Abstention because of Indifference and Alienation, and Its Consequences for Party Competition: A Simple Psychological Model

by

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Summary

The basic idea behind this paper is that voters have to be able to distinguish the positions of the parties. Following Weber’s Law this depends on the relative distance with respect to their own optimal position. Using such a measure a model of voter participation is developed which allows for abstention because of indifference as well as alienation. Two variants of this model are applied on a two parties contest: one where participation is proportional to the relative distance and another one where voters participate if this distance is above a certain threshold. We show that in both situations Nash equilibria exist which do not imply (full) convergence to the median voter’s position.

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1 Introduction

The rational choice approach to politics has often been criticised for being unable to explain several empirically well founded facts. Most prominently figures the ‘Paradox of Non-Voting’, already stated in A. Downs (1957). The ‘pure’ theory of rational voting behaviour assumes that citizens participate in elections because they might decide the election (decision hypothesis). However, this theory is unable to explain why (more than a very small number of) citizens vote at all. Attempts to solve this problem employing this decision hypothesis as, e.g., the game theoretic approach of Th. R. Palfrey and H. Rosenthal (1983) have failed; they have to use rather extreme assumptions which hardly hold in reality. More successful attempts are the introduction of a consumption value into the calculus of voting as already done by W.H. Riker and P.C. Ordeshook (1968), the model of expressive voting as developed by G. Brennan and L. Lomasky (1993), or the mobilisation hypothesis as originally developed by V.O. Key, (1950) which has been employed, e.g., by G. Kirchgaessner and J. Schimmelpfenning (1992). These latter efforts have in common that the closeness of the expected outcome which is the main driving force behind the decision hypothesis has at best a minor and/or indirect impact on voting turnout.

The second and related question is why citizens, if they participate at all, should take their individual voting decision seriously if they realise that it has no impact at all on the collective outcome, because they act behind a “veil of insignificance” as stated by H. Kliemt (1986). The reason might be some kind of moral behaviour by citizens which again is in contrast to the purely instrumental behaviour assumed by the economic approach.

This paper considers two other ‘old problems’ of the economic approach of voting behaviour which remain open even if the two problems stated above are solved. The first is that this model is hardly able to explain abstention because of alienation, at least as long as the usual assumption of decreasing marginal utility (increasing marginal loss) is employed. The further away the optimal position of a citizen is from the actual positions of the (two) parties, the larger is the utility difference caused by a certain difference between these two parties. Consequently, those with optimal positions far away should participate more than the ones close to the two parties. However, it is a well known fact that exactly those abstain to a large extent because they are alienated. One attempt to solve this problem is to assume only quasi-convexity of the utility functions and to skip the assumption of decreasing marginal utility, i.e. to allow first for a region of increasing and only after some point of decreasing marginal utility. Another possibility is to assume that the utility function is convex, as discussed, e.g., in M.J.

1. See, e.g., the philippica by D.P. Green and I. Shapiro (1994).
2. See, also G. Brennan and A. Hamlin (2000, pp. 129ff.).
3. For a similar argument see already J.M. Buchanan (1954).
OSBORNE (1995, p. 275ff.) 5) Though this allows for abstention because of alienation, it is in stark contrast to standard utility theory. Furthermore, without an additional theory it is a purely ad hoc assumption that standard utility theory should hold for economical but not for political choices. The alternative which is proposed in this paper is to assume, that not utility differences, but the ability to distinguish the positions of the different parties dispose citizens to participate or not.

A related problem, associated with the median voter theorem, is that if parties were mobile and had full information about the distribution of the voters’ optimal points they would converge at the median voter’s optimal point. According to R. CALVERT (1985) this result is quite robust. On the other hand, in order to gain votes at all, “parties have to be visibly different” (M. PALDAM and P. SKOTT (1995, p. 161)), and apart from the fact of some observable convergence they are obviously different: There is hardly anybody who would deny, e.g., that the current policy in the U.S. would be different if ALBERT GORE instead of GEORGE BUSH had won the presidential elections. Attempts to resolve this contradiction assume uncertainty about voters’ preferences, ideological preferences of candidates, and/or the impossibility of making binding commitments prior to an election. 6) While these assumptions seem to be quite plausible for single electoral decisions in a static framework, they are less convincing in a dynamic context. Why should a new party not place itself close to the median position and/or old parties slowly move into this direction?

