Endogenous Market Thickness, Prices and
Honesty:
Quality Demand Traps

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Abstract

We study the interaction between product quality, prices and demand
in a dynamic model of asymmetric information. Sellers choose between
producing high quality goods which gives low profits today but increases
probability of future survival in the market and low quality ones which
gives higher returns today but lowers future survival. However, demand
depends on expected quality. Multiple steady states (high demand high
quality, low demand low quality) exist if the present discounted value of
lifetime profits from selling high quality goods exceeds a certain cutoff. We
also characterise the equilibrium price which depends on the distribution
of buyer valuations.

JEL classification: L14, L15, O12, O17.

Keywords: market thickness, endogenous quality, multiple equilibria,
price mechanism.

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1 Introduction

In this paper we use a dynamic model of asymmetric information to study the interaction between demand, quality and prices. The basic idea is as follows: production of high quality goods is costly and thus worthwhile only when present and future demand for the product is expected to be high, while demand itself depends on the expected quality. Thus, if production of high quality goods gives low present returns, it will be undertaken only if sellers (or firms) have optimistic expectations about future market activity. Therefore, an economy may move to a high quality, high demand steady state or fall into a low quality, low demand one. This may provide an explanation for why there may be wide variation in product quality and demand across economies (or markets) having similar primitives. In particular, we predict that ‘thick’ markets should see high quality products on sale, while ‘thin’ markets will have low quality products.

We consider markets where quality of a product is not verifiable before consumption\(^1\) and there are no institutional arrangements (like ISO certification) to provide credible signals. This scenario is possibly most typical in developing, emerging and transition economies where buyers face uncertainty about the quality of the products they buy in the marketplace and sellers face uncertainty about their present and future prospects in the market. Information in these economies about past market transactions is limited and exchange in the anonymous market place is potentially risky. Indeed, transition economies show diverse rates of market transactions and differing product qualities. In Russia and other transition economies, (see McMillan (1997), for instance, which discusses specific examples) it is reported that people get cheated if they buy from anonymous sellers, which is why very few people do. Since sellers have low expectations about meeting buyers in the market they find it optimal to cheat when they find a buyer, justifying the buyers’ fears about transacting in the market. Thus, the economy may be caught in a vicious circle of expectations, which are self fulfilling, even though there may be nothing in the preferences of agents and the technology available to prevent the market from being very active with high quality products being sold.\(^2\)

Of course, the quality, demand uncertainty is not confined to underdeveloped countries. The internet whose growing volume of trade is now widely acknowledged\(^3\) faces this quality uncertainty problem as well, which in turn affects demand. As a recent article in The New York Times (NYT, March 7, 2002) pointed out ‘fraud had become a problem since the first online auctions...and auction fraud is now the most prevalent computer-related crime, according to the Internet Fraud Complaint Center.’ This affects buyers as well. The report mentions reactions of buyers to this quality uncertainty saying that this affects their future entry in online transactions. Interestingly, there seems to

\(^1\)In other words this is the market for what is commonly called an ‘experience good’.

\(^2\)Other examples of this include medical care in various hospitals in India, the large variation in the unorganised sector etc. We discuss this in the concluding section in more details.

\(^3\)Neuman and Medvinsky (1997) note the dramatic growth of users and organizations reachable on the net in the last couple of years.
be some evidence (see Resnick and Zeckhauser (2001)) from eBay that buyers place much less credence on positive messages than on negative ones and indeed this is the model specification we adopt in this paper. Resnick and Zeckhauser’s data suggests that prices and probability of trade do not depend significantly on the number of positive messages received about a seller. However, negative messages seem to matter, and the extreme negative message is a prosecution for fraud. The auction site eBay reports, according to Resnick and Zeckhauser, a small proportion of ‘problem’ trades. Other, less well-known, internet sites might be more prone to such problems. Presumably, the expectation that there will be relatively few problems on eBay is self-fulfilling (at least, this is what this paper will argue).

1.1 An outline of the model and summary of results

We develop a model of two sided asymmetric information. Buyers are unaware of the quality of the sellers product and sellers do not know about the valuation of individual buyers or the total market demand. Hence, expectations about the quality of a product (good or service) induces a certain level of market thickness (by which we mean the relative abundance of buyers to sellers), which in turn induces a certain average level of quality. Market thickness defined in this way is used to measure market activity.

More specifically, buyers face uncertainty about the quality of the product they purchase. Sellers face uncertainty about the valuations that buyers place on the quality of their products. Sellers choose the quality they produce and face a trade-off between producing a high quality product, which gives low one period returns but leads to higher future profits, and a low quality product, which gives higher one period returns but bars the seller from future market activity. Given the uncertainty that buyers face about quality, they may not find it optimal to purchase the product, while sellers facing this demand uncertainty may not find it worthwhile to invest in production of high quality products. Thus, multiple steady states may emerge -some involving low quality and low market thickness and some involving high quality and high market thickness.

Buyers and sellers are randomly matched such that each seller meets the same expected number of buyers. Once matching has taken place sellers announce prices, buyers can accept the price, be rematched at a small cost or exit the market. Each market period ends when all buyers have purchased a product or have exited. The sellers who have produced low quality and the unmatched who quit exogenously at a rate δ are replaced next period and the game continues.

4 Thus demand is endogenously determined unlike most models dealing with how demand can affect product quality (see Rogerson (1982), for example).

5 This distinction is worth noting as the term ‘market thickness’ has been used differently by different people in the literature. (See McLaren (2000) who, in the introduction, discusses the different uses this term has in the literature.)
We now provide a brief summary of our main results. An important observation here is the difference between a static version of our model and the dynamic characterisation: the static version of our model yields the low quality, low market thickness outcome as the unique equilibrium while the dynamic model yields multiple steady states.

In characterising the steady states, we derive conditions under which multiple steady states emerge. It turns out that the patience of sellers and the profits from investing in high quality must reach a certain cutoff value for multiple steady states to emerge. The equilibrium prices depend on the ratio of high valuation to low valuation buyers, if the ratio of low valuation buyers is sufficiently high, the prices are such that both types of buyers are served though the low valuation type has zero consumers surplus. If they are not sufficiently high, only the high type is served. Hence, the presence of the low types in sufficient number acts as an externality for the high type who face a lower price.

A couple of important corollaries come out from this model. In the appendix, we also show that with heterogeneity on both sides of the market, if the relative proportion of high valuation buyers and patient sellers is large compared to low valuation buyers and impatient sellers we get only one steady state apart from the no trade equilibrium. We derive the steady state distribution of different types of sellers as well. As the steady state distribution of the population varies with expectations and time preferences it points out the importance of these factors in determining the survival of new firms. A standard explanation in the literature is in terms of different firms facing different random shocks (which determines their exit). This model may perhaps provide a clue towards an alternate explanation. Another important result we get is that market thickness (which is an index of demand) matters only up to a certain level and beyond that quality is driven by cost conditions and preferences only. This is analogous to the market exhibiting Keynesian features (being demand driven) up to a certain point and then exhibiting classical characteristics (where supply conditions drive the economy). This sharp result is partly an artefact of the specific matching technology used but the general intuition is that market thickness is more important when the market is relatively thin. This may partly explain why even flourishing markets are not invasion free from low quality products.

