon = \frac{(C_t^H)^\rho}{1 - \frac{1}{1 + r_t + 1}}
\]

(2.3)
The optimal labor supply decision is characterized by
\[
\eta L_t^\psi = \frac{W_t}{P_t(C_t^H)^\rho}
\]

(2.4)
Finally, the household’s intertemporal consumption stream is chosen such that
\[
\beta E_t \left( \frac{(C_t^H)^\rho}{(C_{t+1}^H)^\rho} \right) P_t = \frac{1}{1 + r_t + 1}
\]

(2.5)

The optimal first-order conditions of the foreign households are entirely analogous, except that foreign household’s consumption is denoted by \( C_t^* \), as there is only one type of representative agent in the foreign country.

\(^{3}\)For now, \( C_t^H + C_t^T = C_t \), where \( C_t^T \) is the composite consumption of the foreign traders in home country at time \( t \), and \( C_t \) is a index of consumption to be defined below.
2.2 Consumption and Price Indexes

There are a continuum of domestic goods indexed by \( i \in [0, 1] \) and a continuum of foreign goods indexed by \( j \in [0, 1] \), which are imperfect substitutes in consumption. The composite goods \( C_{h,t} \) and \( C_{f,t} \) could be then defined as:

\[
C_{h,t} = \left( \int_0^1 C_{h,t}(i)^{\frac{\theta}{\theta - 1}} di \right)^{\frac{\theta}{\theta - 1}} \tag{2.6}
\]

\[
C_{f,t} = \left( \int_0^1 C_{f,t}(j)^{\frac{\theta}{\theta - 1}} dj \right)^{\frac{\theta}{\theta - 1}} \tag{2.7}
\]

where \( C_{h,t}(i) \) and \( C_{f,t}(j) \) denote date \( t \) domestic consumption of the home good \( i \) and foreign good \( j \) respectively; and \( \theta > 1 \) denotes the elasticity of substitution between different kinds of home (foreign) goods.

The consumption index \( C_t \) is defined as:

\[
C_t = \left[ \omega \gamma \frac{1}{\gamma} C_{h,t}^{\gamma - 1} + (1 - \omega) \frac{1}{\gamma} C_{f,t}^{\gamma - 1} \right]^{\frac{1}{\gamma - 1}} \tag{2.8}
\]

where \( \gamma > 0 \) represents the elasticity of substitution between the two composite goods \( C_{h,t} \) and \( C_{f,t} \), and the weight \( \omega \in (0, 1) \) determines the home representative agents’ bias for the domestic composite good.

Let \( P_{h,t}(i) \) and \( P_{f,t}(j) \) be the home-currency prices of the home good \( i \) and foreign good \( j \) respectively. Given these prices, the consumption-based price index \( P_t \) is defined by:

\[
P_t = \left[ \omega P_{h,t}^{1-\gamma} + (1 - \omega) P_{f,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{2.9}
\]

where the price indexes \( P_{h,t} \) and \( P_{f,t} \) for each composite good are given by:

\[
P_{h,t} = \left( \int_0^1 P_{h,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{2.11}
\]

\[
P_{f,t} = \left( \int_0^1 P_{f,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}
\]

\[
4 The price indexes \( P, P_h \) and \( P_f \) are defined as the minimum expenditure necessary to buy one unit of composite goods \( C, C_h \) and \( C_f \), respectively, taking the prices for individual good \( P_h(i) \) and \( P_f(j) \),
and

\[ P_{f,t} = \left( \int_0^1 P_{f,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \]  

(2.12)

Taking prices for all individual goods as given, the optimal demand function of the consumer for each individual good can be derived, which implies that in each period the consumer allocates a given level of total consumption among the differentiated goods.

\[ C_{h,t}(i) = \omega \left( \frac{P_{h,t}}{P_{h,t}(i)} \right)^{\theta} \left( \frac{P_t}{P_{h,t}} \right)^\gamma C_t \]  

(2.13)

and

\[ C_{f,t}(j) = (1 - \omega) \left( \frac{P_{f,t}}{P_{f,t}(j)} \right)^{\theta} \left( \frac{P_t}{P_{f,t}} \right)^\gamma C_t. \]  

(2.14)

### 2.3 Producers and Market Structure

The production function for each home intermediate good \( i \) is given by

\[ y_t(i) = z_t L_t(i) \]  

(2.15)

where \( L_t(i) \) represents labor input on the production of good \( i \) and \( z_t \) is an aggregate (country specific) productivity shock. Currently I just assume \( z_t = 1, \forall t \). Because all goods are imperfect substitutes in consumption, each individual firm has some market power determined by the parameter \( \theta \).

It is assumed that, due to high costs of arbitrage to consumers, each individual monopolist can price discriminate across countries. Furthermore, I assume local currency pricing: firms set prices (separately) in the currencies of each set of buyers.\(^5\) Finally, prices are

\[ \min_{C_h, C_f} P_h C_h + P_f C_f \]  

subject to \( C = \left[ \omega^{\frac{1}{\gamma}} C_h^{\frac{\gamma-1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} C_f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} = 1, \) for given \( P_h \) and \( P_f \).

\(^5\)See Devereux (1997) for a discussion of evidence on pricing-to-market.
assumed to be set one period in advance and cannot be revised until the following period. That is, the home monopolist sets \( P_{h,t}(i) \) and \( P^*_{h,t}(i) \) optimally at the end of period \( t-1 \), and these prices cannot be changed during time \( t \).

The price setting problem of monopolist \( i \) is to maximize expected profit conditional on \( t-1 \) information, by choosing \( P_{h,t}(i) \) and \( P^*_{h,t}(i) \). That is, firm \( i \) solves

\[
\max_{P_{h,t}(i), P^*_{h,t}(i)} E_{t-1} [D_t \Pi_t(i)]
\]

subject to

\[
z_t L_t(i) = C_{h,t}(i) + C^*_{h,t}(i)
\]

and the downward-sloping demand functions for \( C_{h,t}(i) \) and \( C^*_{h,t}(i) \), as in Equation 2.13 and the foreign analogue.

In Equation 2.16, the term \( D_t \) denotes the pricing kernel used to value date \( t \) profits. Because all firms are assumed to be owned by the domestic households, it follows that in equilibrium \( D_t \) is the intertemporal marginal rate of substitution in consumption between time \( t-1 \) and \( t \):

\[
D_t = \beta \frac{U_c(C^H_t, L_t, \frac{M_t}{P_t})}{U_c(C^H_t, L_{t-1}, \frac{M_{t-1}}{P_{t-1}})} \frac{P_{t-1}}{P_t}
\]

The time \( t \) profit of monopolist \( i \) (in home currency units), \( \Pi_t(i) \), is given by:

\[
\Pi_t(i) = P_{h,t}(i)C_{h,t}(i) + S_tP^*_{h,t}(i)C^*_{h,t}(i) - W_tL_t.
\]

where \( P_{h,t}(i) \) and \( P^*_{h,t}(i) \) are denoted in the home and foreign currency, respectively. \( S_t \) is the nominal exchange rate in period \( t \).

Solving the intermediate goods producer’s problem, the first order conditions w.r.t. \( P_{h,t}(i) \) and \( P^*_{h,t}(i) \) are:

\[
E_{t-1} \left\{ D_t \left[ (1 - \theta)P_{h,t}(i)^{-\theta} \Lambda_{h,t} + \theta W_t P_{h,t}(i)^{-\theta-1} \Lambda_{h,t} \right] \right\} = 0
\]

\[
E_{t-1} \left\{ D_t \left[ (1 - \theta)S_tP^*_{h,t}(i)^{-\theta} \Lambda^*_{h,t} + \theta W_t P^*_{h,t}(i)^{-\theta-1} \Lambda^*_{h,t} \right] \right\} = 0
\]
where

\[ \Lambda_{h,t} = (P_{h,t})^\theta \left( \frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t \quad \Lambda_{h,t}^* = (P_{h,t})^\theta \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \] (2.22)

From Equation 2.20 and Equation 2.21, the optimal price setting schedule of firm \( i \) could be derived,

\[ P_{h,t}(i) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t W_t \Lambda_{h,t} \right]}{E_{t-1} \left[ D_t \Lambda_{h,t} \right]} \] (2.23)

\[ P_{h,t}^*(i) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t W_t \Lambda_{h,t}^* \right]}{E_{t-1} \left[ D_t \Lambda_{h,t}^* \right]} \] (2.24)

Analogously, I could derive the pricing setting equations for the intermediate goods producer \( j \) in the foreign country:

\[ P_{f,t}(j) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t^* W_t^* \Lambda_{f,t}^* \right]}{E_{t-1} \left[ D_t^* \Lambda_{f,t}^* \right]} \] (2.25)

\[ P_{f,t}^*(j) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t^* W_t^* \Lambda_{f,t}^* \right]}{E_{t-1} \left[ D_t^* \Lambda_{f,t}^* \right]} \] (2.26)

where

\[ D_t^* = \beta \frac{U^*_c(C^*_t, L^*_t, \frac{M^*_t}{P^*_t})}{U^*_c(C^*_t, L^*_t, \frac{M^*_{t-1}}{P^*_{t-1}})} \frac{P_{t-1}^*}{P_t^*} \] (2.27)

\[ \Lambda_{f,t} = (P_{f,t})^\theta \left( \frac{P_{f,t}}{P_t} \right)^{-\gamma} C_t \quad \Lambda_{f,t}^* = (P_{f,t}^*)^\theta \left( \frac{P_{f,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \] (2.28)

2.4 Government

The home government issues the local currency, has no expenditures, and runs a balanced budget every period. The nominal transfers are then given by:

\[ T_t = M_t - M_{t-1} \] (2.29)
The stochastic process describing the evolution of the domestic money supply is:

\begin{align}
    M_t^s &= \mu_t M_{t-1}^s \\
    \log(\mu_t) &= \epsilon_{\mu,t}
\end{align}  

(2.30) \hspace{1cm} (2.31)

where \( \epsilon_{\mu,t} \sim N(0, \sigma^2_{\epsilon_{\mu}}) \) is a normally distributed random variable. The stochastic process of money supply in the foreign country is entirely analogous.

2.5 Foreign Exchange Market

2.5.1 Foreign Exchange Traders

Following closely the work of Jeanne and Rose (2002), the foreign exchange traders are modelled as overlapping generations of investors who will decide how many one-period foreign nominal bonds to buy in the first period of their life. Traders have the same tastes, but differ in their abilities to trade in the foreign bond markets. Some of them are able to form accurate expectations on risk and returns costlessly, while others have noisy expectation and must pay an entry cost to invest in the foreign bond market. The former will be referred as the “informed trader” and the latter as the “noisy traders”.

In the foreign exchange market of home country, at each period a generation of foreign exchange traders is born. The continuum of the traders will be indexed by \( i \in [0,1] \). Assume that in each generation of traders, \( N_I \) of them are informed/rational traders, \( 1 - N_I \) are noise traders. At the beginning of the first period, each individual trader will decide whether or not he should enter the foreign exchange market, if enter, he will decide how many foreign currency denominated bonds to buy or sell. In the second period, the return of his investment in terms of home currency is realized and he gets a share \( \alpha \) of the excess return of his investment (over the return of riskless home bond), consumes and dies. Figure 1 could help to explain the timing of the model better.

\footnote{Currently, I will just assume this share \( \alpha = 1 \).}
Action 1: Time $t$ foreign exchange trader $i$ is born; Time $t$ shocks and nominal interest rates are revealed; The time $t$ born trader $i$ decides if he should enter the foreign bond market.

Action 2: He decides how many foreign currency bonds: $B_{h,t+1}(i)$ to purchase based on his expectation about future exchange rate $S_{t+1}$;

Action 3: Time $t+1$ exchange rate $S_{t+1}$ is revealed, so the return of his investment in terms of home currency is realized;

Action 4: He gets the excess return over the riskless home bond return, consumes and dies.

