Unions, Wage Setting, and Economic Growth*

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Abstract

In this paper we analyse the growth effects of union wage bargaining within an expanding product variety growth model. We identify two channels via which unionisation will affect the rate of growth. Firstly, unions capture monopoly profits and thus give rise to a hold-up problem. Firms have less incentive to invest into research. This dampens the rate of growth. Secondly, unionisation changes the “de facto” skill abundance of the economy. This results in a resource reallocation a la Rybczynski which might be growth enhancing. We derive conditions for the dominance of either effect and demonstrate how these will change with the institutional setting of the bargain.

Keywords: Union Wage Bargaining, Growth, Rybczynski effect.

JEL: J51, O31, E13

1 Introduction

Union wage bargaining is the common way of wage determination in continental Europe. Although union membership rates declined substantially over the last two decades, union bargaining still determines approximately 90% of all wage contracts in continental Europe (see, CESifo 2005).

Unionisation will, by means of labour rationing, increase the wage above the non-unionised level. This will result in unemployment, i.e. there will be

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static inefficiencies. Empirically, this effect is quite well documented, see e.g. Nickell and Layard (1999) and - more recently and with data from outside Europe - Beseley and Burgess (2004).

The static efficiency effects, however, are only one part of the story. To get a coherent picture of the economic effects of union wage bargaining, one would also like to know whether unionisation influences the long-run evolution of the economy. Thus, the question begs whether the labour market imperfection affects growth and if so which sign this effect has. Unfortunately, the theoretical arguments as well as the empirical evidence give a mixed picture.

One strand of the literature argues that unionisation will unambiguously decrease the rate of growth of an economy. The basis for this argument is the hold-up problem pioneered by Grout (1984). By bargaining the wage, the union is able to capture some of the (quasi) rents which are generated by an investment of the firm (capital or R&D). Since the firm anticipates this hold-up, it will invest less in the first place, compared to a situation without unionisation. Kemp and van Long (1987), Daveri and Tabellini (2000), Bräuninger (2000) and Kaas and von Thadden (2004) argue along this line.

Unions, however, might also increase the rate of growth in an economy. The wage increase due to unionisation fosters the incentive for firms to invest into labour saving production technologies. R&D investment and thus, the rate of growth increase. This argument is usually put forward within a partial equilibrium industrial organisation framework, see e.g. Ulph and Ulph (2001), Bester and Petrakis (2003) and Haucap and Wey (2004).

A shortcoming of these two strands of literature is that only one of the two effects at a time is analysed. However, we would expect both effects to act simultaneously. As such, one would like to know how these countervailing effects interact and under which circumstances one effect dominates the other.

First steps in this direction are the contributions of Palokangas (1996) and Lingens (2003). In these papers the growth effect of unionisation is analysed within models in which both a growth dampening hold-up effect as well as a potentially growth enhancing reallocation effect exist. Both papers identify the elasticity of substitution between low-skilled and high-skilled labour to be the determinant of which effect is the dominating one.

Although, these papers identify the production structure of the economy to be of major importance for the union effect, they employ a very restrictive framework to model this structure. Firstly, the production functions are of specific functional forms (CES and Cobb-Douglas). Secondly, it is implicitly assumed that the skill intensity in the research sector is infinite since only high-skilled labour is employed there. This is an obvious drawback since
the question begs whether the derived results are only a theoretical curiosity driven by the specify assumptions or whether these also hold in more general settings.

Another shortcoming of the existing literature is that only a very specific institutional bargaining setting is analysed, namely one where only the low-skilled wage is bargained at the firm level. This, however, does not reflect the heterogeneity in bargaining institutions we observe in real world labour markets. The degree of centralisation (see, e.g. Calmfors and Driffield (1988)) as well as the degree of coverage of different skill-groups differs substantially across economies.

Thus, the results derived in the existing literature might not only give a wrong picture of how unionisation affects the evolution of the economy, but this picture is also incomplete. Thus, in order to fully understand the long run effects of union wage bargaining, we are in need of an analysis within a general framework which fully takes institutional heterogeneities into account.

By analysing the growth effects of different bargaining setups, where these setups differ with respect to coverage and centralisation, within a model with a most general production structure, we bridge this gap in the existing literature. We find that the condition for unionisation to be growth enhancing is less restrictive than found in the literature. In addition we are able to show that the degree of centralisation is of major importance for the growth effects of unionisation. Moreover, if low-skilled and high-skilled labour are covered by union wage bargaining the relative rationing effect matters for the growth effect of the labour market imperfection.

In the next section we give a brief overview over the main structure of the model and offer some intuition concerning the union effect. Section 3 presents the model. In section 4 the equilibrium solution of the model is derived. Union wage bargaining is incorporated into the basic model in section 5. The last section concludes.

\section{Overview}

In order to analyse the long-run effects of union wage bargaining we employ an expanding product variety growth model a la Romer (1990). The economy consists of three sectors. A sector where the consumption good is produced using intermediated good varieties, an intermediate goods sector where these varieties are produced using low-skilled and high-skilled labour and a research sector in which blueprints for new intermediate good varieties are produced also using both, low-skilled and high-skilled labour.
Any single variety of the intermediate good is sold monopolistically, thus rents can be earned in this sector. Only by buying a blueprint from the research sector in advance, an entrepreneur has the option to produce a variety of the intermediate good. Thus, the monopoly rent is the incentive for an entrepreneur to invest into buying a blueprint.

The more intermediate good varieties exist, the deeper is the division of labour (see, Ethier (1982)) and the larger is the amount of the consumption good which can be produced. As such, the ultimate source of growth in the economy is the production of blueprints.

The equilibrium in the economy is determined by the allocation of the (exogenous) resource pool between the intermediate sector and the research sector. Thus, when analysing the growth effects of unionisation one has to focus on the change in this equilibrium allocation.

Bargaining institutions are very heterogeneous across countries. In order to get a thorough understanding of the growth effects of unionisation we account for this heterogeneity and consider bargaining set-ups which differ with respect to the skill-groups which are covered in the bargain and to the degree of centralisation. Table 1 gives a taxonomy of the different set-ups and their labour market consequences. We do not analyse the situation in which only high-skilled labour is covered by wage bargaining since this is analytically symmetric to the one in which only the low-skilled wage is bargained. In addition, this situation is empirically implausible.

To give an intuition of the growth effects of union wage bargaining, consider the case in which a central union only bargains the wage for low-skilled labour. The union affects the equilibrium allocation of the resource pool in two different ways. Firstly, the union gives rise to unemployment, thus the economy de-facto becomes more high-skilled abundant. The Rybczynski
effect occurs, i.e. production of the high-skilled intensive good, which we assume to be the research good, will c.p. (i.e. assuming constant prices) increase. Secondly, since we consider a closed economy, also goods prices will adjust due to unionisation. The union captures part of the monopoly profits in the intermediate goods sector. This hold-up decreases the demand for blueprints and thus dampens the blueprints price. As such, blueprints production would decrease.

With the research sector being high-skilled intensive, unionisation will have ambiguous effects on research sector production and hence, on the rate of growth.