2 Related literature

We briefly survey earlier related work and discuss how the present work differs from earlier models of this type. The analysis of quality variation in markets characterised by asymmetric information (leading to market thinness), of course dates back to Akerlof (1970). However, unlike the static lemons model, the distribution of quality and hence the average level of quality gets endogenously determined in our model, which is more representative of certain markets where sellers can typically decide the level of quality they choose to produce. Moreover, the results in Akerlof’s paper are substantively different even if we allow for

\footnote{See Hopenhayn (1992) for instance.}
endogenous quality choice-yielding only one equilibrium. The reason is that the future expected payoffs are what makes costly investment attractive even though one period gains are lower. Hence, a static version of our model yields the low quality low demand equilibrium as the unique outcome.\textsuperscript{7}

Our model is closely related to the models on endogenous quality choice in the Industrial Organization literature (see Shapiro (1983), for example). These models typically use reputation to sustain equilibria involving investment in quality which show one period losses. On the other hand, we analyze an environment where reputation is hard to develop and there is only some kind of negative reputation (or punishment). Hence, in our environment new players are indistinguishable from older ones and the average quality of the market is the only information that potential buyers of the product have. We discuss several markets where this is so.\textsuperscript{9} Moreover, demand is not endogenous in these reputation models and thus one gets a unique outcome in these models. With endogenous demand, we show that the uniqueness result no longer holds.\textsuperscript{9} Again, multiple equilibria models are not new in the literature. In that way, this paper shares some of the features of most multiple equilibria models, in particular, exhibiting strategic complementarities (see Cooper and John (1988) and Cooper (1999)). However, typically these models are static in nature and have an inbuilt increasing returns property either in the matching technology (see for instance Diamond (1982)) or in the payoff functions of players. In other words, these pure co ordination models represent one shot games where future expectations do not determine current actions.\textsuperscript{10} In its focus on multiplicity of equilibria as an explanation of underdevelopment, this also relates to earlier literature, most notably, Murphy, Shleifer and Vishny (MSV, (1989)) but these papers also do not examine the intertemporal maximisation behavior of agents nor can they be used to analyse the interaction of the supply of high quality goods and its demand. Such calculations about the future and comparison of current gains against future losses are however typical of a firm’s optimising behavior—what varies is the value attached by different firms to current as against future payoffs. In this paper, we explicitly model this dynamic optimizing behavior. In the appendix we also model how the distribution of population evolves over time which few of these papers do. (An exception is Banerjee and Newman (1993)

\textsuperscript{7}For variants of this which can yield multiplicity see Mas Colell, Whinston and Green (1998 ) pp. 438-444.

\textsuperscript{8}The Internet has tried to develop a system of reputation but that is at best partially successful, eBay has a system of buyer feedback. The value of positive feedback left by buyers is of doubtful value, often left by friends or associates of the seller and are thus less reliable than negative feedback which closely corresponds with our model. (see the NYT article earlier cit, and Resnick and Zeckhauser (2001) for details. The latter note that in the data set they look at, existing sellers do not enjoy a boost in price over new entrants.)

\textsuperscript{9}There are endogenous quality choice models in the money literature as well but their purpose is to show how money acts as a uniform quality good and its effect in reducing the quality uncertainty and hence on the supply of high quality goods and welfare.(See Williamson and Wright (1994), and Bernsten and Rocheteau (2004) for examples of such work.)

\textsuperscript{10}For a somewhat different type of model focusing on failure of co ordination leading to suboptimal equilibria see Basu (1986). In his model different conjectures lead to different equilibria, showing how expectations crucially determine the equilibrium.
who model how the distribution of population changes. However, their concern is with the interaction between occupational decisions and the distribution of wealth, which is very different from the present work). This interlinkage across time has been captured in a somewhat different context by Ghosh and Ray (1996) where buyers make repeated entry into the market and hence market history matters but their paper deals with the levels of cooperation which can be sustained when bad past conduct of sellers cannot be punished.\textsuperscript{11} Another related work is that of Kranton (1996) which shows how increases in market thickness can cause alternate forms of exchange (like reciprocal exchange) to diminish, while the widespread use of personalized exchange can itself cause markets to remain thin, hence causing such exchanges to persist over time. The questions addressed viz. the interaction between two different institutions for exchange and the model she uses are completely different from this present work. Finally, mention may be made of the important literature which looks at sustaining co operation when there are no official law enforcement agencies (see in particular Tirole (1996) who looks at a scenario where only one party has an incentive to cheat and Dixit (2003) where both parties may cheat). The role of intermediaries while not discussed formally in this paper is an important complementary area of research. Two current papers which examine the role of intermediaries are Liebi (2002) and Quesada and Peyrache (2004).

The paper proceeds as follows. The next section sets up the basic model. In section 4 we do a steady state analysis. Section 5 discusses illustrations and extensions. Appendix 1 analyses a modification of the basic model to consider what happens when there is heterogeneity among sellers and Appendix 2 works out the different distributions of seller types that can occur in steady state.

\section{The model}

This is an infinite horizon, discrete time model. There are two sides of the market, buyers and sellers. There are two kinds of goods on sale-high and low quality. There are two types of buyers, those with a low valuation for the high quality which we denote by $V_L$ and those with a high valuation denoted by $V_H$. We assume that low valuation goods are uniformly valued by all buyers and we normalise it to 0.\textsuperscript{12} Sellers are characterised by a common discount factor $\delta$.\textsuperscript{13} This discount factor is interpreted as a survival probability. This means that there is a probability $1 - \delta$ in each period that a seller will quit the market for exogenous reasons. Each buyer is however assumed to live only one period during which

\textsuperscript{11}For an interesting study of how history can matter and related dynamics see Adsera and Ray (2000). There are also other dynamic models eg. models of repeated purchases (see Hendel and Lizzieri, (1999)) but their concern is very different from the present work.

\textsuperscript{12}The assumption of no gains from trade in the low quality good simplifies the exposition but makes no difference. Thus this assumption is not restrictive. We show that the steady state characterisation remains unchanged even when there is positive surplus in a working paper version of this paper.

\textsuperscript{13}In the appendix we analyse what happens when we have heterogeneity on the sellers side. We need to deal with finding the distribution of seller types in the population in that case.
he either buys a good (at any stage within the market period) or exits without buying. Thus, there are three types of agents in the economy.\textsuperscript{14} Denote by $N$ the total number of buyers, by $n_H$ the number of $V_H$ and by $n_L$ the number of $V_L$ buyers and by $S$ the number of sellers every period.

Cost, technology and endowments are as follows. High quality goods are produced at a higher cost than low quality goods—we normalise the cost of low quality goods to 0 and the high quality is produced at constant cost per unit denoted by $c$. Sellers have excess capacity and can produce goods to demand. Denote the price charged by a seller by $p$. Given constant unit costs, gains per unit from the high quality good is also a constant every period which we denote by $\pi = p - c$.

There is asymmetric information of the following type - buyers cannot distinguish between the low and high quality good before purchase. Sellers also cannot distinguish the two types of buyers and know only the ratio of the two types. The past history of market transactions is summarised by three variables namely the ratio of buyers who purchased a good to the total stock of sellers (market thickness), the prices at which goods were sold (including the number of goods sold at a certain price) every period and the ratio of high quality goods to low quality goods sold every period. The history of individual transactions that have taken place in the past is not known but these summary statistics are known to all agents. Given these, buyers and sellers have to form expectations (or assessments) about the market thickness and quality at the beginning of each period.

Matching is random every market period. In any time period each seller meets $\frac{N}{S}$ buyers.\textsuperscript{15} After matching, each matched seller announces a price to each buyer he is matched with. Buyers can either accept the price, be rematched (randomly as before) or exit the market. If they are rematched they incur an $c$ cost of rematching. The market closes each period when there are no more buyers i.e. buyers have either made a purchase or quit. The sequence of actions within each market period can be summarised as follows:

- Sellers and buyers get matched randomly
- All matched sellers (simultaneously) announce a price to the buyer they are matched with (they announce prices to all buyers simultaneously if they are matched to more than one buyer). Unknown to buyers sellers also make a quality choice.

\textsuperscript{14}It is important to mention that though we are dealing with finite numbers, we assume that no agent in the economy believes that his action has any influence on market outcome.