Let $\varphi^i_t$ denote the dummy variable characterizing the market-entry condition of period $t$ born foreign exchange trader $i$. If $\varphi^i_t = 0$, trader $i$ will not enter the foreign bond market and if $\varphi^i_t = 1$, he will enter. At the beginning of period $t$, trader $i$ will enter the market as long as:

$$E^i_t(U^i_t \mid \varphi^i_t = 1) \geq E^i_t(U^i_t \mid \varphi^i_t = 0)$$ (2.32)

where $U^i_t$ is the expected utility of trader $i$ at time $t$ and $E^i_t$ represents the conditional expectations of the foreign exchange trader $i$ based on the information available at the beginning of time $t$. Note that the expectation operator bears the traders’ index to allow for heterogeneity of the traders.

To a foreign exchange trader who has entered the foreign bond market, his problem
could be written as:

$$\max_{B_{h,t+1}^i} E_t^i \left\{ - \exp\{-aC_{t+1}^T\} \right\}$$  \hspace{1cm} (2.33)

Subject to

$$P_{t+1}C_{t+1}^T = \alpha \left[ B_{h,t+1}^i(i)(1 + r_{t+1}^*)S_{t+1} - B_{h,t+1}^*(i)S_t(1 + r_{t+1}) \right] - P_{t+1}c_i$$  \hspace{1cm} (2.34)

where $B_{h,t+1}^i(i)$ denotes the amount of one-period foreign currency bonds held by the trader $i$ from time $t$ to time $t+1$. $r_{t+1}$ and $r_{t+1}^*$ are the nominal interest rate of the one-period bond in the home and foreign country respectively. Parameter $a$ is the absolute risk aversion coefficient.

The cost $c_i$ reflects the costs associated with entering the foreign bond market. These costs may include the tax issues, information costs for investment in foreign bond market and other problems when investing abroad.\footnote{Given the size of the “home market effect” illustrated in Lewis(1995) and other literatures, these costs might be big. There is a lot of ways to model these costs. In this model it is assumed to be resource-consuming.} It is assumed that the traders differ with respect to their entry costs. There are several ways to formalize this heterogeneity, here I will just follow the specification in Jeanne and Rose (2002). Informed/rational traders are assumed to have a larger stock of knowledge about the economy and so do not need to invest in the acquisition of information. That is to say, their entry costs are assumed to be zero. For noise traders, they do not have a natural ability to acquire and process the information about the economy and therefore have to pay a entry cost greater than zero.

Although the preferences of the noise traders are the same, it is assumed that the noise traders differ in their entry costs. Without loss of generosity, the noise traders could be sorted by increasing entry costs:

$$c_i = 0 \text{ for } i \leq N_I$$  \hspace{1cm} (2.35)

$$\mathcal{L}(c_i) \sim U(0,\bar{c}), \text{ } c_i \text{ increase with } i \text{ for } i > N_I.$$  \hspace{1cm} (2.36)

where $U(0,\bar{c})$ stands for a uniform distribution.
2.5.2 Foreign Exchange Traders Industry

In addition to foreign exchange traders, it is also assumed that there exists a foreign exchange trader industry/firm in the foreign exchange market of home country. The introduction of the trader industry plays an important role in extending Jeanne and Rose (2002) to a dynamic general equilibrium model. The trader industry is an organization owned by home household, acting as a “middle-man” through which the home household and the foreign exchange traders are connected. In each period $t$, it has the following four functions:

1. Borrow from the home bond market to provide the fund needed by time $t$ foreign exchange traders for the purchase of $B_{h,t+1}^*$, so borrow $B_{h,t+1}^*S_t$ from the home bond market;

2. Get the return of time $t-1$ bought foreign bonds in terms of home currency: $B_{h,t}^*S_t(1 + r_t^*)$ from the time $t-1$ born active traders.

3. Give the time $t-1$ born active traders a share $\alpha$ of the excess return $S_tB_{h,t}^*(1 + r_t^*) - S_{t-1}B_{h,t}^*(1 + r_t)$ for their consumption.

4. Pay back the principal and interest of its borrowing from the home bond market at time $t-1$: $S_{t-1}B_{h,t}^*(1 + r_t)$.

Therefore, the profit left in the trader industry is:

$$\Pi_t^T = \left\{ S_tB_{h,t}^*(1 + r_t^*) + S_tB_{h,t+1}^* \right\} - \left\{ S_tB_{h,t+1}^* + \alpha[S_tB_{h,t}^*(1 + r_t^*) - S_{t-1}B_{h,t}^*(1 + r_t)] + S_{t-1}B_{h,t}^*(1 + r_t) \right\}$$  (2.37)

and this profit will be returned to home household, as they own this trader firm. If $\alpha = 1$, $\pi_t^T = 0$. Here we will just assume $\alpha = 1$.

Figure 2 shows the structure of the relationship between home household, foreign exchange traders, traders industry/firm and the bond markets in the home and the foreign country in each period $t$. 

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Figure 2: Home and Foreign Bond Market ($\alpha = 1$)

Home Household → Home Bond Mkt → t born Active Trader

Trader Firm → t-1 born Active Trader

Foreign Bond Mkt → Foreign household

where:

\[ a : B_{t+1} \quad b : (1 + r_t)B_t \quad c : S_t\beta_{h,t+1} \quad d : S_{t-1}\beta_{h,t}(1 + r_t) \quad \] (2.39)

\[ e : S_t\beta_{h,t}(1 + r^*_t) - S_{t-1}\beta_{h,t}(1 + r_t) \quad f : \beta_{h,t}S_t(1 + r^*_t) \quad g : \beta_{h,t+1}S_t \quad \] (2.40)

\[ h : \beta_{h,t}(1 + r^*_t) \quad i : \beta_{h,t+1} \quad j : \beta_t(1 + r^*_t) \quad k : \beta_{t+1} \quad \] (2.41)

### 2.5.3 Optimal demand for foreign bond

Once the traders have decided to enter the market, the optimal demand for foreign bonds of each type of traders could be derived. Substitute Equation 2.34 into Equation 2.33 to get:

\[
\max_{\beta_{h,t+1}(i)} E_t^i \left\{ - \exp \left\{ -a\alpha \frac{\beta_{h,t+1}(i)}{P_{t+1}} [S_{t+1}(1 + r^*_t) - S_t(1 + r_{t+1})] + ac \right\} \right\} \quad (2.42)
\]

and rewrite Equation 2.42 as:

\[
\max_{\beta_{h,t+1}(i)} E_t^i \left\{ - \exp \left\{ -a\alpha \left[ \frac{\beta_{h,t+1}(i)}{P_{t+1}} S_t(1 + r_{t+1}) \frac{S_{t+1}(1 + r^*_t)}{S_t(1 + r_{t+1})} - 1 \right] + ac \right\} \right\} \quad (2.43)
\]

or \(^8\)

\[
\max_{\beta_{h,t+1}(i)} E_t^i \left\{ - \exp \left\{ -a\alpha \left[ \frac{\beta_{h,t+1}(i)}{P_{t+1}} S_t(1 + r_{t+1}) (s_{t+1} + r^*_t - s_t) \right] + ac \right\} \right\} \quad (2.44)
\]

\(^8\)Here an approximation is used: If $\zeta$ is small enough, then $\ln(1 + \zeta) \approx \zeta$. 

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where \( s_t = \log(S_t) \), \( s_{t+1} = \log(S_{t+1}) \), \( \log(1 + r^*_t) \approx r^*_t \), \( \log(1 + r_{t+1}) \approx r_{t+1} \). Let

\[
\rho_{t+1} = s_{t+1} + r^*_t - s_t - r_{t+1}
\]

be the excess return. The foreign currency trader \( i \)'s problem could be rewritten as:

\[
\max_{B_{h,t+1}^i} E_t^i \left\{ -ax \left[ \frac{B_{h,t+1}^i(i)}{P_{t+1}} S_t (1 + r_{t+1}) \rho_{t+1} \right] + ac_i \right\}
\]

From Equation 2.45 it can be seen that the only unknown variable at time \( t \) is the log of time \( t + 1 \) exchange rate: \( s_{t+1} \). It is assumed that \( s_{t+1} \) is normally distributed and both types of traders in the foreign exchange market know that. However, the informed/rational trader can predict \( s_{t+1} \) correctly; while the other type: the noise traders cannot predict the future nominal exchange rate correctly. From now on, the informed trader will be denoted by a superscript \( I \) and the noise trader will be denoted by a superscript \( N \).

For informed traders:\(^{11}\)

\[
E_t^I[s_{t+1}] = E_t[s_{t+1}]
\]

\[
Var_t^I[s_{t+1}] = Var_t[s_{t+1}]
\]

\[
\mathcal{L}_t^I(s_{t+1}) \sim N(E_t[s_{t+1}], Var_t[s_{t+1}])
\]

For noise traders, following closely the work of De Long et al. (1990), I make the

\(^9\)This assumption is true as all the shocks in this model are log-normal, then the solution of the model will take a log-normal distribution.

\(^{10}\)For rational traders, the subject distributions of the random variables are the same as the objective distributions.

\(^{11}\)\(\mathcal{L}_t^I(s_{t+1})\) is the subjective distribution conditional on trader \( i \)'s information set at period \( t \).
following assumptions about noise trader’s subjective distribution over $s_{t+1}$:

\[ E_t^N[s_{t+1}] = E_t[s_{t+1}] + v_t \quad (2.53) \]
\[ Var_t^l[s_{t+1}] = Var_t[s_{t+1}] \quad (2.54) \]
\[ Var(v_t) = \lambda Var(s_t) \quad \text{where } \lambda \in (0, +\infty) \quad (2.55) \]
\[ \mathcal{L}_t^N(s_{t+1}) \sim N(E_t^N[s_{t+1}], Var_t[s_{t+1}]) \quad (2.56) \]

where $v_t$ is assumed to be i.i.d and satisfies $E(v_t) = 0$.

From Equation 2.53 and Equation 2.47, it can be seen that, compared to the rational trader’s expectation, the noise traders’ expectation of $s_{t+1}$ based on time $t$ information is biased from the true conditional expectation by a random error. However, noise trader can correctly forecast the conditional variance of the exchange rate, therefore,

\[ Var_t^l[\rho_{t+1}] = Var_t^l[N] \quad (2.57) \]

From Equation 2.55, another assumption is that the unconditional variance of $v_t$ is proportional to the unconditional variance of the exchange rate itself.\(^{13}\)

If $s_{t+1}$ is normally distributed, the excess return $\rho_{t+1}$ will be normally distributed as well, which implies

\[ \mathcal{L}_t^i(\rho_{t+1}) = N \left( E_t^i[\rho_{t+1}], Var_t[\rho_{t+1}] \right) \quad \forall \ i \in [0, 1] \quad (2.58) \]

\(^{12}\)In Jeanne and Rose (2002), the forecasts of noise traders are:

\[ E_t^N[s_{t+1}] = \bar{p} + v_t \quad \text{where } \bar{p} = E(\rho) \quad (2.50) \]
\[ Var_t^N[s_{t+1}] = Var_t[s_{t+1}] \quad (2.51) \]
\[ Var(v) = \lambda Var(s) \quad (2.52) \]

\(^{13}\)The logic behind this assumption is that the bias in noise traders expectation must be related to the volatility of the exchange rate itself, otherwise noise traders might anticipate the future exchange rate is volatile even under a fixed exchange rate regime.
Then it is well known that maximizing Equation 2.33 is equivalent to maximizing the mean-variance objective function:

\[
\max_{B^*_{h,t+1}(i)} E_i^T(C^T_{t+1}(i)) - \frac{a}{2} Var_i^T(C^T_{t+1}(i))
\]  

(2.59)

From Equation 2.59 I can solve for the optimal bond holding of trader \(i\), which is given by:\(^{14}\)

\[
B^*_h(t+1) = \frac{E^T_i[\rho_{t+1}]}{a \alpha S_t \left(1 + r_{t+1}\right) Var_t[\rho_{t+1}]}
\]  

(2.60)

Therefore, for each informed trader:

\[
B^I_{h,t+1} = \frac{E^T_i[\rho_{t+1}]}{a \alpha S_t \left(1 + r_{t+1}\right) Var_t[\rho_{t+1}]}
\]  

(2.61)

For every noise trader:

\[
B^N_{h,t+1} = \frac{E^T_i[\rho_{t+1}]}{a \alpha S_t \left(1 + r_{t+1}\right) Var_t[\rho_{t+1}]}
\]  

(2.62)

Having derived the optimal demand for foreign bonds of each type of traders, I can analyze the equilibrium condition of the foreign bond market. In the first specification, the number of noise traders in the market is exogenously determined. Then we will discuss the case where the entry of the noise traders is endogenously decided.