3 The Model

3.1 Production

The production function of the consumption good, $Y$, is:

$$Y = \left( \int_0^n x_i^\alpha di \right)^{\frac{1}{\alpha}},$$

(1)

where $\alpha \in [0, 1]$. Output of the consumption good increases with the number of product varieties $n$. The consumption good is the numéraire in the economy, hence $P_Y = 1$. Profit maximization in the consumption good sector leads to the following (inverse) demand function for the intermediate good of variety $i$

$$p_i = \left( \int_0^n x_i^\alpha di \right)^{\frac{1-\alpha}{\alpha}} \cdot x_i^{\alpha - 1},$$

(2)

with $p_i$ denoting the price (in terms of the consumption good) of intermediate good $i$. The elasticity of substitution between two intermediate good varieties is $(1 - \alpha)^{-1} > 1$.

Any intermediate good $x_i$ is produced using both low-skilled and high-skilled labour. In order to keep the model analytically tractable, we assume identical production functions across the $n$ firms in this sector. Dropping the subscript $i$, the production function is given by

$$x = f(\ell_x, h_x),$$

(3)

where $\ell_x$ and $h_x$ denote the amounts of low-skilled and high-skilled labour employed in any intermediate good sector $i$, respectively. The total amount of low-skilled and high-skilled labour employed in the intermediate goods
sector are given by \( n \cdot l_x \) and \( n \cdot h_x \), respectively. \( f(\cdot) \) is homogenous of degree one and fulfills the usual neoclassical properties.

Intermediate good firms face monopolistic competition. Profit maximisation yields:

\[
p_x = \frac{1}{\alpha} (a_{xl} \cdot w_l + a_{xh} \cdot w_h).
\]  

\( w_l \) and \( w_h \) denote the wage rates for low-skilled and high-skilled labour, respectively. The coefficients \( a_{xl} \) and \( a_{xh} \) specify the amount of low-skilled and high-skilled labour needed to produce one unit of \( x \), i.e. \( a_{xl} \equiv \frac{l_x}{x} \) and \( a_{xh} \equiv \frac{h_x}{x} \). The term in parentheses in equation (4) thus gives unit costs. With monopolistic competition, the price \( p_x \) is a mark-up over unit costs. The mark-up is a function of the price elasticity of the intermediate demand function (which in turn is a function of the elasticity of substitution).

The R&D sector produces blueprints of new intermediate good varieties, also by means of low-skilled and high-skilled labour. Blueprints production function is:

\[
\hat{n} = n \cdot g(l_n, h_n) \Rightarrow \hat{n} \equiv \gamma = g(l_n, h_n),
\]  

where \( l_n \) and \( h_n \) denote the amount of low-skilled and high-skilled labour employed in the research sector, respectively. The function \( g(\cdot) \) is also assumed to display homogeneity of degree one and neoclassical properties.

Blueprint production is characterised by a linear spill-over effect. The more varieties already ”produced”, the easier it is to produce additional ones. This assumption ensures that the economy can grow unboundedly although the primary resource base is fixed. The property of these type of growth models is called scale effect and has been questioned recently in the literature, see e.g. Jones (1995) or Jones (1999). Nevertheless, the steady state behaviour of a scale effect growth model can be interpreted as a reasonable proxy for the transitional behaviour in non-scale models, see Lingens (2005). Thus, we think it is worthwhile to analyse union wage behaviour within a scale model.

Blueprints are sold competitively. With perfect competition among research firms, profit maximization ensures prices of new blueprints, \( p_n \), to equal unit costs:

\[
p_n = \frac{1}{n} (a_{nl} \cdot w_l + a_{nh} \cdot w_h),
\]  

where the coefficients \( a_{nj}, j = l, h \) denote the amount of low-skilled and high-skilled labour to produce one unit of \( \gamma \), i.e. \( a_{nl} \equiv \frac{l_x}{\gamma} \) and \( a_{nh} \equiv \frac{h_x}{\gamma} \).
3.2 Demand for Blueprints

Whereas (6) characterizes the supply of new blueprints, we still have to be specific about the demand for blueprints. Entrepreneurs who want to produce a new variety of an intermediate good have to buy a blueprint from the R&D sector at a price \( p_n \). They finance this investment by borrowing funds from private households who will receive an infinite stream of payment \( r \cdot p_n \) per period, where \( r \) denotes the rate of interest. Entrepreneurs will buy blueprints as long as the flow of monopoly profits that can be earned exceeds the interest payment:

\[
r \cdot p_n \leq p_x \cdot x - w_l \cdot l_x - w_h \cdot h_x \Leftrightarrow r \cdot p_n \leq p_x \cdot x(1 - \alpha),
\]

(7)

where the manipulation uses the fact that by (4) \( w_l \cdot l_x + w_h \cdot h_x = \alpha p_x x \).

Competition of entrepreneurs to get blueprints will drive up \( p_n \) until (7) holds with equality.

The supply of funds from households is determined by the time profile of consumption \( C \). Assuming risk neutrality and an exogenous rate of time preference \( \rho \), intertemporal optimization of households implies a consumption path characterized by:

\[
\frac{\dot{C}}{C} = r - \rho,
\]

(8)

where a dot over a variable denotes the derivative of the variable w.r.t. time.

Aggregate consumption must be equal to aggregate consumption good production, \( C = Y \). Thus the growth rate of \( C \) must be equal to the rate of growth of \( Y \). In a symmetric equilibrium, consumption good production is given by (using equation (1)) \( Y = n^\frac{1}{\alpha} x = n^\frac{1}{\alpha} X \), where \( X \) is aggregate intermediate goods production, i.e. \( X \equiv n \cdot x \). Thus, the growth rate of consumption good production is given by:

\[
\frac{\dot{Y}}{Y} = \frac{1 - \alpha}{\alpha} \cdot \gamma + \frac{\dot{X}}{X}.
\]

(9)

A steady state is defined as a constant allocation of the resource pool between the sectors of the economy. As such, \( X \) is constant in a steady state. The steady state interest rate is, hence:

\[
r = \rho + \frac{1 - \alpha}{\alpha} \cdot \gamma.
\]

(10)

Using equation (10), the demand for blueprints reads:

\[
p_n = \frac{(1 - \alpha) \cdot p_x \cdot x}{\rho + \frac{1 - \alpha}{\alpha} \cdot \gamma}.
\]

(11)
The larger the monopoly profits, the larger will be the demand for blueprints and thus, the price entrepreneurs are willing to pay. The higher the rate of interest, the larger are the cost to invest into blueprints. This dampens demand.

### 3.3 Resource Constraints

In order to close the model, we have to specify the economy-wide quantities of low-skilled and high-skilled labour. These are (up to this point) exogenously given and denoted by $L$ and $H$, respectively. The resource constraints thus read:

$$L = n \cdot l_x + l_n = a_{xl} \cdot X + a_{nl} \cdot \gamma$$

and

$$H = n \cdot h_x + h_n = a_{xh} \cdot X + a_{nh} \cdot \gamma.$$  

### 4 Solution of the Model

The key equations of our model are the supply functions for intermediate goods (4) and blueprints (6), respectively, the (implicit) demand for blueprints (11) and the two resource constraints (12) and (13). These five equations determine the five endogenous variables: equilibrium growth $\gamma$, aggregate production of intermediate goods $X$, the relative wage for the two skill groups $\frac{w_l}{w_h}$, the price of blueprints $p_n$ and the price of intermediate goods $p_x$.

