Rebates as Incentives to Exclusivity†

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Abstract

We show how rebates can be used by an incumbent manufacturer in order to deter the entry of a more efficient rival. The ability to exclude depends on both the size of the cost advantage of the entrant and on the degree of competition between retailers. We find that the incumbent, by using rebates, will be able to induce to exclusivity whenever the goods produced by retailers are sufficiently close substitutes. We also find that the incumbent will be an exclusive supplier when the cost advantage of the entrant is small even if it does not offer rebates. This is not due to buyers’ disorganization but rather to the fact that retailers prefer to face a higher wholesale price in order not to be seen as aggressive; that is, retailers, by being exclusive, adopt a ‘fat cat’ strategy.

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1 Introduction

Exclusive dealing can be seen as an agreement between a manufacturer and, for example, a retailer that prohibits the latter from buying from the manufacturer’s rivals. While exclusive dealing agreements can have valid pro-competitive justifications,\(^1\) they can also be used by a manufacturer to prevent the entry of a potential competitor, to force a rival out of the market or to diminish the competitive pressure exercised by rivals.

An example of an exclusive dealing agreement can be found in the Director of Investigation and Research v. The NutraSweet Company. The agreements stipulated (p. 64) that:

“X agrees to use NutraSweet brand aspartame as the sole (or primary) intense sweetening ingredient in its [list of products by name] [diet and sugar-free products] produced in Canada during the duration of this agreement and further agrees to purchase all of its requirements of NutraSweet brand aspartame from NSC.”

Because of their potential to reduce competition, such agreements that lead \textit{directly} to exclusivity are of concern to antitrust authorities. However, antitrust authorities should also be concerned by contracts involving price structures that can achieve exclusivity \textit{indirectly}, through incentives such as rebates.

For instance, when a manufacturer enters into exclusionary vertical agreements with retailers, retailers are prohibited from selling the products of the manufacturer’s present or potential rivals. This imposes an opportunity cost on retailers equivalent to the forgone surplus associated with selling the products of the manufacturer’s rivals. Hence, to induce retailers to deal exclusively with it, the incumbent manufacturer must compensate them for this opportunity cost.

\(^1\)For example, Ornstein (1989) suggests that exclusive dealing could \(i\) avoid free-riding by an entrant on the incumbent’s product quality, image, advertising, and investment in its customers, \(ii\) protect the incumbent’s property rights in product innovation, and \(iii\) insure investments in specific assets.
If it is supposed that the agreements do not generate pro-competitive benefits that could be used as a compensation device (e.g., by sharing the value created by the transaction), then the manufacturer could induce retailers to accept the exclusionary agreements by sharing a portion of the supra-competitive profits generated by the creation, preservation, or enhancement of market power.

The sharing of supra-competitive profits could take either the form of i) a lower wholesale price charged to retailers, ii) lump-sum payments made to retailers, or iii) a combination of i) and ii). More generally, contracts can be designed in such a way that there is one level of rebates paid to retailers as long as they retain the incumbent manufacturer as their sole supplier, and another level of rebates (or none at all) if they decide to deal with an entrant.

Willard et al. (2000) suggest that rebates in the form of lump-sum payments can shift the pattern of purchases because they can be designed in such a way that their entire value is concentrated on the decision to buy marginal units. While these contracts provide the opportunity for an entrant to sell to retailers, they can be structured in a way such that the entrant must at least match the reduction in payments that would result if a retailer chose to deal with it. In effect, viewed from a retailer’s perspective, purchasing a unit from a different supplier, perhaps a new entrant, can be equivalent to facing a tax or penalty in the form of the loss of cumulative rebates. If the alternative supplier is small, then it may be unable to match the absolute dollar value of the discounts offered by the incumbent. This may represent, for a potential entrant, a very large percentage rebate based on its smaller sales volume. In turn, this could make it difficult for the potential entrant to sell enough to reach its minimum efficient scale (MES). Finally, because of the uncertainty surrounding the success of entry, a new supplier may have difficulty convincing retailers to switch to its product.\(^2\)

\(^2\)This uncertainty about an entrant’s potential profitability may arise for many reasons. For example, retailers may not purchase from an entrant if they are likely to face high switching costs (such as the loss of rebates) when
Therefore, discounting packages may represent an exclusionary device by locking in retailers through inducements. Such a pricing structure may not only injure competition, but may also injure end-customers.

In this paper, we formalize and go beyond the intuition developed in Willard et al. (2000). We find that, by offering rebates in the form of lump-sum payments, an incumbent manufacturer can induce retailers to exclusivity and deter efficient entry. This possibility will depend on the degree of competition among retailers and on the size of the cost advantage of the entrant. When exclusivity is achieved, it leads unambiguously to higher retail and wholesale prices. Moreover, when the cost advantage of the entrant is small, exclusivity arises without retailers receiving compensation. This is because retailers, by dealing with the incumbent, adopt a ‘fat cat’ strategy making them look less aggressive on the output market.

This paper is organized as follows. Section 2 reviews briefly the exclusive dealing literature and compares the assumptions used to ours. Section 3 presents the model. Section 4 focuses on the equilibria in the marketplace. Section 5 is devoted to the ability of the incumbent to induce retailers to exclusivity. Section 6 concludes. All proofs are provided in the appendix.

2 Review of the literature

Following the Chicago School critique of the foreclosure argument in exclusive dealing, a number of economists reconsidered the role of exclusive contracts in settings of strategic interactions. It was demonstrated that exclusive contracts could indeed be used for anticompetitive purposes.

they stop being exclusive to the incumbent. This is reinforced if the entrant has a limited production capacity or if it does not offer a full-line of products. Retailers may also value a supplier that has developed a reputation in product quality or time delivery; a new entrant may not enjoy such a reputation initially. For example, in The Director of Investigation and Research v. The NutraSweet Company, the Competition Tribunal recognized (p. 70) that ‘... new suppliers must become sufficiently established so that potential customers are willing to entrust all of their need for a product line to the new supplier.’

See, for example, Whinston (2001) for an exposition of the Chicago School critique.
For example, Mathewson and Winter (1987), in a model with two manufacturers and one retailer, show that the dominant manufacturer, the one with the lowest marginal production cost, can make an exclusive dealing offer that will be accepted by the retailer. Because manufacturers compete in terms of wholesale prices for the right to be chosen by the retailer, retail price, under exclusive dealing, can, for certain values of the parameters, be sufficiently low to increase welfare despite the restriction imposed on the choice set of consumers. However, for other parameter values, they find that exclusive dealing can indeed reduce welfare.