One problem with these attempts is that they assume that all voters participate in elections, i.e. they abstract from abstention, especially from abstention because of alienation. If a party moves, however, too close to a competitor, it might lose (nearly) all its voters: Why should citizens vote at all if they are unable to distinguish the parties from each other? Consequently, even if the parties would converge at any other than the median voter position nobody would participate.

In the following, a model of voter participation is developed which allows for abstention because of indifference as well as of alienation, and which generates only partial convergence of the two parties even if the positions of the citizens are completely known. To keep the model as simple as possible we consider a two parties contest in the usual one-dimensional left-right-setting with a uniform distribution of voters. In the next section we present the measure of relative distance which is the key element of this model (Section 2). To demonstrate the impact of abstention because of alienation on the two-parties competition, we employ two versions of the model: In Section 3 we assume that the probability of participation is a specific continuous monotonic function of the relative distance measure, while in Section 4 we assume

5. M.J. OSBORNE (1995) speaks of pay-off instead of utility functions. But because utility should be closely related to the pay-off this does not change the argument.

that voters participate if this measure is above a certain threshold. In the first version we get
(i) under the assumption of vote maximisation a Nash-equilibrium different from the median
voter’s position, (ii) if parties maximise the margin over the competing party the usual con-
vergence, and (iii) if we combine the two motivations partial convergence. In the threshold
model we do not find any convergence. We conclude with some remarks about possible ex-
tensions (Section 5).

2 The Measure of Relative Distance

The basic idea behind the model is a fact well known from psychology: In order to react on
different signals people must be able to distinguish them. We are well able, e.g., to distinguish
a weight of 30 gram from one of 60 gram, but we are hardly able to distinguish a weight of
1030 gram from one 1060 gramm, even though the absolute difference between the two stim-
uli is the same. According to WEBER’s Law, it is not the absolute but the relative distance
which counts: “A change in a stimulus that will be just noticeable is a constant ratio of the
original stimulus.” 7) Let y be the original stimulus and Δy its change. This change will be
noticeable if its absolute value is larger than a certain threshold ε, ε > 0, i.e. if

\[
\frac{Δy}{y} > ε.
\]

Party programmes can be interpreted as such stimuli. Correspondingly, it is not the absolute
difference between the politics of the parties which counts but their relative difference in re-
lation to the optimal point of the citizen.

Let us consider two parties, I and II, which have optimal positions A and B. The measure of
the relative distance D between the voter’s optimal position x and the optimal positions of the
two parties we use is given by

\[
D(x, A, B) = \frac{|x - A| - |x - B|}{|x - A| + |x - B|}, \quad D(x; A = B) = 0.
\]

i.e. we divide the difference in the (absolute) distances between the positions of the two par-
ties and the voter by the sum of these distances. | · | might denote any norm. This measure of
relative distance reaches its maximum value D = 1 if the voter’s optimal position is one of the
two parties positions and the other party has a different position, and it takes on its minimum

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   This law, also called the WEBER-FECHNER Law, goes back to the German physiologist ERNST HEINRICH
   WEBER (1795 – 1878) and his student GUSTAV THEODOR FECHNER (1801 – 1887). See also H. TEIGEN and
value $D = 0$ whenever the two parties have identical positions ($A = B$) and/or they have exactly the same distance from the voter’s position.\(^8\)

If the threshold $\varepsilon$ is the same for all citizens, i.e. if all citizens participate if their relative distance is greater than $\varepsilon$, than the probability of participation is given by

\[
(3) \quad P(x, A, B) = \begin{cases} 
1 & \text{if } D(x, A, B) \geq \varepsilon, \\
0 & \text{if } D(x, A, B) < \varepsilon, 
\end{cases} \quad 0 \leq \varepsilon \leq 1.
\]

If on the other hand, there is heterogeneity between the citizens the probability to participate in an election might be any positive monotonic transformation $g(D(x, A, B))$ which satisfies the conditions $1 \geq g(D) \geq 0$ for $D > 0$, $g(D; x = A) \leq 1$ and $g(D; x = B) \leq 1$. For simplicity, we use the absolute norm and assume $g(D) = D$.\(^9\) This implies, e.g., that all voters who have exactly the same position as one of the two parties will vote while those who are exactly in between the two parties do not vote. Thus, for the probability of participation, $P$, we get

\[
(4) \quad P(x, A, B) = \begin{cases} 
\frac{B - A}{A + B - 2x}, & x \leq A \\
\frac{A + B - 2x}{B - A}, & A \leq x \leq \frac{A + B}{2} \\
\frac{2x - A - B}{B - A}, & \frac{A + B}{2} \leq x \leq B \\
\frac{B}{2x - A - B}, & B \leq x
\end{cases}
\]

In both cases, we assume that all voters vote ‘sincerely’, i.e. that, if they participate, they vote for the party which is nearest to them.