To solve the model we apply a solution strategy for general equilibrium models proposed by Jones (1965). Basically, this strategy implies to rewrite the model in log-linear form and analysing the comparative statics. Denoting relative changes by a "hat" (e.g. $\frac{dx}{x} \equiv \hat{x}$) and assuming $d\rho = d\alpha = 0$, we get the following five equations:

$$\hat{p}_x = \theta_{xl} \cdot (\hat{a}_{xl} + \hat{w}_l) + (1 - \theta_{xl}) \cdot (\hat{a}_{xh} + \hat{w}_h),$$

$$\hat{p}_n = \theta_{nl} \cdot (\hat{a}_{nl} + \hat{w}_l) + (1 - \theta_{nl}) \cdot (\hat{a}_{nh} + \hat{w}_h) - \hat{n},$$

$$\hat{p}_n - \hat{p}_x = \hat{x} - \frac{r - \hat{\rho}}{\rho} \cdot \hat{\gamma},$$

$$\hat{L} = \lambda_{xl} \cdot (\hat{a}_{xl} + \hat{X}) + (1 - \lambda_{xl}) \cdot (\hat{a}_{nl} + \hat{\gamma}),$$

$$\hat{H} = \lambda_{xh} \cdot (\hat{a}_{xh} + \hat{X}) + (1 - \lambda_{xh}) \cdot (\hat{a}_{nh} + \hat{\gamma}).$$

1For an analysis of the comparative statics effects of the rate of time preference, $\rho$, or $\alpha$, see e.g. Grossman and Helpman (1991).
For notational convenience define \( i \in \{x, n\} \) and \( j \in \{l, h\} \). The coefficients \( \theta_{ij} \) denote cost shares for the primary factors of production \( j \) in sector \( i \), e.g. \( \theta_{xl} = \frac{w_l x_l}{x_l} \) and \( \theta_{nl} = \frac{w_l n_l}{n_l} \). In (4’), we make use of the fact that \( \theta_{hl} = (1 - \theta_{hl}) \). Likewise, \( \lambda_{ij} \) denotes the share of the total factor supply of factor \( j \) that is employed in sector \( i \), e.g. \( \lambda_{xh} = \frac{a_{xh} x_l}{h} \). The resource constraints imply \( \lambda_{xj} = (1 - \lambda_{nj}) \), \( j \in \{l, h\} \).

The above system of equations is still indeterminate, because we did not yet introduce cost minimization, i.e. the optimal factor mix both in intermediate good firms and in the R&D sector. In other words, the change of the four sectoral factor intensities \( a_{ij} \) is endogenous. Applying cost minimization leads to:

\[
\theta_{xl} \cdot \hat{\alpha}_{xl} + (1 - \theta_{xl}) \cdot \hat{\alpha}_{xh} = 0, \quad (14)
\]
\[
\theta_{nl} \cdot \hat{\alpha}_{nl} + (1 - \theta_{nl}) \cdot \hat{\alpha}_{nh} = 0. \quad (15)
\]

The definitions of the sectoral elasticities of substitution between low-skilled and high-skilled labour (in the case of constant return production functions) are given by

\[
\sigma_i = \frac{\hat{\alpha}_{il} - \hat{\alpha}_{ih}}{\hat{w}_l - \hat{w}_h}. \quad (16)
\]

Combining (14),(15) and the two definitions in (16), we arrive at the following solutions for the change of input coefficients:

\[
\hat{\alpha}_{il} = -\theta_{il} \cdot \sigma_i \cdot (\hat{w}_l - \hat{w}_h) \quad \forall i \in \{x, n\}, \quad (17)
\]
\[
\hat{\alpha}_{ih} = \theta_{ih} \cdot \sigma_i \cdot (\hat{w}_l - \hat{w}_h) \quad \forall i \in \{x, n\}. \quad (18)
\]

Subtracting (4’) from (6’), eliminating the change in relative prices by equating the result to (11’) and eliminating the \( a_{ij} \)’s from this equation, (12’) and (13’) with (17) and (18) leads to the following system of equations:

\[
\hat{X} = (\theta_{nl} - \theta_{xl}) \cdot (\hat{w}_l - \hat{w}_h) + \frac{r - \rho}{r} \cdot \hat{\gamma}, \quad (19)
\]
\[
\hat{L} = (1 - \lambda_{xl}) \cdot \hat{\gamma} + \lambda_{xl} \cdot \hat{X} - \delta_l \cdot (\hat{w}_l - \hat{w}_h), \quad (20)
\]
\[
\hat{H} = (1 - \lambda_{xh}) \cdot \hat{\gamma} + \lambda_{xh} \cdot \hat{X} + \delta_h \cdot (\hat{w}_l - \hat{w}_h), \quad (21)
\]

with \( \delta_j = \sum_i \lambda_{ij} (1 - \theta_{ij}) \sigma_i \).

These three equations can be solved for \((\hat{w}_l - \hat{w}_h), \hat{X} \) and \( \hat{\gamma} \). Focusing on the change in the relative factor price and on the rate of growth, we arrive
at:

\[
\hat{\gamma} = \frac{1}{\Lambda} \cdot \left[ (\delta_h + \lambda_{xh}(\theta_{nl} - \theta_{xl}))L + (\delta_l + \lambda_{xl}(\theta_{xl} - \theta_{nl}))H \right], \tag{22}
\]

\[
(\hat{w}_t - \hat{w}_h) = -\frac{1}{\Lambda} \cdot \left[ ((1 - \lambda_{xh}) + \lambda_{xh} \frac{r - \rho}{r})L + ((1 - \lambda_{xl}) + \lambda_{xl} \frac{r - \rho}{r})H \right], \tag{23}
\]

where \( \Lambda > 0 \) is the determinant of the coefficient matrix.

Remember, that with perfectly competitive labour markets, there is neither low-skilled nor high-skilled unemployment. Labour market imperfections will decrease the employable resource base. Equation (22) depicts that the effect of a change in this employable resource base has ambiguous effects on the rate of growth. Changes in the resource base will on the one hand change the skill abundance in the economy. This gives rise to the, from trade theory well-known, Rybczynski effect. On the other hand, the resource base also influences the relative price. Both effects potentially have countervailing directions which make the net effect ambiguous.

5 Unions

5.1 Centralised Wage Setting

After having determined the general set-up of the model and having derived a solution for the case with competitive labour markets, we will discuss in this section the growth effects of union wage bargaining in more detail.

We consider an economy in which only low-skilled labour is covered by union wage bargaining. This union bargains a uniform low-skilled wage with an employer federation. After one period,\(^2\) the bargain is repeated and all unemployed low-skilled agents have again the same probability of becoming employed. This implicitly assumes that unemployed do not loose skills. This assumption highly simplifies the decision making of the union since wage setting has no intertemporal effects.

