The Marginal Cost of Funds from Public Sector Borrowing

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Abstract

An expression for the welfare cost of a marginal increase in the public debt is derived using a simple AK endogenous growth model. This measure of the marginal cost of public funds (MCF) can be interpreted as the marginal benefit-cost ratio that a debt-financed public project needs in order to generate a net social gain. The model predicts an increase in the public debt ratio will have little effect on the optimal public expenditure ratio and that most of the adjustment to an increase in the debt ratio will occur on the tax side of the budget.

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1. Introduction

This paper analyzes the opportunity cost of funds obtained by public sector borrowing. The public debt imposes a burden on an economy because higher taxes have to be levied over the long-term in order to finance an increase in the public debt. The higher tax rate reduces the incentive to save and invest, and therefore reduces the long-term growth rate of the economy. Thus, the “true” burden of the public debt is a lower rate of economic growth. That higher taxes and slower growth are a consequence of a higher public debt is one way of explaining the burden of the public debt. Other commentators, such as Serieux (1999), have argued that reduced spending on social programs and public investment is the way most heavily indebted poor countries have adjusted to higher debt servicing costs.

A simple AK endogenous growth model is used to explore the connections between the public debt, distortionary taxation, and the rate of economic growth. In this model, the growth rate of the economy is proportional to the ratio of investment to total output. It is a model of a closed economy, and the net savings rate, which equals the investment rate, determines the growth rate of the economy. The net savings rate is the difference between the private sector savings rate and the public sector’s deficit ratio. An increase in the public debt affects the growth rate of the economy through its effects on the deficit ratio and on the private sector savings rate. The public sector deficit ratio increases when public debt ratio increases, crowding out investment if the private sector savings rate constant. The decline in the investment rate leads to a decline in the growth rate.
An increase in the public debt ratio can also affect the private sector savings rate.

In the model outlined in this paper, an increase in the public debt has two offsetting effects on the private sector saving rate. There is a *distortionary tax effect* that arises because of the increase in the tax rate that is required to finance interest payments on the additional debt. The tax rate increase reduces the net rate of return on saving, making savings less attractive. There is also a *Ricardian equivalence effect* because in this model, forward-looking individuals, with infinitely long lives, increase their savings rate to pay for the future tax increases that are required to finance the public debt. Thus, an increase in the public debt has an ambiguous effect on the private sector savings rate.

The model predicts that an increase in the public debt will reduce the investment rate, and with it the growth rate, because the Ricardian equivalence effect exactly offsets the increase in the deficit ratio. Therefore, the net savings rate declines because of the distortionary tax effect.

Calculations based on this simple model indicate that a doubling of the debt ratio leaves the private sector savings rate virtually unaffected, and there are only very modest declines in the investment rate and the growth rate. However, even if an increase in the public debt has only a small effect on the long-term growth rate of an economy, it can have large cumulative impacts on aggregate output, and hence future living standards. An expression is derived for the welfare cost of a marginal increase in the public debt using the concept of the marginal cost of public funds. The MCF measures the cost incurred in a society in raising an additional dollar of tax revenue because a tax increases distorts private sector decisions leading to a less efficient allocation of resources. This
MCF can be interpreted as the “hurdle benefit-cost ratio” that a debt-financed public project needs in order to generate a net social gain.

The formula for the MCF from debt financing has two components. One is the inverse of the elasticity of the present value of the government’s net revenue stream (PVNR) with respect to the tax rate. The lower the elasticity of the PVNR, the greater the distortionary effect of a tax increase, and the higher the MCF from raising funds obtained by borrowing. The other component is the change in the present value of private and publicly-provided goods and services due to changes in the growth rate caused induced by a tax rate increase. Even though the public debt has only a very small effect on the growth rate in this model, a public project financed by debt would have to have a marginal benefit-cost ratio of 1.15 (using the base case parameter values) in order to improve social welfare. The model clearly suggests that there is a significant return from using “temporary” public sector surpluses to pay down the public debt even when the growth-retarding effects of the public debt are relatively low.

The model is also used to analyze a government’s intertemporal budget constraint, and its optimal tax and expenditure decisions. It indicates that government’s dynamic Laffer curve, which shows relationship between the tax rate and the present value of the government’s net revenue stream, always has a positive slope if the individuals’ intertemporal elasticity of substitution for consumption is less than 1 (which is the empirically relevant case). The model also predicts an increase in the public debt to output ratio will have little effect on the optimal public expenditure ratio. In other words, this model predicts that most of the adjustment to an increase in the debt ratio occurs on the tax side of the budget. The insensitivity of the program expenditures to an increase in
the debt ratio is surprising given that some commentators, such as Serieux, think that a high public debt crowds out program expenditures.

The paper is organized as follows. Section 2 describes the basic AK endogenous growth model that is used to analyze the effect of an increase in the public debt on the rate of economic growth. In Section 3, we also derive an analytical expression for the marginal cost of public funds obtained by borrowing an additional dollar. In Section 4, the optimal tax and program expenditure rates are derived, given the government’s intertemporal budget constraint and its debt ratio. The limitations of the model and directions for future research are discussed in the final section.

