Investigating Nonlinearity in the Canadian Business Cycle Using STAR Models

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Abstract

The possible nonlinearity of business cycles is an old topic in economics. In this paper, I adopt the Smooth Transition Autoregressive (STAR) models developed by Teräsvirta and Anderson (1992) to investigate nonlinearity in the Canadian business cycle. Using the Canadian Industrial Production Index (IPI) growth rate as an indicator for the business cycle, linearity is rejected when the delay parameter is five. The specification leads to an LSTAR(1) model. The residual variance of the LSTAR(1) model slightly outperforms the AR(1) model, but there is no difference in the post-sample forecasting performance. When I add U.S. information into the transition function of the STAR models to investigate the relationship between the business cycles in Canada and the U.S., I find that linearity is rejected for the growth rate series of U.S. GDP and IPI, but not for the growth rate series of U.S. M1 money supply. Although linearity is rejected for the growth rate series of the U.S. interest rate, the estimated STAR model is not satisfactory. Putting the U.S. and Canadian factors together into the transition function, the empirical results do not indicate clearly which set of factors is driving the Canadian business cycle.

Keywords: business cycles, nonlinearity, smooth transition autoregressive models

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1 Introduction

Nonlinearity of the business cycle, as reflected in the asymmetric characteristics of expansionary and contractionary periods, is an old and interesting topic in economics. Many authors have proceeded by characterizing different features of the potential nonlinearity in the business cycle.

Diebold and Rudebusch (1990) applied a nonparametric test to investigate whether there is duration dependence of the expansions and contractions of the business cycle. They found some evidence for duration dependence in whole cycles and in prewar expansions for the U.S. series. Sichel (1991) used a parametric hazard model and found stronger evidence of duration dependence. Durland and McCurdy (1994) extended Hamilton’s (1989) model to allow state transition to be duration dependent and found duration dependence for recessions but not for expansions.

Brock and Sayer (1988) and Frank, Gencay and Stengos (1988) searched for the presence of deterministic chaos in the macroeconomic data. Deterministic chaos was rejected but both papers found evidence of nonlinearity in some of the macroeconomic data.


Teräsvirta and Anderson (1992) and Teräsvirta (1994) tested linearity against the smooth transition autoregressive (STAR) models using macroeconomic data in different countries. They rejected linearity in favor of the STAR models. For technical linearity tests see Luukkonen, Saikkonen and Teräsvirta (1988) and Saikkonen and Luukkonen (1988).

In this paper, I apply the STAR models as in Teräsvirta and Anderson (1992) to analyze the nonlinearity of business cycle in Canada. The reasons are two folds. First, I use a longer data set to investigate nonlinearity
in Canadian business cycle using the STAR models. Second, because of the close economic relationship between Canada and the U.S., I try to investigate the relationship between the U.S. business cycle and the Canadian business cycle through the STAR models.

The organization of the paper is as follows. I outline the STAR models in Teräsvirta and Anderson (1992) in Section 2. Section 3 presents the empirical results from the STAR models of the Canadian business cycle indicators. In Section 4 I investigate whether Canadian business cycle is driven by the U.S. factors. Conclusions are provided in Section 5.

2 The specification of the STAR models

Using the smooth transition autoregressive (STAR) model to analyze the business cycle allows the business cycle indicator to alternate between two extreme regimes with a smooth transition between them. The two-regime TAR model is nested in the STAR model as a special case.

Consider the following STAR model of order $p$, as in Teräsvirta and Anderson (1992):

$$y_t = \pi_{10} + \pi_1' w_t + (\pi_{20} + \pi_2' w_t) F(y_{t-d}) + \mu_t$$

(1)

where $\mu_t \sim \text{nid}(0, \sigma^2)$, $\pi_j = (\pi_{j1}, \ldots, \pi_{jp})'$, $j = 1, 2$, $w_t = (y_{t-1}, \ldots, y_{t-p})'$, $d$ is the delay parameter. The transition function $F(.)$ is bounded between zero and one. Determination of $d$ is described later. Two different transition functions are considered. The first one is the logistic function

$$F(y_{t-d}) = \frac{1}{1 + \exp[-\gamma(y_{t-d} - c)]}, \gamma > 0$$

(2)

and the second one is the exponential function

$$F(y_{t-d}) = 1 - \exp[-\gamma(y_{t-d} - c)^2], \gamma > 0$$

(3)

For convenience, we rewrite (1) in the following form:

$$y_t = (\pi_{10} + \pi_{20} F) + (\pi_1 + \pi_2 F)' w_t + \mu_t$$

(4)

We can see that with the logistic function in (2) the parameters in (4) change monotonically with $y_{t-d}$. When $\gamma \to \infty$ in (2), the values of $F(y_{t-d})$ switch between zero and one depending on the delay parameter $y_{t-d}$: $F(y_{t-d}) = 0$.
when \( y_{t-d} \leq c \), \( F(y_{t-d}) = 1 \) when \( y_{t-d} > c \). In this situation (1) and (2) turn into a TAR(p) model. When \( \gamma \to 0 \), (1) and (2) turn into a linear AR(p) model. When we combine (1) and (2), we obtain a logistic STAR (LSTAR) model. When we analyze the business cycle using the LSTAR model, the dynamics are such that the contraction and expansion phases of an economy are different, but there is a smooth transition between them.

When (1) is associated with (3), the parameters in (4) changes systematically about \( c \) with \( y_{t-d} \). We call this an exponential STAR (ESTAR) model. We can see that the ESTAR model becomes linear when \( \gamma \to \infty \) as well as when \( \gamma \to 0 \). Applying the ESTAR model to the business cycle analysis, we will observe similar dynamics for the contractions and expansions phases of an economy, whereas the middle regime behaves differently.

We can see that different dynamic processes can be generated from the LSTAR and ESTAR models, so that it is of interest to investigate nonlinearity in the business cycle using the STAR models.

As mentioned above, the linear AR(p) model is nested in the STAR models. We start testing linearity against the STAR models based on the linear AR(p) model. If linearity is rejected in favor of the STAR models, we will choose between the LSTAR and ESTAR models based on the data. The delay parameter is also determined from the data.

As discussed by Teräsvirta and Anderson (1992) and Teräsvirta (1994), the specification of the STAR models consists of the following three stages:

1. Specify a linear AR(p) model as a baseline model for linearity test.

2. Carry out the linearity test by fixing \( d \), the delay parameter, at different values. If linearity is rejected, determine \( d \) in the STAR models.