The utility function is given by:

\[ U^{Union} = Lw_t + (\bar{L} - L)B, \]

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\(^2\)Note that the model is in continuous time, so this period of time is infinitesimally small. We could have also written the model in discrete time. This would not have changed anything.
where $B$ is the unemployment benefit. The union seeks to maximise aggregate utility of all low-skilled agents.\footnote{We assume the unemployment benefit to be financed by a lump-sum tax on low-skilled and high-skilled agents, thus we do not have to worry about the effects of financing the benefit since lump-sum taxes are neutral to the results of the model.}

The goal of the employer federation in the bargaining process is to maximise aggregate profits in the economy. Let $\Pi$ denote this sum of profits, the utility function is given by:

$$U^{\text{Federation}} = \Pi = P_n n + P_X X - w_l L - w_h H.$$  

The bargained wage can in turn be determined by maximising the following Nash product:

$$\Omega = (U^{\text{Union}} - \bar{U}^{\text{Union}})^\beta (U^{\text{Federation}} - \bar{U}^{\text{Federation}})^{1-\beta}, \quad (24)$$

where $\bar{U}^{\text{Union}}$ and $\bar{U}^{\text{Federation}}$ denote the utility of the union and the employer federation in case the bargain fails.

To determine these fall-back positions it is crucial to specify the timing of events. We assume the structure shown in figure 1. With this the fall-back position of the employer federation is $-w_h \bar{H}$ since firms in the economy hire high-skilled labour in advance of the low-skilled bargain.

The fall-back utility of the union is given by $BL$. If the bargain fails the unemployed union members will have no chance of finding an other job in the economy.

Plugging in the utility functions and maximising the Nash product w.r.t. the low-skilled wage and subject to the aggregate labour demand function $L(w_l) = L_X(w_l) + L_n(w_l)$, gives the following first order condition:

$$\frac{\partial \Omega}{\partial w_l} = \left( \frac{\partial L}{\partial w_l} \frac{w_l - B}{L(w_l - B)} + \frac{L}{L(w_l - B)} \right) \frac{\beta}{1-\beta} + \frac{-L}{\Pi - w_h \bar{H}} = 0.$$
Denoting the (positively defined) aggregate labour demand elasticity by $\eta_L$ and the low-skilled wage bill over profits net of high-skilled costs by $s_L$, we get the following wage equation:

$$w_l = \frac{1}{1 - (s_L - \frac{1-\beta}{\beta} + \eta_L)^{-1} B}. \quad (25)$$

The bargained wage is a mark-up over the unemployment benefit. Assuming this mark-up to be (locally) constant yields a wage setting curve which is a horizontal line in the $w_l$-$L$-space.\footnote{This claim will be only exactly true with a Cobb-Douglas specification of the production functions. The "mistake", however, we make with our assumption of the locally constant mark-up is negligible since all of these effects are of second order.} Thus, the bargained wage of the union will not change with overall employment conditions in the economy. This is due to the fact that the central union internalises these overall employment conditions.

The competitive case is nested in the wage equation for $\beta = 0$. With this we can derive the relative change of $w_l$ as a function of the relative change in union power:

$$\hat{w}_l = a_1 \hat{\beta}, \quad (26)$$

where the coefficient is defined as $a_1 \equiv \frac{(1-\beta)s_L}{s_L - \frac{1-\beta}{\beta} + \eta_L - 1} > 0$ and $\hat{\beta} = \frac{d\beta}{1-\beta}$. We can interpret $\hat{\beta}$ as reflecting the relative change in the low-skilled wage when the economy starts from a competitive equilibrium to a unionised one. Using equation (26) and the definition of the (positive) labour demand elasticity, we get the relative employment change in the unionised case as:

$$\hat{L} = -\eta_L \hat{w}_l = -\eta_L a_1 \hat{\beta} \quad (27)$$

compared to the latter section with competitive labour markets, the only difference (concerning the derivation of the equilibrium in the economy) is that low-skilled employment is endogenous in the union case. Thus, using (27) and (22) we get:

$$\hat{\gamma} = \frac{1}{\Lambda} \cdot \left[ -\left( \frac{\delta_t + \lambda_{xh}(\theta_{nt} - \theta_{xt})}{\eta_L a_1 \hat{\beta} + \left( \frac{\delta_t + \lambda_{xh}(\theta_{xt} - \theta_{nt})}{\eta_L a_1 \hat{\beta}} \right)} \right] \cdot \hat{H}. \quad (28)$$

With (28) we can state the following:

**Proposition 1** If the research sector is low-skilled intensive, the rate of growth will unambiguously decline in case of union wage bargaining. If the research sector is high-skilled intensive, the growth effect is ambiguous.
Proof 1 \( \theta_{nl} - \theta_{xl} > 0 \) if the research sector is low-skilled intensive. By (28) this makes the effect of \( \beta \) unambiguously negative for the rate of growth. This effect is, however, ambiguous if the intermediate sector is low-skilled intensive, i.e. \( \theta_{nl} - \theta_{xl} < 0 \).

Consider the ambiguous case in which the research sector is high-skilled intensive. The condition in this case for the growth rate to increase with union wage bargaining is:

\[
\delta_h + \lambda_{xh}(\theta_{nl} - \theta_{xl}) < 0.
\]

Plugging in for \( \delta_h \) from above and doing some rearrangements reveals:

\[
\theta_{nl}\sigma_n + \lambda_{xh}(\theta_{xl}(\sigma_x - 1) + \theta_{nl}(1 - \sigma_n)) < 0. \tag{29}
\]

This equation implies the following:

**Proposition 2** If the research sector is high-skilled intensive, the growth effect of union wage bargaining will depend on the elasticities of substitution between low-skilled and high-skilled labour in both sectors of the economy.

Equation (29) depicts the sufficient condition for the rate of growth to decrease with unionisation. It requires the elasticity of substitution in the production (research) sector to exceed (be less than) unity.

The restriction concerning the elasticity of substitution in the production sector was already derived in the existing literature (see, e.g. Palokangas (1996) or Lingens (2003)). Recent estimates of this elasticity for the manufacturing sector of the economy find high values (see, e.g. Fitzenberger and Kohn (2004)). Hence, the more restrictive models would have pointed to a negative growth effect of union wage bargaining in the real world.

Our result, however, which is derived in a more general setting suggests that not only the level of one elasticity of substitution matters. The important determinant for the growth effect of unionisation is the "interaction" between the elasticities of substitution in the production and in the research sector of the economy. So restricting the model under consideration might give a wrong picture of the growth effects of labour market imperfections.

The model implies that for the impact of unionisation on the rate of growth the skill-intensity and the elasticities of substitution matter. What is the intuition behind these results? Remember that low-skilled unionisation gives rise to two different effects. Firstly, the economy will become more skill-abundant (due to the rationing effect). Secondly, the demand for blueprints and thus the relative price between intermediate goods and blueprints will be affected.
Consider first of all the rationing effect of union wage bargaining. The basic economic effects of unemployment are the same as a decline in the low-skilled resource base, i.e. the transformation curve shifts towards the origin (see figure 2 in the appendix). The impact of a decline in the low-skilled resource base on the resource allocation in the economy is described in the well-known Rybczynski effect (see, e.g. Gandolfo (1998)). Rationing of low-skilled labour will on impact reduce production in the intermediate as well as in the research sector. But with this high-skilled labour would become unemployed. This cannot be an equilibrium since the high-skilled labour market is competitive. To absorb the (initially) unemployed high-skilled agents (with unchanged relative goods prices) the high-skilled intensive sector must expand. Thus, if the research (intermediate) sector is high-skilled intensive, it will expand in the case of union wage bargaining. The rate of growth will increase (decrease).