2. A Model of the Effect of the Public Debt on Economic Growth

Total output at time t is equal to:

\[ Y_t = AK_t \]  

where \( K_t \) is the accumulated factor of production (physical and human capital) and \( A \) is the constant rate of return on this input. We will restrict our attention to the balanced growth path for this economy, where total output is growing at a constant rate \( \gamma \). The capital stock is also growing at the constant rate \( \gamma \) because it is assumed that there is no technological change and no depreciation. This implies that the annual rate of net investment is \( I_t = \gamma K_t \). Substituting back into (1), we obtain:

\[ \gamma = A i \]  

where \( i \) is the investment rate, \( I/Y \). In other words, the growth rate is proportional to the investment rate in the economy. This simple relationship between the growth rate of the
economy and the investment rate is the key feature of this simple endogenous growth model, and there is considerable empirical evidence indicating that countries with higher investments rates also have higher growth rates.¹

The population is normalized to equal one, so all of the stocks and flows can be interpreted as per capita variables. Individuals are identical and are represented by a single individual whose utility at time $t$ is:

$$U_t = \left(\frac{\sigma}{\sigma - 1}\right) C_t^\sigma + \beta \left(\frac{\sigma}{\sigma - 1}\right) G_t^\sigma$$  \hspace{1cm} (3)

where $C_t$ is private consumption, $G_t$ is consumption of publicly-provided goods and services, $\sigma > 0$ is the intertemporal elasticity of substitution, and $\beta > 0$ is a parameter that reflects the relative valuation of private and public consumption. The representative individual takes as given the level of public services, $G_t$, and the tax rate, $\tau$, used to finance them. Each period, the individual chooses his level of consumption and allocates his savings between investment in new capital and purchases of government bonds, $B_t$. The individual’s budget constraint in each time period is:

$$C_t + \dot{K}_t + \dot{B}_t = (1 - \tau)AK_t + (1 - \tau)AB_t$$  \hspace{1cm} (4)

where $\dot{K}_t$ and $\dot{B}_t$ are the rates of change in capital and government bonds. The right-hand side of (4) shows the individual’s current after-tax income from production and interest payments on government bonds. This is a closed economy, and there is no external debt, i.e. the individuals owe the public debt to themselves. The representative

¹ See, for example, McGrattan (1998) and Durlauf and Quah (1999).
individual discounts future utility at the rate $\rho > 0$ and makes consumption-savings decisions to maximize welfare $V$ where:

$$V = \int_{0}^{\infty} U_t e^{-\rho t} \, dt$$

(5)

To simplify the notation, we will omit the time subscript from variables unless it is necessary for their interpretation.

With the optimal consumption plan, private consumption grows at the rate, $\gamma$, where:

$$\frac{\dot{C}}{C} = \sigma ((1 - \tau)A - \rho) = \gamma$$

(6)

An increase in the tax rate will slow the growth rate of consumption because it reduces the net rate of return on savings. The reduction in the growth rate caused by an increase in the tax rate, $\partial \gamma / \partial \tau = -\sigma A$, is proportional to the intertemporal elasticity of substitution, $\sigma$, the key behaviour parameter in the model.

The growth of the public debt is equal to the public sector’s budget deficit, which is given by the right-hand side of (7):

$$\frac{\dot{B}_i}{B_i} = (1 - \tau)AB_i + G_i - \tau Y_i$$

(7)

Along the balanced growth path of the economy, $C$, $B$, $K$, $G$, and $Y$ all grow at the rate $\gamma$, and the public sector’s debt ratio, $b = B/Y$, its program expenditure ratio, $g = G/Y$, and the tax rate, $\tau$, remain constant. Therefore the deficit ratio is equal to $\gamma b$ where:

$$\gamma b = (1 - \tau)Ab + g - \tau$$

(8)
This intertemporal budget constraint can also be written as:

$$\tau - g = [(1 - \tau)A - \gamma]b = \theta b$$  \hspace{1cm} (9)

The government’s primary surplus ratio, which is the left-hand side of (9) assuming for simplicity that interest on the government’s debt is not taxed, has to equal the equilibrium debt ratio multiplied by the difference between the after-tax rate of return on capital and the growth rate of the economy if the debt ratio is to remain constant.

Note that the government’s intertemporal budget constraint does not depend on whether interest payments on government debt are taxed. If interest of the public debt is not taxed, the interest rate on government bonds would be equal to the after-tax return on capital, \((1 - \tau)A\). If interest on the public debt is taxed, the interest rate on the public debt is pre-tax return on capital, and the right-hand side of the (9) would be \((A - \gamma)b\).

However, the left-hand side would be equal to \(\tau(1 + Ab) - g\), and therefore the government’s intertemporal budget constraint would be the same as in the case where interest on the public debt is not taxed. It will be convenient to assume that interest on the public debt is not taxed, because this implies that the public sector and the private sector will discount future income streams using the same discount rate. Thus the present values of tax revenues and program expenditures are based on the after-tax rate of interest, \((1 - \tau)A\), and not the pre-tax rate of return on capital.