2.5.4 Analysis with an exogenous number of noise traders

Assume that in each period \(t\), \(n_t\) noise traders are going to enter the foreign bond market\(^{15}\). Therefore, the aggregate demand for foreign bonds by foreign exchange traders of home country can be denoted as:

\[
B^*_{h,t+1} = N_i B^I_{h,t+1} + n_t B^N_{h,t+1}
\]  

(2.63)

\(^{14}\)Notice that, \(P_{t+1}\) is decided one period ahead, so it is known at time \(t\).

\(^{15}\)In endogenous entry case, \(N_i + n_t \leq 1\), i.e. \(n_t \leq 1 - N_i\), the number of noise traders entering the foreign bond market will be less than the number of existing noise traders.
\[
\begin{align*}
&= N_I [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1}] \\
&\quad + n_t [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1} + v_t] \\
&\quad + \frac{a\alpha S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1}) \\
&= \frac{(N_I + n_t) [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1}] + n_v t}{\alpha \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1})} \\
\Rightarrow s_t = E_t(s_{t+1}) + r_{t+1}^* - r_{t+1} + \frac{n_t}{N_I + n_t} v_t - \alpha \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1}) B_{h,t+1}^* \\
\end{align*}
\]  

Equation 2.66 characterizes the deviation from UIRP when noise traders are present in the market. This deviation consists of two parts: the expectation error of the noise traders, and the excess volatility of \( \rho_{t+1} \) brought by the existence of noise traders.

When there is no entry cost, every trader enters the market, so \( n_t = 1 - N_I \) and Equation 2.66 could be rewritten as:

\[
\begin{align*}
&= N_I [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1}] \\
&\quad + \frac{n_t [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1} + v_t]}{\alpha \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1})} \\
&= \frac{(N_I + n_t) [E_t(s_{t+1}) + r_{t+1}^* - s_t - r_{t+1}] + n_v t}{\alpha \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1})} \\
\Rightarrow s_t = E_t(s_{t+1}) + r_{t+1}^* - r_{t+1} + (1 - N_I) v_t - \alpha \frac{S_t}{P_{t+1}} (1 + r_{t+1}) \text{Var}_t(s_{t+1}) B_{h,t+1}^* \\
\end{align*}
\]  

\section*{2.5.5 Endogenous entry of noise traders}

I now endogeneize the composition of the pool of traders who enter the market in each period.

The entry decision for informed traders is trivial: they bear no entry cost and always enter the foreign bonds market in equilibrium. However, a noise trader enters if and only if Equation 2.32 is satisfied. It is shown in the Appendix A that for trader \( i \) this condition takes the form:

\[
c_i \leq GB_i^N 
\]  

where \( GB_i^N \) could be considered as the gross benefit of entry for noise traders and is given
by 16.

\[ GB_t^N = \frac{[E_t^N(\rho_{t+1})]^2}{2a \text{Var}_t(s_{t+1})} \]  

(2.69)

The gross benefit of entry has an intuitive interpretation. For noise traders, it is increasing with the expected risk premium and decreasing with conditional time \( t + 1 \) exchange rate volatility. Note that in our general equilibrium setting they are both functions of the number of incumbent noise traders.

Now Equation 2.68 will be used to derive the foreign bond market equilibrium condition, let

\[ c_t^* = \frac{[E_t^i(\rho_{t+1})]^2}{2a \text{Var}_t(s_{t+1})} \]  

(2.70)

Since \( c_i \) is uniformly distributed, this implies

\[ \begin{align*}
\text{if } c_j \leq c_t^* & , \quad \varphi_i^j = 1 \\
\text{if } c_j \geq c_t^* & , \quad \varphi_i^j = 0
\end{align*} \] 

(2.71) (2.72)

And the number of incumbent noise traders \( n_t \) is given by:

\[ n_t = \frac{c^*}{\bar{c}} (1 - N_I) = \frac{[E_t^i(\rho_{t+1})]^2}{2a \text{Var}_t(s_{t+1})} \frac{1 - N_I}{\bar{c}} \] 

(2.73)

Apparently, the number of active noise traders on the market is increasing in the squared expected risk premium and number of potential noise traders, and decreasing in the entry cost, risk aversion coefficient \( a \) and the exchange rate volatility. Therefore, this implies that the exchange rate volatility will not increase unboundedly in this model. The risk aversion of the foreign exchange traders will reduce the exchange rate volatilities. The entry of noise traders will have two effects. The presence of more active noise traders creates higher squared expected risk premium and incentives for other noise traders to enter the market, but the extra exchange rate volatility brought by their entry will reduce the gross benefit of entry for noise traders. In equilibrium, the second effect will dominate and no more noise traders will enter.

\(^{16}\)Note that \( \text{Var}_t(\rho_{t+1}) = \text{Var}_t(s_{t+1}) \).
Substitute Equation 2.73 into Equation 2.66, I could derive the equilibrium condition of the foreign bond market when the entry decision of the traders is endogenized:

\[ s_t = E_t(s_{t+1}) + r^*_t + 1 - r_t + 1 + c^*_t \bar{c}(1 - N_I) / N_I + c^*_t \bar{c}(1 - N_I) + v_t - a\alpha \frac{S_t}{P_{t+1}[N_I + \frac{c^*_t}{\bar{c}}(1 - N_I)]} (1 + r_{t+1}) \text{Var}_t(s_{t+1}) B_{h,t+1} \]

\[ \text{First Part} \hspace{2cm} \text{Second Part} \]

\[ (2.74) \]

2.5.6 Interpretation

The last two terms in Equation 2.67 and Equation 2.74 characterize the impact of noise traders on the exchange rate. These two terms work in opposite direction on their effect on the exchange rate volatility.

The first part captures the fluctuations in the exchange rate due to the variation of noise traders’ misperceptions. As one would expect, Equation 2.66 shows that the more numerous noise traders are, the more impact the noise traders’ expectation error will have on the exchange rate. For example, when there is a positive expectation error shock, the noise trader will have a higher demand for foreign bonds and foreign currency, which leads to a domestic currency depreciation.

The second part is the heart of the model. The entry of noise traders will increase the exchange rate volatility. Informed trader would not hold the foreign bonds unless compensated for bearing the risk created by noise traders. So the price of the foreign currency (risky asset) should fall. Both noise traders and rational traders of period \( t \) believe that foreign currency is mispriced, but because \( s_{t+1} \) is uncertain, neither group is willing to bet too much on this mispricing. At the margin, the return from enlarging one’s position in an asset that everyone agrees is mispriced (but different type would have a different idea of the direction of the mispricing) is offset by the additional price risk that must be born.

Therefore, in this sense, like De Long et al. (1990) shows, the noise traders “create their own space”: the uncertainty of the noise traders’ expectations over the future exchange
rate increases the riskiness of return to asset and drives its price down and its return up. In other words, the noise trader can affect prices through this “volatility” term. Friedman (1953) argues that the noise traders who affect prices earn lower returns than the rational traders they trade with, and so in the long run the economic selection will work to eliminate the noise traders from the market. In our model, this need not to be the case. The noise traders’ collective mis-expectation makes the risky asset (foreign currency) riskier and therefore if the noise traders take a large position in the foreign bond, it is possible that they earn a higher ex-post rate of return than the informed traders.

2.6 Equilibrium Condition

From Equation 2.23 and Equation 2.24, it can be seen that the equilibrium is symmetric in that all firms located in the same country will make same pricing decisions.

\[ P_{h,t}(i) = P_{h,t} \quad P^*_h(i) = P^*_{h,t} \quad \forall i \in [0, 1] \tag{2.75} \]

\[ P_{f,t}(j) = P_{f,t} \quad P^*_f(j) = P^*_{f,t} \quad \forall j \in [0, 1] \tag{2.76} \]

Therefore, from Equation 2.13, Equation 2.14 and their counterparts in foreign country,

\[ C_{h,t}(i) = C_{h,t} \quad C^*_h(i) = C^*_{h,t} \quad \forall i \in [0, 1] \tag{2.77} \]

\[ C_{f,t}(j) = C_{f,t} \quad C^*_f(j) = C^*_{f,t} \quad \forall j \in [0, 1] \tag{2.78} \]

Equilibrium for this economy is a collection of 26 sequences \((P_t, P^*_t, P_{h,t}, P^*_{h,t}, P_{f,t}, P^*_{f,t}, C_t, C^*_t, C^H_t, C^*_t, C^*_h, C^*_h, C^*_f, C^*_f, S_t, r_t, \theta^*_t, D_t, D^*_t, W_t, W^*_t, B_t, B^*_t, B^*_{h,t}, L_t, L^*_t)\) satisfying 26 equilibrium conditions. They include the household optimality conditions (Equations 2.3, 2.4, 2.5 and their foreign counterparts), the definition of the

\[ \Pi_t(i) = \omega \left[ (P_{h,t} - W_t) \left( \frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t + (P^*_{h,t} - W_t) \left( \frac{P^*_{h,t}}{P^*_t} \right)^{-\gamma} C^*_t \right] \quad \forall i \tag{2.79} \]
price indexes (Equation 2.9 and its foreign analogy), the definition of the pricing kernel (Equation 2.18 and its foreign analogy), the equation that characterizes the deviation from UIRP (Equation 2.67 or Equation 2.74), the four demand equations:

\[ C_{h,t} = \omega \left( \frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t \]  
\[ C_{f,t} = (1 - \omega) \left( \frac{P_{f,t}}{P_t} \right)^{-\gamma} C_t \]  
\[ C_{h,t}^* = \omega \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \]  
\[ C_{f,t}^* = (1 - \omega) \left( \frac{P_{f,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \]

And the four pricing conditions:

\[ P_{h,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t C_t]}{E_{t-1} [D_t C_t]} \]  
\[ P_{h,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t W_t C_t^*]}{E_{t-1} [D_t C_t^*]} \]  
\[ P_{f,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* C_t]}{E_{t-1} [D_t^* C_t^*]} \]  
\[ P_{f,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} [D_t^* W_t^* C_t^*]}{E_{t-1} [D_t^* C_t^*]} \]

And the four market clearing conditions for the bonds and goods markets:

\[ B_{t+1} = S_t B_{h,t+1}^* \forall t \]  
\[ B_t^* + B_{h,t}^* = 0 \]  
\[ L_t = C_{h,t} + C_{h,t}^* \]  
\[ L_t^* = C_{f,t} + C_{f,t}^* \]

\[ \Pi_t^*(j) = \Pi_t^* = (1 - \omega) \left[ (P_{f,t}^* - W_t^*) \left( \frac{P_{f,t}^*}{P_t^*} \right)^{-\gamma} C_t^* + \left( \frac{P_{f,t}}{S_t} - W_t^* \right) \left( \frac{P_{f,t}}{P_t} \right)^{-\gamma} C_t \right] \forall j \]
Finally, the budget constraint of the foreign exchange traders, for the exogenous entry specification:

\[ P_t C^T_t = \alpha \left[ B_{h,t}^*(1 + r_t^*)S_t - B_{h,t-1}^*(1 + r_t) \right] \]  
(2.93)

For the endogenous entry specification:

\[ P_t C^T_t = \alpha \left[ B_{h,t}^*(1 + r_t^*)S_t - B_{h,t-1}^*(1 + r_t) \right] - P_t \sum_{i=0}^{n_t+N_t} c_i \]  
(2.94)

And the home market clearing condition\(^{18}\):

\[
C_t = C^H_t + C^T_t \quad \text{Exogenous Entry} \quad (2.95)
\]

\[
C_t = C^H_t + C^T_t + \sum_{i=0}^{n_t+N_t} c_i \quad \text{Endogenous Entry} \quad (2.96)
\]

Then the above two equations and the budget constraints of the home households, Equation 2.2, \(^{19}\) can be combined to get:

\[
P_t C_t = W_t L_t + \Pi_t + \alpha \left\{ S_t B_{h,t}^*(1 + r_t^*) - S_t B_{h,t-1}^*(1 + r_t) \right\} + B_t(1 + r_t) - B_{t+1} + \Pi_t^T
= W_t L_t + \Pi_t + B_t(1 + r_t) - B_{t+1} + S_t B_{h,t}^*(1 + r_t^*) - S_{t-1} B_{h,t}(1 + r_t) \]  
(2.97)

Substitute Equation 2.89 and its one-period lag into Equation 2.98, the national budget constraint of home country can be derived:

\[
P_t C_t = W_t L_t + \Pi_t + S_t B_{h,t}^*(1 + r_t^*) - S_t B_{h,t+1}^* \]  
(2.99)

where

\[
\Pi_t = \omega \left[ (P_{h,t} - W_t) \left( \frac{P_{h,t}}{P_t} \right)^{-\gamma} C_t + (P_{h,t}^* S_t - W_t) \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \right] \]  
(2.100)

\(^{18}\)Currently, the entry cost \(c_i\) is modelled as resource-consuming. It consumes the composite consumption good.

