Alibi games: the Asymmetric Prisoner’s Dilemmas

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May 5, 2004

Prepared for
the Meetings of the Canadian Economics Association,
Toronto, June 4-6, 2004

Abstract

We describe Alibi games, a class of asymmetric ordinal games that share the essential features of the Prisoner’s Dilemma but may be asymmetric. Every Alibi game has an equilibrium which is Pareto-dominated by a feasible outcome which is not an equilibrium. The PD is the one symmetric member of the class. Seven of the 144 possible \(2 \times 2\) ordinal games belong to this class.

We examine alternative characterizations of the PD, Alibi games, and social dilemmas, showing how the PD and Alibi games are related to other social dilemmas. We show the PD family lies on the boundary of the topological region [11] containing the twenty-five \(2 \times 2\) social dilemmas.

1 Introduction

The Prisoner’s Dilemma is the most famous \(2 \times 2\) game and the workhorse of introductory game theory. Axelrod called it the \(E.\, coli\)[1] of the social sciences and Elster states that the situation illustrated by the PD is the “fundamental problem in political science.” McCain has claimed the PD may be as important as von Neumann and Morgenstern’s Theory of Games and Economic Behavior.

The Prisoner’s Dilemma is unique among the 144 \(2 \times 2\) games. It is the only symmetric game with a unique, Pareto-dominated dominant strategy equilibrium. If nature were to generate payoffs randomly, lessons derived from the study of the Prisoner’s Dilemma could only be applied with confidence to two thirds of one percent of situations.
The generalizability of the Prisoner’s Dilemma matters to theorists because it provides the most vivid and widely known objection to the most important theorem in all of the social sciences, the First Theorem of Welfare Economics. The First Theorem proves the conjecture embedded in Adam Smith’s famous “invisible hand” metaphor: that, under some conditions, independent rational choice will lead to a “good” allocation. The Prisoner’s Dilemma demonstrates the existence of cases in which independent rational choice leads to a Pareto-inefficient outcome.

In this paper we describe a set of games that are closely related to the Prisoner’s Dilemma. We call them Alibi games because they can be described by varying the standard story to include an alibi for one of the prisoners. There are six Alibi games that share essential features of the Prisoner’s Dilemma. They differ in the efficacy of the alibi and the name of the player with the alibi. Counting the PD as a degenerate case, the Alibi games include every game with an equilibrium which is Pareto-dominated by a feasible outcome which is not a Nash equilibrium.

We describe the Alibi games in Section 2. In Section 3 we provide alternative descriptions of the Prisoner’s Dilemma and discuss the properties of games that are closely related to the Prisoner’s Dilemma in a behavioural sense. Then in Section 4 we sketch a preference-based topology that we have presented elsewhere so that we can examine the topological relationships among the PD, the Alibi games and the Social Dilemmas in Section 5. We show in this section that the Alibi games are related to the set of social dilemmas in a simple and satisfying way. They form the boundary of a topological region containing the twenty-five 2×2 social dilemmas.

After brief remarks on generalizing the PD and its relatives in Section 6 we conclude in Section 7 with comments on the possible importance of the Alibi games. Topological relations among the entire set of 144 2×2 games are presented using order graphs in an appendix.

2 The Alibi game

An Alibi game is an asymmetric variant of the classic Prisoner’s Dilemma story:

**The Prisoner’s Dilemma, Classic**

Two men, charged with a joint violation of the law, are held separately by the police. Each is told that:

1. if neither confesses, both will be given a short sentence on some pretext [3, 3]
2. if one confesses and the other does not, the former will be set free ...and the latter will be given a long sentence [4, 1]
3. if both confess, each will be given a moderate sentence [2, 2]

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1 The result pivots on the presence of reciprocal negative externalities that the First Theorem rules out, making the Prisoner’s Dilemma a complement for the First Theorem rather than a counter-example.

2 The classic version of the story is a descendant of a story Albert Tucker devised to introduce Game theory to Stanford University’s psychology department in May of 1950 [8]

3 Defining the first ordinal as the one corresponding to the lowest real payoff provides the most convenient basis for graphing games in ordinal payoff space as we do in subsequent figures.
If one of the prisoners has an alibi, the game still has a Pareto-dominated Nash equilibrium

**An Alibi game**

Two men, charged with a joint violation of the law, are held separately by the police. Each is told that

1. if neither confesses, both will be given a **short sentence** on some pretext.
   