Using the expression for the equilibrium growth rate of the economy in (6), \(\theta\) is equal to:

$$\theta = (1 - \sigma)(1 - \tau)A + \sigma \rho$$  \hspace{1cm} (10)
A condition for dynamic stability is that $\theta > 0$, or in other words, that the after-tax rate of return on capital exceeds the growth rate of the economy. This condition will hold if:

$$\tau < 1 + \frac{\sigma \theta}{(1 - \sigma)A}$$

(11)

This condition will be satisfied if $\sigma$ is less than one, which is the relevant range of values for $\sigma$ based on econometric studies of savings behaviour.\(^2\)

The government’s intertemporal budget constraint can be interpreted as requiring that the present value of the government’s net revenue stream, PVNR, equal its current debt where:

$$PVNR = (\tau - g) \left( \frac{AK}{\theta} \right)$$

(12)

The present value of the future stream of tax revenues and program expenditures is calculated using the implicit discount rate $\theta$. Note that:

$$\frac{\partial \theta}{\partial \tau} = -(1 - \sigma)A$$

(13)

An increase in the tax rate has two offsetting effects on the present value of the tax/expenditure base. On the one hand, an increase in the tax rate reduces the growth rate of the economy, which lowers the present value of the tax/expenditure base. On the other hand, a higher tax rate lowers the after-tax rate of return on government debt, which increases the present value of the tax/expenditure base. As (13) indicates, when $\sigma < 1$, the latter effect dominates, and $\theta$ declines when $\tau$ increases. In other words, the present value of the tax/expenditure base is increasing in the tax rate when $\sigma < 1$. The
implications of this for the government’s intertemporal budget constraint and the MCF are described in Section 3.

To derive the consumption-capital and the consumption-income ratios along the balanced growth path, we divide both sides of (4) by K.

\[
\frac{C}{K} + \frac{\dot{K}}{K} + \frac{B}{B} \left( \frac{B}{K} \right) = (1 - \tau)A \left( 1 + \frac{B}{K} \right) \tag{14}
\]

Substituting \( \gamma \) for \( \frac{\dot{K}}{K} \) and \( \frac{B}{B} \) in (14), and noting that \( B/K \) is equal to \( Ab \), we obtain the following expression for the consumption rate in the economy:

\[
c = \frac{C}{Y} = \frac{\theta (1 + Ab)}{A} \tag{15}
\]

The model predicts that an increase in the tax rate will reduce the consumption rate if \( \sigma < 1 \) and that an increase in debt ratio will increase the consumption rate since:

\[
\frac{dc}{d\tau} = -(1 - \sigma) (1 + Ab) \tag{16}
\]

\[
\frac{dc}{db} = \theta \tag{17}
\]

As (2) indicated, the growth rate of the economy is proportional to the investment rate, which is equal to the economy’s net savings rate because all debt is held internally. The net savings rate is the difference between the private sector savings rate and the public sector’s deficit ratio. Therefore \( i = s - \gamma b \), where \( s \) represents the private sector’s savings ratio, \( S/Y \). Figure 1 shows how the equilibrium growth rate is determined in this

\[\text{For the case where } \sigma = 1, \theta = \rho > 0, \text{ and therefore the condition for dynamic efficiency is also satisfied.}\]
simple endogenous growth model. The private sector savings rate $s$ is based on individuals’ savings decisions given $A$, $g$, $b$, and $\tau$. If the debt ratio is $b_0$, the deficit ratio will increase as the growth rate increases, and the investment rate will decline because the net savings rate declines. Along the balanced growth path, the investment rate will be $i_0$, and equilibrium growth rate will be $\gamma_0$.

An increase in the public debt ratio affects the economic growth rate through its effect on the deficit ratio and through its effect on the private sector savings rate. Figure 1 shows that an increase in the debt ratio from $b_0$ to $b_1$, holding the private sector savings rate constant, will reduce the investment rate by increasing the deficit rate, and the equilibrium growth rate will decline from $\gamma_0$ to $\gamma_1$. It can be shown that, holding the private sector savings rate constant, the elasticity of the growth rate with respect to the debt ratio is equal to $-\gamma b/s$, which is the deficit ratio divided by the private sector savings rate. In other words, if the government is initially maintaining a constant debt ratio with a deficit ratio equal to one percent of GDP and if the private sector savings rate is 10 per cent, then a 50 per cent increase in the debt ratio will reduce the growth rate by 5 percent.

The model yields the following solutions given $b$, $g$, $A$, $\rho$, and $\sigma$ is:

\[
\tau = \frac{g + [(1 - \sigma)A + \sigma \rho]b}{1 + (1 - \sigma)Ab} \tag{18}
\]

\[
s = \sigma \left( \frac{1 + Ab}{A} \right) \left( \frac{(1 - g - \rho b)A - \rho}{1 + (1 - \sigma)Ab} \right) \tag{19}
\]

\[
\gamma = \sigma \frac{(1 - g - \rho b)A - \rho}{1 + (1 - \sigma)Ab} \tag{20}
\]
We will try to provide an intuitive explanation of the effect of an increase in \( b \) on the growth rate. First note that an increase in debt ratio leads to an increase in the tax rate, assuming that the condition for dynamic stability is satisfied:

\[
\frac{d\tau}{db} = \frac{(1 - g)(1 - \sigma)A + \sigma \rho}{(1 + (1 - \sigma)Ab)^2} > 0
\]  

(21)

The effect of an increase in the debt ratio on the private sector savings rate can be decomposed as follows:

\[
\frac{ds}{db} = \gamma - \sigma (1 + Ab) \frac{d\tau}{db}
\]  

(22)