\textsuperscript{15}The matching technology can be thought of as follows. For the case of less buyers than sellers, assign each buyer to a seller until there are no more buyers. Considering all possible permutations of sellers gives us $\frac{N}{S}$ as the matching probability. When there are more buyers than sellers, first assign the same number of buyers per seller. The remaining buyers are matched as in the case with less buyers than sellers giving us the required result. For a continuum of sellers it is helpful to think of this as some approximation of large discrete numbers. See Binmore and Herrero (1988) who get this result for a continuum. Thus we circumvent the problem that for two continuous interval there is a one to one correspondence between them.
• Buyers decide whether to buy at that price, be rematched or exit the market. If they decide to rematch, all rematched buyers are randomly rematched as before and they (buyers) incur a cost $\epsilon > 0$.

• After all buyers have either purchased or exited the market, all matched sellers producing low quality goods exit, remaining sellers exit at rate $1 - \delta$. The total number of sellers who exit are replaced by the same number of sellers.

• The market opens again the next period.

We now look at the optimisation problem for each agent. Before that we introduce some new notation. Let $n_t \subseteq N$ be those who actually purchase a good in period $t$. Thus market thickness $q_t = n_t/S$. Let $x_t$ be the probability that the buyer of period $t$ assigns to meeting a honest seller. Our solution concept is Perfect Bayesian equilibrium (though we shall only look at a subclass of such solutions)

**Buyers’ objective function**

Buyers are expected utility maximisers and decide for every price whether they are willing to purchase a product. As they do not have information about the product quality before purchase, the choice is based on the expected gains from trade. Thus, a buyer’s decision is based on his valuation and his expectation of acquiring the high quality object. Thus, buyer of type $i$, facing a price $p$ solves the following problem $\max(xV_i - p, 0)$ ($i = H, L$) Hence, given the assessment that a buyer forms about market conditions, his strategy is a mapping from his type, the price asked and the rematching cost into a decision to buy, be rematched or quit. Let us denote his set of strategies by $A$. Formally, $A_t : \{V_t, [h_t^{-1}], \epsilon, p\} \rightarrow \{I_{t1}\} \times \{I_{t2}\} \times \ldots \times \{I_{ts}\}$ s.t. $\sum I_{ts} \leq 1$ where $h_t^{-1}$ denotes the history of market transactions upto period $t - 1$ and $I_{ts}$ indicates the decision of the buyers to buy (1) or not buy (0) with the restriction that he can buy at most one unit of the good. Not we denote his decision to quit as playing 0 forever. Since we look at Perfect Bayesian equilibria, the beliefs are derived at every node from Bayes rule whenever possible.

**Sellers’ objective function**

Sellers are also expected utility maximisers and maximise the present discounted value of lifetime earnings. Each period they can sell low quality goods and face a lower continuation payoff$^{16}$ from then on or sell high quality goods and face a higher continuation payoff$^{17}$. We assume that the future payoffs after selling the low quality good is 0 i.e. the seller has to quit the market.$^{18}$ Thus the sellers’ trade-off is a high one period gain and nothing thereafter to a low one period gain and a probability of future sales in the same market. From now on, selling a low quality object will be referred to as cheating.

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$^{16}$Perhaps in the form of being identified, for example by the buyer who spreads this information or records a complaint against the seller.

$^{17}$If they sell both, we assume they still face a lower continuation payoff.

$^{18}$This is merely a convenient assumption. All we need is that those who produce high quality goods face better future payoffs than those who produce low quality goods.
Thus, in time period \( t \) a generic seller \( j \) maximizes \( V(q_t, q_{t+1}, q_{t+2}, \ldots) = \max(V_C, V_{NC}) \) where the \( q \)'s represent the (point) estimates that the seller makes about future market thickness and subscripts \( C \) and \( NC \) are used to distinguish the discounted payoffs from deciding to be dishonest in period \( t \) (cheat) and honest in period \( t \) respectively. The \( V \) function looks a little different depending on whether \( q \) is less than or greater than 1. (Note that at \( q < 1 \) it is interpreted as a probability of a match)

For \( q \) less than 1, \( V_C = pq_t + (1 - q_t)\delta E(V(q_t)) \) where \( V(q_t) \) is the continuation payoff if an optimal policy is followed from period \( t + 1 \) (\( q' \) is used as a shorthand for the sequence of future market thickness). \( E \) is used to denote the fact that this is the expected continuation payoff of the sellers based on his (point) estimates about future market thickness. This equation is explained very simply— with probability \( q \) a cheat meets a buyer, gets \( p \) units of money and 0 thereafter. With a probability \( 1 - q \) he does not meet a buyer and simply goes through the same optimisation process again based on the expected value of the current market statistic. This is discounted by seller at the rate \( \delta \). Similarly, \( V_{NC} = q_t(\pi + \delta E(V(q_t))) + (1 - q_t)\delta E(V(q_t)) \).

As we can see, the tradeoff comes from the high one period gain in deciding to cheat and getting \( p \) vs. getting \( \pi = p - c \) in period \( t \) but an expected gain in the future.

Thus a seller simply chooses the maximum of these. In the steady state we shall see that he will choose a stationary strategy (or randomise) when called upon to move.

Now, for a seller, a strategy is more complicated as it has to specify his action at every stage in any time period for every time period in a potentially infinite horizon game. The set of possible strategies (which we denote by \( \Omega \)) is essentially a sequence of cheat and not cheat and an associated price vector along with this quality choice. Thus, possible (but not optimal) strategies for a seller are charging a fixed price say \( p \) and being dishonest in period 1, 2, ..., and so on.\(^{19}\)

Formally, \( \Omega_t : \{\delta, [h^{t-1}], \epsilon \} \rightarrow [(0, 1), p] \times [(0, 1), p] \times \ldots \) with the beliefs at every node being derived using Bayes rule whenever possible.

For \( q \) greater than 1 \( V_C = pq_t \) and \( V_{NC} = \pi q_t + \delta E(V(q_t)) \)

Thus given \((n, S, A, \Omega, EU, V, \delta, n_L, S, \sigma, q, q, p)\) we have all the components of a Bayesian game and the natural solution concept to employ is a Perfect Bayesian Nash (with out of equilibrium beliefs appropriately defined). However, in the next section we concentrate on a more restricted class of solutions viz. the steady state where the variables \((q, x, p)\) are unchanging.

\(^{19}\)Note, that while randomising, if a player randomises with probability \( z \) over an action \( A \), we mean that \( z \) fraction of players play \( A \) with probability 1. As we are dealing with large numbers this does not make much difference, though it is, strictly speaking, correct only for a continuum of agents, which we do not have in this paper. However, along with the assumption on matching, it ensures that the market thickness and quality are not random variables and thus we are able to talk of steady state values of the variables rather than steady state distributions of these variables.
4 Steady State Analysis

Here, we analyse the market when \((p, q, x)\) are unchanging over time.

Formally a steady state is defined as follows

**Definition 1** A steady state is a vector \((p, q, x)\), such that

1. Buyers choose buy, rematch or quit decisions every market period to maximize expected utility.
2. Sellers choose a quality decision and a price that will maximize (expected) lifetime discounted payoffs.
3. \(0 \leq x \leq 1\), \(0 \leq qS \leq N\).

Notice that as is usual we write the steady state in terms of the market variables, though they are derived from the individual optimising behaviour of the buyers and sellers. The feasibility constraints simply say that the fraction of honest sellers \(x\) must lie between 0 and 1 and the total number of buyers in the market who actually purchase a product \((qS)\) cannot exceed the pool of potential buyers \((N)\). We now characterise the steady state of the model. To contrast the steady state level of trade in this model with a benchmark where quality is observable, we assume that in the observable case, there are gains from trade for all the types of agents. Hence, we assume the following condition.

**Condition 1** \(V_L > c\)

The following Lemmas gives us the simplified steady state expressions for the buyers' and sellers' optimization problem.