The Rybczynski effect, however, is only one part of the story. This is because it only holds for exogenously given prices, e.g. in a small open economy. Our model economy, however, is closed and as such faces endogenous relative prices. These relative prices are also affected by the labour market imperfection.

Union wage bargaining gives rise to a hold-up effect. The union is able to increase the low-skilled wage and thus, to capture some of the monopoly rents in the intermediate good sector. Potential entrepreneurs who buy blueprints in order to enter the intermediate sector will take this effect into account. Thus, the price they are willing to pay for a blueprint decreases. This implies a decrease in production in the research sector.

The overall effect depends on the sign and on the strength of the Rybczynski effect and on the strength of the relative price effect. Only if the research sector is high-skilled intensive, there exists the possibility of a positive growth effect of unionisation. The endogenous price adjustment will result in a reallocation of resources towards the intermediate sector, dampening or even reversing the original Rybczynski effect (see figure 2). Only if the latter effect was large compared to the price effect, the union could increase the rate of growth in the economy.

This clarifies why the overall effect is a function of $\sigma_n$ and $\sigma_X$. These elasticities determine the relative strength of the two countervailing effects. If $\sigma_n$ is large, i.e. it is easy to substitute expensive low-skilled labour by high-skilled labour, production in this sector expands very much in case of the economy becoming more skill abundant. If $\sigma_X$ is small, the production decrease in the intermediate sector will be low. Thus, the blueprint price decrease will be c.p. low. So the positive union effect will become more likely.
5.2 Decentralised Wage Setting

In the preceding analysis, we have assumed that one central union bargains an economy wide low-skilled wage. However, intersectoral wage differentials are a common feature of labour markets (see, e.g. Edin and Zetterberg (1992)). Moreover, the wage in a majority of OECD countries is not bargained at the central level (see, e.g. CESifo Dice (2005)). As such, the question begs in which way the results are modified when allowing for decentralised bargaining and thus, within group wage heterogeneity.

To keep the analysis comparable to the one of centralised bargaining, we assume the utility functions of the union to be the same as before. 

\[
U^{\text{Union}_X} = L_X (w_X) + (M_X - L_X) AI, \tag{30}
\]

\[
U^{\text{Union}_n} = L_n (w_n) + (M_n - L_n) AI, \tag{31}
\]

where \(M_i\) is the number of union members. We could think of this e.g. as last period’s employment. This, however, would have the effect that the wage bargain of a union has intertemporal effects, i.e. the union would have to solve an intertemporal optimisation problem. Since we want to keep the exposition as simple as possible, we assume membership to be exogenous to the union. \(AI\) denotes the alternative income a union member can get when not finding a job in his/her sector. This consists of the probability of finding a job in another firm in the economy or becoming unemployed. The firm-level union treats \(AI\) as exogenous. At the aggregate level, however, this is endogenous. The bargained wage in the intermediate sector can be derived by maximising the following Nash product w.r.t. the wage and subject to the low-skilled demand of the intermediate sector:

\[
\Omega_X = (L_X(w_X - AI))^{\beta_X} (\Pi_X)^{1-\beta_X}, \tag{32}
\]

where \(\Pi_X\) denotes the profit of a representative intermediate goods firm. The threat points in this bargaining problem are similar to that of the latter section, i.e. in case of a failure of the bargain, the firm can produce nothing and all union members get the alternative income. The Nash product to derive the research sector low-skilled wage is similar to that of the intermediate sector; although we allow for the possibility that the bargaining power of unions might differ between the two sectors of the economy.

\[
\Omega_n = (L_n(w_n - AI))^{\beta_n} (\Pi_n)^{1-\beta_n}. \tag{33}
\]

The first order conditions for the low-skilled wage bargain in both sectors
of the economy read:

\[
\frac{\partial \Omega}{\partial w_{1X}} = \left( \frac{\partial L_X}{\partial w_{1X}} \frac{w_{1X} - AI}{L_X(w_{1X} - AI)} + \frac{L_X}{L_X(w_{1X} - AI)} \right) \frac{\beta_X}{1 - \beta_X} + \frac{-L_X}{\Pi_X} = 0,
\]

\[
\frac{\partial \Omega}{\partial w_{1n}} = \left( \frac{\partial L_n}{\partial w_{1n}} \frac{w_{1n} - AI}{L_n(w_{1n} - AI)} + \frac{L_n}{L_n(w_{1n} - AI)} \right) \frac{\beta_n}{1 - \beta_n} + \frac{-L_n}{\Pi_n} = 0,
\]

which gives eventually the intermediate and research sector low-skilled wage as:

\[
w_{1X} = \frac{1}{1 - (s_{1X} \frac{1 - \beta_X}{\beta_X} + \eta_{1X})^{-1}} \frac{L_X}{L} w_{1X},
\]

\[
w_{1n} = \frac{1}{1 - (s_{1n} \frac{1 - \beta_n}{\beta_n} + \eta_{1n})^{-1}} \frac{L_n}{L} w_{1n},
\]

(34)

(35)

where \(s_i\) and \(\eta_i\) is the part of profits devoted to low-skilled labour in sector \(i\) and the (positive) labour demand elasticity in sector \(i\), respectively.

The alternative income is the same for all low-skilled agents in the economy irrespectively by which union they are represented. As such, the relative sectoral low-skilled wage is only a function of the mark-ups. As already said, the alternative income is endogenous at the aggregate level and it reflects the expected income if not getting employed in any firm in the economy. Thus \(AI\) is given by (see, e.g. Holmlund (1997) for a similar interpretation):

\[
AI = \frac{L_X}{L} w_{1X} + \frac{L_n}{L} w_{1n} + \frac{U}{L} B.
\]

To keep the analysis simple we assume in this section the unemployment benefit, \(B\), to be zero. Note that we could have also assumed (without changing the results) that the unemployment benefit was some fixed ratio of \(w_{1X}\) or of \(w_{1n}\). Using this and the intermediate sector wage equation, we get the following relation between low-skilled employment in both sectors:

\[
1 = \frac{1}{1 - (s_{1X} \frac{1 - \beta_X}{\beta_X} + \eta_{1X})^{-1}} \frac{L_X}{L} + \frac{1}{1 - (s_{1n} \frac{1 - \beta_n}{\beta_n} + \eta_{1n})^{-1}} \frac{L_n}{L},
\]

(36)

which eventually gives:

\[
L_X = \left(1 - (s_{1X} \frac{1 - \beta_X}{\beta_X} + \eta_{1X})^{-1}\right) \frac{L}{L} - \frac{1 - (s_{1X} \frac{1 - \beta_X}{\beta_X} + \eta_{1X})^{-1}}{1 - (s_{1n} \frac{1 - \beta_n}{\beta_n} + \eta_{1n})^{-1}} L_n.
\]
This equation implicitly determines the "resource" constraint for low-skilled labour in the unionised economy (in the non-unionised case this is given by \(L_X = \tilde{L} - L_n\)).