The measure of relative distance (2) is not restricted to the one-dimensional political space: $A$, $B$, and $x$ might be n-dimensional vectors in the political space $\mathbb{R}^n$. However, to keep it simple we restrict the model to the usually applied one-dimensional left-right scale; it runs from the far left −1 to the far right +1. Thus, the political scale is symmetric around zero. If the distribution of voters is also symmetric, the median voter’s position is equal to zero. Without loss of generality, we additionally assume that the position of party $A$ is left of the position of party $B$, $A < B$. Thus, the political System is described by the following

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8. That voters have to be able to discriminate between the alternatives has already been used as an assumption in Social Choice Theory in the discussion of K.J. Arrow’s Impossibility Theorem by L.A. Goodman and H. Markovitz (1952). However, though they are not precise in this respect, the rather use an absolute than a relative measure of distance, because “the significance of a change from one discretion level to the next is the same, no matter what level one starts from.” (p. 260.) – See also the discussion of their approach by K.J. Arrow (1963) and the references given there on p. 115.

9. To derive this formulation from a micro model one might assume that relation (3) holds for every single voter but that the thresholds $\varepsilon$ are equally distributed across the electorate. – As long as we use the one-dimensional political left-right scale the exact norm which is chosen does not matter.
Assumption 1: There are two parties, I and II, with optimal positions A and B, located at a one-dimensional political scale \([-1, 1]\), \(A < B\).

For \(A = -0.3\) and \(B = 0.3\) relation (2) is given in Figure 1. Those voters whose optimal positions are located between the positions of the two parties and do not participate, abstain because of indifference, while those whose positions are left to A or right to B and do not participate, abstain because of alienation. Thus, moving to the centre a party might face a trade-off: It can win additional votes from those located in between the two parties but at the expense of losses from those on the far left or right.

![Figure 1: The measure of relative distance (A = -0.3, B = 0.3)](image-url)

If the two parties have different positions, abstention can only occur if the voters are unable to exactly estimate the distance between their own position and the position of the different parties. Otherwise, if we abstract from voting costs, all voters (with the exception of those who have exactly the same distance to both parties) would participate. If they have only approximate estimates of these distances, they will only participate if they perceive a difference between the two positions. So far it is the same way as the traditional (classical) economic model of voter participation. But while the classical model looked at the absolute distances (or utility differences based on these absolute distances), the approach proposed in this paper takes the relative distances as the relevant measure. There is no basic difference with respect to indifferent voters, as the denominator in the relative difference measure (2) is the same for all voters whose optimal positions are between A and B. On the other hand, for voters at the ends of the political spectrum both models come to quite different conclusions: the further away a voter is located from the two parties the less likely he will participate if the relative distance counts, whereas, if his decision is based on a comparison of the absolute distances, the probability that he will participate is – in any case – not diminished; under the usual as-
In this model, the probability of an individual vote in favor of party I increases the further he is away from the two candidates. Thus, whereas the traditional model cannot explain abstention from alienation (without using additional ad hoc assumptions), this model can.

The expected vote share of party I, $S_I$, is given by

$$S_I(A, B) = \int_{-1}^{A+B/2} P(x, A, B) f(x) \, dx ,$$

where $f(x)$ is the distribution of voters, and the corresponding expected share of party II, $S_{II}$, by

$$S_{II}(A, B) = \int_{A+B/2}^{1} P(x, A, B) f(x) \, dx .$$

To keep the model as simple as possible we assume that there is a uniform distribution of voters. Thus, for $f(x)$ with $-1 \leq x \leq 1$ and $\int_{-1}^{1} f(x) \, dx = 1$ we get

**Assumption 2:**

$$f(x) = \begin{cases} 
0 & \text{if } x < -1 \text{ or } x > 1 \\
0.5 & \text{if } -1 \leq x \leq 1
\end{cases} .$$

In the following, electoral competition between the two parties is first analysed with voter participation according to (4) (*Proportionality Model*) and second according to (3) (*Threshold Model*).