\(^{19}\)From Section 1.5.

\[
\Pi_t^T = \left\{ S_t B_{h,t}^*(1 + r_t^*) + S_t B_{h,t+1}^* \right\} - \left\{ S_t B_{h,t+1}^* + \alpha \left[ S_t B_{h,t}^*(1 + r_t^*) - S_{t-1} B_{h,t}^*(1 + r_t) \right] + S_{t-1} B_{h,t}^*(1 + r_t) \right\}
\]

and \(\pi_t^T = 0\) if \(\alpha = 1\).
3 Analytical solution of the model

In this section we will solve the system by log-linearization around a non-stochastic, symmetric steady state. Appendix C describes the complete solution of the model. Here, we will just outline the important and intuitive steps of the solution. Section 1 of Appendix C gives the detail of log-linearization of the whole system. Given the log-linearized system, the deviations of the exchange rate and the macroeconomic variables from their $t - 1$ expectations are solved in terms of exogenous money supply shocks and the expectation error shocks.

From now on, $\hat{x}_t = \log(x_t) - \log(\bar{x})$, where $\bar{x}$ is the non-stochastic steady state value of variable $x_t$. $x_{t+j} = x_{t+j} - E_{t-1}(x_{t+j}), j \geq 0$. Therefore, the exogenous money supply shocks could be rewritten as:

$$m_{t+1} = \hat{m}_t + \epsilon_{\mu,t}$$
$$m^*_t + 1 = \hat{m}_t + \epsilon^*_{\mu,t}$$

(3.1)

where $\epsilon^*_{\mu,t} \sim N(0, \sigma^2_{\epsilon^*_{\mu}})$ and $\epsilon_{\mu,t} \sim N(0, \sigma^2_{\epsilon_{\mu}})$.

First, from the log-linearization of the pricing equation of firms, we may find that the firms set prices equal to the anticipated marginal costs:

$$\hat{p}_{h,t} = E_{t-1}[\hat{w}_t], \quad \hat{p}^*_{h,t} = E_{t-1}[\hat{w}^*_t] - E_{t-1}[\hat{s}_t]$$

(3.2)

Similarly,

$$\hat{p}_{f,t} = E_{t-1}[\hat{w}^*_t] + E_{t-1}[\hat{s}_t], \quad \hat{p}^*_{f,t} = E_{t-1}[\hat{w}^*_t]$$

(3.3)

Together, these equations give us the price index for home and foreign country:

$$\hat{p}_t = \frac{1}{2}(E_{t-1}[\hat{w}_t] + E_{t-1}[\hat{w}^*_t] + E_{t-1}[\hat{s}_t])$$

(3.4)

$$\hat{p}^*_t = \frac{1}{2}(E_{t-1}[\hat{w}_t] - E_{t-1}[\hat{s}_t] + E_{t-1}[\hat{w}^*_t])$$

(3.5)

Equation 3.4 and 3.5 establish that in an expected sense, PPP holds. It is not surprising as the prices can fully adjust to all shocks after one period.

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Uncovered Interest Rate Parity  To derive the “linearization” of the uncovered interest parity condition (UIRP), we have to be careful. This is because the non-linearities of the UIRP equation play an important role on the dynamics of the economy, so the variance term and some of the non-linear terms will be kept through second-order approximation when the UIRP equation is linearized around the non-stochastic, symmetric steady state.

When there is no entry cost and all the noise traders enter the market, linearizing the UIRP equation 2.67 gives 20:

\[
\hat{s}_t = E_t(s_{t+1}) - \beta (d_{t+1} - d_t) + (1 - N_I)v_t - a\frac{(1 + \bar{r})\bar{S}}{P} Var_t[s_{t+1}]dB^*_h,_{t+1} \tag{3.6}
\]

For the second specification of the model, where the entry of the noise traders is endogenously determined, linearizing the UIRP equation 2.74 gives 21:

\[
\hat{s}_t = E_t(s_{t+1}) - \beta (d_{t+1} - d^*_t) + \frac{1}{N_I} n_t v_t - a\frac{(1 + \bar{r})\bar{S}}{PN_I} Var_t[s_{t+1}]dB^*_h,_{t+1} \tag{3.7}
\]

where \(n_t\), the number of incumbent noise traders is given by:

\[
n_t = \frac{\{E_t(s_{t+1}) - \hat{s}_t - \beta (d^*_t - d_t) + v_t\}^2 (1 - N_I)}{2aVar_t(s_{t+1})} \tag{3.8}
\]

Equation 3.6 and 3.7 imply that in this model, the presence of biased expectation of noise traders introduces a stochastic deviation from uncovered interest rate parity. This deviation is composed by two parts: a direct effect of the noise traders’ expectation errors and an indirect effect that comes from a risk premium term as the foreign exchange traders are risk averse. The direct effect, as discussed intensively in Devereux and Engel (2002), is different from the traditional risk premium term that arises from risk aversion of households. It is quite volatile and tends to increase the exchange rate volatility. The

\[20\] The 2nd order Taylor expansion is used to approximate the variance term in Equation 2.67 as the first order terms are all zero around the steady state and thus these terms determine the dynamics of the model.

\[21\] The 2nd order Taylor expansion is used to approximate \(\frac{n_t}{N_I+1}v_t\) and the variance term in 2.74 as these terms are important for understanding the dynamics of the endogenous entry model.
indirect effect which comes from the risk aversion of foreign exchange traders, is an unique feature of this model. Devereux and Engel (2002) show that the exchange rate volatility can rise without bound under some parameter specification. However, this is not possible in this model because the traders are also risk averse. Their risk aversion limits the unbounded increase of the exchange rate volatility.

**Budget Constraints of Traders** For the first specification of the model, the budget constraint of foreign exchange traders is quite easy:

\[ P_{t+1}C_{t+1}^T = B_{h,t+1}^*[\{1 + r_{t+1}^*\}S_{t+1} - (1 + r_{t+1})S_t] \]  

or

\[ \frac{P_{t+1}C_{t+1}^T}{S_t(1 + r_{t+1})} = B_{h,t+1}^*\rho_{t+1} \]  

Linearizing this equation gives:

\[ dC_{t+1}^T = 0 \]  

For the endogenous entry specification, the budget constraint of foreign exchange traders is:

\[ P_{t+1}C_{t+1}^T = B_{h,t+1}^*[\{1 + r_{t+1}^*\}S_{t+1} - (1 + r_{t+1})S_t] - P_{t+1} \sum_{i=0}^{n_t+N_f} c_i \]  

or

\[ P_{t+1}C_{t+1}^T = B_{h,t+1}^*S_t(1 + r_{t+1})\rho_{t+1} - \frac{P_{t+1}}{2} \frac{n_t^2}{1 - N_f^2} \bar{c} \]  

Linearizing this equation gives:

\[ dC_{t+1}^T = 0 \]  

Notice that here all the second order terms of \( dC_{t+1}^T \) are ignored because of the following reasons: First, introduction of the second order terms of \( dC_{t+1}^T \) will complicate the system and thus make it impossible to focus on the important dynamics of the log-linearized system. Secondly, the income of the traders, as the product of two small deviations from their steady state values (risk premium \( \rho_{t+1} \) and net foreign assets \( dB_{h,t+1}^* \), both variables...
equal to zero at the steady state), should be tiny and therefore \(dC^T_{t+1}\) should not play an important role in deciding the characteristics of the system. Thirdly, when \(C^T_t\) is negative, from \(C_t = C^H_t + C^T_t\), the households will gain from the noise traders’ ‘stupidity’ or irrationality. This welfare implication of the presence of noise traders, might be small and unrealistic. Therefore, by ignoring all the second order terms, we could get rid of this welfare effect and focus on the main mechanism through which the noise traders affect the whole economy.

Relative Expected Consumption and the Net Foreign Assets

The log-linearized resource constraint of home household is:

\[
\hat{p}_t + c^H_t = \frac{1}{2}(\hat{p}^h_{t,t} + c^h_{t,t}) + \frac{1}{2}(\hat{p}^\epsilon_{h,t} + c^\epsilon_{h,t}) + \frac{1}{\beta} \bar{P}\bar{C} dB_t - \frac{1}{\bar{P}\bar{C}} dB_{t+1}
\]  

(3.15)

Appendix C shows that it could be rewritten as:

\[
c^H_t - c^*_t = \frac{dC^T_t}{C} = (\hat{s}_t - E_{t-1}\hat{s}_t) + (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + 2\frac{1}{\beta} \bar{P}\bar{C} dB_t - \frac{2}{\bar{P}\bar{C}} dB_{t+1}
\]  

(3.16)

Take expectation at time \(t - 1\), we can get:

\[
E_{t-1}(c^H_t - c^*_t) = E_{t-1}(\frac{dC^T_t}{C}) + (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + 2\frac{1}{\beta} \bar{P}\bar{C} dB_t - \frac{2}{\bar{P}\bar{C}} dB_{t+1}
\]  

(3.17)

This follows from the fact that (as will hold in equilibrium) in an expected sense, any initial change in net foreign asset is persistent.

Use \(\bar{r} = \frac{1}{\beta} - 1\) and the fact that \(\frac{dC^T_t}{C} = 0\), we could get:

\[
E_{t-1}(c^H_t - c^*_t) = (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + \frac{2\bar{r}}{\bar{P}\bar{C}} dB_t
\]  

(3.18)

Equation 3.18 shows that the relative home consumption is increasing in the changes in the initial net foreign asset position and decreasing in the expected terms of trade, as along as \(\gamma > 1\).
Now we use the goods market clearing conditions in home and foreign to get:

\[ E_{t-1}(\hat{l}_t - \hat{l}_t^*) = -\gamma E_{t-1}(\hat{w}_t - \hat{w}_t^* - \hat{s}_t) \tag{3.19} \]

Log-linearized home labor supply equation and its foreign equivalent imply:

\[ E_{t-1}(\hat{w}_t - \hat{w}_t^* - \hat{s}_t) = \psi E_{t-1}(\hat{l}_t - \hat{l}_t^*) + \rho E_{t-1}(c_t^H - \hat{c}_t^*) \tag{3.20} \]

From Equation 3.19, 3.20 and 3.18 we can get:

\[ E_{t-1}(c_t^H - \hat{c}_t^*) = \frac{2\bar{r}}{\bar{P}} \frac{1}{\bar{C} \sigma} dB_t \tag{3.21} \]

where \( \sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi\gamma} \). Therefore, an increase in the home country net foreign assets lead to an expected increase in the home relative consumption.