The first term on the right-hand side of (22) is the Ricardian equivalence effect. An increase in \( b \) will increase the deficit ratio, \( \gamma b \), and this prompts an individual to increase his savings rate to offset the decline in the public sector savings rate. This forward-looking response arises from our assumption that the economy is composed of infinitely-lived individuals. The second term on the right-hand side of (22) is the distortionary tax effect which arises because the higher tax rate that is required to finance additional debt reduces the net rate of return on saving. These effects push the private sector savings rate in opposite directions. The overall effect of an increase in \( b \) on the growth rate is as follows:

\[
\frac{d\gamma}{db} = A \frac{di}{db} = A \left[ \frac{ds}{db} - \left( \gamma + b \frac{d\gamma}{db} \right) \right]
\]  

(23)

The first term in square brackets is the effect of an increase in \( b \) on the private sector savings rate and the second term is the effect on the deficit ratio. Substituting (22) into (23) yields:

\[
\frac{d\gamma}{db} = - A\sigma \frac{d\tau}{db} < 0
\]  

(24)
An increase in \( b \) causes \( \gamma \) to decline when \( b \) increases, even though an increase in \( b \) has an ambiguous effect on the private sector savings rate, because the Ricardian equivalence effect from the private sector savings response exactly offsets the increase in the deficit rate. Therefore, the total net savings rate declines by the distortionary tax effect, leading to declines in the investment rate and the equilibrium growth rate.

The model can be used to predict the effect of an increase in the debt ratio on \( s \), \( i \), and \( \gamma \). I have calibrated the model so that it produces a balanced growth path for an economy with the public debt and program expenditure ratios for Canada in the mid-1990s. During that period, the ratio of government spending on goods and services to GDP was 0.26 and the debt ratio for all levels of government was 0.83. (See Johnson (forthcoming, Tables 3 and 7)). It is assumed that if the debt ratio was stabilized at 0.83, the long-term growth rate for the economy would be 1.5 percent . I assume the rate of time preference, \( \rho \), to be 0.02 and the intertemporal elasticity of substitution, \( \sigma \), to be 0.25, which is in the middle of the range of values based on econometric studies of savings behaviour.\(^3\) Given \( b = 0.83 \), \( g = 0.26 \), \( \gamma = 0.015 \), \( \rho = 0.02 \), and \( \sigma = 0.25 \), the remaining parameter value, \( A \), can be calculated from (20) as 0.117. Given these parameter values, the model predicts that the consumption ratio would be 0.61 and the investment ratio would be 0.13 which are very close to their average values in the 1992-96 period, 0.601 and 0.162 respectively.

Figure 2 shows that given these parameter values the private sector savings rate would remain almost constant as the debt ratio increases from 0 to 1.5 because the Ricardian equivalence effect almost exactly offsets the distortionary tax effect. Given the
constancy of the private sector savings rate, the investment rate would decline as the
deficit ratio increases. Consequently, the growth rate also declines, but the decline in the
growth rate is very modest. For example, the model predicts that doubling the debt ratio,
from 50 percent of GDP to 100 percent, would reduce the investment rate by 0.8
percentage points and reduce the growth rate by just under a tenth of a percentage point.
Computations using other values of the key parameters (e.g. \( \sigma = 0.4 \) and \( A = 0.082 \) or \( \sigma = 0.10 \) and \( A = 0.278 \)) yield the same qualitative prediction: *the private sector savings rate is virtually unaffected by increases in the debt ratio, and there are only very modest declines in the investment rate and the growth rate when the debt ratio doubles.*

This is a very simple model, and it would not be wise to draw the general
conclusion that the public debt has very little impact on an economy’s economic growth
rate. Still, the model suggests that if the private sector savings rate is relatively constant,
then the impact of increases in the public debt on the growth rate will be minimal. The
model also suggests that if Ricardian equivalence does not hold, then the impact of the
public debt on the growth rate might be much more significant. Some “back-of-the-
envelope” calculations indicate that in the absence of the Ricardian equivalence effect,
the decline in the growth rate would be twice as large, i.e. doubling the debt ratio, from
50 percent of GDP to 100 percent, would reduce the growth rate by two tenths of a
percentage point. While a further tenth of a percentage point reduction in the growth rate
may seem like a small effect, it represents a 1.5 percent reduction in the present value of
future output or approximately $15 billion in 2000. *Thus, even very small changes in the*

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3 Hall (1998) obtained an estimate of 0.1 for \( \sigma \) for the United States. Patterson and Pesaran (1992) obtained an estimate of 0.4 for the United Kingdom.
long-term growth rate of an economy can have large cumulative impacts on aggregate output, and hence future living standards.