### 3 The Proportionality Model

If voter participation is according to (4) proportional to the relative distance, the expected share of party I can be calculated as

$$S_I(A, B) = \int_{-1}^{A} P(x, A, B) f(x) \, dx + \int_{A}^{A+B/2} P(x, A, B) f(x) \, dx$$

$$= S_{I1}(A, B) + S_{I2}(A, B)$$

with
S_{I1}(A, B) = \frac{1}{2} \int_{-1}^{A} \frac{B - A}{A + B - 2x} \, dx = \frac{B - A}{4} \ln \left( \frac{2 + A + B}{B - A} \right),

S_{I2}(A, B) = \frac{1}{2} \int_{A}^{B} \frac{A + B - 2x}{B - A} \, dx = \frac{B - A}{8}.

Thus, \( S_I \) takes the form

(6') \quad S_I(A, B) = S_{I1}(A, B) + S_{I2}(A, B) = \frac{B - A}{4} \left\{ \ln \frac{2 + A + B}{B - A} + \frac{1}{2} \right\}.

Correspondingly, the expected share of party II can be calculated as

(7) \quad S_{II}(A, B) = \int_{A}^{B} \left( \int P(x, A, B) f(x) \, dx \right) + \frac{1}{B} \int P(x, A, B) f(x) \, dx

with

S_{II1}(A, B) = \frac{1}{2} \int_{A}^{B} \frac{2x - A - B}{B - A} \, dx = \frac{B - A}{8},

S_{II2}(A, B) = \frac{1}{2} \int_{B}^{1} \frac{B - A}{2x - A - B} \, dx = \frac{B - A}{4} \ln \left( \frac{2 - A - B}{B - A} \right).

Thus, \( S_{II} \) has the form

(7') \quad S_{II}(A, B) = S_{II1}(A, B) + S_{II2}(A, B) = \frac{B - A}{4} \left\{ \ln \frac{2 - A - B}{B - A} + \frac{1}{2} \right\}.

3.1 Vote Maximisation

The classical hypothesis stated by A. Downs (1957) assumes that parties are interested in vote maximisation, e.g. for party I

(8) \quad \max_{A} S_{I}(A; B)!
Holds. If this party behaves accordingly, we get its reaction function by differentiating $S_I$ with respect to $A$ given $B$. Thus, we get

$$\frac{\partial S_I}{\partial A} = \frac{\partial}{\partial A} \left( \frac{B - A}{4} \left\{ \ln \frac{2 + A + B}{B - A} + \frac{1}{2} \right\} \right)$$

$$= -\frac{1}{8} - \frac{1}{4} \ln \frac{2 + A + B}{B - A} + \frac{1}{2} \left( \frac{B + 1}{A + B + 2} \right).$$

Because of

$$\frac{\partial^2 S_I}{(\partial A)^2} = - \frac{(B + 1)^2}{(B - A)(A + B + 2)^2} < 0,$$

setting (9) equal to zero, we get the unique optimum of the vote share of party I given the position of party II, $B$. If party II is in the median voter’s position, $B = 0$, coming from the left ($B > A$) we get

$$\lim_{A \to 0} \left( \frac{\partial S_I}{\partial A} \right) = -\infty.$$ 

Thus, the median voter’s position is not an equilibrium, and party I can increase its vote share if it deviates from this position. Since the distribution of voters is symmetric (around zero), party II will deviate from this position to the right, and in the unique Nash equilibrium $A + B = 0$ has to hold.

Inserting this into the first order condition (9) and setting it equal to zero we get (as an example) the equilibrium condition

$$0.5 = \ln(-A) - A$$

which gives an optimal position of $A = -0.766$ an of $B = 0.766$. In this situation, both parties get a vote share of 29.3 percent of the total electorate, i.e. the abstention rate is 41.4 percent: While 38.3 percent of the voters have their optimal position between the two parties and abstain because of indifference only 3.1 percent are of the voters are alienated.

