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Competitive Burnout in the Laboratory:
Equilibrium Selection in a Two-Stage Sequential Elimination Game

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Abstract

We examine experimentally equilibrium selection in a two-stage sequential elimination contest in which a group of contestants competes to win a single prize. Only a subset of the participants survives the first stage. In the second stage, the survivors compete once more, with the winner taking home the prize. This game has a continuum of Pareto-rankable equilibria, but only the least efficient of those equilibria satisfies the Coalition-Proof Nash Equilibrium (CPNE) refinement. That equilibrium involves “burning out” by using all of one’s resources in the first stage. We show that CPNE is not a good predictor of behavior when four people compete for two second-stage spots, but that it does predict well when eight people compete for the two available spots. Announcing the successful bids at the end of each stage has little impact on equilibrium selection.

Keywords: all-pay auction, burning out, coalition-proof Nash equilibrium, contests, experiment.

JEL classification: D44, D72.
1. Introduction

Coordination problems are common in many economic contexts. Such situations are particularly interesting when they involve multiple Pareto-rankable equilibria. For example, consider team production. If low effort on the part of one worker reduces the marginal products of other team members, it may not be optimal for a particular worker to exert high effort when the efforts of another are low. In this case, the team may be stuck in a low-effort equilibrium even though all team members would be better off in a high-effort equilibrium. An interesting aspect of this problem is that while both low-effort and high-effort outcomes are Nash equilibria, the latter Pareto-dominates the former. Indeed, there may be a continuum of Pareto-rankable equilibria.

Economists and game theorists have proposed solutions to equilibrium selection in such games. Some of these include focal points (Schelling, 1960), belief-learning (Camerer and Ho, 1999), and Pareto dominance (Harsanyi and Selten, 1988). A growing area of research examines coordination games experimentally in order to shed light on the issue of equilibrium selection (e.g., Van Huyck et al., 1990; Van Huyck et al., 1991; Camerer and Knez, 1994; Van Huyck et al., 2001; Anderson et al., 2001; Berninghaus et al., 2002). Generally, this literature finds that smaller groups reach more efficient equilibria than larger groups, especially when play is repeated with a fixed group of participants.

This paper contributes to this line of research by examining equilibrium selection in a two-stage sequential elimination contest in which a group of contestants compete to win a single prize. Only a subset of the participants survives the first stage. In the second stage,

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1 Chapter 7 of Camerer (2003) provides an excellent summary of this literature.
the survivors compete once more, with the winner taking home the prize. The main point of this game is not cooperation to produce a high return for the group, but competition to win a single valuable prize. However, like the coordination game described above, it has a continuum of Pareto-rankable equilibria. Thus, the equilibrium selected through some process of coordination by group members affects the earnings of the group as a whole even as its members compete for the ultimate prize. Is cooperation to maximize group welfare possible in such a competitive context?

A refinement of Nash equilibrium, in particular the Coalition-Proof Nash Equilibrium (henceforth CPNE) concept (Bernheim et al., 1987), suggests that the answer to this question is no. The unique CPNE involves the exertion of maximum effort to the point of complete competitive burnout during the first stage of the game, leaving no resources to utilize during the second stage. From the perspective of the participants in the experiment, this burning-out CPNE is Pareto-dominated by all of the other equilibria in the game. Since the CPNE refinement produces strikingly different outcomes than Pareto-dominance, this is a challenging context in which to assess the predictive power of the refinement.

In the next section, we describe and analyze the two-stage sequential elimination game. Section 3 presents the experimental design and section 4 discusses the results. Section 5 concludes the paper.

2. A Two-Stage Sequential Elimination Game

In Amegashie (2003), the following game is presented. Consider $N \geq 3$ risk-neutral agents contesting for a prize with valuations commonly known to be $V_1 \geq V_2 \geq \ldots \geq V_{N-1} \geq V_N$.
\( V_N > 0, \) where \( V_i \) is the valuation of the \( i \)-th contestant, \( i = 1, 2, \ldots, N-1, N \). The contest is divided into two stages. In the first stage, the \( F \) contestants with the highest bids or effort levels are chosen to compete in a second stage from which the ultimate winner is chosen, where \( 2 \leq F < N \). Ties are broken randomly in each stage. Formally, the contest success function in stage one is:

\[
P_{ii} = \begin{cases} 
1 & \text{if } e_i \text{ is one of the top } F \text{ effort levels and is not tied with others}, \\
(F - g)/(r + 1) & \text{if } g \text{ contestants bid higher than the } i \text{-th contestant and this contestant ties with } r \\
& \text{other contestants where } g + r + 1 > F \text{ and } 0 \leq g \leq F, 
\end{cases}
\]

where \( P_{ii} \) is the probability of advancing from stage one to stage two and \( e_i \) is the effort level of player \( i \). In stage two, the contestant with the highest bid wins. Note that the contest in each stage is an all-pay auction.\(^2\)

Following Che and Gale (1997, 1998) and Gravious et al. (2002), suppose all contestants face a common budgetary or effort constraint or cap, \( B > 0 \). These papers give examples of caps in contests: caps on campaign contributions, salary caps in US professional sports\(^3\), and caps on how fast Formula 1 racing cars can move. Also, a cap on effort arises because human beings naturally have a limit on much effort they can expend.

Suppose \( B \) can be allocated between the two stages. Let \( e_i \) and \( x_i \) be the bid or effort levels of the \( i \)-th contestant in stages 1 and 2 respectively, where \( e_i + x_i \leq B \). We assume that \( e_i \) and \( x_i \) also represent the cost of expending effort, i.e. the cost function of effort is linear. In each stage, the contestants move simultaneously.

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\(^2\) See Baye et. al. (1996); Hillman and Riley (1989); and Clark and Riis (1998) for analyses of all-pay auctions.

\(^3\) As noted by Gravious et al. (2002), in the year 2000, NFL teams faced a salary cap of $62,172,000. This was a cap on the aggregate amount they could spend on their top 51 salaried players.
Let \( P_{1i}(\tilde{e}) = P_{1i}(e_1, e_2, ..., e_N) \) and \( P_{2i}(\tilde{x}) = P_{2i}(x_1, x_2, ..., x_F) \) be the success probabilities of the i-th contestant in stages 1 and 2 respectively. Denote the equilibrium success probabilities by \( P_{1i}^*(\tilde{e}^*) \) and \( P_{2i}^*(\tilde{x}^*) \) for the i-th contestant.

In stage two, the equilibrium expected payoff of the i-th contestant, conditional on making it to that stage, is

\[
\Pi_{2i}^* = P_{2i}^*(\tilde{x}^*)V_i - x_i^*. 
\]

Focusing on a subgame perfect Nash equilibrium and applying backward induction, the equilibrium payoff to the i-th contestant in stage one is

\[
\Pi_{1i}^* = P_{1i}^*(\tilde{e}^*)\Pi_{2i}^* - e_i^*. 
\]

The solution to this game is summarized in the following proposition:

**Proposition 1:** Consider a two-stage contest where the contest in the first stage is an all-pay auction and the contestants have valuations commonly known to be

\[ V_1 \geq V_2 \geq \ldots \geq V_{N-1} \geq V_N. \]

Suppose that \( V_i \) is such that \( (F/K)(1/F)V_i - B \geq 0 \) for \( i = 1, 2, \ldots, K-1, K \) and that \( (F/K)(1/F)V_i - B < 0 \) for \( i = K+1, K+2, \ldots, N-1, N \). If \( F < K \) contestants are chosen in the first stage to compete in the second stage and all the contestants face a common budget (effort) constraint, \( B \), which can be allocated between the two stages, then there exist a continuum of symmetric pure-strategy Nash equilibria in which the contestants with the \( K \) highest valuations expend \( e^* \) in stage one while the rest of the contestants expend zero where \( 0 \leq e^* \leq B \). In stage two, the \( F \) continuing contestants expend \( x^* = B - e^* \) in stage two.

\[ \text{If } (F/N)(1/F)V_i - B = 0, \text{ a risk-neutral contestant } i \text{ is indifferent between expending } e^* \text{ and expending zero. To simplify the exposition, we have restricted ourselves to describing the equilibria in which such contestants bid } e^*. \]
Proof: The proof is simple. In any equilibrium the expected payoff for the i-th player is
\[ \Pi_i^* = \frac{F}{N}[(1/F)\Pi_i - (B-e^*)] - e^* \geq 0. \]
A player who deviates from this equilibrium by bidding marginally more than \( e^* \) in stage one guarantees entry to stage two, but will then lose in stage two with certainty yielding an expected payoff lower than the equilibrium expected payoff. A player who bids less than \( e^* \) in stage one will lose with certainty in stage one, also yielding an expected payoff lower than the equilibrium expected payoff. Hence there is no profitable deviation from the equilibrium stated in the proposition.

Q.E.D.

All these equilibria can be Pareto ranked by noting that \( \delta \Pi_i^*/\delta e^* = F/N - 1 < 0 \). Hence \( e^* = 0 \) gives the highest payoff and \( e^* = B \) gives the lowest payoff. If we apply the CPNE refinement, which allows for joint deviations, then \( e^* = B \) is the only pure-strategy equilibrium. To see this, consider an equilibrium in which all the contestants in stage one bid \( e^* < B \). Suppose a group of \( M \) contestants deviate by bidding marginally more than \( e \) in stage one. If \( M = F \), then they are all guaranteed entry to stage two. Their payoff will be \( \Pi_i^d = (1/F)\Pi_i - B > 0 \). It is easy to show that \( \Pi_i^d > \Pi_i^* \), if \( \Pi_i \) is sufficiently large for the \( M \) contestants to justify competing for the prize. This deviation by the \( F \) contestants is immune to deviations by sub-coalitions of this group. Recall that a deviation by a single contestant is not profitable if \( e^* < B \) and a deviation by two or more contestants is not feasible at \( e^* = B \). Thus, \( e^* = B \) is the only pure-strategy CPNE if \( \Pi_i \) if sufficiently large.\(^5\)

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\(^5\) See Bernheim et. al (1987) for a discussion of CPNE.
The equilibrium $e^* = B$ is interesting. Amegashie (2003) calls it the burning-out equilibrium because the contestants expend all their energies or resources in stage one, get burnt-out, and thus have nothing to offer in stage two. Amegashie (2003) argues that the game described above captures three properties of sequential-elimination contests which produce burning out: (i) a constraint on aggregate effort across stages or rounds\(^6\) (ii) extremely high-powered incentives in earlier rounds, and (iii) a level playing field where no contestant is outstanding.\(^7\) As in Che and Gale (1998) and Gravious et al. (2002), the third feature is incorporated into the model by placing a common cap, $B$, on effort. Under such circumstances, burning out in stage one is a rational equilibrium behavior even though the ultimate prize is won only if a contestant is successful in both stages.

We experimentally investigate the following issues. First, how does the value of the prize affect the effort or bid level? As indicated in proposition 1, risk-neutral players should bid $e^*$ when $\frac{(F/N)}{(1/F)V_i} - B \geq 0$ and bid zero when $\frac{(F/N)}{(1/F)V_i} - B < 0$. Actual players need not be risk-neutral. Nonetheless, for each player there should be a critical valuation level consistent with their level of risk aversion that would induce a bid of $e^*$ rather than zero.

Second, do we observe burning out despite the fact that it is the least efficient pure-strategy Nash equilibrium? Under what if any circumstances will rational players allocate all their efforts in stage one when there is another stage ahead? Will there be a process of

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\(^6\) Amegashie (2003) discusses applications of burning out in sports, academia, and other contests.

\(^7\) However, the multiplicity of equilibria implies that these conditions are not sufficient for full burning out. They are only necessary conditions.
convergence to the burning-out CPNE over the rounds of a finite repeated game? Will the feedback received between rounds make a difference to the convergence process? Third, how does the number of players affect the equilibrium. Earlier experimental studies on coordination games have shown that coordination on Pareto-superior outcomes are harder to sustain with more players (e.g., Camerer and Knez, 1994). Camerer and Knez (1994) argue that coordination on Pareto-superior outcomes in their minimum-effort coordination game was hard to sustain for more than two players because beliefs about other players’ behavior become ambiguous: while two players only have to worry about each others’ beliefs, the introduction of additional players forces everyone to think about the beliefs that each player has about the others. Interestingly, in our framework the set of pure-strategy Nash equilibria in proposition 1 is independent of the number of players. However, the predictive power of the burning-out CPNE may depend on the number of players, since the higher is the number of players, the more likely it is that some coalition of $F \geq 2$ players will deviate from a non burning-out equilibrium.

3. Experimental Design

We ran twelve sessions with participants who were undergraduate students at the University of Guelph. They were recruited at a table in the University Centre. A thirteenth session was run using economics professors at the University of Guelph. Participants received a $3.00 show-up fee. The rest of their earnings depended on their performance in the game. Average earnings were $13.20, Canadian inclusive of the show-up fee. The sessions lasted about one hour.
Upon entering the room, participants were asked to take a seat and were assigned a player number. Written instructions were distributed. The instructions were then read aloud while participants followed along on their own copies. The experiment lasted for eight periods, each of which was divided into two stages. At the beginning of each period, each participant was asked to draw an envelope containing an information slip from a box held by the experimenter. The randomly selected information slip told each participant his/her potential prize value. There were four different prize values. Participants were also told the prize values assigned to the other players. The potential prize values determined the monetary payoff of each participant if he/she won the prize at the end of stage two. The information slip also indicated that each participant had an endowment of 50 tokens, some or all of which could be used to place bids in stages one and two. Each token was worth two cents Canadian. Any tokens that were not used in either stage could be cashed in at the end of the game.

In stage one, participants were given the opportunity to bid any integer amount of money between zero and their budgetary caps of 50. After writing their bids in the designated space on their information slips, participants raised their hands and the experimenter collected the slips. Once bids were placed, participants understood that the amount bid would not be returned, regardless of whether or not they won the prize. The two participants with the highest bids were then privately informed that they would move on to stage two. Ties were broken randomly by a draw. Other participants were informed privately that they would not be moving on. Their earnings for the period were 50 tokens minus their stage-one bids.
The two participants who reached stage two were then given the opportunity to bid any amount of money from zero up to whatever amount of money remained after their stage-one bids by writing the desired amount in the designated space on their information slips. The participants who had not reached stage two were asked to write zero in the designated space so that it would not be obvious which two players were still in the game. The person who placed the highest stage-two bid was then privately informed that he/she had won the prize, which was worth the amount that had been indicated on his/her information slip. As in stage one, a random draw was used to determine the final winner if both participants bid the same amount.

At the end of each period, the information slips were returned to each participant, indicating his/her payoff for the period. The payoff of the final winner was simply 50 tokens, minus the tokens bid in stages one and two, plus the prize value drawn at the beginning of the period. The payoffs of the other participants were 50 tokens minus the bid or bids placed during the period.

At the beginning of a new period, each participant drew a new information slip containing a new prize value. Tokens from earlier periods could not be used in the new period. Each participant began each period with exactly 50 tokens.

We ran four treatments, which are summarized in Table 1.

**Treatment 1 - Four persons, no announcement of winning bids:** In the first treatment, four persons participated in the game. Participants were informed at the end of stage one whether or not they would advance to stage two. However, they were not given any information about the level of the successful bids. Similarly, at the end of stage two,
continuing participants were informed whether or not they had won the prize. However, they were not told the level of the winning bid.

**Treatment 2 - Four persons, announcement of winning bids:** Once again in treatment 2, four persons participated in the game. However, in this treatment, the two stage-one bids of those moving on to stage two and the stage-two bid of the final winner were publicly announced.

**Treatment 3 - Eight persons, no announcement of winning bids:** In treatment 3, eight persons participated in the game. As in treatment 1, successful wins were not announced.

**Treatment 4 – Eight persons, announcement of winning bids:** In treatment 4, eight persons participated in the game. As in treatment 2, successful bids were announced.

In treatments 1 and 2, the prize values, which were assigned randomly to the four participants, were set at 100, 170, 230 and 300 tokens. Consider a risk-neutral participant who believed the other three participants would also behave as if they were risk-neutral. If such a participant drew the possibility of winning the 100-token prize, proposition 1 indicates that he/she would bid zero in equilibrium since \((F/K)\left[(1/F)V_i\right] – B < 0\) in this case. However, if such a participant drew the possibility of winning one of the other three prizes, proposition 1 indicates that he/she would bid \(0 \leq e^* \leq B\) in equilibrium in stage one and \(x^* = B – e^*\) in stage two since \((F/K)\left[(1/F)V_i\right] – B > 0\) in these cases.

In treatments 3 and 4, the prize values were doubled relative to treatments 1 and 2 in order to hold expected earnings constant across the four- and eight-person treatments. The prize values were accordingly set at 200, 340, 460 and 600 tokens. Each of these
prize values was randomly assigned to two of the eight participants. Employing the same reasoning as above, risk neutrality implies a bid of zero for those drawing the 200-token prize and a bid of $0 \leq e^* \leq B$ in equilibrium in stage one and $x^* = B - e^*$ in stage two for those drawing any of the other three prize values.

Three sessions of each treatment were run using undergraduate student participants and were analyzed in a two-by-two factorial design framework. One session of treatment 2 was run using economics professors. We hypothesized that both announcements of the winning bids and larger numbers of players might facilitate convergence to the burning-out CPNE. In the case of announcements, we guessed that if everyone learned how much those moving on to stage two had bid in stage one, it might encourage attempts to bid even higher. In the case of eight-person versus four-person competitions, we reasoned that more competitors would increase the likelihood of coalition formation and defection, pushing bids higher.

4. Results

We focus our analysis on the stage-one bids. In the Pareto-optimal Nash equilibrium, all participants bid zero. In such a case, while the two participants who move on to stage two will spend their entire endowment in that stage, participants who do not move on to stage two will take home their entire 50-token endowment. The Coalition Proof Nash Equilibrium (CPNE) calls for all participants for whom the prize value is sufficiently large to burn out by bidding their entire 50-token endowment in the first stage. Participants for whom the prize value is not large enough to justify bidding withdraw from the contest by bidding zero. If participants are risk-neutral and their risk neutrality is
common knowledge, those who draw the prize value of 100 (200) tokens will bid zero in the four-person (eight-person) treatments. Of course, any outcome in which all bidding participants bid a common amount in stage one is consistent with a Nash equilibrium. The CPNE is Pareto inferior to all of the other pure-strategy Nash equilibria.

Figures 1 to 5 present the results of five of the 13 experimental sessions, one from each of the student treatments as well as the one session with economics professors as participants. The bars in the figures indicate the bids placed by the individual participants in the first stage of each period. The bars are ordered identically by participant number in each period. Asterisks indicate bids of zero.

As the representative figures indicate, the Pareto-optimal equilibrium was not achieved in any of the experimental sessions. The economics professors playing the four-person announcement treatment, illustrated in Figure 3, came closest, converging to a bid of about 20. Eight-person sessions converged to a bid very close to the CPNE, while four-person sessions did not.

The figures also indicate that some participants placed a bid of zero. However, only in the case of the economics professors did the bidding behavior suggest consistent risk neutrality. In every period, the economics professor who drew the low prize value of 100 bid zero and only in the last period did one professor who had drawn a higher prize value also place a zero bid. In the student sessions, some participants who drew low prize values bid positive amounts, while some who drew higher prize values bid zero.

If participants had different attitudes toward risk, the prize value required to produce a level of expected earnings high enough to warrant a positive bid would differ from person to person. However, one would nonetheless expect the probability of a positive
bid to be higher, the higher the prize value drawn. To examine this issue, we employ a three-level hierarchical logit model and estimate it using the data from the twelve student sessions. The binary dependent variable is equal to one if a positive bid is placed and zero if a zero bid is placed. We hypothesize that the probability of a positive bid will be positively related to the prize value drawn, while controlling for the period of play, possible treatment effects, and random effects related to the actions of individual participants over time and to particular sessions across individuals.

Level 1 is a logit model, defined for each individual participant in every session over the eight periods of play:

$$\log\left[\frac{P}{1-P}\right] = \pi_0 + \pi_1(\text{PER}) + \pi_2(\text{NV})$$

where $P$ is the probability of a positive bid, PER is the period number minus eight, NV is the normalized prize value and the $\pi$s are individual-level coefficients. Subtracting eight from the period number allows the effect of treatment variables that may interact with the period of play to be tested during the last period of the game when convergence to an equilibrium is most likely to have occurred. The prize value is normalized to correspond to the expected earnings it represents by dividing prize values by the number of participants in the session, either four or eight.

The level-2 model takes account of possible individual-specific random effects on the level-1 coefficients:

$$\pi_0 = \beta_{00} + \eta_0$$

$$\pi_1 = \beta_{10} + \eta_1$$

(2)

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8 Raudenbush and Bryk (2002), and Snijders and Bosker (1999) all provide excellent discussions of hierarchical linear and logit models (also called mixed models or random effects models) incorporating both fixed and random effects.
\[ \pi_2 = \beta_{20} + \eta_2 \]

where the \( \beta \)s are session-level coefficients and the \( \eta \)s represent individual-specific random effects.

The level-3 model takes account of possible session-specific treatment and random effects on the level-2 coefficients:

\[
\begin{align*}
\beta_{00} &= \gamma_{000} + \gamma_{001}(NA) + \gamma_{002}(8P) + \mu_{00} \\
\beta_{10} &= \gamma_{100} + \gamma_{101}(NA) + \gamma_{102}(8P) + \mu_{10} \\
\beta_{20} &= \gamma_{200} + \gamma_{201}(NA) + \gamma_{202}(8P) + \mu_{20}
\end{align*}
\]

where the \( \gamma \)s are level-3 coefficients and the \( \mu \)s represent possible session-specific random effects. The treatment dummy variable NA is equal to 0 for sessions in which the winning bids are announced and 1 if they are not announced. The treatment dummy variable 8P is equal to 0 for the four-person treatments and 1 for the eight-person treatments. Combining the three sets of equations, we estimate:

\[
\begin{align*}
\log[P/(1-P)] &= \gamma_{000} + \gamma_{001}(NA) + \gamma_{002}(8P) + \gamma_{100}(PER) + \gamma_{101}(PER\times NA) + \\
&\quad + \gamma_{102}(PER\times 8P) + \gamma_{200}(NV) + \gamma_{201}(NV\times NA) + \gamma_{202}(NV\times 8P) + \eta_0 + \eta_1(PER) + \\
&\quad + \eta_2(NV) + \mu_{00} + \mu_{10}(PER) + \mu_{20}(NV).
\end{align*}
\]

Table 2 reports the results. The prize value is positively related to the probability of a positive bid as hypothesized, rejecting the null hypothesis with a two-tailed p-value of 0.076, which corresponds to a one-tailed p-value of 0.038. We can thus reject the null in the direction of the hypothesized positive relationship. Neither the period variable nor either of the treatment variables or their interactions is significantly related to the probability of a positive bid. Thus, the positive relationship between prize value and the probability of a positive bid appears to be invariant to both the period in which the prize
is drawn and the four treatments. If we drop all of the insignificant variables, maintaining only NV and the individual-specific and session-specific random effects, the two-tailed p-value on NV falls to 0.001, strongly supporting the hypothesized relationship.\(^9\)

We are primarily interested in how close participants came to the burning-out CPNE in the various treatments. The CPNE is consistent with some participants bidding zero in stage one if they determine that the expected gains from bidding are not worth the cost. Of course, if everyone bid zero in stage one, they would be playing the Pareto-optimal Nash equilibrium. However, in the CPNE many participants burn out by bidding their entire 50-token endowment in stage one of the game. Since a bid of either zero or 50 is consistent with the burning-out CPNE, we define \(\text{EQDIST} = \min(50 - \text{Bid}, \text{Bid} - 0)\) as the dependent variable in a three-level hierarchical linear model.

The level-1 model is defined for each individual participant to account for convergence over the course of the game as:

\[
\text{EQDIST} = \pi_0 + \pi_1(\text{PER}) + \epsilon
\]

where \(\epsilon\) is an observation-specific disturbance term. The level-2 model takes into account the possibility of individual-level random effects.

\[
\begin{align*}
\pi_0 &= \beta_{00} + \eta_0 \\
\pi_1 &= \beta_{10} + \eta_1
\end{align*}
\]

The level-3 model introduces the session-specific treatment effects, which are now our primary focus of interest.

\[
\beta_{00} = \gamma_{000} + \gamma_{001}(\text{NA}) + \gamma_{002}(\text{8P}) + \mu_{00}
\]

\(^9\) If the data from the professor treatment is added to the estimation, the two-tailed p-value becomes 0.019 and all the other variables remain insignificant. When the insignificant variables are dropped the two-tailed p-value becomes 0.000.
\[ \beta_{10} = \gamma_{100} + \gamma_{101}(\text{NA}) + \gamma_{102}(\text{8P}) + \mu_{10} \]  

Initially, we included interaction effects between NA, the no-announcement dummy, and 8P, the eight-person dummy at level 3. These effects were highly insignificant and therefore dropped from the model. Combining these equations, we estimate:

\[ \text{EQDIST} = \gamma_{000} + \gamma_{001}(\text{NA}) + \gamma_{002}(\text{8P}) + \gamma_{100}(\text{PER}) + \gamma_{101}(\text{PER} \times \text{NA}) + \gamma_{102}(\text{PER} \times \text{8P}) + \eta_0 + \eta_1(\text{PER}) + \mu_{00} + \mu_{10}(\text{PER}) + \varepsilon. \]  

Table 3 outlines the results. It is important to remember that there are eight periods in the game and that PER is defined as the period number minus eight. Thus, the estimated intercept and coefficients on both NA and 8P are calculated with respect to the last period. NA is highly insignificant, implying that whether or not there was an announcement made no difference to the distance from the burning-out CPNE in the last period. The insignificance of the interaction between PER and NA indicates that whether or not there was an announcement did not affect the speed of convergence to the CPNE either.

In contrast, 8P is negative and highly significant \((p = 0.000)\), indicating that more players push participants significantly closer to the CPNE. Comparing the value of the intercept, an estimate of the distance from the CPNE in the last period of the four-person treatments, and the value of \(\gamma_{002}\), an estimate of the difference between the distance from the CPNE in the last period of the eight-versus the four-person treatments, it is clear that in the eight-person sessions that distance was very small indeed. The coefficient on PER is not significant, implying that in the four-person games, there is no significant movement towards or away from the CPNE. However, the interaction between PER and 8P is negative and highly significant \((p=0.009)\), indicating that in the eight-person
sessions the convergence towards the CPNE was significantly higher than in the four-person case.

How did participants behave in stage two? Table 4 summarizes stage-two bids. In all of the pure-strategy Nash equilibria, both participants who reach stage two after bidding identical amounts as required by all the pure-strategy equilibria in stage one, should bid all of their remaining endowments in the second stage. In 84% of the cases in which the two players entering stage two were tied in stage one, they did in fact both bid all of their remaining endowments in stage two. There were cases, however, in which the two participants entering stage two bid different amounts in stage one despite the fact that such behavior is not part of a pure-strategy Nash equilibrium. In the treatments where successful bids are not announced, a participant moving on would have no way of knowing for sure whether his/her bid were higher, lower or the same as the other successful participant. In 68% of these cases, the two participants both expended all of their remaining resources in stage two. In the case where unequal bids are announced, the situation is less clear. If one stage-one bid was more than one token higher than another, the player who had bid less in stage one could win for sure without using all of his/her remaining resources in stage two. The player who had bid more in stage one might thus give up and bid zero in stage two. If the player who had bid less in stage one knew this might happen, he/she might try to get away with bidding a low amount. On the other hand, if the player who had bid more anticipated this, he/she would not give up after all. In this instance, both players expended all of their resources in just 27% of the cases. In 57% of the cases, one of the players bid all of his/her remaining resources, while the other did not.
5. Conclusion

We have examined equilibrium selection in a two-stage sequential elimination game with a continuum of equilibria in the first stage. These equilibria can be ranked according to the Pareto criterion. This set of equilibria resembles the continuum of Pareto-rankable equilibria in the weak-link coordination game. In that game, groups of two and to a lesser extent three are better able than larger groups to maintain a Pareto-dominant equilibrium over a series of periods in which the game is repeated in a partner protocol. In our game, the main point is not to cooperate, but to win the prize. In addition, the Coalition-Proof Nash Equilibrium refinement rules out all but the least efficient equilibrium in which everyone who bids burns out by bidding all of their resources in stage one. We find that while CPNE predicts quite well for eight-person groups, it does not predict well at all for four-person groups. However, even in the case of the smaller groups, the Pareto-optimal equilibrium has no predictive power at all. Rather small groups seem to coordinate on an equilibrium in between that predicted by the Pareto criterion and that predicted by CPNE. The likelihood of stage-one burnout seems to depend on the number of people competing for entry into the second stage where the possibility exists of winning the prize. When eight people compete for two chances to win one prize, we consistently observe competitive stage-one burnout in the lab.
References


Table 1

Summary of Treatments

<table>
<thead>
<tr>
<th>Without announcement (eight periods)</th>
<th>4-person group</th>
<th>8-person group</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 sessions with students</td>
<td>3 sessions with students</td>
<td></td>
</tr>
<tr>
<td>With announcement (eight periods)</td>
<td>3 sessions with students</td>
<td></td>
</tr>
<tr>
<td>1 session with economics professors (excluded from statistical analysis)</td>
<td>3 sessions with students</td>
<td></td>
</tr>
</tbody>
</table>

Table 2

Positive versus Zero Bid Results


| Independent Variables | Estimate  | $t$ value | $Pr > |t|$ |
|-----------------------|-----------|-----------|--------|
| Intercept             | 0.004332  | 0.004     | 0.997  |
| No Announcement (NA)  | -0.635910 | -0.516    | 0.618  |
| 8 Participants (8P)   | -0.980644 | -0.762    | 0.465  |
| Adjusted Period (PER) | 0.008298  | 0.059     | 0.954  |
| NA × PER              | 0.045866  | 0.331     | 0.748  |
| 8P × PER              | -0.191543 | -1.272    | 0.236  |
| Normalized Valuation (NV) | 0.052823 | 1.999     | 0.076  |
| NA × NV               | 0.037350  | 1.262     | 0.239  |
| 8P × NV               | -0.009759 | -0.320    | 0.756  |
Table 3

Distance from Burning-out CPNE Results

Repeated Measures Three-level Hierarchical Linear Model with Random Effect on Intercept and Adjusted Period using Full Maximum Likelihood.

| Independent Variables | Estimate     | t value | Pr > |t| |
|------------------------|--------------|---------|------|---|
| Intercept              | 14.544341    | 6.053   | 0.000|   |
| No Announcement (NA)   | 0.425206     | 0.155   | 0.881|   |
| 8 Participants (8P)    | -15.001736   | -5.464  | 0.000|   |
| Adjusted Period (PER)  | -0.159603    | -0.474  | 0.646|   |
| NA × PER               | -0.227421    | -0.615  | 0.553|   |
| 8P × PER               | -1.254216    | -3.347  | 0.009|   |

Table 4

Summary of Stage-Two Behavior

<table>
<thead>
<tr>
<th>Announcement</th>
<th>Same Bid in Stage One</th>
<th>Two Participants Burn Out</th>
<th>One Participant Burns Out</th>
<th>No Participants Burn Out</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>21</td>
<td>3</td>
<td>0</td>
<td>24</td>
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<tr>
<td>Yes</td>
<td>No</td>
<td>8</td>
<td>17</td>
<td>5</td>
<td>30</td>
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<tr>
<td>No</td>
<td>Yes</td>
<td>11</td>
<td>2</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>23</td>
<td>9</td>
<td>2</td>
<td>34</td>
</tr>
</tbody>
</table>
Figure 1
4 persons without announcement

Figure 2
4 persons with announcement
Figure 3
4 professors with announcement

Figure 4
8 persons without announcement
Figure 5
8 persons with announcement