Relative Consumption  From the log-linearized money demand equation and its foreign equivalent, if \( \epsilon = 1 \), we have:

\[ \hat{m}_t - \hat{p}_t = \rho \hat{c}_t^H - \frac{\beta}{r} dr_{t+1} \tag{3.22} \]

\[ \hat{m}_t^* - \hat{p}_t^* = \rho \hat{c}_t^H - \frac{\beta}{r} dr_{t+1}^* \tag{3.23} \]

In equilibrium, given the random walk assumption of money supply process and the log money utility function, a very convenient property is that the nominal interest rate will be constant. This is because, if the log of the money stock follows a random walk, so does the log of the term \( P_t(C_t^H)^\rho \). Using this fact, the log-linearized price index equations and the pricing equations, we will get:

\[ \hat{m}_t - \hat{m}_t^* = \rho(\hat{c}_t^H - \hat{c}_t^*) + E_{t-1}(\hat{s}_t) \tag{3.24} \]

Take expectation at \( t - 1 \), we will get:

\[ E_{t-1}(\hat{m}_t - \hat{m}_t^*) = \rho E_{t-1}(\hat{c}_t^H - \hat{c}_t^*) + E_{t-1}(\hat{s}_t) \tag{3.25} \]
Therefore, in an expected sense, the exchange rate is consistent with the standard monetary model. Equation 3.24 minus Equation 3.25 gives the equation which describe the relation between relative money supply and relative consumption:

\[ \tilde{m}_t - \tilde{m}^*_t = \rho(\tilde{c}_t^H - \tilde{c}^*_t) \]  
(3.26)

**Response of Exchange rates to Money shocks**  
First, let us have a look at the wealth effect of an unexpected change in the exchange rate. Equation 3.16 minus Equation 3.18 gives:

\[ \tilde{c}_t^H - \tilde{c}^*_t + \frac{2}{\bar{P}\bar{C}}d\tilde{B}_{t+1} = \tilde{s}_t \]  
(3.27)

or

\[ d\tilde{B}_{t+1} = \frac{\bar{P}\bar{C}}{2}[\tilde{s}_t - (\tilde{c}_t^H - \tilde{c}^*_t)] \]  
(3.28)

The right hand side of Equation 3.27 represents the relative wealth effect of an unanticipated shock to the exchange rate. This relative wealth increase will be then spread between an increase in relative home consumption and net foreign assets accumulation.

Using Equation 3.21 (updated to period \( t+1 \)) and Equation 3.28, we may establish that:

\[ (\tilde{c}_t^H - \tilde{c}^*_t) + \sigma \frac{E_t}{\bar{r}}(c_{t+1}^H - c_{t+1}^*) = \tilde{s}_t \]  
(3.29)

This equation gives us a relationship between current relative consumption, expected period \( t+1 \) relative consumption, and the unanticipated shock to the exchange rate. It represents the constraints on these three variables implied by the intertemporal current account.

Next step is use the intertemporal optimality equations and UIRP condition to get another condition relating these three variables. Home an foreign’s intertemporal optimality conditions imply:

\[ \rho(c_t^H - c_t^*) - \rho E_t(c_{t+1}^H - c_{t+1}^*) - E_t(s_{t+1}^*) = -\beta (d\tilde{r}_{t+1} - d\tilde{r}_{t+1}) \]  
(3.30)
The exogenous UIRP equation 3.6 minus its date $t-1$ expectation gives:

$$-\beta(dr_{t+1} - dr^*_t) = \tilde{s}_t - E_t(s_{t+1}) - (1 - N_I)v_t + a\frac{(1 + \bar{r})\bar{S}}{P}Var_t[s_{t+1}]dB^*_h_{t+1}$$  \hspace{1cm} (3.31)

Therefore, Equation 3.30 and 3.31 give us the condition relating expected relative consumption growth in the home country with an unexpected exchange rate change, the expectation error of the noise traders, and a risk premium term:

$$E_t(c^*_t - c^*_t) = \left(c^*_t - c^*_t\right) - \frac{1}{p}[\tilde{s}_t - (1 - N_I)v_t + a\frac{(1 + \bar{r})\bar{S}}{P}Var_t[s_{t+1}]dB^*_h_{t+1}]$$  \hspace{1cm} (3.32)

From Equation 3.32, expected consumption growth in the home country decreases in response to an unanticipated exchange rate depreciation since this generate an unanticipated real depreciation, therefore reduce the home country’s real interest rate. While a positive “noise” term which represents a positive shock to foreign exchange traders’ expectations of the future exchange rate, will increase the home real interest rate and lead to an increase in relative consumption growth of home country. The last term in the equation, denoting a risk premium term due to the risk-aversion of the foreign exchange traders, will reduce the response of exchange rate to the unanticipated expectation shocks and therefore has a negative effect on home relative consumption growth.

Put Equation 3.26, 3.29, 3.32 and 3.28 together, we get a system of equilibrium conditions that characterizes $\{\tilde{s}_t, \tilde{c}_t - \tilde{c}^*_t, dB_{t+1}\}$. We may solve for the deviation of exchange rate from its $t-1$ expectation: $\tilde{s}_t$, the conditional mean: $E_t(s_{t+1})$ and variance of the future exchange rate deviation: $Var_t(s_{t+1})$ in terms of the exogenous money shock and expectation error of the noise traders about the future exchange rate, which are characterized by the following function:

$$\tilde{m}_t - m^*_t = \frac{\rho\bar{r} + \sigma}{\sigma + \bar{r}}\tilde{s}_t - \frac{\sigma}{\sigma + \bar{r}}(1 - N_I)v_t + \frac{\frac{a(1 + \bar{r})\bar{S}}{P}\sigma}{\sigma + \bar{r}}Var_t(s_{t+1}) \frac{1 + \frac{a(1 + \bar{r})SC}{P}\frac{\sigma}{\rho(\sigma + \bar{r})}Var_t(s_{t+1})}{\frac{P\bar{C}}{2} \left[\frac{\sigma(\rho - 1)}{\rho(\sigma + \bar{r})}\tilde{s}_t + \frac{\sigma}{\rho(\sigma + \bar{r})}(1 - N_I)v_t\right]}$$  \hspace{1cm} (3.33)

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Also, the net foreign assets of home country is given by:

\[ dB_{t+1} = \frac{\tilde{P}C}{2}[\tilde{s}_t - \frac{1}{\rho}(\tilde{m}_t - \tilde{m}_t^*)] \]  \hspace{1cm} (3.34)

**Volatilities of macroeconomic fundamentals**  Now we will find out the response of the macroeconomic fundamentals such as consumption, labor and wage to the exogenous monetary shocks and expectation error shocks. From the log-linearized good market clearing condition, labor supply condition and the money demand condition, we can derive the functions that characterize the response of macroeconomic fundamentals to these shocks. For the detail of the derivation, please refer to Appendix C.

\[ \tilde{w}_t = \frac{\psi}{2\rho}(\tilde{m}_t + \tilde{m}_t^*) + \tilde{m}_t \]  \hspace{1cm} (3.35)

\[ \tilde{l}_t = \frac{1}{2\rho}(\tilde{m}_t + \tilde{m}_t^*) \]  \hspace{1cm} (3.36)

\[ \tilde{c}_t = \frac{1}{\rho}\tilde{m}_t \hspace{1cm} \tilde{c}_t^* = \frac{1}{\rho}\tilde{m}_t^* \]  \hspace{1cm} (3.37)

From the above equations, we could get:

\[ \text{Var}(\tilde{c}_t) = \text{Var}(\tilde{c}_t^*) = \frac{1}{\rho^2}\text{Var}(\tilde{m}_t) \]  \hspace{1cm} (3.38)

\[ \text{Var}(\tilde{w}_t) = \left(\frac{\psi}{2\rho} + 1\right)^2\text{Var}(\tilde{m}_t) + \left(\frac{\psi}{2\rho}\right)^2\text{Var}(\tilde{m}_t^*) \]  \hspace{1cm} (3.39)

\[ \text{Var}(\tilde{l}_t) = \left(\frac{1}{2\rho}\right)^2(\text{Var}(\tilde{m}_t) + \text{Var}(\tilde{m}_t^*)) \]  \hspace{1cm} (3.40)

In other words, the volatilities of the deviations of the macroeconomic fundamentals from their \( t - 1 \) expectations are only affected by the volatility of the relative monetary shock and the value of the parameters, but not by the volatility of the expectation error and the number of incumbent noise traders in the market.

### 3.0.1 Model 2: Endogenous Entry

The endogenous entry case is quite similar to the exogenous entry case except for UIRP equation and the budget constraints of the foreign exchange traders. Therefore, we could
get the following equations analogously:

\[ \tilde{m}_t - \tilde{m}_t^* = \rho(\tilde{c}_t^H - \tilde{c}_t^*) \]

\[ dB_{t+1} = \frac{\bar{P}\bar{C}}{2} [\tilde{s}_t - (\tilde{c}_t^H - \tilde{c}_t^*)] \]

\[ (\tilde{c}_t^H - \tilde{c}_t^*) + \frac{\sigma}{\rho} E_t(\tilde{c}_{t+1}^H - \tilde{c}_{t+1}^*) = \tilde{s}_t \]

\[ \rho(\tilde{c}_t^H - \tilde{c}_t^*) - \rho E_t(\tilde{c}_{t+1}^H - \tilde{c}_{t+1}^*) - E_t(s_{t+1}) = -\beta(d\tilde{r}_{t+1} - dr_{t+1}) \]

Now we use the UIRP equation for endogenous entry: 3.7 and other related equations to derive an equation analogous to Equation 3.33.

Equation 3.7 minus its \( t-1 \) expectation gives:

\[ -\beta(d\tilde{r}_{t+1} - dr_{t+1}) = \tilde{s}_t - E_t(s_{t+1}) - \frac{1}{N_I} n_t v_t + a\frac{(1 + \bar{r})\bar{S}}{PN_I} Var_t[s_{t+1}] dB_{h,t+1}^* \quad (3.41) \]

Equation 3.30 and 3.41 gives us the condition relating home consumption growth with the exchange rate shocks, the expectation error of the noise traders and the risk premium term:

\[ E_t(\tilde{c}_{t+1}^H - \tilde{c}_{t+1}^*) = (\tilde{c}_t^H - \tilde{c}_t^*) - \frac{1}{\rho} [\tilde{s}_t - \frac{1}{N_I} n_t v_t + a\frac{(1 + \bar{r})\bar{S}}{PN_I} Var_t[s_{t+1}] dB_{h,t+1}^*] \]

where \[ n_t = \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2 (1 - N_I)}{2a Var_t(s_{t+1})} \quad (3.43) \]

Equation 3.26, 3.29, 3.28 and Equation 3.42, 3.43 give the solution of the endogenous entry model. After some rearrangements of the terms, we could get:

\[ (\tilde{c}_t^H - \tilde{c}_t^*) = \frac{[1 + \frac{\sigma}{\rho^2} \tilde{s}_t - \frac{\sigma}{\rho^2} \frac{1}{N_I} n_t v_t + a\frac{(1 + \bar{r})\bar{S}}{PN_I} \frac{\sigma}{\rho^2} Var_t[s_{t+1}] dB_{h,t+1}^*]}{1 + \frac{\sigma}{\rho^2}} \]

\[ \text{Here we use the random walk property of shocks and the fact that nominal interest rates are constant, for detailed derivation please refer to Appendix C.} \]
\[
\tilde{m}_t - \tilde{m}_t^* = \frac{\rho}{1 + \frac{\sigma}{\rho}}[(1 + \frac{\sigma}{\rho^2})\tilde{s}_t - \frac{\sigma}{\rho^2} N_t n_t v_t + \frac{a(1 + \tilde{r})S}{PN_t} \tilde{m} \text{Var}_t[s_{t+1}]dB^*_h,t+1]
\]
\[
dB^*_h,t+1 = \frac{\tilde{P}C}{2}[\tilde{s}_t - (\tilde{c}_t^* - \tilde{c}_t^t)]
\]
\[
n_t = \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2 (1 - N_t)}{2a \text{Var}_t(s_{t+1})} \frac{1}{\bar{c}}
\]

Solving this system of equations, we could get the function that relates \(\tilde{s}_t, E_t(s_{t+1})\) and \(\text{Var}_t(s_{t+1})\) to the exogenous money supply shocks and expectation error shocks.

\[
\tilde{m}_t - \tilde{m}_t^* = \frac{\rho}{\sigma + \tilde{r}} - \frac{\sigma}{\rho + \tilde{r}} N_t \frac{\{E_t(s_{t+1}) - \tilde{s}_t + v_t\}^2}{2a \text{Var}_t(s_{t+1})} \frac{1}{\bar{c}} v_t + \frac{a(1 + \tilde{r})S}{PN_t} \frac{\tilde{m} \text{Var}_t(s_{t+1})}{\frac{\sigma}{\rho + \tilde{r}} V \text{ar}_t(s_{t+1})} \frac{2}{1 + \frac{a(1 + \tilde{r})S}{PN_t} \frac{\tilde{m} \text{Var}_t(s_{t+1})}{\frac{\sigma}{\rho + \tilde{r}} V \text{ar}_t(s_{t+1})} \frac{2}{\bar{c}}}
\]

The expression for net foreign assets, consumption, labor and wage are the same as in the exogenous case.