Rasmusen et al. (1991), in a model with one incumbent manufacturer and one potential entrant, show how the incumbent can use market disorganization between a large number of buyers to induce them to exclusivity and deter entry. In their setting, if each buyer thinks that others will buy exclusively from the incumbent, then each buyer may accept the incumbent’s offer because it believes that it does not have the power to influence the final outcome. The buyers’ inability to coordinate their actions leads to the result that the incumbent can convince buyers to sign exclusive contracts at little cost.

O’Brien and Shaffer (1997), in a model of complete information with two manufacturers and one retailer, examine how nonlinear contracts can lead to the exclusion of one of the manufacturers. They find that if a fully integrated retailer would sell only one good, then a manufacturer would be able to exclude its rival with a nonlinear pricing scheme if and only if it would be unprofitable for the other manufacturer to enter even when the incumbent produces its monopoly quantity. They also show that each manufacturer would earn higher profits in non-exclusive equilibria when a fully integrated retailer would sell both goods. In that case, each manufacturer earns its marginal contribution to joint profits with the retailer. Exclusivity is not possible as each manufacturer cannot generate enough profit to compensate the retailer for the lost profit on the other good.

Berheim and Whinston (1998), in a model with perfect information, have shown that manu-
facturers do not need to impose exclusivity to deter the entry of a potential rival. This can be done through nonlinear contracts. However, in the presence of incomplete contracting problems, exclusivity may arise and generate anticompetitive effects. For example, in the case of noncoincident markets where two retail markets develop sequentially and where important economies can be achieved by serving more than one market, the incumbent manufacturer can offer an exclusive contract to the retailer present in the first market. Indeed, as the incumbent manufacturer can outbid the entrant manufacturer and therefore increase its market power in the second market, the retailer in the first market will accept the exclusive contract since it will be able to extract more from the incumbent manufacturer.

Stefanadis (1998), in a model with one incumbent manufacturer and one potential entrant, allows the incumbent to offer favorable and selective (i.e., discriminatory) contracts to some buyers in an oligopolistic industry. The incumbent is able to induce to exclusivity by guaranteeing low prices to the selected buyers who gain a cost advantage over their rivals. By locking up a sufficient number of buyers, the incumbent prevents an entrant from reaching its MES.

Simpson and Wickelgren (2001) propose a model where an incumbent manufacturer has a cost advantage over a potential entrant initially because of learning-by-doing, for example. In order to deter entry, that is to prevent the entrant of making any sale that would lower its cost, the incumbent offers discounts to buyers in the form of a lower unit price. The authors suggest that the degree of downstream competition between the buyers may influence the ability of the incumbent to induce to exclusivity.

Here, we examine a situation where an incumbent manufacturer facing entry sells to retailers competing against each other in an oligopolistic market. The analysis has similar features than the ones in the exclusive dealing literature. For example, one party, the entrant, is absent at the

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4For example, when contracting between a manufacturer and a retailer today has a significant effect on future contract negotiations between the retailer and a new manufacturer that cannot contract today.
contracting stage. Moreover, the entrant has to incur a sunk cost of entry if it decides to enter the market. However, the analysis does not rest on i) uncertainty about the potential entrant’s cost (Aghion and Bolton, 1987), ii) discrimination between customers (Stefanadis, 1998), iii) presence of a future noncoincident market (Berheim and Whinston, 1998), iv) markets served by only one retailer (Mathewson and Winter, 1987), or v) the existence of a large number of small and disorganized buyers (Rasmusen et al, 1991). Rather, the incumbent, facing a potential entrant which has a cost advantage, can exploit competition between retailers to induce them to exclusivity and deter entry by using a non-discriminatory nonlinear pricing scheme. This will always be true when downstream competition is sufficiently strong. When exclusivity is achieved, the nonlinear pricing scheme leads unambiguously to higher retail and wholesale prices.\(^5\)

3 The model

The analysis focuses on pricing schemes used by an incumbent manufacturer that specify one level of payment to retailers as long as they retain the incumbent manufacturer as their sole supplier and another level of payment should they decide to deal with an entrant.

The model consists of three classes of agents: a representative end-consumer, retailers, and manufacturers. We assume that there are two retailers, retailer 1 and retailer 2, that sell symmetrically differentiated products. Furthermore, we assume that there are two manufacturers: an incumbent, denoted by \(I\), and a potential entrant, denoted by \(E\). If both manufacturers are active in the market, then they produce and sell a homogeneous good.

The demand side of the market is described by the preferences of a representative end-consumer over the set of available products in the industry during a given period. Each retailer \(i\) sells a

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\(^5\)Note that the incumbent offers his contract to both retailers simultaneously. This avoids the problem of multilateral vertical contracting (McAfee and Schwartz, 1994).
quantity $q_i$ at a price $p_i$, $i = 1, 2$. The utility function describing the demand side of the market is

$$U(q_1, q_2) = \alpha(q_1 + q_2) - (1/2) \left( q_1^2 + q_2^2 + 2\beta q_1 q_2 \right) + m. \quad (1)$$

In equation (1), $\alpha > 0$ measures the size of the market and $m$ denotes a composite commodity. The parameter $\beta \in (0, 1)$ is an indicator of the degree of substitutability between the products sold by retailers 1 and 2. If $\beta = 0$, the goods are independent; if $0 < \beta < 1$, the goods are imperfect substitutes; and, if $\beta = 1$, the goods are perfect substitutes. Therefore, the degree of substitutability between the two products is increasing in $\beta$.

From the consumer’s maximization problem, it is possible to obtain the demand function for good $i$, $i = 1, 2$, $i \neq j$,

$$q_i = \frac{\alpha(1 - \beta)}{1 - \beta^2} - \frac{p_i}{1 - \beta^2} + \frac{\beta p_j}{1 - \beta^2}. \quad (2)$$

Given these demand functions, assuming that retailer $i$ ($j$) faces a wholesale price $w_i$ ($w_j$), and supposing that retailers compete in price, the following equilibrium price, quantity and profit are obtained for each retailer

$$p_i(w_i, w_j) = \frac{(1 - \beta)(2 + \beta)\alpha + 2w_i + \beta w_j}{4 - \beta^2} \quad (3)$$

$$q_i(w_i, w_j) = \frac{p_i - w_i}{1 - \beta^2} \quad (4)$$

$$\pi_i(w_i, w_j) = \frac{(p_i - w_i)^2}{1 - \beta^2} \quad (5)$$

with $i = 1, 2$, $i \neq j$.\(^6\)

Since it is assumed that retailers transform a manufacturer’s product one for one, the demand for the products of active manufacturers (those present in the market) is $\sum_{i=1}^{2} q_i(w_1, w_2)$.

It is assumed that the incumbent has already incurred a sunk cost of entry and has a constant marginal production cost $c_I = c > 0$. If the entrant decides to come into the market, then it has to

\(^6\)We focus on parameters such that prices and quantities demanded are positive, and second-order conditions are satisfied. Moreover, we assume that the retailers’ only cost is the wholesale price they face.
incur a sunk cost of entry $R$ and has a constant marginal production cost $c_E = c - \varepsilon$, with $0 < \varepsilon < c$. The technology used by the entrant is not ‘drastic’.\footnote{See Kamien (1992).} This assumes that the entrant, acting as a monopolist, would not set a price below the marginal cost of the incumbent. Algebraically, it means that $K > 1$, where $K = (\alpha - c)/\varepsilon$. The sunk cost of entry is sufficiently small such that if the entrant can make a sale at a price above $c_E$ it will enter.\footnote{Having a small but positive sunk cost of entry avoids equilibria where the entrant would come in and produce nothing.} Each manufacturer has the capacity to serve the entire market and there is no fixed cost of production.

The interactions between the incumbent manufacturer, the potential entrant, and retailers are described in five stages. At the first stage, the incumbent offers a contract to retailers that specifies a wholesale price, $w \geq 0$, and a bonus, $x \geq 0$, that takes the form of a fixed payment paid at the end of the last stage of the game if a retailer purchases exclusively from the incumbent. The contract, which is observable by all, is costlessly enforceable by the courts. At the second stage, retailers decide simultaneously whether to accept or refuse the incumbent’s contract. A retailer that accepts this contract, a ‘signed’ retailer, must purchase all of its requirements from the incumbent in order to obtain the bonus. A retailer that refuses the contract, a ‘free’ retailer, can purchase its requirements from any active manufacturer. The incumbent and the retailers, when making their decision in the first two stages, know that a potential entrant exists. It is at the third stage of the game that $E$ decides whether or not to enter. At the fourth stage, active manufacturers set prices for free retailers. It is assumed that free retailers divide themselves equally between active manufacturers charging the same wholesale price. Finally, at the fifth stage, retailers compete in price by offering symmetrically differentiated products and signed retailers obtain their bonus from $I$. 

\textsuperscript{7}See Kamien (1992).\textsuperscript{8}Having a small but positive sunk cost of entry avoids equilibria where the entrant would come in and produce nothing.
4 Equilibria in the marketplace

To determine the contract \((w, x)\) that the incumbent will propose in equilibrium, we must first determine the players’ decisions in the last three stages of the game. That is, we must determine the equilibria of the last three stages of the game for three possible scenarios:

**Scenario 0.** No retailer accepts the incumbent’s contract.

**Scenario 1.** One retailer accepts the incumbent’s contract.

**Scenario 2.** Both retailers accept the incumbent’s contract.

With these equilibria in hand, we will be able to determine the equilibrium decisions at the second and first stages of the game.

4.1 Scenario 0

In scenario 0, both retailers refuse the contract proposed by the incumbent at the first stage of the game. Because of its cost advantage, the entrant can profitably undercut the incumbent. This results in a wholesale price of \(w^0 = c - \Delta\) where \(\Delta < \varepsilon\) is the smallest monetary unit. Letting \(\Delta \to 0\), the equilibrium margin for each retailer is

\[
p^0 - w^0 = \frac{(1 - \beta)K\varepsilon}{2 - \beta}
\]

and the incumbent’s profit is \(\Pi^0_I = 0\). (Note that superscripts denote the scenario and that subscripts denote retailers; subscripts will be suppressed when non-essential.)

4.2 Scenario 1

In scenario 1, only one retailer, say \(i\), accepts the contract proposed by the incumbent at the first stage of the game. Since it is assumed that the sunk cost of entry is sufficiently small for \(E\) to
obtain strictly positive profit if it makes a sale at a price above $c_E$, $E$ enters. However, to make any sales, the entrant has no other choice than to set a price equal to or less than the price charged by the incumbent to the free retailer, retailer $j$. In fact, undercutting $I$ is a strictly dominant strategy for $E$.

In scenario 1, the incumbent has two equilibrium strategies. At the first equilibrium strategy of $I$, labeled $A$, the undercutting process to serve the free retailer, in stage 4, leads the incumbent to charge retailer $j$ a price equal to $c$. As for $E$, it charges a price $c - \Delta$ to retailer $j$. Consequently, the free retailer purchases all its requirement from the entrant. Taking into account that the free retailer incurs a cost of $c - \Delta$ per unit of input, the incumbent sets, at the first stage of the game, a price $w_i^{1A} = \arg \max_{w_i}(w_i - c)q_i(w_i, c - \Delta)$ to the signed retailer.

Letting $\Delta \to 0$, we find that retailers $i$ and $j$ face the following wholesale prices

\[
\begin{align*}
w_i^{1A} &= \frac{(1 - \beta)(2 + \beta)K\varepsilon}{2(2 - \beta^2)} + c \\
w_j^{1A} &= c
\end{align*}
\]

respectively. The margins of the signed and free retailers are

\[
\begin{align*}
p_i^{1A} - w_i^{1A} &= \frac{(1 - \beta)K\varepsilon}{2(2 - \beta)} \\
p_j^{1A} - w_j^{1A} &= \frac{(1 - \beta)[2(2 - \beta^2) + \beta]K\varepsilon}{2(2 - \beta)(2 - \beta^2)}
\end{align*}
\]

respectively, and the pre-bonus profit of the incumbent is

\[
\Pi_I^{1A} = \frac{(1 - \beta)^2(2 + \beta)K^2\varepsilon^2}{4(2 - \beta)(1 - \beta^2)(2 - \beta^2)}.
\]

The incumbent has a second equilibrium strategy, labeled $B$. Instead of trying to undercut $E$ to serve the free retailer in stage 4, the incumbent has an incentive to charge the highest wholesale price possible to retailer $j$. This comes from the fact that the profit the incumbent obtains from the signed retailer is increasing in the wholesale price of the free retailer. (Recall that, in stage
4, the incumbent chooses the wholesale price charged to the free retailer to maximizes its profits from both the free retailer and the signed retailer.) Thus, \( I \) sets a price for the free retailer equal to \( \infty \). As a consequence, the price charged by \( E \) to retailer \( j \) is equal to \( w_j^{1B} \) where \( w_j^{1B} = \arg\max_{w_j} [w_j - (c - \varepsilon)]q_j(w_i, w_j) \). Since \( w_j^{1B} < \infty \), the free retailer purchases all its requirement from the entrant. Taking into account that the free retailer incurs a cost of \( w_j^{1B} \) per unit of input, the incumbent sets, at the first stage of the game, a price \( w_i^{1B} = \arg\max_{w_i} (w_i - c)q_i(w_i, w_j^{1B}) \) to the signed retailer.