### 3.2 Margin Maximisation

Taking into account that parties (candidates) tempt to win the next election, the Downsian assumption of vote maximisation is hardly convincing. In order to be successful they have to get more votes than their competitor. If a party deviates from the vote maximisation Nash...
equilibrium in the direction of the median voter, it will lose votes, but the competing party can lose even more votes if it does not react. Thus, as an alternative to vote maximisation is to maximise the probability of an electoral victory. If we assume that the probability of a victory is a positive monotonic function of the (expected) margin over the competitor, the maximisation of the margin is an alternative which is more plausible than simple vote maximisation. Because \( S_{I2}(A, B) = S_{II1}(A, B) \), the margin of party I over party II, \( DS_I(A, B) \), is given by

\[
(12) \quad DS_I(A, B) = \frac{B-A}{4} \ln\left(\frac{2+A+B}{2-A-B}\right).
\]

The first order condition for party I is that the first derivative

\[
(13) \quad \frac{\partial DS_I}{\partial A} = \frac{\partial DS_I}{\partial A} \left\{ \frac{B-A}{4} \ln\left(\frac{2+A+B}{2-A-B}\right) \right\}
\]

\[
= -\frac{1}{4} \ln\left(\frac{2+A+B}{2-A-B}\right) + \frac{B-A}{(2+A+B)(2-A-B)}
\]

is equal to zero. Because the second derivative

\[
(14) \quad \frac{\partial^2 DS_I}{(\partial A)^2} = \frac{\partial}{\partial A} \left( \frac{8-4AB-4B^2}{(2+A+B)^2(2-A-B)^2} \right) < 0,
\]

there is a unique maximum, given \( B \). However, for the first derivative the following conditions hold:

\[
(13a) \quad B = 0 \quad \Rightarrow \quad \frac{\partial DS_I}{\partial A} = -\frac{1}{4} \ln\left(\frac{2+A}{2-A}\right) + \frac{-A}{(2+A)(2-A)} \gtrless 0 \quad \iff \quad A \gtrless 0.
\]

\[
(13b) \quad B = A \quad \Rightarrow \quad \frac{\partial DS_I}{\partial A} = -\frac{1}{4} \ln\left(\frac{1+B}{1-B}\right) \equiv 0 \quad \iff \quad A \equiv 0.
\]

\[
(13c) \quad B = -A \quad \Rightarrow \quad \frac{\partial DS_I}{\partial A} = \frac{B}{2} \equiv 0 \quad \iff \quad A \equiv 0.
\]

Thus, the unique equilibrium exists when both parties keep the median position, i.e. if \( A = B = 0 \) (13a). If both parties keep the same position different from the median, holding the position of party II constant party I will move away from party II into the direction of the median
position (13b). Consequently, party I will follow. If, on the other hand, both parties, as in the vote maximisation equilibrium, have the same distance from the median position but in different directions, party I will move into the direction of party II and of the median position (13c). Thus, we have again the classic result of convergence at this position when both parties get no votes at all.

3.3 The Generalised Objective Function

A party can hardly accept that it loses all its support from the population, even if this would maximise the probability of being elected. The party (or candidate) might, e.g., not only lose votes but also campaign contributions. Moreover, even if a candidate has no ideological objective at all, he has to seek for some support from ideologically oriented voters (party members) to be nominated or re-elected as a candidate/party leader. Therefore, the objective function might contain both parts, the maximisation of the vote share as well as the margin. Then, the objective function of party I might be written as

\[
W_I = \lambda \cdot S_I(A, B) + (1-\lambda) \cdot DS_I(A, B)
\]

\[
= \frac{B-A}{4} \lambda \left( \ln \frac{2+A+B}{B-A} + \frac{1}{2} \right) + (1-\lambda) \ln \frac{2+A+B}{2-A-B}, \quad 0 \leq \lambda \leq 1.
\]

where \(\lambda\) is the weight given to vote maximisation. For the optimal solution, the first derivative with respect to A.

\[
\frac{\partial W_I}{\partial A} = \lambda \left( -\frac{1}{8} - \frac{1}{4} \ln \frac{2+A+B}{B-A} + \frac{1}{2} \left( \frac{B+1}{A+B+2} \right) \right) + (1-\lambda) \left( -\frac{1}{4} \ln \frac{2+A+B}{2-A-B} + \frac{B-A}{(2+A+B)(2-A-B)} \right),
\]

has to be zero. If we start from the margin maximisation equilibrium \(A = B = 0\), where the second part of (16) is equal to zero, relation (11) shows that a move to the left will improve the situation of party I. On the other hand, starting from the vote maximisation equilibrium where \(A = -B\) and the first part of (16) is equal to zero, because of (13c), with \(B > 0\), a move to the right will improve party I’s situation. Because of (10) and (14), the second derivative of (15) with respect to A is negative, which shows that the problem has a unique equilibrium. Thus, the optimal solution lies between the vote and margin maximisation equilibria.