3. The Marginal Cost of Public Funds from Public Sector Borrowing

We begin by deriving an expression for the equilibrium level of welfare in the economy. Along the balanced growth path, $C_t = zK_0e^{\gamma t}$ and $G_t = gAK_0e^{\gamma t}$ where $K_0$ is the economy’s capital stock at time 0 and $z = \theta(1 + Ab)$. Substituting these values into (3) and (5), the discounted value of the representative individual’s utility stream is:

$$V(\tau, g) = \left(\frac{\sigma}{\sigma - 1}\right) \int_0^\infty \left(\frac{zK_0e^{\gamma t}}{\theta} + \beta \left(\frac{gAK_0e^{\gamma t}}{\theta}\right)^{\frac{\sigma - 1}{\sigma}}\right) e^{-\rho t} dt$$

$$= \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{zK_0}{\theta} + \frac{\beta (gAK_0)^{\frac{\sigma - 1}{\sigma}}}{\theta}\right)$$

$$= \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{AK_0}{\theta} \right)^{\frac{\sigma - 1}{\sigma}} c^{\frac{\sigma - 1}{\sigma}} + \beta \left(\frac{gAK_0}{\theta}\right)^{\frac{\sigma - 1}{\sigma}}$$

(25)

since $\gamma((\sigma - 1)/\sigma) - \theta = -\theta$. This expression indicates that the representative individual’s welfare depends on the shares of income devoted to private consumption and government services and the present value of the stream of “potential utility”, $(AK_0)^{\frac{\sigma - 1}{\sigma}}$, calculated at the “implicit” discount rate, $\theta$, which is the same implicit discount rate used to calculate the present value of the government’s tax revenues and program expenditures. Welfare also depends on $\tau$ because $\theta$ and $c$ are functions of the tax rate. In other words, the implicit discount rate used to calculate the representative individual’s welfare level
depends on the rate of taxation because it reduces the after-tax rate of return on savings and because it lowers the rate of economic growth.

For future reference, the marginal benefit from an increase in the program expenditure ratio, $MB_g$, will be defined as:

$$MB_g = \frac{1}{\lambda_0} \frac{\partial V}{\partial g} = \beta \left( \frac{AK_0}{\theta} \right)^{\frac{1}{\sigma}}$$

(26)

where $\lambda_0 = (cAK_0)^{1/\sigma}$ is the marginal utility of consumption at time 0. $MB_g$ is a money measure of the gain from a permanent increase in the proportion of output devoted to public program expenditures, measured at the initial marginal utility of income.

The marginal cost of public funds is the cost to a society in raising an additional dollar of tax revenue. A tax rate increase usually induces tax avoidance and evasion behaviour that causes the government’s tax base to shrink. The shrinkage of the tax base is a reflection of the loss of economic efficiency caused by the distortion in the allocation of resources in the economy, and the marginal cost of funds is usually greater than one.\(^4\)

In static models, the MCF is usually defined as $(-1/\lambda)(\partial V/\partial \tau)/(\partial R/\partial \tau)$ where $R$ is tax revenue. However, Liu (2002) has shown that when the cost of government programs is affected by the tax rate, it is more appropriate to define the MCF as $(-1/\lambda)(\partial V/\partial \tau)/(\partial NR/\partial \tau)$ where $\partial NR/\partial \tau$ is the rate of change in the government’s net revenues, i.e. the difference between its tax revenues and program expenditures. In a dynamic model, the definition of the MCF should be based on the rate of change in the

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\(^4\) See Dahlby (forthcoming) for a survey on the concept and measurement of the MCF.
present value of the government’s net revenue stream. Thus we will define the MCF for a tax rate increase as:

\[
MCF_i = \left( -\frac{1}{\lambda} \right) \begin{pmatrix} \frac{\partial V}{\partial \tau} \\ \frac{\partial PVNR}{\partial \tau} \end{pmatrix}
\]  

(27)

where the \( V \) is the present value of the representative individual’s utility stream given by (25), and PVNR is the present value of the government’s net revenue stream.

Taking the partial derivative of (25) with respect to \( \tau \), and the following expression for the social cost of a tax increase can be obtained:

\[
- \frac{1}{\lambda_0} \frac{\partial V}{\partial \tau} = \frac{AK_0}{\theta} \left[ 1 + \sigma \beta \left( \frac{c}{g} \right)^{1-\sigma} \right] (1 + Ab)
\]  

(28)

This money measure of the harm caused by a tax rate increase depends on the strength of the preference for publicly-provided services, \( \beta \), and the relative provision of private and public consumption goods \( (c/g) \) because an increase in the tax rate changes the present value of the stream of utility from the public services as well as private consumption. In particular, public program expenditures are assumed to be a constant proportion of output, and therefore a slower rate of economic growth, caused by a tax rate increase, means the level of public services is lower than it otherwise would be. This loss depends on the strength of the preference for the public services and the \((c/g)\) ratio.

Taking the derivative of PVNR in (12) with respect to \( \tau \), we obtain:
\[
\frac{\partial PVNR}{\partial \tau} = \frac{AK_0}{\theta} - (\tau - g) \left( \frac{AK_0}{\theta^2} \right) \frac{\partial \theta}{\partial \tau} \\
= \frac{AK_0}{\theta} \left[ 1 - b \frac{\partial \theta}{\partial \tau} \right] = \frac{AK_0}{\theta} [1 + (1 - \sigma)Ab]
\]

(29)

Based on (13) and the fact that \( \tau - g = \theta b \) along the balanced growth path. Note that the government’s PVNR Laffer curve always has a positive slope for the empirically relevant case where \( \sigma < 1 \), and that the slope of this Laffer curve increases with the tax rate. Thus it is not possible to increase the present value of the government’s net revenues by lowering the tax rate\(^5\).