Let us define \( \Omega \) and \( K(\beta) \) in the following way

\[
\Omega = 2(2 - \beta^2)^2 - \beta^2
\]
\[
K(\beta) = \frac{\beta(2 - \beta^2)}{\Omega - \beta(2 - \beta^2)}
\]

where \( K(\beta) \geq 1 \) if and only if \( \beta \geq \hat{\beta} \) with \( \hat{\beta} = 0.887 \).

We will focus only on situations where \( K > \max\{1, \overline{K}(\beta)\} \). Indeed, we want to consider only situations where the innovation is non-drastic, i.e., \( K > 1 \), and where both retailers can obtain non-negative sales and profits, i.e., \( K \geq K(\beta) \).

We find that retailers \( i \) and \( j \) face the following wholesale prices

\[
w_i^{1B} = \begin{cases} 
\frac{[\Omega - \beta(2 - \beta^2)]K\varepsilon - \beta(2 - \beta^2)\varepsilon}{2\Omega} + c & \text{if } K > \overline{K}(\beta) \\
c & \text{if } K = \overline{K}(\beta)
\end{cases}
\]

\[
w_j^{1B} = \begin{cases} 
\frac{(2 - \beta^2)(2\Omega - \beta^2)(K + 1)\varepsilon - \beta\Omega K\varepsilon}{4(2 - \beta^2)\Omega} + (\bar{c} - \varepsilon) & \text{if } K > \overline{K}(\beta) \\
\frac{(2 - \beta^2)(K + 1)\varepsilon - \beta K\varepsilon}{2(2 - \beta^2)} + (\bar{c} - \varepsilon) & \text{if } K = \overline{K}(\beta)
\end{cases}
\]
respectively. The margins of the signed and free retailers are

\[ p_i^{1B} - w_i^{1B} = \begin{cases} 
\frac{[\Omega - \beta(2 - \beta^2)]K\varepsilon - \beta(2 - \beta^2)\varepsilon}{4(4 - \beta^2)(2 - \beta^2)} & \text{if } K > K(\beta) \\
0 & \text{if } K = K(\beta)
\end{cases} \]  

(10)

\[ p_j^{1B} - w_j^{1B} = \begin{cases} 
\frac{(2 - \beta^2)(2\Omega - \beta^2)(K + 1)\varepsilon - \beta\Omega K\varepsilon}{4(4 - \beta^2)\Omega} & \text{if } K > K(\beta) \\
\frac{(2 - \beta^2)(K + 1)\varepsilon - \beta K\varepsilon}{2(4 - \beta^2)} & \text{if } K = K(\beta)
\end{cases} \]  

(11)

respectively, and the pre-bonus profit of the incumbent is

\[ \Pi_i^{1B} = \begin{cases} 
\frac{\{[\Omega - \beta(2 - \beta^2)]K - \beta(2 - \beta^2)\}^2 \varepsilon^2}{8(1 - \beta^2)(4 - \beta^2)(2 - \beta^2)\Omega} & \text{if } K > K(\beta) \\
0 & \text{if } K = K(\beta)
\end{cases} \]  

(12)

4.3 Scenario 2

In scenario 2, both retailers accept the contract proposed by the incumbent at the first stage of the game. In this case, \( E \) does not enter as it cannot make positive profit. Because the incumbent offers the same contract to both retailers, the retailers’ prices, quantities, and profits are given by (3), (4), and (5), respectively, where \( w_i \) and \( w_j \) are set equal to \( w \). Facing a demand of \( \Sigma_{i=1}^{2} q_i(w, w) \), a profit-maximizing incumbent manufacturer charges a wholesale price equal to

\[ w^2 = \frac{K\varepsilon}{2} + c. \]  

(13)

The pre-bonus equilibrium margin for each retailer is

\[ p^2 - w^2 = \frac{(1 - \beta)K\varepsilon}{2(2 - \beta)} \]  

(14)
and the incumbent manufacturer’s profit, pre-bonus, is

\[ \Pi_I^2 = \frac{K^2\varepsilon^2}{2(2 - \beta)(1 + \beta)}. \] (15)

### 4.4 Acceptance or rejection of the incumbent’s contract

In order to determine the acceptance or rejection of the incumbent’s contract, we only consider situations where the entrant’s technology is non-drastic and both retailers have strictly positive sales; that is, we only consider situations where \( K > \max\{1, K(\beta)\} \). Second, we suppose that whenever a retailer obtains the same profit by accepting or rejecting \( I \)'s contract, it accepts the contract. Third, we need to rank the equilibrium profits, pre-bonus, for the retailers and the incumbent.

**Lemma 1.** Assuming that, in scenario 1, retailer \( i \) (\( j \)) is a signed (free) retailer and that the incumbent manufacturer adopts its equilibrium strategy \( A \), then

\[ \pi^A_j > \pi^0 > \pi^2 = \pi^A_i. \] (16)

Lemma 1 indicates that if the incumbent adopts a strategy that leads to equilibrium \( A \) in scenario 1, then a retailer’s profit is greater when it is the only one to refuse the contract proposed by \( I \). This comes from the fact that it has a cost advantage over its rival. On the other hand, a retailer obtains its lowest level of profit when \( i \) both retailers accept the incumbent’s contract, or \( ii \) it is the only one to accept the incumbent’s contract.