Expressing the union resource constraint in relative changes and using the fact that from the low-skilled demand equations in both sectors of the economy the following holds:

\[
\dot{w}_lX - \dot{w}_l = -\eta_{lX} \dot{L}_X + \eta_{lL} \dot{L}_n
\]

we can determine low-skilled employment in the economy with decentralised bargaining as a function of bargaining power in both sectors. The relative (overall) employment change is given by (see, the appendix):

\[
\dot{L} = c_X \beta_X + c_n \beta_n.
\] (37)

Plugging this into equation (22) we get the effect of decentralised wage setting on the rate of growth:

\[
\hat{\gamma} = \frac{1}{\Lambda} \left[ (\delta_h + \lambda_{xh}(\theta_{nl} - \theta_{xl}))(c_X \beta_X + c_n \beta_n) + (\delta_l + \lambda_{xl}(\theta_{xl} - \theta_{nl})) \hat{H} \right] (38)
\]

If the economy starts from an initial situation in which both labour markets were perfectly competitive (this implies \(c_X \) and \(c_n \) are negative) decentralised unionisation will have the same effects as centralised wage setting, i.e. this equation basically resembles the results derived in the preceding section. The intuition for this result is that the decentralised union rations resources and increases the wage the same way as the centralised did.

The assumption that the economy starts from an initial perfectly competitive situation, however, suppresses the most interesting novelty when considering decentralised bargaining: the existence of intersectoral wage differentials (see, Jones (1971)). With initial wage differentials, decentralised wage setting could result in positive efficiency effects compared to an initial situation with unionised labour markets. This is due to the fact that unionisation could result in a reallocation such that resources are shifted into the more competitive sector. Thus, overall (low-skilled) employment increases (see, Kolm (1998) for a similar argument).

Once we consider these effects, the symmetry of results between the centralised and decentralised bargaining case breaks down. For brevity, we only analyse the situation in which in the initial situation the low-skilled wage in

\(^5\)Note, that unionisation does not only shift the resource constraint to the origin, but also changes the slope. At least if the mark-up is different in both sectors. This implies that unemployment depends on the allocation of low-skilled labour between the two sectors, see Kolm (1998) for this argument.
the intermediate sector exceeds that of the research sector. This could be e.g. due to higher unionisation in the intermediate sector.\textsuperscript{5}

The initial wage differential (if large enough) could result in $c_X > 0$. Higher intermediate sector unionisation would lead to higher low-skilled employment. On impact, the wage hike in the intermediate sector will increase unemployment. Wage moderation follows and employment in both sectors increases. The latter effect could overcompensate the impact effect if the wage differential was large enough. Thus, in case of increased intermediate sector unionisation, the economy will become less skill-abundant. In the case of the research sector being high-skilled intensive, the Rybczynski effect results in a contraction of the research sector. The hold up-problem is the same as in the preceding section. As such, unionisation would unambiguously decrease the rate of growth. As such, decentralised bargaining has different growth effects than centralised bargaining.

Besides changing the sign of the Rybczynski effect, an initial wage differential could also change the reallocation effect due to price changes (see Jones (1971)). The existence of within group wage differentials could change the form of the production schedule.

This is reflected in the sign of the determinant $\Lambda$ which might change. Consider $(\theta_{xl} - \theta_{nl})$. This can be rewritten as (see, the appendix):

$$(\theta_{xl} - \theta_{nl}) = \frac{w_h w_{lX} L_x H_x}{H_x \alpha X P_{X} \tilde{n} P_{n}} \left( \frac{H_n}{H_x} - \frac{w_{ln} L_n}{w_{lX} L_x} \right) \Lambda \leq 0$$

With no initial wage differential, this expression would be positive as long as the research sector employs relatively more high-skilled than low-skilled labour. In this case $(\lambda_{xh} - \lambda_{xl})$ was negative which was the argument for $\Lambda$ being positive (see, the appendix).

With an initial wage differential, however, things turn out to be different. Consider e.g. the research sector is high-skilled intensive, but that in the initial situation $\frac{w_{ln}}{w_{lX}} \gg 1$. With this, $\Lambda$ could be positive implying that compared to the centralised case the opposite holds true for the effect of union wage bargaining (see, e.g. Jones (1971)). The intuition for this is similar to the one mentioned above. Unionisation decreases the relative blueprint price and thus, results in a reallocation towards the intermediate sector. The intermediate sector is however, the more competitive. Low-skilled employment could increase with unionisation. Again, the results from the preceding section would be turned upside down.

\textsuperscript{6}The opposite situation in which the low-skilled wage in the research sector exceeds that of the intermediate sector basically has symmetric effects, i.e. we do not need to discuss both cases.
Initial within group wage differentials matter for the growth effects of union wage bargaining. Without an initial low-skilled wage differential, the growth effect of decentralised bargaining has been the same as in the case with centralised bargaining. Considering initial wage differentials, unionisation in one sector of the economy could result into a situation in which low-skilled employment increases (compared to the initial situation). This substantially modifies the results derived in the case with centralised bargaining.

5.3 Comprehensive Wage Setting

Up to this point we have considered situations in which only low-skilled was covered by union wage bargaining. This could be justified by the observation that with increasing education the probability of being a union member declines (see, e.g. Schnabel (2003)). Nevertheless, assuming only low-skilled agents to be unemployed seems too extreme. We know that union wage coverage rates are relatively high (see, CESifo DICE (2005)) and that in most countries also the high-skilled wage is being bargained. To get a complete picture of the effects of unionisation, we consider in this section a situation in which the low-skilled as well as the high-skilled wage is determined by (centralised) union-firm bargaining.

When analysing the bargaining outcome we assume the low-skilled and the high-skilled wage to be bargained simultaneously. Thus, we derive the reaction functions of the two unions with which we can determine a Nash equilibrium for the two labour markets.

The bargaining of the low-skilled union is the same as in section (5.1). Also in this case, the union considers high-skilled employment as being fixed. As such, the low-skilled wage setting curve is given by:

\[ w_l = \frac{1}{1 - (s_L L + \eta L)^{-1}} B, \]  \hspace{1cm} (39)

\(s_L\) is in this case the wage bill over profits.

The federation cannot produce anything in case of no agreement with either union. But since both wages, the low-skilled as well as the high-skilled, are bargained simultaneously, the federation, i.e. the firms, have to bear no costs in case of non-agreement. The utility function of the high-skilled union is given by:

\[ U_{\text{Skill-Union}} = H w_h + (\bar{H} - H) B \]  \hspace{1cm} (40)

Preferences of both skill-groups as well as the unemployment benefit are identical. The only heterogeneity is the skill-level.
The employer federation which bargains the wage with both unions is identical to the one in section (5.1). Thus, concerning the utility of the federation the same holds as in the above given section. With the changed bargaining situation, the Nash product in case of simultaneous bargaining reads:

\[ \Omega^H = (H(w_h - B))^{\epsilon}(\Pi)^{1-\epsilon} \]  

Maximising this Nash product w.r.t. \( w_h \) and subject to the high-skilled demand function, we get the following f.o.c.:

\[ \frac{\partial \Omega^H}{\partial w_h} = \left( \frac{\partial H}{\partial w_h} \frac{w_h - B}{H(w_h - B)} + \frac{H}{H(w_h - B)} \right) \frac{\epsilon}{1 - \epsilon} + \frac{-H}{\Pi} = 0, \]

using this, we get the high-skilled wage equation:

\[ w_h = \frac{1}{1 - (s_H \frac{1-\epsilon}{\epsilon} + \eta_H)^{-1}} B, \]

where \( \eta_H \) is the positively defined aggregate high-skilled demand elasticity and \( s_H \) is the high-skilled wage bill over aggregate profits. Using the two wage equations for low-skilled and high-skilled labour, we get the wage setting curves in relative changes:

\[ \hat{w}_l = a_1 \hat{\beta}, \]
\[ \hat{w}_h = a_4 \hat{\epsilon}, \]

where \( a_1 \) is identical to the definition in section 5.1 and \( a_4 \equiv \frac{(1-\epsilon)s_H}{(s_H \frac{1-\epsilon}{\epsilon} + \eta_H-1)^2} > 0 \) and as above \( \hat{\epsilon} = \frac{dx}{x} \).