Combining (28) and (29), the following formula for the MCF can be obtained:

\[
MCF_t = \frac{-1}{\lambda_0} \frac{\partial V}{\partial PVNR} = \left[ \left( 1 + \sigma \left( \frac{c}{g} \right)^{\frac{1 - \sigma}{\sigma}} \right) (1 + Ab) \right] \left( \frac{1}{1 + (1 - \sigma)Ab} \right)
\]

(30)

This formula indicates that the MCF from debt financing has two components. The component in round brackets is the inverse of the elasticity of the PVNR with respect to the tax rate. The lower the elasticity of the PVNR, the greater the distortionary effect of a tax increase, and the higher the MCF for debt financing. This component of the MCF will be higher the higher the ratio of interest payments on the public debt to total output, \( Ab \), and the greater the intertemporal elasticity of the substitution because this makes the tax base more sensitive to tax rate increases.

The other component in square brackets is the social loss caused by the reduction in private and public service consumption. Given the ratio of government program
expenditures are a constant proportion of GDP, the slower rate of economic growth, caused by a tax rate increase, means that there is a loss of welfare from a lower level of public services. The magnitude of this loss depends on $\beta$ and the $(c/g)$ ratio because they determine the $MB_g$.

Note also that the MCF approaches 1.00 as when $\sigma$ approaches 0 and the tax is non-distortionary. For $0 < \sigma \leq 1$, the MCF is greater than one, but decreasing in $\tau$ for $0 < \sigma < 1$ because an increase in the tax rate causes $c$ to decline. Normally, we expect the MCF to be increasing in the tax rate because the deadweight loss from tax distortions increases with the square of the tax rate. One way of explaining this anomalous feature of the MCF is that the slope of the PVNR Laffer curve is increasing in the tax rate for $0 < \sigma < 1$, and therefore marginal tax revenues (in present value terms) are increasing as the tax rate increases, thereby lowering the cost of raising additional revenues. Finally, note that the MCF is increasing in $b$, holding the $(c/g)$ ratio constant.

We have derived this expression for the MCF for a tax rate increase, but it can also be interpreted as the marginal cost of public funds from public sector borrowing as is shown below:

$$
MCF_b = \frac{-1}{\lambda_0} \frac{\partial V}{\partial b} \frac{db}{(AK_0)db} = \frac{-1}{\lambda_0} \frac{\partial V}{\partial \tau} \frac{d\tau}{db} = \frac{\left(-1 \frac{\partial V}{\partial \tau}\right)}{\lambda_0} \frac{\theta}{1 + (1 - \sigma)Ab} \left(AK_0\right) = \left[1 + \sigma \beta \left(\frac{c}{g}\right)^{1 - \sigma} \left(\frac{1 + Ab}{1 + Ab - \sigma Ab}\right)\right]
$$

(31)

Intuitively, the $MCF_b$ is the same as the $MCF_r$ because, if the government borrows an extra dollar, the present value of its net revenue stream must also increase by one dollar. In the remainder of this paper, we will simply refer to this common value as the MCF.

Based on the parameter values used in the previous section to replicate a balanced growth path with Canada’s mid-1990s debt level, the computed value of the MCF is 1.15 if $\beta = 0.0375$, which is the value of $\beta$ that is required to make $g = 0.26$ the optimal program expenditure ratio at the mid-1990s debt level. (The condition for the optimal program expenditure ratio is analyzed in the next section.)

This value for the MCF is somewhat lower than the MCF of 1.4 for a general federal personal income tax rate increase in Canada computed by Dahlby (1994) based on a model of the economy where only labour income is taxed. However, that study incorporated the progressivity of the personal income tax system into the computation of the MCF and that is largely responsible for the higher MCF calculated in that study. Figure 3 shows that the MCF increases as the debt ratio increases, but that the rate of increase is very modest, in line with the predicted impact of the debt ratio on the growth rate in this model. With parameter values that reflect a more responsive savings rate to changes in the net rate of return on savings ($\sigma = 0.4$, $A = 0.082$, $\beta = 0.19$), the MCF is equal to 1.27. In the low savings response case, with parameter values of $\sigma = 0.10$, $A = 0.278$, and $\beta = 0.00006$, the MCF is equal to 1.06.

These calculations mean that a public project financed by debt would need a benefit-cost ratio of 1.15 (using the base case parameter values) in order to improve social welfare. Alternatively, the 1.15 value for the MCF can be viewed as indicating the
return that our governments can achieve from increasing their primary surpluses and reducing our public debt by one dollar.

4. The Effect of the Public Debt on Optimal Public Expenditures

The analysis to this point has assumed that ratio of government public expenditures to output remains constant when the public debt level increases, and so all of the fiscal adjustment to a higher debt level occurs on the tax side of the budget. However, many observers feel that higher interest payments on the public debt crowd out program spending. In the following section we derive the condition for the optimal level of public program spending in order to analyze the effects of a higher debt level.