Suppose now that the incumbent adopts its equilibrium strategy \( B \) in scenario 1. Let us define \( K_1(\beta) \) such that for all \( K \geq K_1(\beta) \) we have \( \pi_i^{1B} \geq \pi^0 \), and \( K_2(\beta) \) such that for all \( K \geq K_2(\beta) \) we have \( \pi^2 \geq \pi_j^{1B} \) whenever \( K_2(\beta) \) is strictly positive. \( K_1(\beta) \) and \( K_2(\beta) \) are given by

\[
K_1(\beta) = \frac{2 - \beta^2}{(1 - \beta)(2 + \beta)},
\]

\[
K_2(\beta) = \frac{(2 - \beta^2)(2\Omega - \beta^2)}{(1 - \beta)(2 + \beta)[(2\Omega - \beta^2) - 2\beta(2 - \beta^2)]}.
\]
Lemma 2. Assuming that, in scenario 1, retailer \(i\) is a signed (free) retailer and that the incumbent manufacturer adopts its equilibrium strategy \(B\), we have the following:

i) For all \(\beta \in (0, \tilde{\beta})\) with \(\tilde{\beta} = 0.887\), we have i) \(K_2(\beta) > K_1(\beta) \geq 1 > K(\beta) > 0\), ii) \(\pi^2 \geq \pi_j^{1B} > \pi_i^{1B} > \pi^0 \forall K \geq K_2(\beta)\), iii) \(\pi_j^{1B} > \pi^2 > \pi_i^{1B} \geq \pi^0 \forall K \in [K_1(\beta), K_2(\beta))\), and iv) \(\pi_j^{1B} > \pi^2 > \pi_i^{1B} \forall K \in (1, K_1(\beta))\);

ii) For all \(\beta \in [\tilde{\beta}, 1)\) with \(\tilde{\beta} = 0.951\), we have i) \(K_2(\beta) > K_1(\beta) > K(\beta) \geq 1\), ii) \(\pi^2 \geq \pi_j^{1B} > \pi_i^{1B} > \pi^0 \forall K \geq K_2(\beta)\), iii) \(\pi_j^{1B} > \pi^2 > \pi_i^{1B} \geq \pi^0 \forall K \in [K_1(\beta), K_2(\beta))\), and iv) \(\pi_j^{1B} > \pi^2 > \pi^0 > \pi_i^{1B} \forall K \in (1, K_1(\beta))\);

iii) For all \(\beta \in [\tilde{\beta}, 1)\), we have i) \(K_1(\beta) > K(\beta) > 1 \geq 0 \geq K_2(\beta)\), ii) \(\pi_j^{1B} > \pi^2 > \pi_i^{1B} \geq \pi^0 \forall K \geq K_1(\beta)\), and iii) \(\pi_j^{1B} > \pi^2 > \pi^0 \geq \pi_i^{1B} \forall K \in (K(\beta), K_1(\beta))\).

In order to rank the incumbent’s equilibrium profits pre-bonus, let us define \(\overline{K}_1(\beta)\) such that for all \(K \geq \overline{K}_1(\beta)\) we have \(\Pi_i^{1B} \geq \Pi_i^{1A}\) where

\[
\overline{K}_1(\beta) = \frac{(2 - \beta^2)(4 + \beta - 2\beta^2) + \sqrt{24\beta}}{(1 - \beta)(2 + \beta)(8 + 3\beta - 4\beta^2)}.
\]

Lemma 3. For all \(\beta \in (0, 1)\), we have that

i) \(\Pi_k^2 > \Pi_k^{1B} \geq \Pi_k^{1A} \geq \Pi_k^0\) whenever \(K \geq \overline{K}_1(\beta)\),

ii) \(\Pi_k^2 > \Pi_k^{1A} > \Pi_k^{1B} \geq \Pi_k^0\) whenever \(K \in (\max\{1, K(\beta)\}, \overline{K}_1(\beta))\).

Figure 1 illustrates the values of \(K\) and \(\beta\) mentioned in claims 2 and 3.

Using Lemmas 1 to 3, we can now determine the Nash equilibria of the subgame starting at the second stage of the game. These are given in the following proposition.

Proposition 1. The Nash equilibria of the subgame starting at the second stage of the game are:

i) \(\{\text{Accept, Accept}\}, \text{Stay out}, \{p^2, p^2\}\) if \(K \geq K_2(\beta)\) and \(\beta \in (0, \tilde{\beta})\);

ii) \(\{\text{Accept, Refuse}\}, \text{Enter}, \{w_j^{1B}, w_j^{1B} - \Delta\}, \{p_i^{1B}, p_i^{1B}\}\) and \(\{\text{Refuse, Accept}\}, \text{Enter}, \{w_j^{1B}, w_j^{1B} - \Delta\}, \{p_j^{1B}, p_i^{1B}\}\) if \(K \in [K_1(\beta), K_2(\beta))\) and \(\beta \in (0, \tilde{\beta})\) or if \(K \geq K_1(\beta)\) and \(\beta \in [\tilde{\beta}, 1)\);
Part i) of proposition 1 states that when the cost advantage of the entrant is small \((K \geq K_2(\beta))\) and the retailers’ goods are not too close substitutes \((\beta \in (0, \tilde{\beta}))\), both retailers accept the contract proposed by the incumbent. As a consequence, the entrant stays out. Part ii) of proposition 1 asserts that when the cost advantage of the entrant is intermediate \((K \in [K_1(\beta), K_2(\beta)])\) and the retailers’ goods are not too close \((\beta \in (0, \tilde{\beta}))\), or when the retailers’ goods are very close substitutes \((\beta \in [\tilde{\beta}, 1])\) and the entrant’s cost advantage is not too large \((K \geq K_1(\beta))\), one retailer refuses \(I’s\) contract and one retailer accepts \(I’s\) contract. In this circumstance, \(E\) enters, and the incumbent adopts its equilibrium strategy \(B\) when competing for the free retailer. Part iii) of proposition 1 mentions that when the cost advantage of the entrant is large \((K \in [\bar{K}_1(\beta), K_1(\beta)])\), both retailers refuse \(I’s\) contract whatever the degree of differentiation between their products. In that case, \(E\) enters and charges a price just below the wholesale price of the incumbent who chooses its strategy \(B\) in scenario 1. Finally, part iv) of proposition 1 stipulates that when the cost advantage of the
entrant is very large \((K \in (\max\{1, K(\beta)\}, \overline{K}_1(\beta))\), both retailers refuse \(I\)'s contract whatever the degree of differentiation between their products. In that case, \(E\) enters and charges a price just below the marginal cost of the incumbent who chooses its strategy \(A\) in scenario 1.

5 Rebates as incentives to exclusivity

In this section, we show whether the incumbent can offer a bonus, \(x\), such that it can profitably induce both retailers to exclusivity at a subgame perfect Nash equilibrium. A retailer will accept the contract proposed by the incumbent if and only if the profit it obtains as a signed retailer is greater than, or equal to, the profit it achieves as a free retailer. Similarly, the incumbent manufacturer will offer a bonus to retailers in order to deter entry whenever the profit it obtains is greater than, or equal to, the profit it achieves when it does not induce exclusivity.

As proposition 1 illustrates, this will depend not only on the degree of product differentiation between retailers but also on the size of the cost advantage of the entrant. Indeed, the space \(K > \max\{1, K(\beta)\} \times \beta \in (0, 1)\) can be broken down in three regions: region 1 corresponds to part \(i)\) of proposition 1, region 2 corresponds to part \(ii)\) of proposition 1, and region 3 corresponds to part \(iii)\) and \(iv)\) of proposition 1.

5.1 Region 1

In region 1, the cost advantage of the entrant is small and the retailers’ goods are not too close substitutes. In this case, we obtain the following.

Claim 1. When \(K \geq K_2(\beta)\) and \(\beta \in (0, \tilde{\beta})\), the incumbent will be the exclusive supplier of both retailers without having to induce them to exclusivity.

Indeed, as shown in proposition 1, both retailers will accept the \(I\)'s contract without receiving any compensation. This result is similar to the one obtained by Rasmusen et al. (1991) but
disorganization among retailers is not necessary. Given that retailers compete in price, a strategic complement, they prefer to face a higher wholesale price. Indeed, the incentive to accept the contract offered by the incumbent can be illustrated by the total derivative of \( \pi_i(K_i, p_i^*(K_i), p_j^*(K_i)) \) with respect to \( K_i \), where \( K_i \) reflects the decision of retailer \( i \) to accept \( I \)'s contract, and where \( p_i^*(K_i) \) and \( p_j^*(K_i) \) denote the equilibrium prices of retailers \( i \) and \( j \) respectively. As Fundenberg and Tirole (1984) show, the sign of the strategic effect (the influence of firm \( i \)'s decision to accept \( I \)'s contract) on firm \( j \)'s pricing decision is given by

\[
\text{sign} \left( R_j(p_i^*) \frac{\partial p_i^*}{\partial K_i} \frac{\partial \pi_j}{\partial p_i} \right).
\]

Since the slope of the reaction curve, \( R_j(p_i^*) \), is positive, that the acceptance of \( I \)'s contract increases the equilibrium price of firm \( i \), \( \partial p_i^*/\partial K_i > 0 \), and that firm \( j \)'s profit is increasing in \( p_i \), the strategic effect is positive. Therefore, firm \( i \) should accept the incumbent’s contract. As the same reasoning applies to firm \( j \), we obtain that both firms, by accepting \( I \)'s contract, adopt a ‘fat cat’ strategy to make them look less aggressive and to induce their rival to be also less aggressive.

5.2 Region 2

Region 2 considers situations where the cost advantage of the entrant is intermediate (\( K \in [K_1(\beta), K_2(\beta)] \)) and the retailers’ goods are not too close (\( \beta \in (0, \bar{\beta}) \)), or where the cost advantage of the entrant is not too large (\( K \geq K_1(\beta) \)) and the retailers’ goods are very close substitutes (\( \beta \in [\bar{\beta}, 1) \)).

In these cases, since ‘Accept’ is a retailer’s best response to the other firm’s ‘Reject’ (see part \( ii \) of proposition 1), the incumbent only has to ensure that ‘Accept’ is a retailer’s best response to the other firm’s ‘Accept’. In other words, each retailer will accept to become exclusive to \( I \) if and
only if:

\[ x \geq \pi_j^{1B} - \pi^2. \]  \hspace{1cm} (17)

Equation (17) implies that when a firm has accepted \( I \)'s contract, then the other firm will also accept \( I \)'s contract if and only if the bonus \( x \) is greater than, or equal to, the gain of refusing \( I \)'s contract given that the other retailer has accepted it.

In turn, the incumbent will induce retailers to exclusivity if and only if it is profitable to do so, that is if and only if it obtains more by deterring entry and compensating both retailers for exclusivity than it can obtain by not inducing exclusivity and facing entry

\[ \Pi_f^2 - \Pi_f^{1B} \geq 2x. \]  \hspace{1cm} (18)

Using (17) and (18), we obtain the following.

**Claim 2.** When \( K \in [K_1(\beta), K_2(\beta)] \) and \( \beta \in [0, \tilde{\beta}) \), or when \( K \geq K_1(\beta) \) and \( \beta \in [\tilde{\beta}, 1) \), the incumbent will profitably induce both retailers to exclusivity if and only if

\[ (\Pi_f^2 - \Pi_f^{1B}) - 2(\pi_j^{1B} - \pi^2) \geq 0. \]  \hspace{1cm} (19)

Using (5), (11), (12), (14), and (15), the expression on the left hand side (LHS) of equation (19), denoted by \( f(\beta, K) \), is quadratic in \( K \). It is also convex in \( K \). Therefore, since \( \partial f(\beta, K)/\partial K \geq 0 \) for all \( \beta \in (0, 1) \) when evaluated at \( K = K_1(\beta) \), \( f(\beta, K) \) is increasing in \( K \) for all \( K \geq K_1(\beta) \) and for all \( \beta \in (0, 1) \).

Figure 2 depicts \( f(\beta, K) \) when \( K = K_1(\beta) \) and when \( K = K_2(\beta) \). On the interval \( \beta \in (0, 1) \), we find, through numerical simulations, that \( f(\beta, K_2(\beta)) \geq 0 \) for all \( \beta \in [0.716, \tilde{\beta}] \), and that \( f(\beta, K_1(\beta)) \geq 0 \) for all \( \beta \in [0.803, 1) \).

Therefore, when the retailers sell sufficiently ‘substitutable’ products and when the cost advantage of the entrant is not too large, the incumbent manufacturer will be able to profitably offer
rebates as inducements to exclusivity. As the cost advantage of the entrant decreases from \( K_1(\beta) \) to \( K_2(\beta) \), the degree of substitutability of the retailers’ products that will entice the incumbent to offer profitably rebates as inducements to exclusivity decreases as well.

The intuition is simple. Whenever \( \beta \to 1 \), the profit for any retailer at any equilibrium tends to zero because of price competition, i.e., price tends to marginal cost. Since there is no surplus available to retailers, there are no gain to be had by refusing the contract proposed by \( I \). Indeed, any retailer prefers to accept the incumbent’s contract and obtain a profit at least equal to the bonus \( x \).

Since the profit of the incumbent is greater when entry is deterred, it can profitably compensate retailers for exclusivity. However, when \( \beta \to 1 \), this compensation tends to zero.