Due to the homogeneity in preferences and the similar bargaining situation, both unions behave symmetrically. This is reflected in the symmetry of the wage setting equations. Using this and the labour demand relations, we can solve for the employment effects of both unions:

\[ \hat{L} = -\eta_L \hat{w}_l = -\eta_L a_1 \hat{\beta} \]
\[ \hat{H} = -\eta_H \hat{w}_h = -\eta_H a_4 \hat{\epsilon} \]

Plugging equations (43) and (44) into the equilibrium determining equation (22), we can analyse the growth effects of simultaneous wage bargaining:

\[ \hat{\gamma} = \frac{1}{\Lambda} \left[ -(\delta_h + \lambda_{xl}(\theta_{nl} - \theta_{xl})) \eta_L a_1 \hat{\beta} - (\delta_l + \lambda_{xl}(\theta_{xl} - \theta_{nl})) \eta_H a_4 \hat{\epsilon} \right] \]

The growth effect of high-skilled union wage bargaining is similar to that of low-skilled bargaining, however with opposite signs. This is obvious, since the Rybczynski effect will move in different directions.
The main focus of this section will be on the question what happens when the low-skilled and the high-skilled labour market will move simultaneously from a competitive to a non-competitive situation.

To analyse the growth effect of simultaneous bargaining in a general case is quite diverse, because the effects depend on the relative employment decrease of both factors of production and on the reaction of the relative factor price.

Thus, in order to structure our thinking, we focus on two special cases: in the first one we assume parameters to be such that high-skilled and low-skilled employment decreases to the same extend. Unionisation will in this case only change the relative factor price but not the skill content of the economy. The second one is that the wage hike will be of the same extend, i.e. the relative wage will not change and unionisation will only change the skill content.

Consider the case in which unionisation only changes the relative factor price. This implies:

**Proposition 3** If unionisation decreases high-skilled and low-skilled employment to the same extent, the rate of growth will always decrease.

**Proof 3** The assumption in this case is \( \eta_L a_1 \beta = \eta_H a_4 \epsilon \). Using this we get from (45) the coefficient showing the growth effect of unionisation as:

\[
-\delta_h - \lambda_{2h}(\theta_{nl} - \theta_{xl}) - \delta_l - \lambda_{2l}(\theta_{xl} - \theta_{nl}) \geq 0
\]

\[
\iff -\delta_l - \delta_h - \{(\theta_{nl} - \theta_{xl})(\lambda_{xl} - \lambda_{zh})\} < 0
\]

*The expression in curly brackets is always positive.* As such, an increase in the bargaining power of either union will decrease the rate of growth. $\blacksquare$

With low-skilled and high-skilled labour declining by the same (relative) amount, the skill intensity in the economy does not change. Thus, there will be no Rybczynski effect. The resource rationing effect will be symmetric in both sectors. The price of blueprints, however, will decline and will lead to a reallocation of resources towards the intermediate sector. The rate of growth will unambiguously decline.

If unionisation is such that the relative factor price is constant, i.e. \( a_1 \beta = a_4 \epsilon \), the union will affect the skill abundance in the economy. Thus, the following holds:

\[7\]The same argument applies as for the proof that \( \Lambda \) is always positive. If the research sector is low-skilled (high-skilled) intensive \( (\theta_{nl} - \theta_{xl}) > (<)0 \) and \( (\lambda_{xl} - \lambda_{zh}) > (<)0 \). Thus, the product will always be negative.
Proposition 4 If the research sector is high-skilled intensive (extensive) the rate of growth will unambiguously decline if unionisation makes the economy less (more) skill abundant. If the opposite holds, the growth rate might increase.

Proof 4 The coefficient which depicts the effect of unionisation is in this case given by:

\[-\delta_h \lambda_{xh}(\theta_{nl} - \theta_{xl}) \eta_L - (\delta_l + \lambda_{xl}(\theta_{xl} - \theta_{nl})) \eta_H \leq 0\]

\[-\delta_h \eta_L - \delta_l \eta_H - \left( \eta_H \left( \lambda_{xl} - \lambda_{xh} \frac{\eta_L}{\eta_H} \right) (\theta_{nl} - \theta_{xl}) \right) \leq 0\]

Consider the research sector being high-skilled intensive. This implies that \((\theta_{nl} - \theta_{xl}) < 0\). Moreover if \(\eta_L < \eta_H\), \((\lambda_{xl} - \lambda_{xh} \frac{\eta_L}{\eta_H}) \) will be unambiguously positive, making the overall coefficient which depicts the growth effect negative. Similar arguments will hold true for the research sector being low-skilled intensive. ■

The intuition for this proposition is similar to the intuition for the results in chapter 5.1. \(\eta_L < \eta_H\) implies that with no change in the relative wage, high-skilled employment declines by a larger amount than low-skilled employment. The economy becomes less skill abundant. The ”generalised” Rybczynski theorem (see, Jones (1965)) then implies that with the research sector being high-skilled intensive overall production in this sector will decline. Hence, the rate of growth will unambiguously decline.

A prerequisite for the possibility of a positive growth effect of unionisation is (if the research sector is high-skilled intensive) an increase in the skill abundance of the economy. Only in this case, research sector output increases and might overcompensate the negative hold-up effect. As such, the growth effects of unionisation crucially depends on the skill abundance effect (i.e. relative rationing) of union wage bargaining.

What do we expect concerning the relative rationing effect of unionisation? Freeman and Schettkat (2001) show that the skill-distribution of the unemployed is very similar to that of the employed. The relative rationing effect between low-skilled and high-skilled will by and large be identical. As such, we would expect union wage bargaining covering high-skilled and low-skilled agents rather to depress economic growth.

6 Summary and Conclusion

In this paper we have analysed the growth effects of unionised labour markets. We considered a general two sector, two factor endogenous growth model a
la Romer (1990) as the basic framework. This framework was then in turn amended by different institutional bargaining settings.

In a first attempt we analysed the growth effects of a central union bargaining the low-skilled wage. The consequence of this assumption is that low-skilled agents become unemployed and that there is no intersectoral low-skilled wage differential. Unionisation then has two (possibly countervailing effects). Firstly, the economy will become more skill-abundant which gives rise to a Rybczynski effect. The skill-intensive sector (the research sector in our case) will c.p. expand in absolute terms. This fosters the rate of growth. Secondly, the union gives rise to a hold-up problem. Investing into research will become less attractive. This dampens the rate of growth. Which of the two effect dominates depends on the elasticities of substitution in the economy.