**Lemma 1** In steady state and for \(q < 1\) the sellers’ decision problem simplifies to the following rule:

Seller \(j\) is honest if

\[
\frac{(p-c)q}{1-\delta_j} - \frac{pq}{1-(1-q)\delta_j} \geq 0
\]

is indifferent if this holds with equality.

**Proof.** In steady state \(q_t = q\). Given this, a seller of type \(j\) chooses to maximize \(V = \text{Max} (V_C, V_{NC})\). Putting \(q_t = q\) for all \(t\) we get

\[
V_C = \frac{pq}{1-(1-q)\delta_j}
\]

and

\[
V_{NC} = \frac{(p-c)q}{1-\delta}
\]

Thus, the seller chooses the maximum of the two. ■

Notice that in steady state if a seller is honest today, he will want to be honest forever, if he cheats today, he will cheat forever. Market thickness \(q\) acts
as an additional discount factor, the lower is $q$, the less is the incentive for the seller to be honest. Hence, we see how expectations about market thickness influence the decision about quality choice.

When $q$ is greater than (or equal to) 1 the steady state equations get simplified. Now there is no uncertainty about meeting a buyer-$q$ is simply the expected number of buyers every period. Thus, dishonest sellers die with probability 1. Hence, the seller’s decision choice is given by

**Lemma 2** In steady state and for $q \geq 1$ the sellers’ decision problem simplifies to the following rule:

Seller $j$ is honest iff

$$\frac{(p - c)q}{1 - \delta} \geq pq$$

is indifferent if this holds with equality) i.e.

$$p\delta > c$$

**Proof.** In steady state, $q_t = q$. Given this, a seller chooses to maximize $V = \max (V_C, V_NC)$. Putting $q_t = q$ for all $t$ we get $V_C = pq$ and $V_NC =$

$$\frac{(p - c)q}{1 - \delta}$$

Thus, the greater of the two is chosen giving us the lemma.

Now, we characterise the conditions for multiplicity. There are a few cases to consider viz: (i) $S \geq N$ (ii) $N > S \geq n_H$ (iii) $S \leq n_H$. As is evident from Lemmas 1 and 2, this will lead to slightly different conditions for multiplicity. The following proposition formalises this.

**Proposition 1** Assume condition 1 holds. Multiple equilibria exists if and only if the following conditions hold for the different cases. Under case (i)

$$\max\left\{\frac{p_L - c}{1 - \delta} - \frac{p_L}{1 - \delta + \delta \frac{S_L}{S}}, \frac{p_H - c}{1 - \delta} - \frac{p_H}{1 - \delta + \delta \frac{n_H}{S}}\right\} \geq 0$$

where $p_L \leq V_L$, $p_H \leq V_H$, under case (ii)

$$\max\left\{\delta p_L - c, \frac{p_H - c}{1 - \delta} - \frac{p_H}{1 - \delta + \delta \frac{n_H}{S}}\right\} \geq 0$$

and under case (iii)

$$\max\{\delta p_L - c, \delta p_H - c\} \geq 0$$

**Proof.** It is easy to see that there is a no trade equilibrium such that sellers believe $q = 0$, in which case everyone produces low quality, given that $x = 0$.

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20 Given that $V_H > V_L$, trade with type $L$ necessarily implies trade with type $H$ so we do not need to separately consider what happens if $n_L \leq S$. 

11
the optimal \(q\) is indeed 0. We now need to show that under the various cases the conditions given do imply the existence of another equilibrium. In case (i), substitute \(q = N\) and \(q = n_H\). Given that

\[
\max \left\{ \frac{p_L - c}{1 - \delta}, \frac{p_L}{1 - \delta + \delta \frac{N}{S}}, \frac{p_H - c}{1 - \delta}, \frac{p_H}{1 - \delta + \delta \frac{n_H}{S}} \right\} \geq 0
\]

from lemma 1 we get that for either \(q = N\) or \(q = n_H\), \(V_{NC} > V_C\). Hence, there exists an equilibrium with high quality products and market thickness \(q = N\) or \(q = n_H\).

The only if part can be proved by considering what happens if

\[
\max \left\{ \frac{p_L - c}{1 - \delta}, \frac{p_L}{1 - \delta + \delta \frac{N}{S}}, \frac{p_H - c}{1 - \delta}, \frac{p_H}{1 - \delta + \delta \frac{n_H}{S}} \right\} < 0
\]

From Lemma 1, we know that for \(V_C > V_{NC}\) at \(q = n_H\) if

\[
\frac{p_H - c}{1 - \delta} - \frac{p_H}{1 - \delta + \delta \frac{n_H}{S}} < 0
\]

and \(p_H\). Notice that \(V_{NC} - V_C\) increases in \(q\) (gains from not cheating rises at a lower rate than gains from cheating as market thickness increases) so \(V_C > V_{NC}\) at \(q < n_H\). For \(q = N\) and \(p = p_L\), \(V_C > V_{NC}\) since

\[
\frac{p_L - c}{1 - \delta} - \frac{p_L}{1 - \delta + \delta \frac{N}{S}} < 0
\]

Hence, \(V_C > V_{NC}\) for \(q < N\) as well. The different \(p's\) are such that it either satisfies the \(V_L\) or the \(V_H\) type’s incentive constraint. The different cases are similar, the difference in the expression occurs because \(q\) is either less or more than unity. The corresponding expressions can be derived from Lemmas 1 and 2.

**Remark 1** If \(q \geq 1\) then the equilibria are entirely dependent on the gains from trade \((p - c)\) and the discount factors \((\delta)\). The intuition is that beyond a certain level of market thickness \((q = 1)\) gains from being honest and cheating go up by the same factor so whatever was optimal for a seller at some market thickness (greater than unity) still remains optimal.

What we now show is that given the particular mechanism we described earlier, the equilibrium prices can be characterised for different parameter values. We describe the sequence again.

Buyers are matched with sellers randomly. Sellers announce prices. A buyer has information only about the price announcement of the seller with whom he is matched. A buyer then decides whether to buy or not. If a buyer does not buy, he can be rematched at a small cost \(c\). We assume that in every match each type of buyer will have a cutoff value, if the announced price is above that he will want to be rematched (or leave the market) otherwise he will buy. We also assume a monotonic belief structure, if at any price a buyer believes that a seller is producing a high quality product, he will continue to do so at a higher price.
Proposition 2 For steady states with positive levels of trade to exist the steady state equilibrium price \( (p^*) \) must be the same for all sellers.

Proof. First note that different buyers cannot be charged different prices as sellers cannot distinguish types, hence price announcements cannot be conditioned on buyer types. Now suppose \( p^* \) was different i.e. the equilibrium price configuration had different sellers charging different prices. Any price which differs by \( \epsilon \) cannot be an equilibrium since either the seller charging the lower price should increase the price and increase his profit or the seller charging the higher price should decrease his price to increase his expected profit. This is because at price \( p \) if the buyer is buying from a seller so will he at price \( p - \epsilon \). However, at price \( p \), if no one is willing to buy, clearly sellers must lower price for trade to occur. Now consider any set of arbitrary prices differing by more than \( \epsilon \). Consider any but the highest price at which the buyer is willing to trade in this configuration of prices. Call this arbitrarily selected price charged by a seller \( p_a \). Under monotone beliefs, he should also be willing to trade at \( p_a + \epsilon \) because of the rematching cost. Thus, for the same types of sellers and buyers (i.e. discount factor and valuation) the argument in Diamond (1971) holds.

Proposition 3 The equilibrium price \( p^* = V_L \) or \( V_H \) depending on whether

\[
NV_L \geq n_H V_H
\]

or vice versa.