In contrast, when \( \beta \to 0 \), retailers’ goods become poor substitutes and price competition tends to vanish. Retailers now become more concerned about the level of wholesale price they faced since it affects the surplus available to them. Indeed, as retailers’ profits are decreasing in their own wholesale price, they prefer to deal with the firm offering the lowest wholesale price and the incumbent cannot generate profit sufficiently high to induce both retailers to exclusivity.

Figure 2: \( f(\beta,K_1) \) and \( f(\beta,K_2) \)
5.3 Region 3

Whenever the cost advantage of the entrant gets larger ($K \in (\max\{1, K(\beta)\}, K_1(\beta))$), both retailers refuse $I$’s contract whatever the degree of differentiation between their products. As a result, $E$ enters and charges a price just below the wholesale price set by the incumbent. In that case, two situations arise.

In the first situation, $K \in [K_1(\beta), K_1(\beta))$. For such a value of $K$, the incumbent, along the equilibrium path that leads to the Nash equilibrium described in part iii) of proposition 1, would choose its strategy $B$ in the event that a retailer would accept its contract offered at stage 1 of the game.

Therefore, to induce both retailers to exclusivity profitably, the incumbent would have to offer a discount that respects the following conditions

\[ \pi^2 + x \geq \pi^1_j, \quad (20) \]
\[ \pi^1_i + x \geq \pi^0. \quad (21) \]

These two equations merit some discussion. In (20), $\pi^1_j$ represents the gain of refusing $I$’s contract given that the other retailer has accepted it. In (21), $\pi^0$ represents the gain of refusing $I$’s contract given that the other retailer has refused the contract. To induce both retailers to exclusivity, the incumbent must not only ensure that ‘Accept’ is a retailer’s best response to the other firm’s ‘Accept’, but must also ensure that ‘Accept’ is a retailer’s best response to the other firm’s ‘Reject’. Therefore, using (20), (21), and lemma 2, we find that each retailer will be exclusive to $I$ if and only if

\[ x \geq \max\{\pi^1_j - \pi^2, \pi^0 - \pi^1_i\} = \pi^1_j - \pi^2. \quad (22) \]

In turn, the incumbent will induce both retailers to exclusivity if and only if it is profitable to do so. This will be true if and only if it obtains more by deterring entry and compensating both
retailers for exclusivity than it can obtain by not inducing exclusivity and facing entry; that is, if

$$\Pi_2^I \geq 2x.$$  \hspace{1cm} (23)

Note that (23) is different than (18) as the incumbent’s profit equals zero if it does not induce retailers to exclusivity when the cost advantage of the entrant is large. Now, combining (22) and (23), we obtain the following.

**Claim 3.** For all $\beta \in (0,1)$ and $K \in [\overline{K}_1(\beta), K_1(\beta))$, the incumbent will profitably induce both retailers to exclusivity if and only if

$$\Pi_2^I - 2(\pi_j^{1B} - \pi_2^B) \geq 0.$$  \hspace{1cm} (24)

Using (5), (11), (14), (15), we find that the expression on the LHS of equation (24), denoted by $g(\beta, K)$, is quadratic and convex in $K$. Since $\partial g(\beta, K)/\partial K \geq 0$ for all $\beta \in (0,1)$ when evaluated at $K = 1$, $g(\beta, K)$ is decreasing in $K$ for all $K \geq 1$ and for all $\beta \in (0,1)$.

On the interval $\beta \in (0,1)$, we find, through numerical simulations, that $g(\beta, K_1(\beta)) \geq 0$ for all $\beta \in [0.667, 1)$, and that $g(\beta, \overline{K}_1(\beta)) \geq 0$ for all $\beta \in [0.795, 1)$.

Therefore, even when the cost advantage of the entrant is large, i.e., $K \in [\overline{K}_1(\beta), K_1(\beta))$, the incumbent can induce retailers to exclusivity if they sell sufficiently ‘substitutable’ products. The intuition is the same as the one developed for region 2.

In the second situation, $K \in (\max\{1, \overline{K}(\beta)\}, \overline{K}_1(\beta))$. For such value of $K$, the incumbent, along the equilibrium path that leads to the Nash equilibrium described in part iv) of proposition 1, would choose its strategy $A$ in the event that a retailer would accept its contract offered at stage 1 of the game. By an analysis similar to the one above, we obtain the following.

**Claim 4.** For all $\beta \in (0,1)$ and $K \in (\max\{1, \overline{K}(\beta)\}, \overline{K}_1(\beta))$, the incumbent will profitably induce
both retailers to exclusivity if and only if

\[ \Pi_1^2 - 2(\pi_j^{1A} - \pi^2) \geq 0. \]  

(25)

Using (5), (8), (14), (15), we find that the expression on the LHS of equation (25), denoted by \( h(\beta) \), is independent of \( K \). Through numerical simulations, we find that \( h(\beta) \geq 0 \) for all \( \beta \in [0.781, 1) \).

Therefore, even when the cost advantage of the entrant is large, the incumbent can induce retailers to exclusivity if they sell sufficiently ‘substitutable’ products.

6 Conclusion

The model presented here shows how an incumbent manufacturer can use rebates, in the form of lump-sum payments, to induce retailers to exclusivity. Moreover, the paper tries to limit as much as possible any asymmetries that create an advantage to an incumbent. We find that when the retailers’ goods are sufficiently substitutable, the incumbent, by exploiting downstream competition, will be able to achieve exclusivity even if it faces a more efficient entrant. The paper also shows that pricing schemes that involve rebates can be injurious to both competition and to end-customers. In effect, they can deter entry by creating high switching costs, in the form of lost rebates, leading to higher retail prices. Finally, when the entrant has a small cost advantage, we find the solution obtained by Rasmussen et al. (1991), i.e., that compensation is not necessary to induce to exclusivity and deter entry. However, neither a large number of buyers nor economies of scale are necessary to obtain this result.
References


Appendix

A Proof of lemma 1

Since the profit of any retailer is given by (5), it is easy to rank the retailers’ profits by inspection using (6), (7), (8), and (14).

B Proof of lemma 2

Since the profit of any retailer is given by (5), we can use equations (6), (10), (11), and (14) to rank the retailers’ profits.

1. It is straightforward to verify that $p^2 - w^2 > p^0 - w^0 \forall K \geq 0$ and $\beta \in (0,1)$.

2. Since

$$\frac{\varepsilon \{\beta (2-\beta^2) + K [3(1-\beta)(1-\beta^2) + 3(1-\beta^2) + 2(1-\beta^2)^2] \}}{4(2-\beta)(2+\beta)(2-\beta^2)} > 0$$

$\forall K \geq 0$ and $\beta \in (0,1)$, then $p^2 - w^2 \geq p^1_B - w^1_B$.