   One of the players has an alibi, however, which
   
   protects him from the short sentence
   
   **2a** if the one with the alibi confesses and the other does not,
   
   the former will be **given a token sentence** ...and the latter will be **given a long sentence**
   
   **2b** if the one without an alibi confesses and the other does not,
   
   the former will be **released** ...and the latter will be
   
   **3** if both confess, each will be **given a moderate sentence**

The payoff matrix for this Alibi game is produced from the payoff matrix for the Prisoner’s Dilemma by exchanging the 3 and 4 for the column player, as illustrated in Figure 1<sup>4</sup>. We denote this operation $C_{34}$.

![Figure 1: Changing a PD (111) into an asymmetric Alibi game (412) using $C_{34}$](image)

There are two more patterns for the Alibi game. They differ only in that the the player with the alibi receives increasingly large sentences for a unilateral confession. The second is produced from the first by exchanging the payoffs 2 and 3 for the column player, and operation we denote by $C_{23}$. The third is produced from the second by exchanging payoffs 1 and 2 for the column player, written $C_{12}$. Obviously, there are equivalent operations for the row player, $R_{34}$, $R_{23}$, and $R_{12}$.

The key feature of the Alibi games is that having an alibi removes the incentive to confess for the player with an alibi, providing that player is convinced that the other will not confess. It does not, however, remove the incentive for the other player to defect. Confessing is still a dominant strategy for the player without an alibi.

Since the player with the alibi knows that confessing is a dominant strategy for the other player, confessing is still unambiguously a best choice. The resulting Nash

<sup>4</sup>The 2 × 2 games are numbered according to a tabulation described elsewhere.[13]
equilibrium is Pareto-inferior as in the PD. The player with the alibi does not have a dominant strategy, but the game is still dominance solvable. In fact it provides a useful example of dominance solvability.

The Alibi games reveal several features that are not present in the PD. Experimental studies show consistently that people have a bias in favour of symmetric outcomes. Games 221 and 412 (see Figure 5), for example, have rank symmetric equilibria despite their asymmetric payoff matrices. Game 231, 413, 241, and 414, on the other hand, have asymmetric Nash equilibria. It is not obvious how people will play such games. Conclusions about behaviour in games with inferior Nash equilibria are unlikely to be robust if they are based solely on the symmetric Prisoners Dilemma, the only Alibi game that has been studied.

We have made a brief investigation of the Alibi games using evolutionary models. Beaufils, Delahaye and Mathieu [2] described a small round robin tournament of the evolutionary PD game following seven typical strategies through 1000 generations. Four strategies, led by ”Tit for Tat,” survived. We have replicated the experiment, and then applied it to the Alibi games. Because the games are asymmetric, the tournament was run between a row-playing population and a column-playing population. They began as identical uniform distributions of the seven strategies but evolved separately based on their relative success against the opposing population. The results were distinct from the PD. Tit for Tat, for example, did not survive for either population in any Alibi game except the PD. Again, the implication is that generalizing from the Prisoner’s Dilemma to asymmetric cases may be risky.

It is convenient to have a graphical representation, and we offer for this purpose the order graphs on the right in Figure 1. In an order graph, the ordinal payoff pairs in each column of the payoff matrix are connected with a solid line and the pairs in the rows are connected with a dashed line. With the lines indicating whether the pairs belong to a row or a column in the payoff matrix, the figures are complete strategic form representations of the $2 \times 2$ games presented in matrix form on the left$^5$.

### 3 Key features of the PD and the Alibi games

The ordinal Prisoner’s Dilemma has three notable features:

<table>
<thead>
<tr>
<th>symbol</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>the game is rank-symmetric</td>
</tr>
<tr>
<td>$PI$</td>
<td>the outcome is Pareto-inefficient</td>
</tr>
<tr>
<td>$2DS$</td>
<td>both players have dominant strategies</td>
</tr>
</tbody>
</table>

The presence of a Pareto-dominated equilibrium is clearly the most interesting for economists and other social theorists. Symmetry and the strong equilibrium concept make the Prisoner’s Dilemma an elegant, compelling and teachable example. Rank symmetry was a feature of the “non-cooperative pair” in the experiment conducted

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$^5$An order graph is simply the discrete case of the strategic form represented in payoff space. (The familiar matrix representation is in strategy space.) The lines joining payoff pairs correspond to the Inducement Correspondence for the Nash Situation, as described by Greenberg [6] If the Inducement Correspondences are not identified in the figure, the graph is not a complete representation of the strategic form.
by Merrill Flood and Melvin Dresher in January of 1950 and first described in print by Flood in a RAND memorandum in 1952 [5]. Tucker’s version for the Stanford University department of Psychology in May of 1950 was cardinally symmetric [15].