Proof. Consider some other price \( p' \) different from \( p^* \). Now if \( p' \leq p^* \) a seller \( j \) should be able to raise it by \( \epsilon/k \) \( (k \geq 1) \) and it would still be optimal for the buyer to buy (since buying at the next round of matching at \( p' \) is equivalent to buying in the present round at \( p' + \epsilon \)). Again \( p' \geq p^* \) lowers profits in expected terms. The price is \( V_L \) if the gains from selling at \( V_L \) to all buyers exceed that from selling at a higher price \( V_H \) to only the high type of buyer depending on the population distribution of the two types. Since \( \frac{1}{k} V_L \) and \( \frac{1}{k} V_H \) gives the expected revenue from the two types. Thus, the greater of the two determines the equilibrium price.

This mechanism is, of course, not the only way to endogenise prices. This however gives a particularly sharp prediction for the prices. In particular, the rematching costs and the rematching technology play a role in sustaining the equilibrium prices. A possible alternative to this would be for sellers to post prices, buyers look at prices and choose the seller with the lower price (hence beliefs are not monotonic)-randomly if prices are the same. In this case it is not difficult to see that the sellers compete away all their surplus. However, this cannot be an equilibrium since at the level where sellers are robbed of all surplus they would prefer to produce the low quality good. Buyers correctly assess this leading to only low quality goods sold at a price of zero (or no trade) as the only sustainable steady state.\(^{21}\) This provides one more example of how

\(^{21}\)This is similar to what happens in a symmetric Bertrand game.
competitive pricing can be harmful and can justify government intervention in regulating prices even when it cannot regulate quality. The debate about the effect of competitive pricing on the quality of LASIK surgery (see, for instance, The Washington Post, February 22, 2000 for a chatty piece summarising these fears) seems to be based on this type of argument. With monotonic beliefs, a whole range of prices can be sustained even with this sequence of events, from the price which satisfies at least one of the sellers incentive constraints to the monopoly price which robs the high type of all the surplus. Hence, our price prediction is contained in the equilibrium set of prices even under this mechanism.

5 Extensions and Applications

We have already discussed the quality issue in internet transactions in the introduction. Several other markets come to mind where the quality of product varies, we discuss some more examples where our model may provide some insight and discuss some extensions to the basic model.

5.1 Illustration

Medical services in India: Kidney transplant in East and South Indian Hospitals

Individual buyers of medical services usually try to assess the quality of the service being offered at various centers but are unlikely to have detailed knowledge of individual providers and depend on the reputation of the center or ‘group’ with which the provider is associated. It is part of ‘folk knowledge’ that various cities in India offer medical facilities which show wide variation in quality, especially as perceived by consumers (patients). Some of the ‘Centers of excellence’ enjoy a market share which does not seem to stem from any fundamental difference in primitives i.e. the technology of medical centers and the qualifications of doctors. As perceived quality is hard to measure, there has been no rigorous study of this. However in detailed case studies undertaken by UNDP (see Bandyopadhyay and Gupta 1997 a and b) certain illuminating facts about the organs transplant scenario came out which are worth noting.

There is a wide variation in the number of transplants carried out in East India and South India - comparing big cities in the South, the average number of transplants in 2 major centers in Chennai (a big city in the South) total over 300 and there are quite a few other centers in Chennai where the yearly average is around 100. In contrast, in Kolkata, the major city in the East where such surgery is carried out, the total number in the two major centers in ten years adds to less than four hundred (other centers contribute a negligible amount - transplants are occasional features there.) Thus the ‘market thickness’ is vastly different with Kolkata having, on an average, customers (transplant recipients) of about thirty five in the two major centers as against over three hundred in Chennai. However, the qualifications of doctors are equivalent in both cities and the Hospitals have comparable facilities (both centers in Kolkata satisfy the
rigorous specifications required to obtain licence under The Transplantation of Human Organs Rules, 1995). It also appears from the study that the quality, as measured by patient satisfaction, is low in Kolkata. From detailed questioning of patients about quality of care, behaviour of doctors and nursing staff etc. the authors note widespread satisfaction in patients who have undergone transplants in Madras as opposed to Kolkata. Indeed they conclude ‘The phenomenon of transplant migration has been continuing over the years as more and more patients in need of a kidney move to other parts of India, notably Chennai, Vellore and Bangalore ...hospitals (in Kolkata) have suffered because of charges of negligence ...in health care.’ Thus, the hypothesis of quality being directly related to market thickness seems borne out.

Table 1 shows the differences in market thickness and customer satisfaction in the two cities:

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Location</th>
<th>Average no. of transplants/year (1988-96)</th>
<th>Patient satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apollo</td>
<td>Chennai</td>
<td>100-120</td>
<td>High</td>
</tr>
<tr>
<td>Willingdon</td>
<td>Chennai</td>
<td>100</td>
<td>High</td>
</tr>
<tr>
<td>Belle Vue</td>
<td>Kolkata</td>
<td>20</td>
<td>Low</td>
</tr>
<tr>
<td>Woodlands</td>
<td>Kolkata</td>
<td>15</td>
<td>Low</td>
</tr>
</tbody>
</table>

What is more interesting is that following the passage of The Transplantation of Human Organs Act, 1994, regulating transplants in the country, it has become more difficult to get permission in the South for non local patients causing a shift towards getting transplanted in their nearest locality. This increased demand has been also matched by increases in the level of quality service provided by the above two centers in Kolkata which has recently seen high turnover coupled with higher patient satisfaction. In fact, this increased demand has caused another center (Wockhardt Medical & Research Centre) to start doing kidney transplants in Kolkata, clearly showing how markets are responding to demand. It is difficult to identify a factor which systematically explains this which is why we think that our multiplicity explanation is convincing.

Taxicabs at day and night.

An interesting phenomena that has been observed is the different service provided by taxicabs at day and night at various airports and railway stations. At night, when traffic is thin cabs tend to provide poor quality, cheating customers by overcharging, taking longer routes and even indulging in outright robbery. At day the service is noticeably better when the flow of passengers is larger. Passengers, in turn, seem to respond to this by relying on taxicabs more heavily during the day and less during the night. This seems to be because the future gains from not cheating are higher at day in the form of getting more passengers than at night. At night time many more tourists make arrangements by asking friends and relatives to meet them or even waiting at the Airport till day before taking a cab. Assuming that the distribution of tourist types and cab drivers are the same in the day and night this seems a good example of demand and quality reinforcing each other. Data on the ratio of tourists who use taxicabs in the day as against those who use it at night should bear this out. Note, that this seems to be a worldwide phenomenon and attempts are made to deal with this by having prepaid cab service available at airports and railway stations where
a centralized agency keeps track of passengers assigned to cabs whose license numbers are noted. (This is based on personal experience, conversations with several people and newspaper reports, thus no specific source is cited.)

**Transition economies**

Russia and several erstwhile communist countries underwent a transition and from centralized planning moved towards a market economy. Hence the potential to switch to high quality was created with a move to a market based structure. However, as noted in McMillan (earlier cit.) ‘New firms find it difficult to sell their products...(due to the) problem of finding potential customers’. Thus, there was inadequate market information and in most of Vietnam private enterprise ‘Most firms ..sold only to local customers’ causing the market to remain thin. To get out of it McMillan discusses the role of ‘chambers of commerce, credit bureaus, and trade organizations which seems consistent with the policy recommendations our model has to offer. The fact that this did in fact occur in various economies as a result of conscious policy bears out our argument that there was nothing missing in technology or preferences which was keeping these economies at low levels of trade with dubious quality on sale.

### 5.2 Extensions and policy issues

We would like to extend the model in several directions. Here, we consider the effects of reputation, technology and exogenous shocks.