3. Since

$$\frac{\varepsilon [(2-\beta^2)(2\Omega - \beta^2) + \beta K(1-\beta)(2 + \beta)(4 + \beta - 2\beta^2)]}{4(2-\beta)(2+\beta)\Omega} > 0$$

$\forall K \geq 0$ and $\beta \in (0,1)$, then $p^1_{jB} - w^1_{jB} \geq p^0 - w^0$.

4. Since

$$\frac{\varepsilon (1+\beta)[(2-\beta^2)(16 - 4\beta - 17\beta^2 + 2\beta^3 + 4\beta^4) + \beta^2 K(2 - \beta - \beta^2)]}{4(2-\beta)(2+\beta)\Omega} > 0$$

$\forall K \geq 0$ and $\beta \in (0,1)$, then $p^1_{jB} - w^1_{jB} \geq p^1_i - w^1_i$. 
5. \( p_i^{1B} - w_i^{1B} \geq p^0 - w^0 \) if and only if

\[
K \geq \frac{2 - \beta^2}{(1 - \beta)(2 + \beta)} = K_1(\beta).
\]

Otherwise, \( p_i^{1B} - w_i^{1B} < p^0 - w^0 \).

6. \( p_j^{1B} - w_j^{1B} \geq p^2 - w^2 \) if and only if

\[
K \leq \frac{(2 - \beta^2)(2\Omega - \beta^2)}{(1 - \beta)(2 + \beta)(16 - 4\beta - 19\beta^2 + 2\beta^3 + 4\beta^4)} = K_2(\beta).
\]

For all \( \beta \in [\tilde{\beta}, 1) \) where \( \tilde{\beta} = 0.951 \), \( K_2(\beta) \leq 0 \) as \( 16 - 4\beta - 19\beta^2 + 2\beta^3 + 4\beta^4 \leq 0 \). This implies that \( p_j^{1B} - w_j^{1B} \geq p^2 - w^2 \) since \( K \) must be greater than 0. For all \( \beta \in [0, \tilde{\beta}) \), \( K_2(\beta) \geq 0 \). Therefore, for all \( K \leq K_2(\beta) \), \( p_j^{1B} - w_j^{1B} \geq p^2 - w^2 \) and for all \( K > K_2(\beta) \), \( p_j^{1B} - w_j^{1B} < p^2 - w^2 \).

7. It is easy to check that i) \( K_2(\beta) \geq 1 \) for all \( \beta \in (0, \tilde{\beta}) \), ii) \( K_2(\beta) > K_1(\beta) \) for all \( \beta \in (0, \tilde{\beta}) \), iii) \( K_2(\beta) \geq K(\beta) \) for all \( \beta \in (0, \tilde{\beta}) \), iv) \( K_1(\beta) \geq 1 \) for all \( \beta \in (0, 1) \), \( K_1(\beta) \geq K(\beta) \) for all \( \beta \in (0, 1) \), and \( K(\beta) \geq 1 \) for all \( \beta \in [\tilde{\beta}, 1) \) where \( \tilde{\beta} = 0.887 \).

Using points 1 to 7, we obtain the results of lemma 2.

### C Proof of lemma 3

In order to rank the incumbent’s equilibrium profit, we use equations (9), (12), and (15). We also use the fact that in scenario 0, \( \Pi_l^0 = 0 \).

1. Obviously, the incumbent’s profits in scenarios 1 and 2 are greater than, or equal to, \( \Pi_l^0 = 0 \), the incumbent’s profit in scenario 0.

2. It is easy to verify that \( \Pi_l^2 > \Pi_l^{1A} \forall K \geq 0 \) and \( \beta \in (0, 1) \).
3. $\Pi_1^B - \Pi_1^A$ is quadratic and convex in $K$ for all $\beta \in (0, 1)$. The two roots are

$$K_1(\beta) = \frac{(2 - \beta)(4 + \beta - 2\beta^2) - \sqrt{2\Omega}}{(1 - \beta)(2 + \beta)(8 + 3\beta - 4\beta^2)}$$

$$\overline{K}_1(\beta) = \frac{(2 - \beta)(4 + \beta - 2\beta^2) + \sqrt{2\Omega}}{(1 - \beta)(2 + \beta)(8 + 3\beta - 4\beta^2)}$$

It is easy to show that $\overline{K}_1(\beta)$ is greater than, or equal to, both 1 and $K(\beta)$ for all $\beta \in (0, 1)$ while $K_1(\beta) < \overline{K}(\beta)$ for all $\beta \in (0, 1)$. Hence, we need to focus only on $K_1(\beta)$. For $K \in [\max\{1, K(\beta)\}, \overline{K}_1(\beta))$ we have $\Pi_1^B < \Pi_1^A$, and for $K \geq K_1(\beta)$ we have $\Pi_1^B \geq \Pi_1^A$.

4. $\Pi_1^2 - \Pi_1^B$ is quadratic and convex in $K$ for all $\beta \in (0, 1)$. The two roots are

$$K_2(\beta) = \frac{\beta(2 - \beta)(4 + \beta - 2\beta^2) - \sqrt{(1 - \beta)(96 + 32\beta - 158\beta^2 + 12\beta^4 - 32\beta^5)}}{(1 - \beta)(2 + \beta)(32 - 50\beta^2 + 25\beta^4 - 4\beta^6)}$$

$$\overline{K}_2(\beta) = \frac{\beta(2 - \beta)(4 + \beta - 2\beta^2) + \sqrt{(1 - \beta)(96 + 32\beta - 158\beta^2 + 12\beta^4 - 32\beta^5)}}{(1 - \beta)(2 + \beta)(32 - 50\beta^2 + 25\beta^4 - 4\beta^6)}$$

It is easy to see that $K_2(\beta) < 0$ for all $\beta \in [0, 1]$. While, $\overline{K}_2(\beta) > 0$ for all $\beta \in (0, 1)$, $\overline{K}_2(\beta) < \overline{K}(\beta)$ for all $\beta \in (0, 1)$. Hence, for all $K \geq \overline{K}(\beta)$, and for all $\beta \in (0, 1)$, we have that $\Pi_1^2 > \Pi_1^B$.

Using points 1 to 4, we obtain the results of lemma 3.