4 Simulation Results

4.1 Undetermined Coefficient Method

Although Equation 3.33 and 3.45 are too complicated to be solved analytically as in Devereux and Engel (2002), we can use numerical undetermined coefficient method to show the relationship between the variance of macroeconomic fundamentals, the number of noise traders and the volatility of exchange rates. The detail of the method is given in Appendix D, here I will just briefly introduce it.

First we guess a functional form for \(\tilde{s}_t: \tilde{s}_t = \alpha_0 + \alpha_1 \tilde{m}_t + \alpha_2 \tilde{m}^*_t + \alpha_3 v_t + \text{Second order terms}\). Given that, we could get \(E_t(s_{t+1})\) and \(\text{Var}_t(s_{t+1})\) easily. Then we could solve for \(\tilde{s}_t\) from
3.33 Given any exogenous shocks $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. Is our initial guess a good guess? We can do some simulation and regress the $s_t$ we get from above process on $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. If the coefficients ($\alpha$'s) are equal to the initial guess, we stop. Otherwise, we just repeat the procedure. This method is actually an undetermined coefficient method, it is also known as “parameterized estimation approach” in numerical methods.

Using this method requires calibration of the model. Table 1 gives the parameter values that are used in the simulation. These values are just taken from the new open economy macroeconomic literature as the purpose of this paper is to do some qualitative assessments.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exogenous Case</strong></td>
<td></td>
</tr>
<tr>
<td>Preferences</td>
<td>$\beta = 0.96$, $\rho = 2$, $\epsilon = 1$, $\eta = 0.3$, $\psi = 1$</td>
</tr>
<tr>
<td>Final good technology</td>
<td>$\theta = 5$, $\gamma = 0.5$, $\omega = 0.5$</td>
</tr>
<tr>
<td>Money Growth Process</td>
<td>$\text{corr}(\varepsilon_\mu, \varepsilon_\mu^<em>) = 0$, $\sigma^2_{\varepsilon_\mu} = \sigma^2_{\varepsilon_\mu^</em>} = 0.01$</td>
</tr>
<tr>
<td>Foreign exchange traders</td>
<td>$a = 1$, $\bar{c} = 0$, $N_I$ variable, $\lambda$ variable, $n = 1 - N_I$</td>
</tr>
<tr>
<td>Steady State Values</td>
<td>$\mu_{ss} = \mu_{ss}^* = 1$, $M_{ss} = M_{ss}^* = 2$</td>
</tr>
<tr>
<td><strong>Endogenous Case</strong></td>
<td></td>
</tr>
<tr>
<td>Foreign exchange traders</td>
<td>$\bar{c}$, $N_I$ variable, $\lambda = 2$</td>
</tr>
</tbody>
</table>

aOther parameters in the endogenous case are the same as in the exogenous case.

4.2 Exogenous Case

I first solve for the simpler case when the entry cost is zero - so all the potential noise traders will enter the market and the noise component of the market is exogenously determined by the number of existing noise traders on the market.

Table 2 and Table 3 give the results of the undetermined coefficient method, for different value of the magnification coefficient $\lambda$. They show the changes in the volatilities
of the exchange rate and the net foreign assets when the number of noise traders in the foreign exchange market is increasing. Three important implications from Table 2 and 3 are: 1. The exchange rate volatility is increasing when the number of noise traders is increasing, which is consistent with Jeanne and Rose (2002)’s model. 2. The impact of fundamental monetary shocks on the exchange rate is decreasing in the number of noise traders. Meanwhile, the effect of expectation error on exchange rate is increasing when there is more noise traders on the market. 3. The exchange rate volatility is higher when the magnification coefficient $\lambda$ increases.

These findings seem not surprising, a volatile shock is just introduced into the whole system, so intuitively the volatility of exchange rate should increase. However, this argument is not quite accurate. First, this random error has no effect on the volatilities of other macroeconomic variables, as stated in Section 3. Secondly and more importantly, the erroneous stochastic belief of the noise traders (a deviation from rational expectation in the model) is not a pure random shock. Instead, the unconditional volatility of the stochastic belief is proportional to the unconditional volatility of the nominal exchange rate. In other words, if the exchange rate volatility is zero, then the volatility of noise traders’ expectation errors is also zero - there is no noise at all. The noise traders have erroneous stochastic belief only because the exchange rate is itself volatile. Therefore, the causal relationship could not be simply described as “stochastic belief causes volatile exchange rate”. A more appropriate way to describe their relationship is that they are interdependent. This proves that the noise traders’ role in the market is not for adding some noise. They affect the whole system in an intrinsic way.

Therefore, the critical finding of this model is that the exchange rate volatility exceed that of other relative price and aggregate variables. In short, it suggests a ‘disconnection’ between the exchange rate and the macroeconomic fundamentals.

The last three rows in Table 2 and Table 3 report the volatilities of these macroeconomic fundamental variables, given the calibrated parameter values.

---

23Please refer to Appendix B for the details of the method used in the simulation to make $\text{Var}(v_t) = \lambda \text{Var}(s_t)$. 

39
From Table 2 and Table 3, we can see that there does exist disconnection between exchange rate volatility and macroeconomic fundamental volatility in this model. Thus, the presence of noise traders in foreign exchange market, combined with local currency pricing, implies a degree of exchange rate volatility that may be much higher than the underlying fundamental shocks. In other words, this finding implies that it is possible to explain the "exchange rate disconnect puzzle" by the approach suggested in this paper.

To understand the intuition why the disconnect puzzle can be solved in such a model, we should look at the case where there is no noise traders. Obviously, the presence of local currency pricing tends to remove the expenditure-switching or substitution effects of exchange rate movements. However, with just local currency pricing, the dynamic model will not generate a highly volatile exchange rate and the disconnection as an exchange rate shock also affects the home real interest rates through interest rate parity condition.

Rewrite Equation 3.32, omitting the expectation error and the risk premium term, we will have:

\[ \rho E_t(c_{t+1}^H - c_{t+1}^*) = \rho (c_t^H - c_t^*) - s_t \]  \hspace{1cm} (4.1)

Equation 3.29 and 4.1 illustrate why the exchange rate volatility is limited without noise traders. When there is a depreciation of home currency, the domestic currency value of foreign sales will increase and this results in an increase in home wealth. Equation 3.29 indicates that this positive wealth effect will increase both current and future relative consumption of home country. Meanwhile, as an arbitrage condition, the the interest parity condition 4.1 implies that a depreciation of home currency today will reduce the relative real interest rate in the home country and change the path of consumption so that the current home consumption will increase, relative to the expected future consumption (holding foreign consumption constant). If the changes in exchange rate is large and we need to have a disconnect between consumption and exchange rate (that means the changes in current consumption has to be small), the only possible way suggested by Equation 4.1 is that the expected future consumption drops a lot. However, this contradicts to the implication of the wealth effects of the home currency depreciation.
When the noise traders are introduced into the UIRP equation, we see from Equation 3.32 that now a large increase of exchange rate and a small change in current consumption do not necessarily imply a large drop of expected future consumption, because the presence of the expectation errors and the risk premium term of noise traders also drive wedges between home and foreign real interest rates. This could be called as the “level effects” created by the noise traders.

The noise traders also create some “volatility effects”. The first volatility effect comes from the assumption that the volatility of \( \nu_t \) itself is proportional to exchange rate volatility. This assumption implies that the nominal exchange rate volatility is not limited by the response of current account, and also “magnifies” the response of exchange rate volatility to the expectation error of noise traders. When the nominal exchange rate volatility increases, so is that of the expectation error, this further increase the exchange rate volatility until the system reaches an equilibrium where \( \text{Var}(\nu_t) = \lambda \text{Var}(s_t) \). That’s why we can call the parameter \( \lambda \) magnification parameter, the higher is \( \lambda \), the higher is the exchange rate volatility.

Another volatility effect comes from the presence of conditional exchange rate volatility in the risk premium term. This term works in the opposite direction of the magnification effect. It prevents the exchange rate volatility from increasing unboundedly. In this sense, the theoretical prediction of this model is more realistic.

Therefore, we can get volatile nominal exchange rate in this model. But why the high volatility is not transferred to other macroeconomic variables (except for the net foreign assets)?

Normally, there are two channels through which the exchange rate affects other macroeconomic variables: “expenditure-switching(substitution) effects” and “wealth effects”. The setting of local currency pricing in this model shuts off the “expenditure-switching” channel.

As to the “wealth effects”, we can see from Equation 3.29 that, the relative wealth increase from an unexpected depreciation will be spread between an increase in relative
home consumption and net foreign asset accumulation. But from Equation 3.26, the home consumption is limited by the relative money shocks. In other words, the net foreign asset will absorb most of the wealth increase. This is actually shown in Table 2 and Table 3: when the volatility of the nominal exchange rate increases, so is the volatility of the net foreign assets. However, the magnitude of the volatility of the net foreign asset and the expected future relative consumption are quite small compared to that of the exchange rate. When we increase the steady state value of price and consumption ($\bar{P} \bar{C}$), the volatility of net foreign asset will increase, however, the volatility of the expected future consumption will still be small. In other words, the wealth effect is quite small here. How is that possible? From Equation 3.32, we see that this is because $\sigma$ is quite big. (For current calibration, it is $1.4 \times 10^{-4} = 35$). Recall that $\sigma = 1 - \frac{1-(1-\gamma)\rho}{1+\psi \gamma}$, so it is decided by the elasticity of substitution between home and foreign composite goods, the elasticity of labor supply and the coefficient of risk aversion of households. This implies that the consumption smoothing behavior of households limits the wealth effect in this model. When there is a temporary shock that leads to a temporary exchange rate depreciation, the net foreign asset will absorb most of the wealth effect. The increase in the net foreign asset will be spread over many future periods because the household want to smooth their future consumptions. Therefore, the increase in the expected consumption of next period is quite small.

### 4.3 Endogenous Entry

The ‘exogenous entry’ specification of my model gives important implications of the model. However, will the exchange rate disconnection still holds when the size of the noise component is endogenously determined by introducing heterogenous entry costs for traders? Moreover, when there exists excess volatility due to stochastic, erroneous beliefs of traders, what are the welfare implications? And what can the monetary authorities do to get rid of this excess volatility in nominal exchange rate? To answer these questions, we need to look at the second specification of the model: endogenous entry model. The study of endogenous entry will help us to do policy evaluation, especially for policies that aim at
controlling the non-fundamental risk.

Table 4 illustrates the findings from the endogenous entry specification: 1. The exchange rate disconnection still holds in this specification. 2. Given the number of informed trader: \( N_I \), increasing the entry cost \( \bar{c} \) will reduce the noise component on the foreign exchange market (the number of incumbent noise trader: \( n \)), and this helps to reduce the exchange rate volatility.

The first finding is not surprising, all the theoretical analysis for the exogenous entry case holds in this model as well. However, the second finding is really important. It shows that the exchange rate policies that aim at the non-fundamental risk can be justified theoretically. Also, it suggests possible approaches the monetary authorities might apply to reduce the excess exchange rate volatility - discourages the entrance of noise traders by increasing the entry cost or ‘educates’ the market to reduce the number of potential noise traders on the foreign exchange market. Moreover, it imply that the fixed exchange rate regime might help to reduce this kind of excess exchange rate volatility because of the commitment to low exchange rate volatility by monetary authorities. This effect is not emphasized in the current theoretical literature.