To determine the government’s optimal tax and expenditure program (holding the government’s debt ratio constant), we maximize (25) with respect to $\tau$ and $g$ subject to the government’s intertemporal budget constraint in (9). Let the Lagrangian for this problem be:

$$\Lambda = V(\tau, g) + \mu[\tau - g - \theta b] \quad (32)$$

where $\mu$ is the Lagrange multiplier on the government’s intertemporal budget constraint. The first-order conditions for this problem are:

$$\frac{\partial V}{\partial \tau} + \mu \left[ 1 - b \frac{\partial \theta}{\partial \tau} \right] = 0$$
$$\frac{\partial V}{\partial g} - \mu = 0 \quad (33)$$

Using (26), (28), and (29), the condition for optimal program expenditures has the form:
The left-hand side of (34) is the ratio of the marginal benefit to the marginal cost of an increase in the program expenditure ratio, where \( MC_g = \frac{(AK_0)}{\theta} \), and the right-hand side is the MCF. Thus the optimality condition is the equivalent of the static Atkinson-Stern condition for optimal public expenditures financed by distortionary taxation for a public good that does not affect tax revenues.

The optimal \((\tau, g)\) combination satisfies (9) and (34). It is not possible to get a general closed-form solution for \(\tau\) and \(g\), but some insights can be gained from examining the solution when \(\sigma = 1\):

\[
MB_g \equiv \beta \left( \frac{c}{g} \right)^{\frac{1}{\sigma}} = 1 + \sigma \beta \left( \frac{c}{g} \right)^{\frac{1-\sigma}{\sigma}} \frac{(1 + Ab)}{1 + (1 - \sigma)Ab} \equiv MCF
\]  

(34)

With \(\sigma = 1\), the optimal program expenditure ratio is independent of the level of the public debt. An increase in the debt ratio increases the tax rate and the consumption rate by the personal rate of time preference, \(d\tau/db = dc/db = \rho\). The reason why the optimal program expenditure ratio is independent of the public debt ratio when \(\sigma = 1\) is shown in Figure 4. For a given level of the public debt \(b_0\), the MCF is \((1 + \beta)(1 + Ab_0)\) and
therefore independent of the level of the program expenditure ratio and the tax rate, while
the ratio $\frac{MB_g}{MC_g}$ is decreasing in $g$ and independent of the tax rate. The optimal public
expenditure ratio is $g_0$ when the debt level is $b_0$. An increase in the debt ratio, increases
both the MCF and the $\frac{MB_g}{MC_g}$ ratio by the same proportion, and therefore has no effect
on the optimal level of $g$. Thus the key reason why the optimal $g$ is independent of $b$ is
that a higher debt ratio raises the marginal benefit from $g$ in the same proportion as it
increases the MCF. This effect arises with the preferences specified in (3) because
private consumption and public consumption are complementary, and a higher debt ratio
leads to a higher consumption ratio.

The effect of an increase in the debt ratio on the optimal program expenditure
ratio for $0 < \sigma < 1$ is more difficult to nail down. I have not been able to sign $dg/db$ when
for $0 < \sigma < 1$. However, calculations with a wide range of parameter values indicates that
the optimal $g$ is (slightly) increasing in $b$ when $\sigma < 1$. For example, with the base case
parameter values ($A = 0.117$, $\rho = 0.02$, $\sigma = 0.25$, and $\beta = 0.0375$), the optimal $g$ is 0.260
when $b = 0.83$. If the public debt were eliminated, the optimal $g$ would decrease very
slightly to 0.257. If the public debt doubled to $b = 1.66$, the optimal $g$ would increase
very slightly to 0.263. In other words, the optimal $g$ is virtually constant over a wide
range of values for the debt ratio.

In Figure 5, I try to explain why the optimal $g$ is so unresponsive to the debt ratio.
The optimal $(\tau, g)$ combination is the solution to equation (34), which we will label the
optimization condition (OC), and equation (9), which is the government’s intertemporal
budget constraint (BC). In the absence of the public debt, BC is the 45 degree line from
the origin. If $b > 0$, the BC intercept on the $g$ axis is $-[(1-\sigma)A + \sigma \rho]b$, and the slope of
the BC is \([1 + (1-\sigma)Ab]\) which is greater than one if \(\sigma < 1\). I have not been able to derive a closed-form solution for \(g\) from the OC, but computations indicate that the locus of \((\tau, g)\) combinations that satisfy the OC has a negative slope. In other words, when \(g\) increases, \(\tau\) has to decline for the OC to hold. Figure 5 shows the OC locus computed using the base case parameter values for \(b = 0\) and \(b = 0.83\). The upward shift in the OC locus, as result of the increase in \(b\), offsets the downward shift in the BC such that the optimal \(g\) remains virtually constant, and almost all of the adjustment to the higher debt ratio occurs on the tax side of the budget. The reason why the OC shifts up is that an increase in \(b\) increases the \(\text{MB}_g/\text{MC}_g\) ratio more than the MCF. To restore equality, holding \(\tau\) constant, \(g\) must increase since an increase in \(g\) reduces the \(\text{MB}_g/\text{MC}_g\) ratio proportionately more than it reduces the MCF.

Consequently, the model predicts that public debt does not crowd out spending on government services as a proportion of GDP. However, it can be shown that an increase in the public debt will crowd out program spending in the sense that the \((c/g)\) ratio is increasing in \(b\) if \(\sigma \geq 1\). In other words, the model predicts that a higher public debt will reduce public service consumption relative to private consumption.