Taking the results with the proportionality model together, we get the following
Theorem 1: Given the political Two-Party System described in Assumption 1, the uniform distribution of voters (Assumption 2), and that voter participation is according to the Proportionality Model (4), then the following results hold:

(i) The position of the two parties do not converge if the two parties follow the strategy of vote maximisation.

(ii) The position of the two parties do converge if the two parties follow the strategy of margin maximisation.

(iii) The position of the two parties do not converge if the two parties follow the generalised strategy which is a convex combination of vote and margin maximisation.

Proof: (i) (Vote maximisation; proof by contradiction): Because of (10) there is a unique equilibrium solution. If the two positions of the two parties converge, because of (9) \( \delta S_v/\delta \Lambda = -\infty \). This violates the first order condition \( \delta S_v/\delta \Lambda = 0 \).

(ii) (Margin maximisation) Because of (14) there is a unique equilibrium solution. If the two positions of the two parties converge, because of (13b) \( \delta S_v/\delta \Lambda = 0 \). This satisfies the first order condition.

(iii) (Generalised objective function, proof by contradiction): Because of (10) and (14), there is a unique equilibrium solution. If the two positions of the two parties converge, because of (16) \( \delta S_v/\delta \Lambda = -\infty \). This violates the first order condition \( \delta S_v/\delta \Lambda = 0 \).

q.e.d.

If we impose the symmetry condition \( \Lambda = -B \), the condition for a maximum reduces to

\[
-2A + \lambda (\ln(-A) + 0.5 + A) = 0.
\]

With, as an example, equal weights for both parts of the objective function, i.e. for \( \lambda = 0.5 \), we get \( A = -0.270 \) and \( B = 0.270 \) and a vote share of 0.244 for both parties. In this situation 13.5 percent of the electorate abstain because of indifference but 37.6 percent because of alienation.

4 The Threshold Model

If voters participate as long as the relative distance is larger than a certain threshold and if all voters have the same threshold \( \varepsilon \), the vote share of party I is given by
\[(17)\quad S_I(A, B) = \int_{-\epsilon^-}^{\epsilon^+} f(x) \, dx \]

with \( C = \text{Max}(-1, \frac{A+B}{2} - \frac{B-A}{2\epsilon}) \). Thus, we get

\[(17')\quad S_I(A, B) = \begin{cases} \frac{B-A}{4} \left( \frac{1}{\epsilon} - \epsilon \right) & \text{for } A > \frac{1}{1+\epsilon} (B(1-\epsilon) - 2\epsilon) \\ \frac{1}{2} + \frac{A+B}{4} - \frac{\epsilon}{4} (B-A) & \text{elsewhere} \end{cases} \]

Correspondingly,

\[(18)\quad S_{II}(A, B) = \int_{-\epsilon^+}^{\epsilon^-} f(x) \, dx \]

\[(18')\quad S_{II}(A, B) = \begin{cases} \frac{B-A}{4} \left( \frac{1}{\epsilon} - \epsilon \right) & \text{for } B < \frac{1}{1+\epsilon} (A(1-\epsilon) + 2\epsilon) \\ \frac{1}{2} + \frac{A+B}{4} - \frac{\epsilon}{4} (B-A) & \text{elsewhere} \end{cases} \]

Thus, if a corner solution is excluded, the two parties will always get the same vote share.