**Reputation**

A particular asymmetry that we have in the information structure is worth relaxing. If dishonest sellers can be identified it might seem natural that we should also be able to identify honest sellers which means that existing sellers should enjoy a market advantage over new entrants. This is missing in the simple matching technology we have outlined. However, if old sellers charge higher prices, we can find a distribution of prices according to how well established a seller is. An equilibrium configuration would have different subgroups of buyers being catered to by different sellers—thus we would have price variation and market segmentation as we so often do in the real world. That, however, is a topic for a separate line of research which we do not pursue here. In our information structure only dishonesty is identified—the remaining market players are either honest or new entrants who cannot be distinguished. One can think of this as a society where complaints are recorded—a lack of complaints indicates either good behavior or past inactivity and are indistinguishable for the agents at the beginning of market activity each period. This seems a reasonably good approximation for transition and emerging economies where past history of sellers are hard to come by. There is punishment for bad conduct (perhaps in the form of public announcements. See Greif, Milgrom and Weingast,(1994) for one example of such an authority where complaints could be recorded) but the emerging marketplace has no effective mechanism to record past individual activities. We can also justify this for cases where information is available for pooled samples only, as in the market for milk, where milk from different farmers
is mixed and can only be traced back when there are complaints.\footnote{As noted earlier, in eBay both good and bad comments can be recorded but good comments have less credence as sellers often leave good comments to boost each other’s market. Complaints or bad comments have to be verified.}

We would also like to extend the model to cover cases where reputation matters and try to explain promotion and hiring within firms. Consider different tiers of jobs. Labour starts from the lowest rung (say working at McDonald’s at minimum wages), if they shirk they are dismissed by their employer but they can costlessly enter a similar job at the lowest rung. If they perform well, they are rewarded by being assigned jobs in a higher rung next period. Given differing time preferences, there is an equilibrium with labor employed at different rungs. Now consider the effect of improved monitoring. (say different branches of McDonald’s can develop a network to identify past cheats.) In general, the equilibrium flow of people moving to different jobs will now vary. We expect that a society with a similar structure but different information processing environments will have differing levels of welfare—it would be interesting to study the evolution of a society as it’s information system improves. The ability of agents to signal their types and the costs of signalling could also be important determinants of why such economies could differ.

**Technology and market segmentation**

We have noted earlier that even when the low quality good has surplus, there cannot be a steady state where anyone reveals that they are producing a low quality good. In reality, such separation exists.\footnote{In some models, see Bandyopadhyay, Chatterjee and Vasavada (2001) and Wolinsky (1983) for example, prices lead to separation/semi-separation.} Some sellers are known to produce low quality good at low prices and others produce high quality and sell at high prices. In ongoing work we characterise a separating equilibrium showing how technologies which involve decreasing costs over time (perhaps because of ‘learning by doing’) can use prices to effectively separate high good producers from low good producers, causing market segmentation and resolving the quality uncertainty. Another area of future research is to study the impact of foreign competitors in this environment when there is scope for technology diffusion.

**Exogenous shocks**

We think our simple model can be readily modified to capture the idea that there are good and bad pockets in an economy and that the transition from one to the other is often the result of an exogenous change in the environment. The static lemons model is often used to explain the presence of a suboptimal mar-
ket size when increased trade is beneficial to every agent. This model provides a somewhat different justification for this inoptimality and provides a dynamic framework showing that a central authority facing nearly the same informational constraints can do better by breaking the initial coordination problem. It can successfully explain transitions (why a Mafia ridden state may re-emerge as a tourist attraction spot following stern law enforcement) while the static lemons model would predict that informational constraints will continue to inhibit the market. Moreover changing conditions are likely to affect perceptions about the future - thus it maybe worthwhile looking at how $\delta$ evolves over time. Explicitly modelling of transitions and learning and looking at the effect of exogenous shocks to the system (like better law enforcement, shift of policy from protecting domestic markets to liberalising trade) seems an interesting area of future research.

A brief policy discussion seems appropriate. This framework suggests that markets with fairly similar fundamentals can converge to quite different steady states. Thus the question arises about what policies can take the economy out of a ‘bad’ steady state? The obvious answer is changing expectations but it is not quite clear what that means in real terms.\(^{24}\) It might therefore make sense to talk of ‘small’ changes in parameter value, say a one shot marginal increase in law enforcement. A temporary change in a parameter value matters because by perturbing expectations it can take an economy towards a ‘good’ steady state. As a practical matter, setting up quality certification boards would help-if producers have to go through a quality check they will no longer be able to produce low quality goods removing the uncertainty that buyers face. Thus buyers would no longer hesitate to come to the market-effectively once this process starts we would in fact no longer require such boards-we would have self sustaining system. The market (like a brand name) would be trusted for its quality products and it would continue to live up to that to maintain its future prospects. Thus, in our model quality certification boards only have a temporary role to play\(^{25}\). It is also interesting to note that in a framework like ours, the quality of domestic products in an emerging economy can become worse via foreign competition. Buyers having the option of switching to a known foreign brand would leave the existing market reducing market thickness further and worsen the incentives for sellers to invest in quality.\(^{26}\) A case in study is Russia post-Perestroika when Russians can travel to Europe and acquire good quality second hand cars-the

\(^{24}\) An interesting possibility is the use of local currency to signal demand to local investors who have to decide whether to invest in a costly technology while facing uncertain demand. This is a possibility suggested by Jayaraman and Oak (2001). However, there are several limitations as pointed out by the authors themselves. In particular, apart from the credibility issue of introducing such currency, for this to work in our set up, consumers must have no uncertainty regarding whether they want a locally produced good.

\(^{25}\) See the papers by Liebi (2002) and Quesada and Peyra\-che (2004) for a formal modelling of certification.

\(^{26}\) The fact that in a market with imperfect information about quality, late entrants may be dissuaded from entering if firms already in the market enjoy a reputation has been recognized before (see Mayer (1984) and Grossman and Horn (1988)). Thus, it seems natural to believe that a firm with an established reputation may capture the market in an environment characterized by imperfect information.
Russian car industry continues to produce cars of inferior quality. Thus, if helping the domestic industry to grow is an objective, the entry of firms with established reputation may be harmful. However, liberalisation will make the consumer better off (by lowering prices) at the cost of the domestic producer and welfare analysis is not possible on an a priori basis. This merely points out that the introduction of foreign competition is not unambiguously beneficial. The evaluation of welfare requires a general equilibrium analysis which our model does not permit. A policy prescription which does come out is that any liberalising policy which allows access to cheaper technology for producing high quality goods is unambiguously beneficial as it increases the incentives for producers to invest in quality.

In conclusion, this simple framework has permitted us to formally study how demand, quality and prices interact. This may perhaps explain why we see a fair amount of quality dispersion in the real world which are not reflected in price dispersion. The model suggests that this leads to varying demand for these products, often leading to a suboptimal market size. We hope that the illustrations we have provided will spur empirical work to see how well our predicted relationship between market size and quality is borne out. This, together with many of the extensions suggested will hopefully enrich our understanding of the market for experience goods.

27 The important thing to note is that here, used cars seem to have better reputation than new cars, perhaps because the counteracting institutions of the type mentioned by Akerlof (earlier cit.) may have developed. An empirical study by Bond (1982) notes the absence of a market for lemons in the used trucks industry. Thus, what is important is not the type of product in question but whether there are credible ways to signal quality.
Appendix 1

Here we look at what happens when there is heterogeneity on the sellers side as well. We work out the scenario with two sellers but as will be evident from the equations, this can be easily generalised to any arbitrary types (for both buyers and sellers).

As before, there are two sides of the market, buyers and sellers. Each period the market opens and two types of objects are on sale—a high quality type and a low quality type. Buyers are again of two types, characterised by $V_L$ and $V_H$. Now, assume that sellers are also of two types, those with a low discount factor $\delta_l$ and those with a high discount factor $\delta_h$. (This discount factor can be interpreted again as a survival probability and indeed this will be important in this case.) Thus, there are four types of agents in the economy.