To my knowledge, there are no empirical studies of the extent to which public debt crowds out government spending on goods and services. Some impressions of the relationship between government spending and public debt are contained in Figure 6, which plots the average debt ratios and consumptive government spending ratios over the period 1990-98 for 22 industrialized countries. In this cross-section data, the debt ratio does not have a statistically significant effect on the ratio of consumptive government sending to GDP, as the model predicts. On the other hand, Figure 7 plots the average
(g/c) ratio and the debt ratio over the 1990-98 period for 22 industrial countries, and there is no statistically significant relationship between these variables, even though the model predicts there should be a positive relationship. This is of course only a very superficial analysis, and a more detailed empirically analysis is required to test the hypotheses regarding the effect of debt on government spending, but

5. Conclusion

In this paper, I have used a simple AK endogenous growth model to illustrate the inter-relationships between the public debt, distortionary taxation and economic growth. The higher tax rate that is required to finance the interest payments on a higher public debt reduces the growth rate of the economy by lowering the net savings and investment rate. Although the predicted reduction in the growth rate appears to be quite modest—doubling the debt ratio from 50 percent of GDP to 100 percent only reduces the growth rate by about a tenth of a percentage point—it represents a significant social loss because of the cumulative forgone public and private consumption. A formula for the marginal cost of funds from public sector borrowing has been derived which indicates that an additional dollar of debt imposes a social cost of $1.15 with the base case parameter values. We have shown that the optimal program expenditure ratio is relative insensitive to increases in the public debt ratio.

The model has the merit of providing a simple, intuitive framework for analyzing the impact of the public debt on the rate of economic growth. It allows us to obtain closed-form solutions for the key endogenous variables, such as the growth rate, and a formula for the MCF so that we do not have to rely on the “black box” simulations that are necessary from more complex endogenous growth models. However, the simplicity
of the model also imposes a number of limitations. One of the most important is that the model only incorporates the aggregate tax rate, and it treats all taxes as if they taxed the return to financial and human capital. In practice, the tax mix may be more important than the level of taxation in determining the rate of economic growth, with taxes on the return to savings having a bigger impact on the growth rate than consumption taxes. It clearly would be very useful to incorporate a wider range of tax instruments in the model. It should be noted, however, that even if payroll and consumption taxes do not affect the rate of economic growth, they could affect the level of economic activity insofar as they reduce people’s incentive to supply labour. These “level effects” are also a burden of the public debt. It would be very useful to extend the model to include a wider range of tax instruments and to include both the growth and the level effects of higher taxes in the measuring the burden of the public debt.

Second, the model represents a closed economy, and there is no foreign-held debt. Many countries borrow abroad, either directly or indirectly, in order to finance a public sector deficit. A higher public debt can impose a burden on the economy either by increasing the interest rate that foreigners require in order to finance the debt or by putting downward pressure on the exchange rate. Van der Ploeg (1996) and Turnovsky (1997) have developed open economy endogenous growth models with foreign borrowing. In these models, a higher level of foreign indebtedness increases the interest rate charged by foreign lenders which reduces investment and the rate of economic growth. Thus, the predicted effects of an increase in debt are similar, in qualitative terms, in open and closed endogenous growth models, but it would be interesting to have an analysis of the relative costs of public debt in open and closed economies.
A third limitation of the model is that it assumes that private sector savings
behaviour is based on identical, forward-looking, infinitely-lived individuals, and this
gives rise to the Ricardian equivalence effect. While I have reservations about the
empirical importance of the Ricardian equivalence, I have adopted it in this model for
two reasons. First, it greatly simplifies the modeling of savings behaviour and aggregate
social welfare. Second, as Elmendorf and Mankiw (1999) note, Ricardian equivalence
provides a useful benchmark, or a “natural starting point,” in constructing a model of
government debt. In the current context, it shows how a single departure from the
conditions for strict Ricardian equivalence—in this case the use of distortionary taxes—
will affect the results of the model. Our model suggests that distortionary taxes do not
push the growth rate very far from its equilibrium value under strict Ricardian
equivalence, although they have a significant effect on the MCF from debt financing. It
obviously would be useful to study the effects of the public debt on economic growth in
models that do not assume Ricardian equivalence behaviour. Some steps have been made
in this direction by Saint-Paul (1992), van der Ploeg (1996), and Scarth (forthcoming),
but in these models taxes are non-distortionary. Incorporating public debt, financed by
distortionary taxes, in a non-Ricardian endogenous growth model would be a very useful
direction for future research.
References


Figure 1

The Effect on the Growth Rate of an increase in the Debt Ratio with a Constant Private Sector Saving Rate

\[ \gamma = A_i \]

\[ i = s - \gamma b_0 \]

\[ i = s - \gamma b_1 \]

Growth Rate

Savings and Investment Rates
Figure 2
The Effect of an increase in the Debt Ratio on
Savings, Investment and Growth Rates

Figure 3
The Effect of Increases in the Debt Ratio on the
MCF from Public Sector Borrowing
Figure 4
The Optimal Program Spending Ratio when $\sigma = 1$

$$MCF = (1 + Ab_0)(1 + \beta)$$

$$\frac{MB_g}{MC_g} = (1 + Ab_0) \left( \frac{p\beta}{Ag} \right)$$

$g_0$ program expenditure ratio

Figure 5
The Optimal Program Spending Ratio and Tax Rate when $0 < \sigma < 1$
Figure 6
The Government Consumption Expenditure vs Debt in 22 Industrialized Countries

Figure 7
The Ratio of Private to Public Consumption Expenditures in 22 Industrialized Countries