Symmetry (condition S), however, is not necessary to select the PD and only the PD from among the 144 $2 \times 2$ games. The Prisoner’s Dilemma is the only $2 \times 2$ game that has both a Pareto-inferior outcome and a dominant strategy equilibrium. Our initial description therefore contains a certain redundancy.

The presence of a dominant strategy equilibrium is not necessary either. We can replace the presence of a dominant strategy equilibrium with the weaker requirement that the game be dominance-solvable

\[ 1DS \quad \text{one player has a dominant strategy} \]

Note that $2DS \subset 1DS$.

We can also abandon dominance altogether, describing the Prisoner’s Dilemma as a symmetric game with a unique Nash equilibrium

\[ U \quad \text{the game has a unique Nash equilibrium} \]

that is Pareto-dominated. We now have three descriptions with successively weaker equilibrium concepts that select the PD and only the PD from among the $2 \times 2$ games.

\[
P I + 2DS \Rightarrow PD 
\]

(1)

\[
P I + 1DS + S \Rightarrow PD 
\]

(2)

\[
P I + U + S \Rightarrow PD 
\]

(3)

The three equilibrium concepts are nested: $2DS \subset 1DS \subset N$, where

\[ N \quad \text{the game has at least one Nash equilibrium} \]

For two person games $1DS \Leftrightarrow U$, but the equivalence does not hold for games with more than two players. Weakening the equilibrium concept requires additional restrictions. Symmetry serves in the second and third definitions to ensure that a single game is chosen\(^6\).

The Alibi games can be similarly characterized:

\[
P I + 1DS \Rightarrow Alibi 
\]

(4)

\[
P I + U \Rightarrow Alibi 
\]

(5)

which allows us to write

\[
Alibi + S \Rightarrow PD 
\]

(6)

\(^6\)There is in addition a fourth characterization in terms of externalities that imposes conditions on the inducement correspondences and provides a natural link to larger games.
We can conclude, therefore, either that the Prisoner’s Dilemma is a symmetric Alibi game, or that the Alibi games are asymmetric PDs. Since the Alibi games are the larger class, it would be more sensible to adopt the first convention, but since the PD is already an established brand, it is not likely common sense will prevail.

4 The topological framework

Being a bit more rigorous about relationships between games will allow us to relate the Alibi games, including the PD to a still larger class, the so-called social dilemmas.

Every $2 \times 2$ game is related to every other in the sense that there is some transformation that converts the payoff structure for one into the payoff structure for the other. Some games are more closely related in the sense only a small change is required to turn one into the other. Developing a satisfactory notion of closeness allows us to define a **neighbourhood**, which is the key to a rigorous topological approach. Preferences, as we think of them in economics, provide enough structure to induce a topology.

Since games are characterized by the payoff matrix, similar games must have similar payoff matrices. To define meaningful neighbourhoods, we need to characterize the smallest significant change in the payoff matrix. Obviously a change affecting the payoffs of one player is smaller than a change affecting two players. The closest neighbouring games are therefore those games that differ only in that a small change has been made in the sequence of the four numbers describing the ordinal payoffs for one player.

To create our first Alibi game we used one of the six smallest possible changes. We used the five others, sometimes by concatenation, to produce five more Alibi games. Changes like these in the payoffs might result from small changes in information, preferences, or technology, or might result from small errors in identifying games. A player might, for example, receive a very small amount of new information. She might then reconsider the outcome she had originally ranked 1, and decide that it is slightly better than she first thought, and that it is superior to the outcome she had previously ranked 2. She would naturally relabel the two outcomes, resulting in a different payoff matrix, and hence a different game. The new game is close to the old game in that it is reached as a result of a small perturbation in one player’s information set. The game is also close in the sense that it might be mistaken for the original game or it might evolve into the other as a result of a small exogenous change in the underlying technical conditions.

The six minimal exchanges,

$$X = \{X_{ij} | i \in \{1..3\}, j = i + 1, X \in \{R, C\}\}$$

$$= \{R_{12}, R_{23}, R_{34}, C_{12}, C_{23}, C_{34}\}$$

$X_{ij}$ changes the rank of the outcome originally ranked $i$ by the player $X$ to rank $j$ and the rank of the outcome originally ranked $j$ to rank $i$. When $X = R$, we call it a **row swap** and if $X = C$ it is a **column swap**.

Games can be characterized as close or distant neighbours depending on how many swaps are necessary to transform one into the other. We define the **primary neighbourhood of a game** as the set of games that can be reached by a single swap. Neigh-
Neighbourhoods are thus defined strictly in terms of the preferences of the players. The seven-game neighbourhood of the Alibi game in Figure 1 is shown in Figure 2.

The neighbourhood of our first Alibi game, \( g_{412} \), includes several well known games. We already knew that the Prisoner’s Dilemma is one of the neighbours. There is a second symmetric game, \( g_{322} \), or Chicken, in the neighbourhood. It is possible therefore to make one symmetric game, the Prisoner’s Dilemma, into another, Chicken, by the sequence of operations \( R_{34}, C_{34} \). We call a combined swap operation in which the same swap is made for the row and the column players, a symmetric operation, \( S_{ij} \).

There are three games in the neighbourhood with strongly Pareto-dominated, unique Nash equilibria. Equivalently, there is a connected region containing games like the Prisoner’s Dilemma.

Row swaps from \( g_{412} \) yield additional insights. One neighbour, \( R_{12}(g_{412}) = g_{422} \) has no Nash equilibrium in pure strategies\(^7\). Another neighbour, \( R_{34}(g_{412}) = g_{322} \), has two Nash equilibria. It belongs to another group which we identify with the general class of social dilemmas.

\(^7\)It turns out that the games with no Nash equilibria also form a connected subset. Furthermore, the Prisoner’s Dilemma family of games with inferior equilibria lies on the boundary of this important set of games.

Figure 2: The Neighbourhood of an Alibi game
5 The graph of the $2 \times 2$ games

The topology sketched here induces a graph with 144 nodes, each representing one of 144 distinct games\(^8\) in which every node has six edges emanating from it. This graph is connected but not complete: it is possible to start with any game and apply some sequence of transformations to produce any other $2 \times 2$ game, but it requires more than one step to reach most games.

Conditions $2DS, 1DS, PI, S$ and $U$ all correspond to regions in the space of $2 \times 2$ games. Combinations of conditions correspond to the intersections of regions. It is useful to examine the relationships among the regions graphically. The graphical treatment that follows illustrates the intersection of the regions described by the various conditions, but it also reveals the relationships between games in each region and across the regions.

Fully describing the graph that represents the topology of the $2 \times 2$ games is beyond the scope of this paper\(^9\). Instead we describe only those relationships necessary to present the relationships between the Alibi games, the PD, and the Social Dilemmas.

First we need the concept of a layer. Thirty-six games can be produced from the Prisoner’s Dilemma using only $R_{12}, R_{23}, C_{12}, C_{23}$. These games are close to each other in the sense that the position of the most preferred outcome for each player is constant. Adjacent games are related thorough one of the four operations.

The thirty-six games form a layer, a proper subspace of the $2 \times 2$ games\(^10\). There are four such subspaces, or layers. In Figures 8-11, the 144 games are organized by layer. Adjacent games differ by one swap\(^11\). As stated, four operations are used within each layer while the remaining two, $R_{34}, C_{34}$, link games on different layers. The four layers can be stacked to locate the neighbours across the layers.

In Figure 3 the games within a layer are arranged according to whether players

\begin{itemize}
  \item Column has dominant strategy
  \item Both have dominant strategies
  \item Row has dominant strategy
  \item Neither has a dominant strategy
\end{itemize}

Figure 3: A layer organized by dominant strategy

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\(^8\)Rapoport and Guyer [9] offer an exhaustive catalog of only 78 games that has become conventional. Their count is based on treating games which are reflections of each other in a special sense as identical. In effect they assume players are always identical. Their notion of equivalence completely ignores the underlying preference structure of the games.

\(^9\)For a complete treatment of the topology of the $2 \times 2$ games see Robinson and Goforth [13], forthcoming.

\(^10\)We describe the subspaces in detail in [12].

\(^11\)readers can easily verify that adjacent games are separated by a single swap.
have dominant strategies. Games with a dominant strategy for both players appear in the lower left quadrant in Figure 3. The lightly shaded squares indicate games for which one player has a dominant strategy. These games are also dominance solvable. The nine games in the upper right quadrant of each layer have no dominant strategy. All games with either zero or two Nash equilibria are in this region. This block includes almost all of the most interesting $2 \times 2$ games. The Prisoner's Dilemma and the Alibi games are the most intriguing games that fall outside the region.

The eighteen games with zero equilibria appear in the upper right quadrants on layers 2 and 4 in Figure 4 and in Figures 8 and 11 in the appendix. Games without Nash Equilibria may be constant rank-sum games, but they need not be. The games in this region can be seen as the Rift Valley of game theory - mixed strategies were devised to deal with the challenge of such games. Proof of the existence of a mixed strategy was the event from which all game theory developed.

The eighteen games in the corresponding regions on layers 1 and 3 in Figures 4, 9 and 10 are Battle of the Sexes games and Coordination games respectively, both with two equilibria. Strictly ordinal Coordination games must have one inferior equilibrium. Coordination games therefore share an important feature with the PD and the Alibi games.

Coordination games do not have the property that rational individual choice automatically results in an inferior outcome. Rapoport and Guyer [9] actually classify the $2 \times 2$ Coordination games as no-conflict games for which the solutions are obvious. Their treatment obscures the relationships between the Coordination games and the PD. The solution appears obvious, but it remains the case that the inferior equilibrium is, like any Nash equilibrium, stable. Furthermore, the solution is not obvious in the general Coordination games because outcomes are not in general strictly ordered.

The Prisoner's Dilemma is in the third row and column from the lower left in layer 1, far from the Coordination games in the upper right of layer 3. The apparent distance is essentially an accident of the representation. It turns out that the Alibi games provide a direct link between the PD and its cousins on layer 3. If we relax

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12 Rapoport and Guyer [9] made their primary classification according to whether zero, one or two players have a dominant strategy.
Condition $U$ to $N$ in relation 5, accepting all games that have Pareto-inefficient Nash equilibria, we get

$$PI + N \Rightarrow PD + Alibi + Coordination games$$

2DS reduces the set to the PD. 1DS reduces it to the Alibi games including the PD. Either 1DS or $U$ will exclude Coordination games, which have two Nash equilibria.

Alibi games are produced from the Prisoner’s Dilemma by two series of swaps, one series for the column player and one for the row player. Since the first step in each series employs a 3-for-4 swap, the three games in each series must be on a different layers from the PD. Alibi games for the column player are on layer 4. The equivalent sequence of row swaps produces games on layer 2. The location of the Alibi games and the PD are shown in Figure 4. The order graphs for the entire set of Alibi games are shown in Figure 5.

Although it is not entirely obvious in this Figure 4, the PD and the Alibi games lie in a connected region that includes every $2 \times 2$ game that satisfies equations 4 or 5. The Prisoner’s Dilemma is simply an Alibi game that also satisfies Conditions S and 2DS.

Alibi games are neighbours of both zero- and two-equilibrium games. Alibi games all have unique Pareto-inefficient equilibria that are delicate in a particular sense. Swapping payoffs 3 and 4 introduces a new upper equilibrium, producing a game with two equilibria. The 1-2 swap removes the inferior equilibrium, creating a game with no

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13The payoffs for the row player are constant along any row of a layer and the columns represent constant payoffs for the column player. The Alibi games therefore lie in two strips, since they are produced by varying only one player’s payoffs.

10
Nash solution. Both the 3-4 and the 1-2 swap are available at the seam surrounding the upper right quadrant.

Figure 6 presents the relationships among the PD, the other Alibi games, the Coordination games, and the Battles of the Sexes. Since the payoff patterns represented by the games in Figure 6 can all yield socially sub-optimal outcomes under individual rational choice, and since they constitute, as Figure 6 shows, a connected region, we suggest that the entire collection of games is precisely the group that various authors have been groping for when they speak of social dilemmas.\footnote{The games are not a proper subspace of the $2 \times 2$ games however.}

Figure 7, which is easily extracted from Figure 6, presents the PD, the Alibi games, and the Coordination games in a way that emphasizes their contiguity. The figure is one of the more directly teachable and useful by-products of the topological treatment of the $2 \times 2$ games.

Notice that the positive diagonal of the figure consists of symmetric games - The PD, Stag and Hare, and two versions of the Coordination game. The six games remaining can be seen as asymmetric Coordination games. In the Rapoport and Guyer terminology \cite{9} games above the diagonal are reflections of games below the diagonal.

\section{Generalizing the Prisoner’s Dilemma}

The Prisoner’s Dilemma is the minimal game that demonstrates the Hobbesian dilemma, in which actions that are individually beneficial are socially harmful. Most real-world
Figure 7: The PD family and its Coordination cousins

situations described as PD’s - arms races, common property problems, free-rider problems, public goods - are actually multi-person or multi-stage games.

Descriptions that are equivalent for $2 \times 2$ games (relations 1 to 3), are not equivalent for larger games. For $3 \times 3$ games the set of symmetric dominance solvable games with inferior equilibria is larger than the set of symmetric games with inferior dominant strategy equilibria.

It is possible to construct $3 \times 3$ games with multiple Pareto-dominated equilibria. A larger game with multiple, more-or-less equally unattractive equilibria would be interesting in the same way that the PD is, while a game with multiple but Pareto-ranked equilibria would not. It would be more closely related to the Coordination game. The presence of a dominant strategy rules out multiple equilibria, but it is possible to construct games with a multiply dominated dominant strategy equilibrium.

What is lacking is a coherent notion of how to generalize the PD so that its relationship with other problematic games is precisely described and so that the social and economic implications are clearly connected to specific features of the payoff function. The analysis here provides the basis for generalizing the PD systematically. Since there is no agreement about which descriptions is the correct one however, it is likely that there will be many admissible generalizations.

7 Conclusions

In our view the significant feature of the PD, the feature that can be used to identify an economically interesting class, is not the equilibrium concept, but the presence of
a Pareto-dominated outcome of rational choice. The PD is compelling for two reasons: the outcome is inferior in a strong sense, (strong Pareto-inefficiency), and it is supported by an equilibrium concept that is difficult to resist (a dominant strategy equilibrium). Symmetry is essentially an expository convenience.

The Alibi games can be seen as PDs with various alibi structures. Alternatively the Prisoner’s Dilemma can be seen as the degenerate symmetric case of the Alibi game - the only one in which neither player has an alibi. Each of the seven Alibi games (including the PD) is equally likely to appear in a randomly generated payoff matrix. Unless the process that generates payoffs for real situations has a bias in favour of symmetry, only one in seven Alibi games will be symmetric. The Prisoner’s Dilemma is only one of 144 strict ordinal games, but the existence of asymmetric games like the Prisoner’s Dilemma suggests that the problems it illustrates may be even more common than they appear.

It is difficult to say whether Alibi games are important until we search for real world examples. The Prisoner’s Dilemma itself was originally seen as a curiosity. It took time to recognize that multi-person versions like the common property resource problem, the free-rider problem, and multi-period escalation games like the arms race are both common and important.

Since symmetric games are easier to construct and to explain, it is possible that the main reason social situations corresponding to the Alibi games have not been described is that no one has been looking for them. It is also possible that some asymmetric Alibi games have been misidentified as PDs. The apparent real-world ubiquity of the PD may even be illusory - all those situations with nasty equilibria may seem to be PDs because we haven’t looked closely. It is certainly possible that the multi-person and multi-period social dilemmas that are seen as PD analogues may be mixed cases.

The Alibi games provide more than a generalization of the PD, however. They also provide the missing link between the Prisoner’s Dilemma and the wider class of social dilemmas. Study of the Alibi games may help to bring order to our essentially anecdotal approach to the 2 × 2 games.

References


A class of trust games has been describes as an asymmetric Prisoner’s Dilemma [4], [7], [14]. These games are analyzed in extensive form and are not in fact 2 × 2 games in strategic form.


Figure 8: The $2 \times 2$ games by layer: layer 2

Figure 9: The $2 \times 2$ games by layer: layer 1
Figure 10: The $2 \times 2$ games by layer: layer 3

Figure 11: The $2 \times 2$ games by layer: layer 4