The population dynamics of the sellers are more complicated with heterogeneous sellers. Every period sellers who are matched and cheat are thrown out (i.e. dishonest sellers who are matched die with probability 1) and a fraction of the unmatched sellers (if $q$ is less than 1) die. The rate at which they die can be calculated from their survival probabilities. We work out the steady states in detail for the case of more sellers than buyers. At the same time new sellers of either type enter in an exogenously fixed proportion. Thus, given the fixed ratio at which sellers are replaced, the existing distribution of the two types every period together with the market thickness and the decisions of the two types determine the ratio of the two types (the state variable) in the population next period. In other words, the distribution of population evolves endogenously.

Births equal deaths and the ratio in which the two types are born is exogenous and given by $a$ and $1-a$. In steady state, given this inflow, the distribution of the two types of sellers in the population are unchanged.

We now characterise the steady state for the four type agent case.

Let $x$ be the probability of meeting an honest seller and let $\gamma$ and $1-\gamma$ be the (steady state) fractions of the two types of sellers $\delta_h$ and $\delta_l$ in the market. We use $s_i$ to denote the probability that type $i$ seller is honest. ($i = l, h.$) Thus, $x = \gamma s_h + (1-\gamma)s_l$. For a buyer, denote by $b_i$ the probability of entry in the market and by $n_i$ the number of type $i$ buyers. ($i = L, H$). Given our assumption on buyers’ valuations ($V_L < V_H$) and sellers’ discount factors, ($\delta_l < \delta_h$) the admissible randomizations for buyers and sellers can be divided into the following cases:

**Buyer**
- $b_L = b_H = 0$
- $b_L = 0 < b_H < 1$
- $b_L = 0 < b_H = 1$
- $b_L = b_H = 1$

**Seller**
- $s_L = s_H = 0$
- $s_L = 0 < s_H < 1$
- $s_L = 0 < s_H = 1$
- $s_L = s_H = 1$

Steady state equations giving us the fraction of each type of seller is obtained by equating total births and deaths. Births are in an exogenously fixed ratio. In a steady state, the fraction of each type of seller in the market will be unchanged.

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28 We take that to be equal to the proportion of high quality goods sold which is a fair approximation for a large population. Since we are doing ex ante maximization this seems reasonable.
When there are fewer buyers than sellers the following two equations provide exact expressions for these quantities:

$$S\gamma ((1-q)(1-\delta_h) + q(1-s_h + s_h (1-\delta_h))) = aB \quad (1)$$

$$S(1-\gamma)((1-q)(1-\delta_l) + q(1-s_l + s_l (1-\delta_l))) = (1-a)B \quad (2)$$

$$D \equiv B \quad (3)$$

Here, $a$ denotes the exogenously fixed ratio of type $\delta_h$ sellers entering the population, $D$ the number of deaths each period and $B$ denotes the total number of births in that period. Note that $D = B$ always holds where $D$ is obtained from adding the first two equations. This is because we have assumed that the total number of births is always equal to total number of deaths (fixed number of sellers in the market). In addition, in steady state the above two equations hold i.e. in steady state the number of deaths of each type of seller is equal to the number of births of each type. The equation can be explained quite simply—the unmatched fraction of sellers $(1-q)$ die at a rate $1-\delta$. The fraction of matched sellers is $q$. They are honest with a probability $1-s$ in which case they die at their natural rate of $1-\delta$. The dishonest fraction $(qs)$ die with probability $1$.\(^{30}\)

From this we can find out the steady state ratio of sellers for the different types of randomizations. Together with the buyers' and sellers' incentive constraints we can get the possible equilibria in our model. The following Lemma gives us the simplified steady state expressions for the buyers' and sellers' optimization problem.

**Lemma 3** In steady state and for $q < 1$ the sellers’ decision problem simplifies to the following rule:

Seller $j$ is honest iff

$$\frac{(p - c)q}{1-\delta_j} \geq \frac{pq}{1 - (1-q)\delta_j}$$

is indifferent if this holds with equality).

**Proof.** In steady state $\gamma_t = \gamma$, $q_t = q$ and $x_t = x$. Given this, a seller of type $j$ chooses to maximize $V = \text{Max}(V_C, V_{NC})$. Putting $q_t = q$ for all $t$ we get

$$V_C = \frac{pq}{1 - (1-q)\delta_j}$$

\(^{29}\)Note that the distribution of sellers in the population will not, in general, be equal to the distribution of new born sellers. This is because impatient sellers survive for shorter periods which is why the exogenous ratio of births must have higher proportion of impatient types than in the population.

\(^{30}\)Of course, there is an integer problem here in that the number of sellers may be fractions. Since we are dealing with large numbers we ignore this. In any case this is not germane to our analysis.
and

$$V_{NC} = \frac{(p - c)q}{1 - \delta_j}$$

Thus, the seller chooses the maximum of the two. □

When $q \geq 1$ the steady state equations get simplified. Now there is no uncertainty about meeting a buyer, $q$ is simply the expected number of buyers every period. Thus, dishonest sellers die with probability 1 and the steady state equations reduce to

$$S(1 - \gamma)(1 - s_l + s_l(1 - \delta_l)) = (1 - a)D \quad (5)$$

We can work out the different types of steady states using the admissible values of buyers’ and sellers’ randomizations. Together with the incentive constraints we can find out what steady states can be supported for different parameter values. Even without the detailed algebra, however, we can understand what is going on quite clearly by drawing graphs of how the thickness of markets change (because of the buyers’ entry decision) with the steady state level of honesty and how the level of honesty changes with the market thickness. The intersection of the two curves gives us the possible steady states.

For certain parametrisations which yield multiplicity, we plot two graphs illustrating this. In figure 1 we plot the level of honesty ($x$) as a function of market thickness ($q$). Note that more thickness can never induce less honesty so the curve will never slope down.\(^{31}\) In figure 1 upto $q_h$ both types prefer to be dishonest so $x$ is zero. At $q_h$ the sellers with discount factor $\delta_h$ are indifferent to being honest and dishonest. So they randomise as $s_h$, which is the randomisation probability of type $\delta_h$ being honest rises, so does the steady state level of honesty. Beyond $q_h$ type $\delta_h$ strictly prefers to be honest and type $\delta_l$ still prefers to be honest. However if $q$ is less than 1 it increases the probability of matching and impatient sellers get knocked out faster which means that the steady state level of the impatient type gets lowered thereby raising the level of honesty. (The mathematics showing the convex shape is worked out in appendix 2). At $q$ the $\delta_l$ type is indifferent and beyond that both types prefer to be honest. Note that this is not the only possible shape of the curve. There are some conditions on the minimum amount of patience and gains from trade for sellers to decide to be honest at some level of market thickness. Moreover, when the market thickness is such that $q$ is unity or more we will see how these conditions entirely determine sellers behavior (beyond $q = 1$ market thickness has no effect on the level of honesty). Thus this curve can be thought of as the sellers response curve.

In figure 2 we plot the response of buyers (and hence market thickness) to the level of honesty. The interpretation is similar but there is no convex portion because buyers live only 1 period and thus their steady state is not endogenously determined. We call this the buyers’ response curve.

\(^{31}\) The sufficient condition for multiplicity ensures this.
In figure 3 we plot the two curves together and their intersection gives us the steady states. In general, we get multiple steady states. We now present some additional propositions with two types of sellers. They are essentially about the existence of multiple non zero trade steady states and bounds on market thickness ($q$) which induce this.

**Proposition 4** Non degenerate multiple equilibria exist if the conditions in Proposition 1 are satisfied and in addition

$$\frac{p}{V_H} < x^* < \frac{p}{V_L}$$

and

$$q_h < \frac{n_H}{S} < q_l$$

where $x^*$=the maximum value of $x$ when $q = q_h$, where

$$q_l = \frac{c(1 - \delta_l)}{(p - c) \delta_l}$$

and

$$q_h = \frac{c(1 - \delta_h)}{(p - c) \delta_h}$$

Proof. Figure 3 makes this quite clear. The first intersection on the positive quadrant is ensured by the given condition. We now need to show at least a second intersection exists. If the configuration is as shown in figure 3 it is obvious. Otherwise by proposition 1 we know that at $q_{max}$ there is an equilibrium with full honesty and full entry.