3. Distinguish between the LSTAR and ESTAR models.

2.1 Specification of a linear autoregressive model

Because the linear AR(p) model is the baseline model for testing linearity against the STAR models, the first step is to determine the optimal lag for the linear model. In this paper, I use the Akaike Information Criterion (AIC) to choose the optimal lag. A possible problem here is that the residuals of the estimated AR model are serially correlated for a low value of \( p \). In this case linearity is rejected due to the serial autocorrelation (see discussions of Teräsvirta (1994)). So we have to check for autocorrelation for the AR
2.2 Testing linearity and determining the delay parameter \( d \)

First we look at the linearity test for the LSTAR model. Using a third-order Taylor series approximation to the logistic transition function, Teräsvirta (1994) used a LM-type statistic for linearity based on the following auxiliary regression (for more details see Luukkonen, Saikkonen and Teräsvirta (1988), Saikkonen and Luukkonen (1988) and Teräsvirta (1994)):

\[ y_t = \beta_0 + \beta'_1 w_t + \beta'_2 (w_t y_{t-\delta}) + \beta'_3 (w_t y_{t-\delta}^2) + \beta'_4 (w_t y_{t-\delta}^3) + \nu'_t \] (5)

where \( \nu'_t \sim \text{nid}(0, \sigma^2_{\nu}') \), \( \beta_j = (\beta_{j,1}, \cdots, \beta_{j,p})' \), and \( j = 1, \cdots, 4 \). The null hypothesis is

\[ H'_0: \beta_2 = \beta_3 = \beta_4 = 0 \] (6)

The linearity test can be carried out by an ordinary F test (see Teräsvirta (1994)).

For the ESTAR model, Teräsvirta (1994) modified the LM-type test from Saikkonen and Luukkonen (1988) to obtain the following auxiliary regression

\[ ^1y_t = \gamma_0 + \gamma'_1 w_t + \gamma'_2 (w_t y_{t-\delta}) + \gamma'_3 (w_t y_{t-\delta}^2) + \nu''_t \] (7)

The null hypothesis is

\[ H''_0: \gamma_2 = \gamma_3 = 0 \] (8)

which can also be carried out as an ordinary F-test.

So the a general linearity test against the STAR (both LSTAR and ESTAR) models can be obtained by running the following auxiliary regression

\[ ^2y_t = \delta_0 + \delta'_1 w_t + \delta'_2 (w_t y_{t-\delta}) + \delta'_3 (w_t y_{t-\delta}^2) + \delta'_4 (w_t y_{t-\delta}^3) + \nu_t \] (9)

where \( \nu_t \sim \text{nid}(0, \sigma^2_\nu) \), the null hypothesis is

\[ H_0: \delta_2 = \delta_3 = \delta_4 = 0 \] (10)

\[ ^1\text{The symbols used here are analogous to (5).} \]

\[ ^2\text{The symbols used here are analogous to (5).} \]
So far the F tests are carried out assuming the delay parameter $d$ is known, but in practice we need to determine it from the data. For the discussion of choosing $d$ see Teräsvirta (1994), who borrowed his ideas from Tsay (1989). Following his suggestion, an appropriate range of values $1 \leq d \leq D$ is considered, the linearity tests are carried out for different values of $d$. The delay parameter $d$ is chosen such as $\hat{d} = \arg\min_p p(d)$, where $p(d)$ is the $p$-value for the linearity test. His argument is that the test has maximum power for a correctly specified $d$.

### 2.3 Distinguish between the LSTAR and ESTAR models

If we reject the linear AR model in favor of the STAR models and determine the delay parameter $d$, we move to the next stage of choosing between LSTAR and ESTAR models. The choice can be made through a sequence of ordinary F tests based on (10) for the following sequences of hypotheses:

\[
\begin{align*}
H_{04} : & \delta_4 = 0 \\
H_{03} : & \delta_3 = 0 | \delta_4 = 0 \\
H_{02} : & \delta_2 = 0 | \delta_4 = 0, \delta_3 = 0
\end{align*}
\]

The rejection of (11) leads to a LSTAR model. An ESTAR model will be selected if we accept (11) but reject (12). If we accept (11) and (12) but reject (13), we choose a LSTAR model$^3$.

To distinguish between the LSTAR and ESTAR models, we can carry out the three F tests mentioned above and note which hypotheses are rejected in the sequence. To avoid erroneously selecting the ESTAR or LSTAR model, Teräsvirta (1994) suggested comparing the relative strengths of the rejections. If the model is an ESTAR model, then typically $H_{03}$ is rejected more strongly than $H_{02}$ and $H_{04}$, while the opposite situation would apply for an LSTAR model. He proposed the following decision rule. After rejecting linearity in favor of the STAR models, carry out the three F tests. We select an ESTAR model if the $p$-value of $F_3$ (the test of $H_{03}$) is the smallest of the three; if this is not the case, we choose a LSTAR model.

$^3$For the inconclusive case when both (12) and (13) are rejected, see Teräsvirta and Anderson (1992).
3 Investigating nonlinearity in the Canadian business using the STAR models

In this section, I apply the STAR models to investigate nonlinearity in the Canadian business cycle using Gross Domestic Product (GDP) and the Industrial Production Index (IPI). The GDP series is seasonally adjusted in constant 1992 prices. The IPI series is taken from the quarterly International Financial Statistics, and is adjusted so that the base year is 1980.

The two series are made stationary by taking the first difference of their logarithms. Define \( g_t = \log(GDP_t) - \log(GDP_{t-1}) \) and \( y_t = \log(IPI_t) - \log(IPI_{t-1}) \). The sample size used in estimation is from 1961 (I) to 1995 (IV), with an additional 12 observations reserved for post-sample forecasting. Summary statistics are reported in Table 1 for \( g_t, y_t \) and the relevant series for the U.S.

3.1 Specification of a linear AR(p) model

Based on the Akaike information criterion (AIC), I specify a linear AR(p) model first. For each AR(p) model, I check for autocorrelation in the residuals using LM-type tests. The results indicate that there is no AR(1) and AR(4) process in the residuals. The AIC suggests an AR(1) model for both the GDP and IPI growth rate series by setting the maximum lag length of ten. The regression results for the two AR(1) models are as follows.

For the GDP growth rate series:

\[
\hat{g}_t = 0.0062 + 0.3087 g_{t-1} \tag{14}
\]

\[
(0.0011)(0.0807)
\]

\( N = 138 \quad R^2 = 0.0971 \quad \hat{\sigma}^2 = 0.85492 \times 10^{-4} \)

For the IPI growth rate series:

\[
\hat{y}_t = 0.0052 + 0.3872 y_{t-1} \tag{15}
\]

\[
(0.0017)(0.0788)
\]

\( N = 138 \quad R^2 = 0.1508 \quad \hat{\sigma}^2 = 0.3015 \times 10^{-3} \)

where \( N \) is the number of observations, and the figures in parentheses are the estimated standard errors.
3.2 Investigating nonlinearity in the Canadian business cycle using the STAR models

Based on the AR(1) model we have specified using the AIC criterion, I carry out the linearity tests using the ordinary F tests discussed in Section 2 for different delay parameters. The artificial regression is as follows:

\[ z_t = \alpha_0 + \alpha_1 z_{t-1} + \alpha_2 z_{t-1} z_{t-d} + \alpha_3 z_{t-1} z_{t-d}^2 + \alpha_4 z_{t-1} z_{t-d}^3 + \nu_t \] (16)

where \( z_t = g_t \) or \( y_t \). The null hypothesis is \( H_0 : \alpha_2 = \alpha_3 = \alpha_4 = 0 \). I set the maximum delay parameter to be 10. The test results for linearity are listed in Table 2.

Using 0.05 as a rather arbitrary threshold \( p \)-value, we note that linearity is not rejected for the GDP time series, but it is rejected for the IPI series very strongly when the delay parameter is five. I also apply the LM-type test for linearity as derived in Teräsvirta (1994). The results are similar to the F test results and are not reported here. The lack of evidence of nonlinearity for the GDP series may be due to the fact that GDP contains some sluggish components as suggested by Teräsvirta and Anderson (1992). For example, there may be a consumption smoothing effect, i.e., the consumers seem to smooth their consumption over different time periods. Or the nonlinearity may be of a different form to the STAR models. Furthermore, the sample size may be too small for the test to have good power.

After rejecting linearity for the IPI series, I carry out the three F tests as discussed in section 2 to choose between the LSTAR and ESTAR models. The results in Table 3 are for the following three null hypotheses based on the artificial regression of (16):

\[ H_{04} : \alpha_4 = 0(F_4) \]
\[ H_{03} : \alpha_3 = 0 | \alpha_4 = 0(F_3) \]
\[ H_{02} : \alpha_2 = 0 | \alpha_4 = \alpha_3 = 0(F_2) \]

From Table 3 we can see that \( F_4 \) is rejected most strongly. According to the arguments of Section 2, I apply a LSTAR(1) model to the IPI series. We rewrite the LSTAR(1) model as follows:

\[ y_t = \alpha_{10} + \alpha_{11} y_{t-1} + (\alpha_{20} + \alpha_{21} y_{t-1})(1 + \exp[-\gamma(y_{t-d} - c)])^{-1} + \nu_t \quad \gamma > 0 \] (17)

To ensure that \( \gamma > 0 \), I replace the \( \gamma \) with \( \theta^2 \) in the estimation. After trying different starting values, I obtain the following LSTAR(1) model by
maximum likelihood estimation.

\[
\hat{y}_t = 0.0040 + 0.4474 y_{t-1} + \\
(0.0045)(0.0758) \\
(0.0237 - 1.0753 y_{t-1})(1 + exp[-60.078 * 52.63 * (y_{t-5} + 0.0328)])^{-1} \\
(0.0166) (0.7215) \quad (137.97) \quad (0.0018) \\
N = 138 \quad LLF = 359.6262 \quad \hat{\sigma}^2 = 0.2732 * 10^{-3}
\]

where $LLF$ is the log-likelihood value. Note that $\gamma$ has been scaled by $\hat{\sigma}_{y_{t-5}}$, where $\frac{1}{\sigma_{y_{t-5}}} = 52.63$. The standard deviation of $\gamma$ is obtained by the Delta method based on the Taylor approximation.\(^4\)

The residual variance for the LSTAR(1) model is about 90% of the corresponding residual variance for the AR(1) model, which implies improvement in the in-sample explanatory power. The parameters in the transition function of the LSTAR(1) model are $\hat{\gamma} = 60/\hat{\sigma}_{y_{t-5}}$ and $\hat{c} = -0.0328$. The value of $\hat{\gamma} = 60/\hat{\sigma}_{y_{t-5}}$ indicates a very quick transition from one regime to the other regime, and the model behaves very similar to a TAR (Threshold Autoregressive) model. The standard deviation of $\hat{\gamma}$ is very large, which implies that accurate estimation of $\gamma$ is a problem. The parameter $\hat{c} = -0.0328$ is quite low in value. In our sample, only two of the observations are below this value, and there are not many observations around this value. This also implies that estimation of $\gamma$ is not accurate since we need a lot of observations around $c$ to obtain a precise estimate of $\gamma$. Thus with a small sample size, these estimates should be interpreted with caution. Furthermore, $\hat{c}$ marks a point during recession. The delay parameter is estimated as $\hat{d} = 5$. This, combined with $\hat{c}$, describes the dynamic process of the LSTAR(1) model. When $y_{t-5} + 0.0328 > 0$, the value of the transition function approaches one, and the economy is performing in a high growth period; when $y_{t-5} + 0.0328 < 0$, the value of the transition function approaches zero, and the economy is running in a low growth period. Consider the two extreme regimes: when the value of the transition function is zero, the performance of the economy follows an AR(1) process such as $\hat{y}_t = 0.0040 + 0.4474 y_{t-1}$; when the value of the transition function is one, the economy follows another AR(1) process such as $\hat{y}_t = 0.0276 - 0.6279 y_{t-1}$. We can see that the expansion and recession of

\(^4\)By the Delta method, if $\gamma = g(\theta)$, then $Var(\gamma) = |g'(\theta)|^2 * Var(\theta)$, where $g'(\theta)$ is the first derivative of $g(\theta)$ w.r.t. $\theta$. Here $\gamma = g(\theta) = \theta^2$, so that $S.E.(\gamma) = 2\theta * S.E.(\theta)$. 
the economy have different dynamics when we use a LSTAR model, but the transition between them is continuous.

To examine the performance of the LSTAR(1) and the AR(1) model, I plot the fitted values from the two models and the real IPI growth rates in Figure 1. Examining Figure 1, we can see that some expansions of the economy are captured better by the LSTAR(1) model than by the AR(1) model. But both models have poor prediction performance during recessions.

An interesting question is the source of the linearity in this LSTAR(1) model. We want to investigate whether nonlinearity is due to the constant term $\alpha_{20}$ or the coefficient on the lagged variable $\alpha_{21}$ in the bracket in (17). We can test the hypotheses using a likelihood ratio (LR) test. First I restrict $\alpha_{20} = 0$ in the LSTAR(1) model. The LR test indicates that we reject the null hypothesis of "$H_0: \alpha_{20} = 0$" at the 1% significance level. When I restrict $\alpha_{21} = 0$, the null hypothesis of "$H_0: \alpha_{21} = 0$" is not rejected at a 5% significance level. This implies that nonlinearity is due to the constant term of the two regimes.

3.3 Diagnostic tests for the nonlinear model

The diagnostic statistics are presented in Table 4. There is evidence of negative skewness. There is no ARCH(4) process in the residuals. The Jarque-Bera normality test failed at 5% significance level, but not at 1% level. Since nonlinear least squares is used for estimation, the coefficients obtained are consistent and asymptotically normal when certain regularity conditions are satisfied even if the innovations follow other distributions (see Davidson and MacKinnon, 1993, pp.145-157).

Another way to evaluate the model is to investigate its long-run properties as discussed by Teräsvirta and Anderson (1992). Consider the following equation:

$$y_t = f(y_{t-1}, y_{t-5}; \hat{\theta})$$

(19)

where $\hat{\theta}$ are the estimated coefficients. We can start (19) with various starting values $y_{t-1}^0$ and $y_{t-5}^0$, then extrapolate the process with noise suppressed with $t \to \infty$. It may converge to a stable point, exhibit a limit cycle, or diverge. We will reject the estimated model if the process diverge since this kind of behavior is not observed in the original series. We may also observe chaotic behavior in the process: when the starting values are perturbed by a small
magnitude, we will observe quite different solution paths for the process.

I use three groups of starting values to generate the process. Series 1 uses the first 6 observations from the original series, Series 2 uses the first 6 observations of the original series in ascending order (representing a recession period), and Series 3 uses the first 6 observations of the original series in descending order (representing an expansion period). The simulation indicates that the model converges to a stable point of about 1.7% using the three sets of starting values. Using the data after the oil shock in 1981 as the starting values does not change the results. Based on this analysis, this model has a stable growth equilibrium point in the long run.

### 3.4 Our-of-sample forecasting of the linear and nonlinear models

We can evaluate the estimated nonlinear model using out-of-sample forecasting, although the forecasting performance relies on what happens in the time series during the prediction period. Generally we need a wide range of values over the prediction period to efficiently compare the forecasting performance between the linear and nonlinear models. From a comparison of Figure 1, we can see that if the prediction period does not contain a clear expansion, then the linear and nonlinear model will behave similarly. The prediction period used for our forecasting is rather ‘normal’. The biggest expansion is about 1.3% for the quarterly data. Table 5 presents the root mean squared errors (RMSE) of the twelve quarterly one-step-ahead forecasts for the linear and nonlinear models. The RMSE of the nonlinear model is slightly smaller than that of the linear model.

We can applied the test in Granger and Newbold (1986, pp.278) to see whether there is any difference in the forecasting performance between the linear and nonlinear models. Suppose two competing procedures produce errors $e_1^t$ and $e_2^t$, $t = 1, 2, \ldots, N$. The test statistic $r$, which is based on the sample correlation coefficient, is calculated in the following way

$$
r = \frac{\sum_{t=1}^{N}(e_1^t + e_2^t)(e_1^t - e_2^t)}{\left[\sum_{t=1}^{N}(e_1^t + e_2^t)^2 \sum_{t=1}^{N}(e_1^t - e_2^t)^2\right]^{1/2}}
$$

The null hypothesis is $H_0 : r = 0$, with the one-sided alternative as $H_1 : r > 0$. We assume that the nonlinear model has superior forecasting performance
over our alternative. Under the null, we have the following approximate distribution

\[ w = \frac{1}{2} \ln \left( \frac{1 + r}{1 - r} \right) \sim N \left( 0, \left( \frac{1}{N - 3} \right) \right) \]

where \( N \) is the number of post-sample forecasts (here \( N = 12 \)). The null hypothesis is not rejected in our case. So the forecasting performance in our prediction period does not give us any new evidence whether against or in favor of the estimated LSTAR(1) model.

### 3.5 Examining the differences between the LSTAR(1) and ESTAR(1) model

In order to see the properties of the STAR family of models, I also fit an ESTAR(1) model to the IPI series. The estimated results are

\[
\hat{y}_t = 0.0028 + 0.48y_{t-1} + \\
(-0.0021 + 0.0969y_{t-1})(1 - e^{[-0.3732 \times 2.771 \times (y_{t-5} - 0.0024)^2]}) \\
(0.0037) (0.1729) (0.2349) (0.0034)
\]

\[ N = 138 \quad LLF = 359.6262 \quad \hat{\sigma}^2 = 0.2732 \times 10^{-3} \]

Comparing this result with the LSTAR(1) model, we have identical log likelihood function value and residual variance. Following the suggestion of Teräsvirta (1994), the \( \gamma \) parameter has been scaled by \( \hat{\sigma}_{y_{t-5}} \). The value of \( \hat{c} = 0.0024 \) marks the halfway point between the recession and the expansion, and it is very close to zero. Although \( \hat{\gamma} = \frac{0.6109}{\hat{\sigma}_{y_{t-5}}} \) seems to be excessively large, after multiplying by the term \((y_{t-5} - 0.0024)^2\), the values of the transition function become quite reasonable. To compare the properties of the LSTAR(1) and ESTAR(1) models, I sort the IPI series into ascending order and in Figure 2 plot the values of the transition function against the values of the sorted series to examine their differences.

From Figure 2, we can see that the LSTAR(1) model performs like a two-regime TAR model, while the ESTAR(1) model exhibits a smooth transition between recessions and expansions.

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5So in the calculation of \( r \), \( e^1_t \) are the forecasting errors from the linear model while \( e^2_t \) are those from the nonlinear model.
The form of the transition function of the LSTAR(1) model can be attributed to the fact that $\hat{c} = -0.0328$, which marks a point during recession. Also there are only two observations below this value, which means only a large negative economic shock can bring the economy into recession. Combined with the fact that $\hat{\gamma} = 60/\hat{\sigma}_{y_t-5}$ is large, the transition from one regime to the other is very quick. Only eight of the values of the transition function are equal to zero, while the other values of the transition function are equal to one. Thinking of the long-run properties of the LSTAR(1) model we have discussed previously, this implies that unless a large negative economic shock can bring the Canadian economy into a recession, the economy will run in a stable equilibrium with a positive economic growth rate.

On the other hand, the form of the transition function of the ESTAR(1) model derives from the fact that $\hat{c} = 0.0024$, which marks a halfway point between recession and expansion and results in a symmetric transition function. Also the transition from one regime to another is quite fast.

The plots of the transition function show that the LSTAR(1) model describes an economy where the contractions and expansions have different dynamics while the transition between them is abrupt. The ESTAR(1) model describes a situation where the contractions and the expansions have rather similar dynamics, whereas the middle ground has different dynamics.

3.6 Comparing the empirical results with a previous study

Based on a comparison with the empirical results of Teräsvirta and Anderson (1992), my findings have some similar and some different aspects. Using four-quarter differences of the logarithm of the Canadian industrial production index taken from the OECD Main Economic Indicators, Teräsvirta and Anderson (1992) specify an LSTAR(5) model for the series. The sample size was from 1960 (I) to 1986 (IV), and the observations from 1987 (I) to 1988 (IV) were used for forecasting. The regression result is reported as (see Teräsvirta and Anderson, 1992, pp.132)

Note that Teräsvirta and Anderson (1992) used four-quarter differences of the data and I use first differences of the data, which may be why we obtain different lag structures for the LSTAR model.
\[ \hat{y}_t = 1.29y_{t-1} - 0.64y_{t-3} - 0.35y_{t-4} + 0.55y_{t-5} + \\
(0.064) (0.12) (0.12) (0.089) \\
(0.027 - 0.56y_{t-1} + 0.64y_{t-3} - 0.34y_{t-5}) [1 + \exp(-37 * 19.8(y_{t-2} - 0.060))]^{-1} \\
(0.014) (0.15) (0.12) (0.10) (50) (0.042) \]
\[ S = 0.0179 \quad S^2/S^2_L = 0.80 \]

where \( S \) is the residual standard deviation, and the \( S_L \) is the corresponding statistic for the linear model. We can see that \( \hat{\gamma} \) in the transition function is large and accurate estimation of it is difficult, which is also the case in our results. The threshold value of \( \hat{c} = 0.06 \) is quite high, which marks a point in an expansion of the economy, in contrast to \( \hat{c} = -0.0328 \) in my results, which marks a point during a recession. Note that the sample period for Teräsvirta and Anderson (1992) is from 1960-1986, while I have additional observations from 1987-1995. During this period there is a recession for the Canadian economy. Combined with the different lagged variable in the transition function, maybe these differences explain why we obtain a quite different threshold value. An unreasonable high or low threshold value may imply that the accuracy of the estimation of the threshold value is affected by the accuracy of the estimation of \( \gamma \). In both cases, the estimated \( \gamma \) seems to be large and accurate estimation of it is difficult. The delay parameter specified by Teräsvirta and Anderson (1992) is two, which implies the state of the economy is governed by the difference between the two period lagged IPI growth rate and the threshold value. In both cases, we find that the residual variance of the nonlinear model is smaller than that of the linear model, which implies improvement in estimation. In Teräsvirta and Anderson (1992)’s paper, the \( p \)-value of the linearity test is only 0.071 when the maximum delay parameter is set to five, which implies linearity can not be rejected at the 5% significance level they adopted. In my paper, I find strong evidence against a linear model.

4 Adding U.S. information into the STAR model of the Canadian business cycle

Since Canada has a close economic relationship with the U.S., it is natural to expect a dependence between the two countries’ business cycles. Because
of the differences in country size, it is also natural to expect that the U.S. economy is driving the Canadian one rather than the other way around. I hypothesize that it is the U.S. factors that drive the Canadian business cycle. This is tested by investigating the effects of different U.S. variables in the transition function of the STAR models to see how the U.S. economy affects the Canadian economy. In this section, I assume that the U.S. variables are weakly exogenous in the STAR model, where weak exogeneity is defined in Engle et al. (1983, pp.278).7

I still adopt the IPI series as the indicator of the Canadian business cycle in order to compare the results obtained here with those reported in Section 3. The U.S. variables I consider are quarterly GDP (in 1992 dollars), the IPI index (1987=100), the one-year Treasury Bill (TB) rate and the M1 money supply.

The economic mechanism whereby the U.S. economy influences the Canadian one is as follows. When the U.S. economy fluctuates, the growth rate of GDP will change correspondingly. This will affect the demand for the Canadian commodities in the U.S. Of course, we will expect a time lag for this influence. Hence we put a delayed U.S. variable into the transition function to see whether the U.S. business cycle influences the Canadian one. The GDP and IPI indexes are common indicators of business cycles. The one-year TB rate is an important base interest rate in the economy. If money is not neutral, then M1 will affect the economy directly. These two indicators can be viewed as signs of the business cycle.

We transform the four time series using the first differences of their logarithms. The transformed series (i.e., their growth rates) are stationary examined by the augmented Dickey-Fuller test.

4.1 Adding the U.S. GDP growth rate into the transition function

The first variable I try in the transition function is the U.S. GDP growth rate. The baseline linear model is still the AR(1) model according to the results of Section 3. So I test linearity using the baseline AR(1) model. The linearity tests are carried out using the ordinary F-test and the results are

---

7Essentially, a variable $z_t$ in a model is defined to be weakly exogenous for estimating a set of parameters $\lambda$ if inference on $\lambda$ conditional on $z_t$ involves no loss of information.
reported in Table 6.

From Table 6 we can see that linearity is rejected most strongly when the delay parameter is five. We set the delay parameter of the U.S. GDP growth rate at five. To choose between the LSTAR and ESTAR models, I apply the ordinary F-tests for the three nested hypotheses outlined in Section 2. The three test statistics and their p-values are presented in Table 7. Since \( H_0^3 \) is rejected most strongly, we choose an ESTAR model based on the arguments in Section 2. The estimated ESTAR(1) model is

\[
\hat{y}_t = 0.0047 + 0.5095y_{t-1} + \\
(0.0016)(0.0997) \\
(-0.0004 + 0.0591y_{t-1})(1 - exp[-0.4615 \times 11060 \times (yu_{t-5} - 0.0026)^2]) \\
(0.0008) (0.1344) (0.3187) (0.0016) \\
N = 134 \quad LLF = 359.1203 \quad S = 0.01659 \quad S^2/S^2_L = 91\%
\]

where \( yu_t \) is the U.S. variable, \( S \) is the standard deviation of the estimated residuals (the same notations are used for the subsequent regressions). Since \( S^2/S^2_L < 1 \), we can see slight improvement in the estimation using the nonlinear model over the linear one. The estimated \( \gamma \) is very large (after multiplying by 11060, i.e., \( 1/\hat{\sigma}^2(yu_{t-5}) \)), implying that the economy described by the ESTAR model changes very rapidly between the two regimes. The parameter \( \hat{c} = 0.0026 \) is close to zero, indicating the halfway point between recession and expansion (i.e., \( \hat{c} \) is close to the mean of \( yu_{t-5} \)). The delay parameter is five, implying that the U.S. GDP growth rate affects the Canadian IPI growth rate five quarters later.

We can investigate the source of nonlinearity of this ESTAR(1) model as before using a LR test. When I restrict \( \alpha_{20} = 0 \) in the ESTAR(1) model, the test statistic is not significant at a 1% level. When I restrict \( \alpha_{21} = 0 \), the LR test statistic is significant at 1% level. This indicates that the nonlinearity is mainly due to the coefficient on the lagged IPI growth rate term in the bracket.

The diagnostic tests are presented in Table 8. There is no ARCH(4) process in the residuals. The Jarque-Bera normality test can not be rejected at 1% level. Table 8 also contains the post-sample forecasting performance of the estimated ESTAR(1) model. The RMSE of the ESTAR(1) model is 0.00643, which is almost identical to that of the linear model. The test based on the sample correlation coefficient discussed before indicates that there is no difference between the two forecasting performance of the linear and non-
linear models.

To check the long-run properties of the model, I assign different starting values for the ESTAR(1) model to generate the process with the noise suppressed. The process converges to a stable point of about 0.96%.

4.2 Adding U.S. IPI growth rate into the transition function

The second U.S. business cycle proxy is the U.S. IPI growth rate. The linearity test results are reported in Table 9 by setting the maximum lag to ten. From the table, we can see that the linearity is rejected most strongly when the delay parameter is six.

After rejecting linearity in favor of the STAR models, I carry out the three F-tests to choose between the LSTAR and ESTAR models. The test results in Table 10 indicate that $H_{03}$ is rejected most strongly. Thus I select an ESTAR model in this case.

When I estimate the ESTAR(1) model, convergence is difficult. I try to estimate the model by a grid search of $\gamma$. The value of $\gamma$ is determined with the maximum value of the log likelihood function. After scaling by the sample variance of the delay variable, the estimate of the scaled $\gamma$ does not seem to be too large according to our empirical results. Thus I set the first round grid search for $\gamma$ from 10 to 200 with an increment of 10 at each step.

The results are presented in Table 11 for the first round of grid search with AIC. From the table, I suspect $\gamma$ to be around ten, since the AIC is the smallest at ten (which also means that the value of the log likelihood function is the biggest here). Then I set the second round of grid search for $\gamma$ by setting the range from 1 to 20 with the increment of 1 for each step. The results for the second round grid search are presented in Table 12. From the table, we can see that AIC is smallest when $\gamma$ is fixed at 17. Thus we get the estimated $\gamma$ as 17.

When we re-estimate the ESTAR(1) model with $\gamma$ fixed at 17, the regres-
The estimated $\gamma$ is very large in the transition function so that the value of the transition function changes very rapidly between the two regimes. When we explore the source of the nonlinearity in this ESTAR(1) model, the empirical results indicate that nonlinearity is mainly due to the coefficient on the lagged IPI growth rate term in the bracket.

The diagnostic tests are reported in Table 13. We can see that both the ARCH(4) and the Jarque-Bera normality test pass at the usual significance level. For the post-sample forecasting performance, the RMSE of the non-linear model is slightly smaller than that of the linear model, and the test statistic indicates that there is no difference in the forecasting performance.

In Figure 3, we can see that due to the large value of the estimated $\gamma$ in the transition function, the value of the transition function is changing rapidly around the threshold value. For a point far away from the threshold, the transition function becomes one, where the ESTAR(1) model becomes a linear model.

The long-run properties of this model are investigated in the same way as before, and different starting values seem to drive the economy to a stable point of about 0.68%, although with some small fluctuations around this point.

### 4.3 Adding the growth rate of the U.S. interest rate into the transition function

The third variable I put into the transition function is the growth rate of the interest rate in the U.S. Interest rates are a business cycle indicator because when the economy is in expansion, the interest rate seems to be high due to the investment opportunities and because the demand for money is high. Furthermore, when the economy is expanding, the inflation rate is
usually going up, and the central bank usually increases the interest rate to
fight against inflation.

As before, we start by testing the linearity against the STAR models. The
ordinary F-tests are presented in Table 14 with the lag up to 10. Linearity
is rejected most strongly when the delay parameter is nine. So we set the
delay parameter to be nine. This delay is quite large, meaning that the
Canadian economy is affected by the growth rate of the interest rate in the
U.S. two years ago. Since the interest rate affects the economy with a lag
and since the U.S. economy affects the Canadian economy with a lag, this is
not unreasonable.

After I reject linearity in favor of the STAR models, I distinguish between
the LSTAR and ESTAR models with three nested F-tests. The results are
reported in Table 15. I choose a LSTAR model since the rejection to $F_4$ is
the strongest.

The estimated LSTAR(1) model is reported as

\[
\hat{y}_t = 0.0039 + 0.4340y_{t-1} + \\
(0.0016)(0.0736)
(0.0145 - 1.3474y_{t-1})(1 + exp[-44.89/0.10387 * (yu_{t-9} - 0.2302)])^{-1} \\
(8.4432) (255.75) (123322.38) (3.3915)
\]

\[
N = 130 \quad LLF = 346.1938 \quad S = 0.01687 \quad S^2/S^2_L = 97\%
\]

We notice that standard deviation of $\gamma$ is very large, implying that accurate
estimation of it is difficult. The residual variance of this nonlinear model is
almost identical with the linear model. The threshold value in the transition
function appears to be very large, about 23%. But this is not unreasonable,
since the growth rates of the interest rate in the U.S. seem to have a wide
range of values, ranging from -0.32 to 0.28, with a standard deviation of
about 0.1038. So the threshold marks a point during the expansion.

The diagnostic tests of the LSTAR(1) model is reported in Table 16.
There is no ARCH(4) process in the residuals. But the normality test can
not pass at 5% level but pass at 1% level. For the post-sample forecasting,
there is no difference between the linear and nonlinear models.

Because of the large threshold value in the transition function and large
$\gamma$, the value of the transition function becomes zero when the delayed U.S.
interest rate growth rate is low. With different starting values, the long-run
stationary point of this LSTAR(1) model is about 0.69%.
4.4 Adding U.S. M1 growth rate into the transition function

The last variable I try to put into the transition function is the growth rate of the U.S. M1 money supply. This implies that we assume money is not neutral here, but has a direct effect on the economy. So we assume that the growth rate of M1 affects the GDP of the U.S. and further affects the demand for Canadian commodities and services, thus affecting the business cycle of Canada in an indirect way.

First I carry out the linearity test using the ordinary F-test. Table 17 provides the results up to the maximum lag of 10. We can see that linearity is not rejected at the 5% significance level. The smallest p-value corresponds to the delay equal to six. I reject the linearity at the 10% significance level and set the delay parameter to six.

To choose between the LSTAR and ESTAR models, the three F-tests are presented in Table 18. We can see that the three F-tests indicates no rejection at a 5% significance level. This confirms our finding that linearity is not rejected in favor of the STAR models when the delayed U.S. M1 growth rate is put into the transition function. A possible reason might be that money has a small real effect on the U.S. economy.

4.5 Exploring the dominant factors in the transition function

In Section 3, we have assumed that the Canadian IPI growth rate is driven by its lagged value. From the study of Section 4, we investigate whether the Canadian business cycle is driven by the U.S. factors instead of Canadian factors. To test this hypothesis, we can put both the Canadian and U.S. delay variables into the transition function and see what differences occur in the estimation. That is, the transition function now becomes $F(-\gamma_1(y_t - d - c) - \gamma_2(y^*_t - d - c^*))$, where the superscript * indicates the U.S. variable. However there is an identification problem for $c$ and $c^*$, so I restrict $c^*$ to zero in the estimation to eliminate this problem. We want to see if the Canadian business cycle is totally driven by the U.S. factors, i.e., to test whether $\gamma_1 = 0$ while $\gamma_2 \neq 0$.

Recall that in Section 3 we specified a LSTAR(1) model for the Canadian IPI series. To simplify model selection, I put different U.S. delay variables
into the transition function of the LSTAR(1) model together with the Canadian variable to test the hypothesis.\(^8\)

First I put the growth rate of the U.S. GDP into the transition function of the LSTAR(1) model together with the Canadian IPI growth rate. In Table 6 we can see that linearity is rejected strongly when the delay parameters are five and six when only the U.S. GDP growth rate is used in the transition function. Here I vary the lag of the U.S. GDP growth rate from one to six in the transition function and re-estimate the LSTAR(1) model again to choose the optimum lag for the U.S. By choosing the maximum log likelihood value, I find that the delay parameter should be set at four. The result of the estimation is

\[
\hat{y}_t = 0.0314 - 0.7143y_{t-1} +
\]

\[
(0.0225)(1.6388)
\]

\[
(-0.0277 + 1.1516y_{t-1})* \quad (26)
\]

\[
(0.0226)(1.6278)
\]

\[
(1 + \exp[-7.7757/0.019 * (y_{t-5} - 0.1381) - 43.0375/0.0094 * y_{t-4}])^{-1}
\]

\[
(6.5602) \quad (0.0238)(46.0153)
\]

Judging from the t-ratios of the estimated \(\gamma_1\) and \(\gamma_2\), neither \(\gamma_1\) nor \(\gamma_2\) is significant at a 10% significance level. We can not reach a conclusion based on the empirical results.

When I put the U.S. IPI growth rate into the transition function of the LSTAR(1) model together with the Canadian delay variable, I also vary the delay parameter of the U.S. variable from one to six and re-estimate the LSTAR(1) model. The convergence is very sensitive to the starting values and the estimations are not satisfactory, with very large standard deviation for the estimated \(\gamma_1\) and \(\gamma_2\). The results are not reported here.

We can see that using the LSTAR(1) model as a baseline model to characterize the nonlinearity of the Canadian business cycle, the empirical results do not indicate clearly whether the Canadian business cycle is driven by the Canadian specific factor or by the U.S. factor. Further research on this aspect should be conducted to explore the dominant factor that is driving the

\(^8\) We specify an ESTAR(1) model when we put U.S. GDP and IPI growth rate into the transition function, so it may not be appropriate to put different U.S. variables into the transition function of the LSTAR(1) model. The results in this subsection are very preliminary.
Canadian business cycle.

5 Conclusion

In this paper, I apply the STAR models to capture the nonlinearity of the Canadian business cycle. First I specify an AR(p) model using the AIC criterion. Using the growth rate of GDP and IPI as the indicators for the business cycle, I specify an AR(1) model for both series. When I apply the linearity test for the two series based on the AR(1) model, linearity is not rejected for the GDP series, while it is rejected for the IPI series. A possible reason for this might be that GDP contains some sluggish components and is not as volatile as the IPI. For the IPI growth rate, the three F-tests for choosing between the LSTAR and ESTAR models lead to the specification of an LSTAR(1) model. Using a LR test, the empirical results indicate that nonlinearity is mainly due to the constant term of the AR(1) process. The final estimation indicates that the estimated $\gamma$ in the transition function is quite large and the model behaves like a two-regime TAR model. The large standard deviation of $\gamma$ also implies that accurate estimation of it is difficult. The threshold value is -0.0328, which marks a low point in recession. Judging from the residual variance, there is slight improvement in the estimation of the nonlinear model relative to the linear model. However, there is no difference in the post-sample forecasting performance. The long-run property of the LSTAR(1) model is that the economy will stay in a stable growth position.

When I put delay U.S. variables into the transition function to investigate the influence of the U.S. business cycle on the Canadian one, we find different results for different variables. When the GDP growth rate is tried in the transition function, we specify an ESTAR(1) model with the delay up to five quarters. The threshold value in the transition function is 0.26%, which marks a halfway point between recession and expansion. But the estimated $\gamma$ is large. The long-run property of the ESTAR(1) model seems to converge to a stable point. Use of the growth rate of the U.S IPI series also leads to the specification of an ESTAR(1) model, but convergence is difficult to achieve. When a grid search for $\gamma$ is is tried, the estimated $\gamma$ seems to be around 17. The delay is up to 6 quarters and the threshold value is 2.7%. The long-run process of this ESTAR(1) model indicates convergence to a stable
point with some small fluctuations around it. Using the growth rate of the U.S. interest rate as the delay variable leads to a specification of a LSTAR(1) model, with the delay up to nine quarters, and the threshold value of 23.02%. But the estimation is not satisfactory with very large standard deviations for the nonlinear part of the model. Finally, applying the growth rate of M1 money supply as the delay variable results in linearity not being rejected at a 5% significance level. When I put the U.S. delay variable together with the Canadian one in the transition function, the results do not indicate which factor is driving the Canadian business cycle.
Table 1: Summary statistics for the Canadian and U.S. series

<table>
<thead>
<tr>
<th>NAME</th>
<th>N</th>
<th>MEAN</th>
<th>ST. DEV</th>
<th>VARIANCE</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDPG($g_t$)</td>
<td>151</td>
<td>9.08E-03</td>
<td>9.44E-03</td>
<td>8.91E-05</td>
<td>-1.39E-02</td>
<td>3.34E-02</td>
</tr>
<tr>
<td>IPIG($y_t$)</td>
<td>151</td>
<td>8.60E-03</td>
<td>1.81E-02</td>
<td>3.29E-04</td>
<td>-5.05E-02</td>
<td>5.05E-02</td>
</tr>
<tr>
<td>GDPGU</td>
<td>151</td>
<td>8.12E-03</td>
<td>9.04E-03</td>
<td>8.17E-05</td>
<td>-2.43E-02</td>
<td>3.73E-02</td>
</tr>
<tr>
<td>IPIGU</td>
<td>151</td>
<td>9.14E-03</td>
<td>1.83E-02</td>
<td>3.34E-04</td>
<td>-9.90E-02</td>
<td>5.38E-02</td>
</tr>
<tr>
<td>IRGU</td>
<td>151</td>
<td>2.90E-03</td>
<td>0.10217</td>
<td>1.04E-02</td>
<td>-0.32921</td>
<td>0.28876</td>
</tr>
<tr>
<td>M1GU</td>
<td>151</td>
<td>1.35E-02</td>
<td>1.12E-02</td>
<td>1.26E-04</td>
<td>-2.01E-02</td>
<td>4.39E-02</td>
</tr>
</tbody>
</table>

Note: N is the number of observations (we lose one observation due to calculation of the growth rate). GDPGU is the GDP growth rate in the U.S., IPIGU is the IPI growth rate in the U.S., IRGU is the interest rate growth rate in the U.S., M1GU is the M1 money supply growth rate in the U.S.

Table 2: $p$ values of the linearity tests at different values of the delay parameter for the two time series

<table>
<thead>
<tr>
<th>Delay Parameter</th>
<th>Time series</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_t$</td>
<td></td>
<td>0.35</td>
<td>0.34</td>
<td>0.49</td>
<td>0.31</td>
<td>0.4</td>
<td>0.92</td>
<td>0.73</td>
<td>0.63</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>$y_t$</td>
<td></td>
<td>0.17</td>
<td>0.59</td>
<td>0.29</td>
<td>0.52</td>
<td>0.00168</td>
<td>0.9</td>
<td>0.33</td>
<td>0.43</td>
<td>0.28</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 3: $p$ values of the fourth-order, cubic and quadratic linearity tests when the delay parameter is five for Canadian IPI series

<table>
<thead>
<tr>
<th></th>
<th>$p(F_4)$</th>
<th>$p(F_3)$</th>
<th>$p(F_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-test</td>
<td>0.00046</td>
<td>0.0991</td>
<td>0.76503</td>
</tr>
<tr>
<td>LM tests</td>
<td>0.00011913</td>
<td>0.089941</td>
<td>0.76201</td>
</tr>
</tbody>
</table>
Table 4: $p$ values of ARCH(4), Jarque-Bera normality test, skewness and excess kurtosis measures of residuals from the estimated LSTAR(1) model for Canadian IPI series

<table>
<thead>
<tr>
<th></th>
<th>Other Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(4) test</td>
<td>Jarque-Bera Normality test</td>
</tr>
<tr>
<td>0.73</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

Table 5: Root mean squared errors of the forecasts for the AR(1) and LSTAR(1) models and test of the forecasting performance of the two models

<table>
<thead>
<tr>
<th>Root Mean Squared Errors</th>
<th>p-value of the test for forecasting performance $H_0 : r = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>LSTAR(1)</td>
</tr>
<tr>
<td>0.006455</td>
<td>0.006374</td>
</tr>
<tr>
<td></td>
<td>0.3937</td>
</tr>
</tbody>
</table>

Table 6: p-values of the linearity tests using the U.S. GDP growth rate in the transition function

<table>
<thead>
<tr>
<th>Delay</th>
<th>p-value of $F_L$</th>
<th>Delay</th>
<th>p-value of $F_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60</td>
<td>6</td>
<td>0.033</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
<td>7</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>0.19</td>
<td>8</td>
<td>0.83</td>
</tr>
<tr>
<td>4</td>
<td>0.51</td>
<td>9</td>
<td>0.61</td>
</tr>
<tr>
<td>5</td>
<td>0.0166</td>
<td>10</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 7: Three F-test statistics and their p-values for choosing between the LSTAR and ESTAR models with U.S. GDP growth rate

<table>
<thead>
<tr>
<th></th>
<th>$F_4$</th>
<th>$F_3$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test statistics</td>
<td>0.59</td>
<td>10.05</td>
<td>0.08</td>
</tr>
<tr>
<td>p-value</td>
<td>0.4437</td>
<td>0.0019</td>
<td>0.8933</td>
</tr>
</tbody>
</table>
Table 8: Diagnostic tests and forecast performance of the estimated ESTAR(1) model with U.S. GDP growth rate

<table>
<thead>
<tr>
<th>p-values</th>
<th>Other statistics</th>
<th>RMSE</th>
<th>Test of forecast performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(4) test</td>
<td>Jarque-Bera Test</td>
<td>Skewness</td>
<td>Excess Kurtosis</td>
</tr>
<tr>
<td>0.99</td>
<td>0.023</td>
<td>0.072</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Table 9: p-values of the linearity tests using the U.S. IPI growth rate in the transition function

<table>
<thead>
<tr>
<th>Delay</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value of $F_L$</td>
<td>0.67</td>
<td>0.96</td>
<td>0.0173</td>
<td>0.32</td>
<td>0.92</td>
</tr>
<tr>
<td>Delay</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>p-value of $F_L$</td>
<td>0.0167</td>
<td>0.42</td>
<td>0.83</td>
<td>0.61</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 10: Three F-test statistics and their p-values for choosing between the LSTAR and ESTAR models with U.S. IPI growth rate

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>$F_3$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test statistics</td>
<td>0.3366</td>
<td>5.8496</td>
</tr>
<tr>
<td>p-value</td>
<td>0.56</td>
<td>0.0017</td>
</tr>
</tbody>
</table>
Table 11: AICs for the ESTAR(1) model with U.S. IPI growth rate for the first round grid search of $\gamma$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>110</td>
<td>120</td>
<td>130</td>
<td>140</td>
<td>150</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>160</td>
<td>170</td>
<td>180</td>
<td>190</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 12: AICs for the ESTAR(1) model with U.S. IPI growth rate for the second round grid search of $\gamma$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 13: Diagnostic tests and forecast performance of the estimated ESTAR(1) model with U.S. IPI growth rate

<table>
<thead>
<tr>
<th>p-values</th>
<th>Other statistics</th>
<th>RMSE</th>
<th>Test of forecast performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(4) test JARQUE-BERA TEST</td>
<td>Skewness</td>
<td>Excess Kurtosis</td>
<td>Nonlinear model</td>
</tr>
<tr>
<td>0.55</td>
<td>0.35973</td>
<td>-0.23</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Table 14: p-values of the linearity tests using the U.S. interest rate growth rate in the transition function

<table>
<thead>
<tr>
<th>Delay</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value of $F_L$</td>
<td>0.43</td>
<td>0.88</td>
<td>0.14</td>
<td>0.0371</td>
<td>0.078</td>
</tr>
<tr>
<td>Delay</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>p-value of $F_L$</td>
<td>0.31</td>
<td>0.83</td>
<td>0.0434</td>
<td>0.0027</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 15: Three F-test statistics and their p-values for choosing between the LSTAR and ESTAR models with the growth rate of U.S. interest rate

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>$F_4$</th>
<th>$F_3$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.0012</td>
<td>0.0563</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 16: Diagnostic tests and forecast performance of the estimated LSTAR(1) model with U.S. interest rate

<table>
<thead>
<tr>
<th>p-values</th>
<th>Other statistics</th>
<th>RMSE</th>
<th>Test of forecast performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(4)test</td>
<td>Skewness</td>
<td>Excess Kurtosis</td>
<td>Nonlinear model</td>
</tr>
<tr>
<td>Jarque-Bera Test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>0.0159</td>
<td>-0.29</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 17: p-values of the linearity tests using the U.S. M1 growth rate in the transition function

<table>
<thead>
<tr>
<th>Delay</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value of $F_L$</td>
<td>0.75</td>
<td>0.26</td>
<td>0.77</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>Delay</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>p-value of $F_L$</td>
<td>0.0638</td>
<td>0.19</td>
<td>0.65</td>
<td>0.25</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Table 18: Three F-test statistics and their p-values for choosing between the LSTAR and ESTAR models with the U.S. M1 growth rate

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.068</td>
<td>0.054</td>
<td>0.89</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Figure 1: Fitted values from the AR(1) and LSTAR(1) models for Canadian IPI growth rate
Figure 2: The values of the transition functions of the LSTAR(1) and ESTAR(1) models for Canadian IPI growth rate
Figure 3: The values of the transition functions of the ESTAR(1) model with U.S. IPI growth rate.
Data Description

Canadian Data
GDP is taken from CANSIM. It is the seasonally adjusted series in 1992 constant dollars.
IPI is taken from the quarterly International Financial Statistics (1961-1998). It is adjusted so that the base year is 1980.

U.S. Data
All the U.S. data is downloaded from the website http://www.economagic.com. GDP is in billions of chained 1992 dollars. The base year for IPI is 1980. M1 is expressed in billions of U.S. dollars. The one-year TB rate uses the average of the monthly TB rates within a quarter as the quarterly index.
References


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