These issues are all worth further investigation. Besides, the explicit welfare specification in this model provides a good context under which rigorous policy analysis and welfare evaluation can be performed. The main purpose of this paper is to show the integration of the new open macro approach and the microstructure approach could help to explain the exchange rate disconnect puzzle, so we will leave these topics for future discussion.

5 Conclusion and Subsequent Research

In this paper we have presented a model of exchange rate determination which combines the new open macroeconomics approach and the noise trader approach on exchange rate behavior. This model emphasizes the interaction of the macroeconomic fundamentals of exchange rate and the microstructure channel through which exchange rates are deter-
mined. The latter factor is often ignored by the conventional macroeconomic research
on exchange rate and policy evaluation literature. So it has important implications for
understanding exchange rate behaviors and exchange rate policies.

Two important and promising findings of this model are: 1. Models that take both the
macroeconomic and microeconomic factors of exchange rate determination into consider-
ation can explain the “exchange rate disconnect puzzle”. 2. The policymakers’ concern
about irrational market behavior or non-fundamental shocks could be justified in this kind
of model. Therefore, as Jeanne and Rose (2002), this paper also rationalizes the exchange
rate policies that control the exchange rate volatilities by affecting the composition of the
foreign exchange market.

Thus, subsequent research should focus on the policy implication of this model. What
kind of exchange rate regime is better when there exist non-fundamental shocks to ex-
change rates, flexible or fixed? If the real exchange rate volatility is primarily affected by
non-fundamental factors and most exchange rate volatility is useless, then is it therefore
ture that we will be better off with fixed exchange rate regime or a single currency area?
The presentation of explicit utility and profit maximization problem in my model allows
for the possibility of answering these questions based on rigorous analysis.

Meanwhile, this model could also be used to evaluate such exchange rate policies as
Tobin’s tax or other policies that discourage the entry of noise traders. These policies
are discussed widely, but due to the lack of a fully articulated model of exchange rate
determination, have not been evaluated on welfare basis. The new open economy macroe-
conomic framework of my model provides a rigorous context for the welfare analysis of
these policies.

Although this model could help to explain the exchange rate disconnect puzzle, it is
still not a full developed model that could be used to explain all the empirical features
of exchange rate. To explain the persistence of the real exchange rate, we might need
more persistent price setting or ‘sticky’ information of traders. For example, we could
change the information structure of the noise traders such that the expectation error is
more persistent.

Another interesting direction for future research would be the empirical studies suggested by this model. We have developed testable hypothesis about the nature of exchange rate volatility and exchange rate disconnect. The higher the degree of local currency pricing, the greater will be the disconnection between exchange rate and macroeconomic fundamentals. Also, our model implies the following predictions about the features of foreign exchange market: The deviations from UIRP will be greater and the trading volume of foreign exchange will be higher, when there is more noise traders on the market.
<table>
<thead>
<tr>
<th>No. of Noise Traders</th>
<th>( \tilde{s}_t )</th>
<th>( \text{Var}(\tilde{s}_t) )</th>
<th>( \text{Increase of } \text{Var}(\tilde{s}_t) )</th>
<th>( \text{Var}(dB_{t+1}) )</th>
<th>( \text{Increase of } \text{Var}(dB^*_{t+1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \tilde{s}_t = 0.8963\tilde{m}_t - 0.8963\tilde{m}^*_t )</td>
<td>0.0157</td>
<td>0.00%</td>
<td>2.5496E-06</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.1</td>
<td>( \tilde{s}_t = 0.8963\tilde{m}_t - 0.8963\tilde{m}^*_t + 0.0793v_t )</td>
<td>0.0159</td>
<td>1.27%</td>
<td>2.6851E-06</td>
<td>5.10%</td>
</tr>
<tr>
<td>0.2</td>
<td>( \tilde{s}_t = 0.8962\tilde{m}_t - 0.8962\tilde{m}^*_t + 0.1585v_t )</td>
<td>0.0163</td>
<td>3.82%</td>
<td>3.0052E-06</td>
<td>18.04%</td>
</tr>
<tr>
<td>0.3</td>
<td>( \tilde{s}_t = 0.8961\tilde{m}_t - 0.8961\tilde{m}^*_t + 0.2377v_t )</td>
<td>0.0171</td>
<td>8.92%</td>
<td>3.5442E-06</td>
<td>38.82%</td>
</tr>
<tr>
<td>0.4</td>
<td>( \tilde{s}_t = 0.8959\tilde{m}_t - 0.8959\tilde{m}^*_t + 0.3167v_t )</td>
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<td>15.29%</td>
<td>4.3433E-06</td>
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<tr>
<td>0.5</td>
<td>( \tilde{s}_t = 0.8956\tilde{m}_t - 0.8956\tilde{m}^*_t + 0.3956v_t )</td>
<td>0.0196</td>
<td>24.84%</td>
<td>5.4900E-06</td>
<td>115.29%</td>
</tr>
<tr>
<td>0.6</td>
<td>( \tilde{s}_t = 0.8952\tilde{m}_t - 0.8952\tilde{m}^*_t + 0.4743v_t )</td>
<td>0.0217</td>
<td>38.22%</td>
<td>7.1300E-06</td>
<td>179.61%</td>
</tr>
<tr>
<td>0.7</td>
<td>( \tilde{s}_t = 0.8946\tilde{m}_t - 0.8946\tilde{m}^*_t + 0.5525v_t )</td>
<td>0.0247</td>
<td>57.32%</td>
<td>8.1500E-06</td>
<td>219.61%</td>
</tr>
<tr>
<td>0.8</td>
<td>( \tilde{s}_t = 0.8937\tilde{m}_t - 0.8937\tilde{m}^*_t + 0.63v_t )</td>
<td>0.0292</td>
<td>85.99%</td>
<td>1.3170E-05</td>
<td>416.47%</td>
</tr>
<tr>
<td>0.9</td>
<td>( \tilde{s}_t = 0.8923\tilde{m}_t - 0.8923\tilde{m}^*_t + 0.7061v_t )</td>
<td>0.0363</td>
<td>131.21%</td>
<td>1.9010E-05</td>
<td>645.49%</td>
</tr>
<tr>
<td>1.0</td>
<td>( \tilde{s}_t = 0.8914\tilde{m}_t - 0.8914\tilde{m}^*_t + 0.7827v_t )</td>
<td>0.0490</td>
<td>212.10%</td>
<td>2.9390E-05</td>
<td>1052.55%</td>
</tr>
</tbody>
</table>

Consumption \( \text{Var}(\tilde{c}_t) = \text{Var}(\tilde{c}^*_t) = 0.0025 \)

Home wage \( \text{Var}(\tilde{w}_t) = 0.0163 \)

Home Labor \( \text{Var}(\tilde{l}_t) = 0.0013 \)
<table>
<thead>
<tr>
<th>No. of Noise Trader</th>
<th>( \hat{s}_t )</th>
<th>( \text{Var}(\hat{s}_t) )</th>
<th>Increase of ( \text{Var}(\hat{s}_t) )</th>
<th>( \text{Var}(dB_{t+1}) )</th>
<th>Increase of ( \text{Var}(dB_{t+1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \hat{s}_t = 0.8963\bar{m}_t - 0.8963\bar{m}_t^* )</td>
<td>0.0157</td>
<td>0.00%</td>
<td>2.5000E-06</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.1</td>
<td>( \hat{s}_t = 0.8963\bar{m}_t - 0.8963\bar{m}_t^* + 0.0793v_t )</td>
<td>0.0160</td>
<td>1.91%</td>
<td>2.8000E-06</td>
<td>12%</td>
</tr>
<tr>
<td>0.2</td>
<td>( \hat{s}_t = 0.8961\bar{m}_t - 0.8961\bar{m}_t^* + 0.1585v_t )</td>
<td>0.0169</td>
<td>7.64%</td>
<td>3.4000E-06</td>
<td>36%</td>
</tr>
<tr>
<td>0.3</td>
<td>( \hat{s}_t = 0.8959\bar{m}_t - 0.8959\bar{m}_t^* + 0.2375v_t )</td>
<td>0.0184</td>
<td>17.20%</td>
<td>4.6000E-06</td>
<td>84%</td>
</tr>
<tr>
<td>0.4</td>
<td>( \hat{s}_t = 0.8954\bar{m}_t - 0.8954\bar{m}_t^* + 0.3163v_t )</td>
<td>0.0208</td>
<td>32.48%</td>
<td>6.5000E-06</td>
<td>160%</td>
</tr>
<tr>
<td>0.5</td>
<td>( \hat{s}_t = 0.8946\bar{m}_t - 0.8946\bar{m}_t^* + 0.3946v_t )</td>
<td>0.0249</td>
<td>58.60%</td>
<td>9.7000E-06</td>
<td>288%</td>
</tr>
<tr>
<td>0.6</td>
<td>( \hat{s}_t = 0.8931\bar{m}_t - 0.8931\bar{m}_t^* + 0.4718v_t )</td>
<td>0.0321</td>
<td>104.46%</td>
<td>1.5600E-05</td>
<td>524%</td>
</tr>
<tr>
<td>0.7</td>
<td>( \hat{s}_t = 0.8901\bar{m}_t - 0.8901\bar{m}_t^* + 0.5461v_t )</td>
<td>0.0473</td>
<td>201.27%</td>
<td>2.8000E-05</td>
<td>1020%</td>
</tr>
<tr>
<td>0.8</td>
<td>( \hat{s}_t = 0.8822\bar{m}_t - 0.8822\bar{m}_t^* + 0.6116v_t )</td>
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<td>452.87%</td>
<td>6.0500E-05</td>
<td>2320%</td>
</tr>
<tr>
<td>0.9</td>
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<td>0.1915</td>
<td>1119.75%</td>
<td>1.4720E-04</td>
<td>5788%</td>
</tr>
<tr>
<td>1.0</td>
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<td>0.3623</td>
<td>2207.64%</td>
<td>2.8890E-04</td>
<td>11456%</td>
</tr>
</tbody>
</table>

**Consumption**

\( \text{Var}(\hat{c}_t) = \text{Var}(\hat{c}_t^*) = 0.0025 \)

**Home wage**

\( \text{Var}(\hat{w}_t) = 0.0163 \)

**Home Labor**

\( \text{Var}(\hat{l}_t) = 0.0013 \)
<table>
<thead>
<tr>
<th>$N_I = 0.2$</th>
<th>$\bar{c} = 1$</th>
<th>$\bar{c} = 1.5$</th>
<th>$\bar{c} = 2$</th>
<th>$N_I = 0.3$</th>
<th>$\bar{c} = 1$</th>
<th>$\bar{c} = 1.5$</th>
<th>$\bar{c} = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Var(\tilde{s}_t)$</td>
<td>0.1027</td>
<td>0.0512</td>
<td>0.0371</td>
<td>$Var(\tilde{s}_t)$</td>
<td>0.045</td>
<td>0.0287</td>
<td>0.0235</td>
</tr>
<tr>
<td>$Var(dB_{t+1})$</td>
<td>8.13E-05</td>
<td>3.79E-05</td>
<td>2.57E-05</td>
<td>$Var(dB_{t+1})$</td>
<td>3.26E-05</td>
<td>1.85E-05</td>
<td>1.37E-05</td>
</tr>
<tr>
<td>$Var(v_t)$</td>
<td>0.2055</td>
<td>0.1024</td>
<td>0.0741</td>
<td>$Var(v_t)$</td>
<td>0.0901</td>
<td>0.0573</td>
<td>0.047</td>
</tr>
<tr>
<td>$Mean(n)$</td>
<td>0.1392</td>
<td>0.1275</td>
<td>0.1219</td>
<td>$Mean(n)$</td>
<td>0.1811</td>
<td>0.1688</td>
<td>0.1574</td>
</tr>
<tr>
<td>$N_I = 0.4$</td>
<td>$\bar{c} = 1$</td>
<td>$\bar{c} = 1.5$</td>
<td>$\bar{c} = 2$</td>
<td>$N_I = 0.5$</td>
<td>$\bar{c} = 1$</td>
<td>$\bar{c} = 1.5$</td>
<td>$\bar{c} = 2$</td>
</tr>
<tr>
<td>$Var(\tilde{s}_t)$</td>
<td>0.0274</td>
<td>0.022</td>
<td>0.0197</td>
<td>$Var(\tilde{s}_t)$</td>
<td>0.0214</td>
<td>0.0185</td>
<td>0.0169</td>
</tr>
<tr>
<td>$Var(dB_{t+1})$</td>
<td>1.73E-05</td>
<td>1.20E-05</td>
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<td>$Var(dB_{t+1})$</td>
<td>1.08E-05</td>
<td>8.06E-06</td>
<td>6.48E-06</td>
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<tr>
<td>$Var(v_t)$</td>
<td>0.0548</td>
<td>0.0441</td>
<td>0.0395</td>
<td>$Var(v_t)$</td>
<td>0.0428</td>
<td>0.0371</td>
<td>0.0338</td>
</tr>
<tr>
<td>$Mean(n)$</td>
<td>0.2169</td>
<td>0.1931</td>
<td>0.1739</td>
<td>$Mean(n)$</td>
<td>0.2251</td>
<td>0.1953</td>
<td>0.1729</td>
</tr>
</tbody>
</table>

**Consumption**  
$Var(\tilde{c}_t) = Var(\tilde{c}_t') = 0.0025$

**Home wage**  
$Var(\tilde{w}_t) = 0.0163$

**Home Labor**  
$Var(\tilde{l}_t) = 0.0013$
APPENDIX

A Entry Condition of Noise Traders

This appendix derives the entry condition (Equation 2.71) for noise traders to enter the foreign bond market. The noise trader \( i \) will enter the foreign bond market if and only if:

\[
E^i_t(U^i_t | \varphi^i_t = 1) \geq E^i_t(U^i_t | \varphi^i_t = 0) \tag{A.1}
\]

Without entry, trader \( i \)'s expected utility is given by:

\[
E^i_t(U^i_t | \varphi^i_t = 0) = -\exp(-a(0)) = -1 \tag{A.2}
\]

While if he enters, it is given by:

\[
E^i_t(U^i_t | \varphi^i_t = 1) = E^i_t \left\{ \max_{B^*_{h,t+1}(i)} \left\{ -\alpha \left[ \frac{B^*_{h,t+1}(i)}{P_{t+1}} S_t(1 + r_{t+1})\rho_{t+1} \right] + ac_i \right\} \right\} \tag{A.3}
\]

Substitute the optimal demand for foreign bonds, \( B^*_{h,t+1}(i) = \frac{E^N_t[\rho_{t+1}]}{a\alpha \frac{E_t}{r_{t+1} + (1 + r_{t+1})Var_t[\rho_{t+1}]}} \), into above equation to get:

\[
\varphi^i_t = 1 \iff E^i_t \left\{ \exp \left[ -\frac{E^N_t[\rho_{t+1}]\rho_{t+1}}{Var_t[\rho_{t+1}]} + ac_i \right] \right\} \leq 1 \tag{A.4}
\]

Notice that for noise traders:

\[
\mathcal{L}^N_t(\rho_{t+1}) \sim N(E^N_t[\rho_{t+1}], Var_t[\rho_{t+1}]) \tag{A.5}
\]

Therefore, Equation A.4 is equivalent to:

\[
\int_{-\infty}^{+\infty} \exp \left[ -\frac{E^N_t[\rho_{t+1}]\rho_{t+1}}{Var_t[\rho_{t+1}]} + ac_i \right] \frac{1}{\sqrt{2\pi} \sqrt{Var_t[\rho_{t+1}]}} \exp \left[ -\frac{(\rho_{t+1} - E^N_t[\rho_{t+1}])^2}{2Var_t[\rho_{t+1}]} \right] d\rho_{t+1} \leq 1 \tag{A.6}
\]
or:
\[
\int_{-\infty}^{+\infty} \exp \left[ - \frac{\mu_t^2}{2 \text{Var}_t[\rho_t+1]} + ac_i \right] \frac{1}{\sqrt{2\pi \text{Var}_t[\rho_t+1]}} d\rho_{t+1} \leq 1
\]  
(A.7)

\[
\Rightarrow \int_{-\infty}^{+\infty} \exp \left\{ - \frac{\rho_{t+1}^2 + (E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} + ac_i \right\} \frac{1}{\sqrt{2\pi \text{Var}_t[\rho_t+1]}} d\rho_{t+1} \leq 1
\]  
(A.8)

\[
\Rightarrow \int_{-\infty}^{+\infty} \exp \left[ - \frac{\rho_{t+1}^2 + (E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} + \mu_0 \right] \frac{1}{\sqrt{2\pi \text{Var}_t[\rho_t+1]}} d\rho_{t+1} \leq 1
\]  
(A.9)

where \( \mu_0 = -\frac{(E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} + ac_i \).

By the property of normal distribution\(^{24} \), it could be shown that:
\[
\varphi^i_t = 1 \iff \exp \left[ - \frac{(E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} + ac_i \right] \leq 1
\]  
(A.11)

or:
\[
\varphi^i_t = 1 \iff \exp \left[ - \frac{(E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} \right] \leq \exp(-ac_i)
\]  
(A.12)

or:
\[
\varphi^i_t = 1 \iff \exp \left[ \frac{(E_t^N[\rho_{t+1}])^2}{2 \text{Var}_t[\rho_t+1]} \right] \geq \exp(ac_i)
\]  
(A.13)

As exponential functions are strictly increasing on \((-\infty, +\infty)\), Equation 2.68 could be derived:
\[
\varphi^i_t = 1 \iff c_i \leq \frac{(E_t^N(\rho_{t+1}))^2}{2a\text{Var}_t(\rho_{t+1})}
\]  
(A.14)

B The simulation of \( \text{Var}(v_t) = \lambda \text{Var}(s_t) \)

First, for a given distribution of fundamentals: \( L_0(\varepsilon_{\mu,t}, \varepsilon^*_t) \), I calculate the variance of the exchange rate when the expectation errors of the noise traders are zero. It is called \( \sigma_{S_0}^2 \).

\[
\int_{-\infty}^{+\infty} \exp \left[ - \frac{\rho_{t+1}^2}{2 \text{Var}_t[\rho_{t+1}]} \right] \frac{1}{\sqrt{2\pi \text{Var}_t[\rho_{t+1}]}} d\rho_{t+1} = 1
\]  
(A.10)
Then I assume that the distribution of the stochastic process: $v_t$ is as follows:

$$v_t = \sqrt{\lambda \sigma^2 S_0} \varepsilon_t$$  \hspace{1cm} (B.1)

where $\varepsilon_t$ is a random variable which satisfies the following three conditions:\(^{25}\)

$$Cov(\varepsilon_t, \varepsilon_{\mu,t}) = Cov(\varepsilon_t, \varepsilon_{\mu,t}^*), \quad \sigma^2 = 1$$  \hspace{1cm} (B.2)

Notice that this implies $\sigma^2_v = \lambda \sigma^2 S_0$, then I compute the variance of exchange rate: $\sigma^2_S$ given $L_0(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*)$ and the distribution of $v_t L(v_t)$ defined above. It is called: $\sigma^2_{S_1}$.

Compare $\sigma^2_{S_1}$ and $\sigma^2_{S_0}$, if

$$|\sigma^2_{S_0} - \sigma^2_{S_1}| = \epsilon, \quad \epsilon \to 0$$  \hspace{1cm} (B.3)

I will stop. Otherwise, I will redefine the stochastic process of $v_t$ and let it be:

$$v_t = \sqrt{\lambda \sigma^2 S_1} \varepsilon_t$$  \hspace{1cm} (B.4)

Notice that now $\sigma^2_v = \lambda \sigma^2 S_1$, use this distribution and $L_0(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*)$, unconditional exchange rate volatility could be computed again and we call it $\sigma^2_{S_2}$. If $|\sigma^2_{S_1} - \sigma^2_{S_2}| = \epsilon$, and $\epsilon \to 0$, the procedure stops here, otherwise, the procedure described above will be repeated to get $\sigma^2_{S_3}$, \ldots, $\sigma^2_{S_n}$, $\sigma^2_{S_{n+1}}$ until $\sigma^2_{S_{n+1}} - \sigma^2_{S_n} = \epsilon$.

### C Solve the model analytically

Due to the length of this section, it will be provided upon request.

### D Numerical Undetermined Coefficient Method

This section gives the detail of the undetermined coefficients method I use to solve for the functional form of $\tilde{s}_t$.

\(^{25}\)Note that $Cov(\varepsilon_{\mu,t}, v_t) = Cov(\varepsilon_{\mu,t}^*, v_t)$ must equal to zero, as $v_t$ is some noise and should not have any fundamental content.
First let us guess a functional form for $\tilde{s}_t$:

$$
\tilde{s}_t = \alpha_0 + \alpha_1 \tilde{m}_t + \alpha_2 \tilde{m}_t^* + \alpha_3 v_t + \alpha_4 \tilde{m}_t^2 + \alpha_5 \tilde{m}_t^{*2} + \alpha_6 v_t^2 + \alpha_7 \tilde{m}_t v_t + \alpha_8 \tilde{m}_t^* v_t + \alpha_9 \tilde{m}_t \tilde{m}_t^* \quad (D.1)
$$

Given that, we could get $E_t(\tilde{s}_{t+1})$ and $Var_t(\tilde{s}_{t+1})$ easily. Use the facts that $\tilde{m}_t = \epsilon_{\mu,t}$, $\tilde{m}_t^* = \epsilon_{\mu,t}^*$ and $Cov(\epsilon_{\mu,t}, \epsilon_{\mu,t}^*) = 0$, recall that $v_t$ is noise and should not have any fundamental content, we could get:

$$
Cov(\tilde{m}_t, \tilde{m}_t^*) = Cov(\tilde{m}_t, v_t) = Cov(\tilde{m}_t^*, v_t) = 0 \quad (D.2)
$$

Therefore,

$$
E_t(\tilde{s}_{t+1}) = \alpha_4 \sigma^2_{e_{\mu}} + \alpha_5 \sigma^2_{e_{\mu}^*} + \alpha_6 \sigma^2_v \quad (D.3)
$$

To get the conditional variance of the exchange rate, we should use the properties of the normally distributed variables and the fact that the three random variables are independent.  

$$
Var_t(\tilde{s}_{t+1}) = \alpha_4^2 Var(\tilde{m}_t) + + \alpha_5^2 Var(\tilde{m}_t^*) + \alpha_6^2 Var(v_t) + \alpha_7^2 Var(\tilde{m}_t^2) + \alpha_8^2 Var(\tilde{m}_t^{*2}) + Covariance \ terms
$$

$$
= \alpha_4^2 \sigma^2_{e_{\mu}} + \alpha_5^2 \sigma^2_{e_{\mu}^*} + \alpha_6^2 \sigma^2_v + 2\alpha_4 \sigma^2_{e_{\mu}} + 2\alpha_5 \sigma^2_{e_{\mu}^*} + 2\alpha_6 \sigma^2_v + 2\alpha_7 \sigma^2_{e_{\mu}} \sigma^2_v + 2\alpha_8 \sigma^2_{e_{\mu}^*} \sigma^2_v + \alpha_9 \sigma^2_{e_{\mu}} \sigma^2_{e_{\mu}^*} \quad (D.5)
$$

Then we could get $\tilde{s}_t$ from 3.33 and 3.45 given any exogenous shocks $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. Is our initial guess of $\alpha$ a good guess? We can do some simulation and regress the $\tilde{s}_t$ we get from above process on $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. If the coefficients($\alpha$'s) are equal to the initial guess, we stop. Otherwise, we will just repeat the above procedure. This method is actually an undetermined coefficient method, it is also called as “parameterized estimation approach” in numerical methods.

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26Notice that, if $x_t$ is normally distributed with variance $\sigma^2_x$, then

$$
E[(x_t)^{2k}] = (2k - 1)(\sigma^2_x)^k \quad E[(x_t)^{2k+1}] = 0 \quad \text{where} \ k = 1, 2, \cdots, n \quad (D.4)
$$

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References


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