This proposition sets conditions on the proportion of the two types on either side of the market. Intuitively, this means that if there are too many patient sellers or high valuation buyers the only steady state involving positive trade will have the high quality goods on sale with all buyers entering. The intuition is clear from figure 3. As the steady states can be Pareto ranked we can speak of non degenerate states as first, second etc. in ascending order of quality and demand. Given the conditions in this proposition, we can characterize the first non degenerate steady state quite sharply i.e. we can calculate the randomisations by buyers and sellers that support this. Corollary 1 formalises this.

**Corollary 1** If Proposition 4 holds then the first non zero steady state involves randomization by type $H$ buyers and type $h$ sellers with the level of honesty at $x_h$ and the market thickness at $q_h$. Thus, the corresponding randomisations can be calculated.

$^{32}x^*$ can be calculated using the steady state equations. In case 2 worked out in Appendix 2 putting $s_h = 1$ we can find $\gamma$ and hence $x^*$.  

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Proof. The proof is clear from figure 3. Given condition 2 $x_H$ must lie to the left of $x^*$ and $q_h$ must lie below $n_H/S$. Thus, this gives us the first intersection of the 2 curves beyond the origin. The buyers randomization is got by simply equating $\alpha n_H = q_h$, ($\alpha$ is the randomization probability of buyers of valuation $V_H$). For the seller we calculate this from the steady state equation (case 2 in Appendix 2).

Appendix 2
Here, we work out the various possible steady state market thickness, quality choice and population distribution of the sellers. For more buyers than sellers the expressions differ but the analysis is simpler—we briefly discuss that after working through the various cases for $q < 1$.

Case 1: $s_l = s_h = 0$

The steady state ratio

$$\frac{\gamma}{1 - \gamma} = \frac{(1 - \delta_l)a}{(1 - \delta_h)(1 - a)}$$

when $q < 1$

(otherwise the steady state equations is simply $\frac{\gamma}{1 - \gamma}$.) However in equilibrium $q$ is 0 since $x = 0$ and the optimal response is $b_L = b_H = 0$.

Case 2: $s_l = 0 < s_h < 1$

The steady state ratio is found by equating the ratio of deaths of the two types to the exogenously given births (or inflow). Thus we have

$$\frac{\gamma((1 - q)(1 - \delta_h) + q((1 - s_h) + s_h (1 - \delta_h)))}{(1 - \gamma)(q + (1 - q)(1 - \delta_l))} = \frac{a}{1 - a}$$

Now we can solve for $q$ by looking at the seller's decision problem. For the seller of type $\delta_h$ to be randomizing it must be that he is indifferent to cheating and being honest. Hence the payoffs from cheating and being honest must be equal. Hence we get

$$\frac{(p - c)q}{1 - \delta_h} = \frac{pq}{1 - (1 - q)\delta_h}$$

where the left hand side represents the gains from being honest and the right hand side the gains from cheating. Solving for $q$ we get

$$q = \frac{c(1 - \delta_h)}{(p - c)\delta_h}$$

Substituting in the buyers entry problem we can find out the number of buyers who enter and from that $\gamma$ and hence $s_h$ can be solved. More precisely since $S$ (the total stock of sellers) is known from $q$ we can calculate $n$ (those buyers who enter in that period), If $n$ is greater than $N$ (the population of potential buyers) then there is no solution. If $n$ is less than the type of $V_H$ buyers then we calculate the randomization $\alpha$ so that only $n$ buyers enter. At this it must be that these buyers are indifferent hence $x_{V_H} = 1$ giving us the value of $x$ and hence $\gamma$. Thus $q_h$ can be calculated. If $n$ equals the number of type $V_H$
buyers then $1/V_H \leq x < 1/V_L$ and hence admissible ranges of $\gamma$ and hence $s_h$ can be found. (this would correspond to a continuum of equilibria). For $n$ such that type $V_H$ always enters and type $V_L$ is indifferent $x$ can again be precisely calculated and hence $\gamma$ and $s_h$ can be solved. Finally if $n = N$ then we can again solve for the admissible ranges of steady states and seller randomisations.

Case 3: $s_l = 0 < s_h = 1$

The steady state equation is given by

$$\frac{\gamma(1-\delta_h)}{(1-\gamma)(q + (1-q)(1-\delta_l))} = \frac{a}{1-a}$$

and

$$\frac{c(1-\delta_h)}{(p-c)\delta_h} < q < \frac{c(1-\delta_l)}{(p-c)\delta_l}$$

must hold. Now for each $q$ in this range find $\gamma$ and hence $x$. This gives us the level of honesty induced by the different values of $q$. (this gives us the sellers response curve). Now for each of this $x$ so found find the level of $q$ this induces by looking at the buyers maximization problem. This gives the buyers response curve. If the $x$ induced by a value of $q$ in turn induces the same $q$ we have an equilibrium point.

Case 4: $0 < s_l < s_h = 1$

With the impatient sellers randomizing the steady state equation is given by

$$\frac{\gamma(1-\delta_h)}{(1-\gamma)((1-q)(1-\delta_l) + q(1-s_l) + s_l(1-\delta_l))} = \frac{a}{1-a}$$

The value of $q$ is found by equating the gains from honesty and cheating for the $\delta_l$ type of seller giving us

$$q = \frac{c(1-\delta_l)}{(p-c)\delta_l}$$

Plugging in the buyers maximization problem we get $x$ and substituting these values in the steady state equation gives us the equilibrium randomization for the seller.

Case 5: $s_l = s_h = 1$

With both types being honest the steady state becomes

$$\frac{\gamma(1-\delta_h)}{(1-\gamma)(1-\delta_l)} = \frac{a}{1-a}$$

Note this must induce both types of buyers to enter (since $x = 1, V_i > 1$ for $i = L, H$ by assumption ) -with only high quality objects on sale all potential buyers must find it optimal to enter given condition 1.
When \( q \geq 1 \) we know that sellers behavior is independent of \( q \). Thus their choice is dependent only on whether \( p\delta > c \) or the converse. Depending on that each type of sellers behavior is determined. As there is no more uncertainty about matching we do not have the part with \( 1 - q \). The analysis is similar except that if \( s = 0 \) for any type at \( q \geq 1 \) it is always 0.

To show that the steady state level of honesty rises as market thickness increases and the inequalities in case 3 hold.

**Proof.** The steady state equation is

\[
\frac{\gamma(1 - \delta_h)}{(1 - \gamma)(q + (1 - q)(1 - \delta_l))} = \frac{a}{1 - a}
\]

which gives us

\[
\frac{\gamma}{1 - \gamma} = b(q + (1 - q)(1 - \delta_l))
\]

where \( b \) is a constant

\[
\frac{a}{(1 - a)(1 - \delta_h)}.
\]

This simplifies to

\[
\gamma = 1 - \frac{1}{1 + b(1 - \delta_l + q\delta_l)}
\]

Taking derivatives we have

\[
\frac{d\gamma}{dq} = \frac{\delta_l}{(1 + b - b\delta_l + bq\delta_l)^2} > 0
\]

showing that it is increasing in \( q \). The second derivative is

\[
\frac{d^2\gamma}{dq^2} = -\frac{2\delta_l^2}{(1 + b - b\delta_l + bq\delta_l)^3} < 0
\]

justifying the shape of the curve. \( \blacksquare \)
References:


Fig 1

Sellers’ response curve
Fig 2: Buyers’ response curve
Equilibrium