If we have an interior solution and two parties maximise their vote share we get for party I

\[(19)\quad \frac{\partial S_I}{\partial A} = -\frac{1}{4} \left( \frac{1}{\epsilon} - \epsilon \right) < 0 \]

Thus, this party has always an incentive to go to the left until it reaches the corner solution (which then is the optimal solution) with

\[(20)\quad A = \frac{B(1-\epsilon) - 2\epsilon}{1+\epsilon} \]

and the vote share

\[(21)\quad S_I(A, B) = \frac{(1+B)(1-\epsilon)}{2} \]
Correspondingly, the optimal solution for party II is the corner solution with

\[ B = \frac{A(1-\varepsilon) + 2\varepsilon}{1+\varepsilon} \]  

and the vote share

\[ S_{II}(A, B) = \frac{(1-A)(1-\varepsilon)}{2}. \]

Thus, the Nash equilibrium is \( A = -\varepsilon, B = \varepsilon \) with vote shares of

\[ S_I(A, B) = S_{II}(A, B) = \frac{1-\varepsilon^2}{2}. \]

For, e.g., \( \varepsilon = 0.3 \) (as in Figure 1) this gives for both parties vote shares of 45.5 percent and a voter turnout of 91 percent. Since this is a corner solution there is only abstention because of indifference. Nevertheless, the threat of possible abstention because of alienation forces the two parties not to converge.

The picture does hardly change if the parties follow a strategy of margin maximisation. If neither party I nor party II is in a corner solution, then because of (17') and (18') \( DI_I(A, B) = 0 \) irrespective of the positions A and B. Thus, maximising the margin does not provide any incentive to deviate from the actual position, irrespective what the reason for taking this position was. This also holds if \( B = \varepsilon \) and \( A > -\varepsilon \). If, on the other hand, \( B = \varepsilon \) and \( A < -\varepsilon \), then both parties are in corner solutions, and the margin is

\[ DI_I(A, B) = \frac{A+B}{2} < 0 \]

with

\[ \frac{\partial DI_I}{\partial A} = \frac{1}{2} > 0, \]

i.e. A will move to the right (at least) until \( A = -\varepsilon \) holds. Thus, while there is no reason why the equilibrium in the case of margin maximisation should be the same as the vote maximisation equilibrium, there is also no convergence in this case.

Finally, if parties follow the generalised objective function there is no incentive to deviate from the vote maximisation equilibrium. Thus, contrary to the proportionality model, there is no convergence at all in the threshold model.
**Theorem 2:** Given the political Two-Party System described in *Assumption 1*, the uniform distribution of voters (*Assumption 2*), and that voter participation is according to the *Threshold Model* (3), then the positions of the two parties do not converge, whether they follow the strategy of vote maximisation, of margin maximisation, or follow the generalised strategy.

**Proof:**

(i) (Vote maximization): Because of (19) \( \delta S / \delta A < 0 \) and because by assumption \( B > A \), the left party will increase the distance to the right party as long as it is not in the corner position. Thus, starting from a point where the two parties hold the same position the two parties will diverge instead of converge.

(ii) (Margin maximisation): If neither party is in a corner position, the margin will always be zero. The same holds if both parties are in their respective corner positions. Thus, there is no incentive to converge.

(iii) (Generalised objective function): From (i) and (ii) it follows that with the generalised strategy both parties will end up in their respective corner positions. Thus, there is also no incentive to converge.

q.e.d.

5 **Concluding Remarks**

This paper shows that the possibility of abstention because of alienation might prevent parties and/or candidates from convergence to the median voter position. Thus, we do not need ideological preferences of the candidates nor uncertainty about the distribution of voters to explain the empirical observable lack of convergence in two party systems. This is not to deny that such factors exist and can be effective, but according to the results of R. CALVERT (1985) it is questionable how effective they are.

The psychological assumption behind our model is quite similar to the assumption of convex utility functions as discussed in M.J. OSBORNE (1995). However, while convexity of a utility function is at odds with conventional utility theory, the psychological assumption employed in this paper is quite consistent with standard psychological theory. Thus, the argument of employing an add-hoc assumption is no longer valid.

One of our basic assumption was a uniform distribution of voters. This presumption is advantageous because we do not need a special distribution of voters to explain the non-convergence of the two parties’ positions. For the proportionality model, the qualitative results should not change when using other, even asymmetric distribution functions. On the other hand, the results of the threshold model, especially with respect to margin maximisation,
clearly depend on the assumption of a uniform distribution. Thus, an obvious extension to this research will be to investigate the threshold model for other distribution functions.

A further meaningful extension would be the application of this concept to systems with three (or more) parties. However, this implies the use of a multidimensional distance measure like, e.g., an entropy measure, and/or a stepwise decision procedure of the voters. This, however, goes far beyond the scope of this paper.
References:


