The Comovement Between Output and Prices: Evidence from Canada

Jim Lee
Texas A&M University-Corpus Christi
Corpus Christi, Texas, USA 78412
jlee@cob.tamucc.edu

January 2004

Abstract
This paper employs a multivariate dynamic conditional correlation GARCH model to detect the timing and nature of plausible changes in the comovement between Canadian output and prices beginning in 1920. The conditional correlation between output and prices varies periodically and, particularly in periods after the 1960s, it changes from positive to negative. Both output and price series exhibit structural breaks in conditional means, but allowing for such breaks does not alter the major findings on their comovement behavior.

JEL Classification: E3, C5

Keywords: Comovement, GARCH, Dynamic Conditional Correlation, Canada
1. **Introduction**

Much of the development of business cycle theories is predicated on the assumption that the overall price level is procyclical, meaning that output and prices move in the same direction. Yet beginning with Friedman and Schwartz (1982), many economists, including Cooley and Ohanian (1991), and Backus and Kehoe (1992), have challenged the generality of this empirical feature, which appeared to hold only until recent decades.

Studies concerning plausible changes in the comovement between output and prices in other industrialized countries are limited, however. We intend to partially fill this gap with Canadian data. To accomplish this objective, we derive measures of the time-varying (conditional) variances of output and the prices and their correlation using Engle’s (2002) dynamic conditional correlation (DCC) GARCH model. This model allows us to pinpoint precisely the timing and nature of plausible changes in the time series’ comovement.

As illustrated below, this DCC-GARCH model appears to adequately capture the volatility behavior of Canadian output and prices as well as their comovement over the past century. The empirical results support that our specified DCC-GARCH model provides an adequate characterization of the volatility behavior of Canadian output and prices, and thus variation in the cyclical property of the overall price level over time. Moreover, allowing for the possibility of structural breaks in both the conditional means and variances does not alter the major findings on the conditional correlation behavior.

The remainder of the paper is organized as follows. The second section discusses the empirical methodology. The third section presents the estimation results. The fourth section evaluates the possibility of structural breaks. The last section contains concluding remarks and suggestions for future research.
2. Methodology

To illustrate Engle’s (2002) dynamic conditional correlation model for our purposes, let $y_t \equiv [y_{1t}, y_{2t}]'$ be a $2 \times 1$ vector containing the output and price series in a conditional mean equation as a reduced-form vector autoregression (VAR):

$$A(L)y_t = \varepsilon_t,$$

where $A(L)$ is a polynomial matrix in the lag operator $L$, and $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]'$ is a vector of disturbances in the VAR with a conditional variance-covariance matrix $H_t \equiv \{h_{it}\}$ for $i = 1, 2$.

The DCC-GARCH framework can be easily understood by first rewriting the conditional variance-covariance matrix as:

$$H_t = D_t R_t D_t$$

where $D_t = \text{diag} \left\{ \sqrt{h_{it}} \right\}$ is a $2 \times 2$ diagonal matrix of time-varying standard deviations from univariate GARCH models; and $R_t \equiv \{\rho_{ij}\}_t$ for $i,j = 1, 2$, which is a correlation matrix containing conditional correlation coefficients. The elements in $D_t$ follow the univariate GARCH($p,q$) processes in the following manner:

$$h_{it} = \omega_i + \sum_{p=1}^r \alpha_p \varepsilon_{it-p}^2 + \sum_{q=1}^q \beta_q h_{it-q} \quad \forall \ i = 1, 2.$$  \hfill (2)

Engle’s (2002) particular DCC($m,n$) structure can be written as:

$$R_t = Q_t^{-1} Q_t' Q_t^{-1},$$

where

$$Q_t = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}.$$
\[ Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)Q + \sum_{m=1}^{M} a_m (\xi_{t-m} \xi_{t-m}^\top) + \sum_{n=1}^{N} b_n Q_{t-n} \]

where \( \xi_t = \varepsilon_t / \sqrt{h_t} \), which is a vector containing standardized errors; \( Q_t = \{q_{ij}\}_t \) is the conditional variance-covariance matrix of the standardized errors with its time-invariant (unconditional) variance-covariance matrix \( \overline{Q} \) obtained from the first stage of estimation; and \( Q_t^* \) is a diagonal matrix containing the square root of the diagonal elements of \( Q_t \):

\[
Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 \\ 0 & \sqrt{q_{22}} \end{bmatrix}.
\]

The key element of our interest in \( R_t \) is \( \rho_{12,t} = q_{12,t} / \sqrt{q_{11,t}q_{22,t}} \), which represents the conditional correlation between output and prices. This DCC-GARCH framework (2)-(3) can be estimated using the maximum likelihood method in which the log-likelihood can be expressed as:

\[
L = -\frac{1}{2} \sum_{t=1}^{T} \left\{ 2 \log(2\pi) + 2 \log|D_t| + \log|R_t| + \xi_t^\top R_t^{-1} \xi_t \right\}.
\]

(4)

The estimation process involves two stages. In the first stage, \( R_t \) is replaced by a 2×2 identity matrix, which reduces equation (4) to the sum of log-likelihoods of the univariate GARCH equations in (2). In the second stage, the DCC parameters in equation (3) are estimated using the original likelihood in equation (4) conditional on the first stage GARCH parameter estimates.

While many GARCH parameterizations enforce positive definiteness in the variance-covariance matrix, which reduces equation (4) to the sum of log-likelihoods of the univariate GARCH equations in (2). In the second stage, the DCC parameters in equation (3) are estimated using the original likelihood in equation (4) conditional on the first stage GARCH parameter estimates.
matrix, this restriction is not imposed in our DCC model such that the conditional correlation coefficients can vary freely between positive and negative values.

3. Estimation Results

In this section, we explore plausible variation in the comovement between Canadian output and prices using the DCC-GARCH model described in the preceding section. We measure output by the log level of Canadian industrial production index, and the overall price level by the log level of Canadian Consumer Price Index. The data are observed monthly and cover the period 1920:1 to 2000:12. The data are made available from Statistics Canada.

The innovation series for the DCC-GARCH model are the residuals of a bivariate VAR of output and prices. Based on the Bayesian information criterion (BIC), we select an autoregressive order of 6 for both output and price series such that the conditional mean equation (1) is represented by a sixth-order bivariate VAR.

Table 1 shows some diagnostic statistics for the innovation series. The first panel displays results of Ljung-Box tests for serial correlation using the residuals and squares of the residuals. The \( Q \)-statistics for an order of 20 clearly indicate the presence of serial correlation in both output and price residuals. The second panel displays test results on the variance-covariance matrices: The first test is Engle’s (1983) Lagrange multiplier (LM) test for ARCH with 20 lags and the second is Breusch and Pagan’s (1980) test for heteroskedasticity.\(^1\) These test statistics offer overwhelming evidence for the presence of conditional heteroskedasticity in the output and price innovations. These findings lend support to the use of ARCH-type models and, in particular, GARCH, to capture the volatility behavior of the output and price series.
The second panel of Table 2 shows results of three tests for normality. There is strong evidence against the null hypothesis of normality. More precisely, the output residuals are found to have a sharper central peak, while the price residuals are found to have both a fat tail and a sharper central peak than the standard normal distribution. This finding calls for the use of Bollerslev and Wooldridge’s (1992) quasi-maximum likelihood method to generate consistent standard errors that are robust to non-normality.

A comparison of the log-likelihood values among alternative lag specifications of the DCC-GARCH model suggests that the data are best represented by a DCC(1,1) with each of the conditional variances captured by a univariate GARCH(1,1) model, i.e., $M=N=P=Q=1$. The bottom of Table 1 shows some diagnostic statistics for the model estimation. Given the scant evidence of remaining serial correlation or conditional volatility in residuals, this parsimonious specification appears to be an adequate characterization of the volatility behavior of Canadian output and prices.

Table 2 displays estimation results for the DCC(1,1)-GARCH(1,1) model. In particular, the sums of $\alpha_i$ and $\beta_i$ in both GARCH equations are fairly close to one, indicating rather high persistence in the conditional variances.\(^2\) The mean value of the conditional correlation coefficient ($\bar{\rho}_{t\tau}$), which reflects the unconditional correlation, appears to be rather small in absolute size. However, two test results indicate that the assumption of constancy in the correlation coefficient over time might lead to spurious inference. The first is a likelihood ratio (LR) test statistic, which is in favor of the DCC structure over the alternative constant conditional coefficient \textit{a la} Bollerslev (1990). The second is a $\chi^2$ statistic developed by Engle and Sheppard (2001) for testing the null of constant correlation $R_t = R$.\(^3\) The test results indicate significant variation in the correlation coefficient over the observation period.
Figure 1 plots over time the conditional variances of the output and price series and their covariance. In line with the conventional view that the Canadian economy has become more stable, the size of output conditional variance is considerably smaller in the post-World War II period than before, particularly the interwar period (1920-1939). Similar to that for the output series, the conditional variance of the price series is noticeably higher in the prewar period than in the postwar period. Within recent subperiods, prices appear to be more volatile during the late 1980s, but price volatility quickly falls to historically low levels after Bank of Canada began targeting inflation in 1991.

The evidence of increased overall economic stability over the postwar period is in line with the experience of other countries, particularly the U.S., as observed by Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Koop and Potter (2000). Some studies (e.g., Clarida, Gali and Gertler, 2000) attribute the reduced variability of both output and prices to improvement in monetary policy, while others (e.g., Ahmed, Levin and Wilson, 2001) focus on an overall reduction in the size of the shocks buffeting the economy. Still, McConnell and Perez-Quiros (2000), and Kahn, McConnell and Perez-Quiros (2002) explain this postwar phenomenon by improvement in information technology and inventory management.

The bottom panel illustrates the pattern of conditional correlation coefficient between output and prices, $\rho_{12}$. In periods after the 1960s, the conditional variance not only reduces in size, but it also turns from positive to negative. Figure 2 further shows how the conditional correlation evolves over time. The covariance determines the sign of the conditional correlation coefficient, which is largely positive before the 1960s. Since then, the correlation coefficient
becomes predominantly negative. The mean of the correlation estimate for the periods before 1960 is 0.10 and that for the periods after 1960 is –0.02.

The finding of a negative correlation in the postwar period is consistent with several earlier studies based on U.S. data (e.g., Kydland and Prescott, 1990; Cooley and Ohanian, 1991; Backus and Kehoe, 1992), but in contrast with den Haan’s (2000) finding. A common interpretation for a negative output-price comovement is that the economy is dominated by aggregate supply versus aggregate demand shocks. However, Ball and Mankiw (1994), Judd and Trehan (1995), and Rotemberg (1996) show that this empirical finding is consistent with the prediction of a sticky price model with only aggregate demand shocks.

4. Structural Breaks

Underlying most GARCH modeling results are correctly specified conditional means. Arguably, the time-varying patterns of variances and correlation coefficient observed in Figures 1 and 2 might have been an artifact of parameter instability in the conditional mean equations. In other words, a structural break in equation (1) can possibly induce ARCH-type behavior and thus misleading inference. For this reason, it makes sense to evaluate the robustness of the empirical results previous reported with the consideration of possible structural breaks.

**Conditional Means**

To test for parameter constancy in the conditional mean equations, we apply a method suggested by Bai (1997), and Bai and Perron (1998, 2003). The method involves a sequential procedure that tests for multiple breakpoints with *a priori* unknown breakdates. We begin with testing for a single structural break on each of the two equations in (1). If the test rejects the null
hypothesis of no structural break, then the sample is split into two based on the breakdate identified. The structural break test is reapplied to each subsample. This sequence continues until each subsample test fails to reveal evidence of a break. Two Wald-type statistics developed by Andrews and Ploberger (1994) are adapted for this sequential testing procedure. The family of Wald statistics test for a single structural break at an unknown point within the middle 90 percent of a data sample. This version of the tests is robust to possible conditional heteroskedasticity.

Table 3 reports $Sup-W_T$ and $Exp-W_T$ statistics, which denote the maximum Wald statistics and exponentially weighted average Wald statistics, respectively. We begin the structural break test over the entire estimation period of 1920:1-2000:12. Both Wald statistics strongly reject the null hypothesis of structural stability in both conditional mean equations of output and prices. The locations of the maximum Wald statistics confirm a structural break at 1935:11 for the output equation and 1948:3 for the price equation.

To detect additional breakpoints, we repeat the Andrews-Ploberger test for subsamples delineated by the breakdates previously identified. For the output equation, we test for possible structural shifts within the 1920:1-1935:11 and 1935:12-2000:12 subperiods. The results show scant evidence of parameter instability within those two subperiods. For the price equation, we test for possible breakpoints within the 1921:1-1948:3 and 1948:4-2000:12 subperiods. The Andrews-Ploberger test statistics indicate an additional structural break at 1972:6. By repeating the stability test over the subsamples delineated by 1972:6, we find an additional break at 1991:1.

In light of the above evidence of parameter instability, it is instructive to allow for the identified structural breaks in the conditional mean equations. Since different breakpoints have

Using the spliced residual series of output and prices, we repeat the DCC-GARCH model estimation again. The estimation results are similar to those in Table 2 and thus are not reported here. As shown in Figure 1, the conditional variance and covariance series of the spliced series (dashed lines) follow closely those without the allowance for structural breaks. Figure 2 shows that the allowance for structural breaks in the conditional mean equations smoothes out the correlation coefficient as expected, but it is also apparent that the overall pattern of the correlation estimate remains similar to that without the allowance for structural breaks. We therefore conclude that our earlier findings on the comovement between output and prices are robust to the presence structural breaks in conditional means.

**Conditional Variances**

Other than instability in the conditional mean equations, failure to detect and thus ignore parameter instability in the conditional variance equations can also lead to spurious inference. For this reason, we apply Chu’s (1995) method to test for parameter instability in the output and price conditional variance equations, as captured by the univariate GARCH(1,1) models.\(^5\) The procedure involves a recursive sequence of LM tests for parameter constancy in the GARCH process against the alternative of a one-time parameter shift at an *a priori* unknown date. The
specific test procedure allows for the violation of the normality assumption, as evident in Table 1. The LM statistics are calculated for all points within the middle 90 percent of the observation period.

The maximum of the LM sequence for the output series is 1.293 and that for the price series is 2.091. Both LM statistics are well below the critical values reported by Chu (1995, Table I), meaning that the null hypothesis of parameter constancy cannot be rejected. The results for the innovation series with the allowance for structural breaks in conditional means are also qualitatively similar. Such results indicate that the observed time variation in the estimated conditional correlation and particularly the sign switching are not the outcomes of structural shifts in either the conditional means or variances of the two time series.

5. Concluding Remarks

In this paper, we have employed a dynamic conditional correlation GARCH model to explore the evolution of the conditional variances of output and prices as well as their conditional correlation using historical Canadian data. The estimation results confirm that the output-price comovement varies considerably over much of the 20th century. In particular, the correlation coefficient turns negative in periods after 1970.

Structural breaks have been detected in the conditional means of both output and price series but none in their conditional variances. Allowing for structural instability, however, does not alter the major findings on the dynamic behavior of output-price comovements. This further implies that the GARCH(1,1) model along with a parsimonious dynamic conditional correlation structure, DCC(1,1), well captures the volatility behavior of Canadian output and prices over the relatively lengthy observation period.
Our findings for Canada have been observed in the U.S. as well and have been subject to different interpretations. Since theoretical models *per se* offer limited guidance on how to ascertain the underlying sources of variation in the output-price correlation, it would be interesting for future research to evaluate these competing theories using our correlation measures.
Endnotes

1 The Breusch-Pagan tests are based on regressing the VAR residuals on the explanatory variables and their squares in addition to an intercept term. The statistics, which are computed as the number of observations times $R^2$ of the estimation, are distributed as $\chi^2$ with 16 degrees of freedom.

2 The overall model estimation results remain the same as those reported in Table 2 even if the restriction of integration, i.e., $\alpha_t + \beta_t = 1$, is imposed.

3 The null hypothesis $H_0$: $R_t = R$ is tested against the alternative hypothesis $H_1$: $\text{vec}(R_t) = \text{vec}(R) + \phi_1\text{vec}(R_{t-1}) + \ldots + \phi_p\text{vec}(R_{t-p})$. Four lagged values are used in this test. The test is conducted as $2TR^2$, which has a $\chi^2(4)$ distribution.

4 The structural break statistics test the null hypothesis $H_0$: $A_t(L) = A(L)$ against the alternative $H_1$: $A_t(L) = A(L), t \leq k$, and $A_t(L) = A^*(L), t > k$, where $k$ is an a priori unknown date, $1 \leq k \leq T$. The statistics considered here are Wald- or $F$-type statistics: $\text{Sup-WT} = \max_{k \in \{t_0, T-t_0\}} F_T(k)$, and $\text{Exp-WT} = \ell n\{(T-2t_0)^{-1}\sum_{k=t_0}^{T-t_0} \exp[F_T(k)/2]\}$, where $t_0$ reflects the middle 90 percent of the sample for which the test is evaluated.

5 We have also employed the recursive CUSUM-type procedures suggested by Andreou and Ghysels (2002) to test for structural breaks in the conditional variances of the output and price series as well as their covariance. The general findings are essentially the same as those using Chu’s (1995) test.
References


Table 1: Diagnostic Test Results.

<table>
<thead>
<tr>
<th>VAR Residuals</th>
<th>Output</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autocorrelation Tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box Q(20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residuals</td>
<td>71.576 (0.000)</td>
<td>67.072 (0.000)</td>
</tr>
<tr>
<td>Residuals Squared</td>
<td>1265.815 (0.000)</td>
<td>504.211 (0.000)</td>
</tr>
<tr>
<td><strong>Heteroskedasticity Tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARCH (20) LM Test</td>
<td>255.981 (0.000)</td>
<td>118.688 (0.000)</td>
</tr>
<tr>
<td>Breusch-Pagan $\chi^2$ Test</td>
<td>545.939 (0.000)</td>
<td>166.930 (0.000)</td>
</tr>
<tr>
<td><strong>Normality Tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.003 (0.973)</td>
<td>0.507 (0.000)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.469 (0.000)</td>
<td>4.347 (0.000)</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>637.454 (0.000)</td>
<td>779.845 (0.000)</td>
</tr>
<tr>
<td><strong>DCC-GARCH Residuals</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Autocorrelation Tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box Q(20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residuals</td>
<td>1.301 (0.201)</td>
<td>11.618 (0.369)</td>
</tr>
<tr>
<td>Residuals Squared</td>
<td>22.867 (0.295)</td>
<td>3.863 (0.999)</td>
</tr>
<tr>
<td>ARCH (20) LM Tests</td>
<td>1.301 (0.172)</td>
<td>0.383 (0.993)</td>
</tr>
</tbody>
</table>

*Note: P-values are indicated in parentheses.*
Table 2: DCC-GARCH Model Estimation Results, 1920-2000.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>0.000 (45.670) ***</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0.000 (31.544) ***</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.140 (18.034) ***</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.101 (29.826) ***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.847 (10.912) ***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.854 (8.923) ***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.014 (23.146) ***</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.983 (9.343) ***</td>
</tr>
</tbody>
</table>

$\rho_{12}$ | -0.053 |
Log-likelihood | 8065.439 |
LR Test: DCC vs. Constant CC | 26.460 ** |
$\chi^2$ Test: $R_t = R$ | 911.423 *** |

Notes: Estimation is based on the DCC-GARCH model:

$$h_i = \omega_i + \alpha_i \epsilon_{t-1}^2 + \beta_i h_{i-1} \quad \forall \ i = 1,2,$$

$$Q_t = (1-a_1-b_1)\bar{Q} + a_1(\xi_{t-1}\xi_{t-1}) + b_1 Q_{t-1}.$$  

Absolute t-ratios are indicated in parentheses.

** and *** denote statistical significance at the 5% and 1% levels, respectively.
### Table 3: Structural Break Test Results.

<table>
<thead>
<tr>
<th>Period for Output Equation</th>
<th>$Sup-W_T$</th>
<th>$p$-value</th>
<th>$Exp-W_T$</th>
<th>$p$-value</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920:1-2000:12</td>
<td>16.484</td>
<td>(0.001)</td>
<td>5.396</td>
<td>(0.000)</td>
<td>1935:11</td>
</tr>
<tr>
<td>1920:1-1935:11</td>
<td>2.290</td>
<td>(0.714)</td>
<td>0.174</td>
<td>(0.845)</td>
<td>1926:3</td>
</tr>
<tr>
<td>1935:12-2002:4</td>
<td>3.972</td>
<td>(0.378)</td>
<td>0.971</td>
<td>(0.205)</td>
<td>1946:1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period for Price Equation</th>
<th>$Sup-W_T$</th>
<th>$p$-value</th>
<th>$Exp-W_T$</th>
<th>$p$-value</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920:1-2000:12</td>
<td>8.183</td>
<td>(0.061)</td>
<td>3.155</td>
<td>(0.012)</td>
<td>1948:3</td>
</tr>
<tr>
<td>1921:1-1948:3</td>
<td>2.289</td>
<td>(0.571)</td>
<td>1.117</td>
<td>(0.135)</td>
<td>1935:2</td>
</tr>
<tr>
<td>1948:4-2000:12</td>
<td>13.045</td>
<td>(0.006)</td>
<td>3.708</td>
<td>(0.005)</td>
<td>1972:6</td>
</tr>
<tr>
<td>1948:4-1972:6</td>
<td>1.495</td>
<td>(0.909)</td>
<td>0.308</td>
<td>(0.635)</td>
<td>1959:12</td>
</tr>
<tr>
<td>1972:7-2000:12</td>
<td>19.631</td>
<td>(0.000)</td>
<td>5.792</td>
<td>(0.000)</td>
<td>1991:01</td>
</tr>
</tbody>
</table>

**Notes:** The $Sup-W_T$ and $Exp-W_T$ are respectively the maximum and exponentially weighted average of the Wald statistics for testing structural breaks within $0.10 \leq T \leq 0.90$ of the specified period. The tests are described in Andrews and Ploberger (1994) with $p$-values reported in Hansen (1997). The dates on the last column indicate the locations of the maximum Wald statistics.
Figure 1: Conditional Variances and Covariance.

- **Output Variance**
  - Values: 0.0000, 0.0025, 0.0050, 0.0075

- **Price Variance**
  - Values: 0.00000, 0.00005, 0.00010, 0.00015, 0.00020

- **Output-Price Covariance**
  - Values: -0.00002, 0.00000, 0.00002, 0.00004, 0.00006

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- **Full sample residuals**
- **Spliced residuals**
Figure 2: Conditional Correlation.

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Full sample residuals —- Spliced residuals