To get a more thorough understanding of the growth effects of union wage bargaining, we extended the basic model to allow for a decentralised bargaining institution. As before, only low-skilled agents are covered by the wage bargain, however, there will be no uniform low-skilled wage, but within group wage inequality. The growth effects of unionisation in this case are basically the same as in the economy with centralised bargaining. There will be low-skilled unemployment, making the economy more skill abundant and the price for blueprints will decline.

However, this symmetry of results between the centralised and the decentralised bargaining breaks down in the case that the initial situation was characterised by large wage differentials (i.e. unionisation was already very large). If this is the case, unionisation could result in a reallocation of resources towards the less unionised sector. Thus, employment could increase compared to the initial (unionised) situation. The economy will become less skill-abundant and the Rybczynski effect will have the opposite sign compared to the centralised bargaining situation. Wage differentials matter for the growth effect of unionisation.

With union coverage rates of around 90% in continental Europe, it is hard to justify the assumption that only low-skilled agents are covered by union wage bargaining. Thus, in order to reflect the situation in Europe, we considered a situation in which the low-skilled as well as the high-skilled wage is determined by (central) union firm bargaining. Besides low-skilled agents facing unemployment, in this situation there will also be high-skilled unemployment.

The basic effect of unionisation in this case is again the same as in the benchmark case. The hold-up effect dampens the rate of growth and the Rybczynski effect could foster growth. However, in the case with low-skilled and high-skilled unemployment, it is a priori not clear whether the economy
will become more or less skill abundant. As such, the direction of the Rybczynski effect is unclear, too. If the economy becomes more skill abundant the same results as in the case of only low-skilled bargaining will arise. Opposite results hold in the case in which the skill abundance in the economy decreases. Thus, in a comprehensive bargaining setting, relative rationing matters for the growth effects of unionisation.

Summarising, unionisation affects the rate of growth via two different channels. On the one hand, there will be a hold-up problem which decreases the incentive to invest into research. The rate of growth would c.p. decline. On the other hand, unionisation alters the factor intensity in the economy. This gives rise to a potentially growth enhancing Rybczynski effect. The sign of the latter effect, however, depends on the institutional bargaining framework. Thus, when empirically analysing the growth effects of unionisation one has to take the different wage setting systems into account.

The focus of this paper was only on the positive growth effects of unionisation. We did not analyse whether from a social welfare point of view unions are desirable or not. As such, no normative analysis has been done. This, however, is an important aspect. This is especially true because the type of growth model under consideration is characterised by many externalities. The rate of growth could to be too high or too low, depending on the form of these externalities. Thus, unionisation could adjust the rate of growth to the social optimal level. This would come at the cost of unemployment, begging the question whether the potentially welfare enhancing intertemporal effect dominates the welfare decreasing unemployment effect. We leave this question for future research.
7 Appendix

7.1 The equilibrium System

Equations (19), (20) and (21) determine the equilibrium values of the growth rate $\dot{\gamma}$, intermediate goods production (therewith consumption) $\hat{X}$ and of the wage differential $(\hat{w}_l - \hat{w}_h)$. In matrix notation, this system reads:

$$\begin{bmatrix}
1 & (\theta_{xl} - \theta_{nl}) & -\frac{r-\rho}{r} \\
\lambda_{xl} & -\delta_l & (1 - \lambda_{xl}) \\
\lambda_{xh} & \delta_h & (1 - \lambda_{xh})
\end{bmatrix}
\begin{bmatrix}
\hat{X} \\
\hat{w}_l - \hat{w}_h
\end{bmatrix}
= \begin{bmatrix}
0 \\
\hat{L} \\
\hat{H}
\end{bmatrix}.$$  \hspace{1cm} (46)

At this point of argumentation, we assume the amount of primary resources $L$ and $H$ as exogenously.

To get solutions for the endogenous variables we have to determine the determinant of the coefficient matrix:

$$|A| := -\delta_l(1 - \lambda_{xh}) + (1 - \lambda_{xl})\lambda_{xh}(\theta_{xl} - \theta_{nl}) - \frac{r-\rho}{r}\lambda_{xl}\delta_n - \lambda_{xh}\delta_l\frac{r-\rho}{r} - \delta_n(1 - \lambda_{xl}) - \lambda_{xl}(1 - \lambda_{xh})(\theta_{xl} - \theta_{nl})$$

$$\Leftrightarrow -\delta_l(1 - \lambda_{xh}) - \frac{r-\rho}{r}\lambda_{xl}\delta_n - \lambda_{xh}\delta_l\frac{r-\rho}{r} - \delta_n(1 - \lambda_{xl}) + (\lambda_{xh} - \lambda_{xl})(\theta_{xl} - \theta_{nl}) < 0$$

This determinant is unambiguously negative. Consider e.g. the case in which the intermediate goods sector is high-skilled (low-skilled) intensive. Then $(\lambda_{xh} - \lambda_{xl})$ is positive (negative) and $(\theta_{xl} - \theta_{nl})$ is negative (positive).

Using the definition $|A| := -\Lambda$, we can derive solutions for (the relative change of) the rate of growth and the low-skilled wage differential and intermediate goods production:

$$\dot{\gamma} = \frac{1}{\Lambda} \cdot \left[ (\delta_h + \lambda_{xh}(\theta_{nl} - \theta_{xl}))\hat{L} + (\delta_l + \lambda_{xl}(\theta_{xl} - \theta_{nl}))\hat{H} \right]$$  \hspace{1cm} (47)

$$(\hat{w}_l - \hat{w}_h) = -\frac{1}{\Lambda} \cdot \left[ ((1 - \lambda_{xh}) + \lambda_{xh}\frac{r-\rho}{r})\hat{L} + ((1 - \lambda_{xl}) + \lambda_{xl}\frac{r-\rho}{r})\hat{H} \right]$$  \hspace{1cm} (48)

$$\hat{X} = \frac{1}{\Lambda} \cdot \left[ (\delta_h\frac{r-\rho}{r} + (1 - \lambda_{xh})(\theta_{xl} - \theta_{nl}))\hat{L} + (\delta_l\frac{r-\rho}{r} + (1 - \lambda_{xl})(\theta_{nl} - \theta_{xl}))\hat{H} \right]$$  \hspace{1cm} (49)

Since the reaction of intermediate goods production and the rate of growth is quite symmetric, we will focus in the text on the latter one.
7.2 An expression for \((\theta_{xl} - \theta_{nt})\)

\[
\begin{align*}
(\theta_{xl} - \theta_{nt}) & \iff (\theta_{xl}(1 - \theta_{nl}) - \theta_{nt}(1 - \theta_{xl})) \\
& \iff (\theta_{xl}\theta_{nh} - \theta_{nt}\theta_{eh}) \\
& \iff \left( \frac{w_{lX}L_x}{\alpha xP_x} \frac{w_hH_n}{\hat{n}P_n} - \frac{w_{ln}L_n w_hH_x}{\alpha xP_x} \right) \\
& \iff \frac{w_hw_{lX}L_xH_x}{\alpha xP_x\hat{n}P_n} \left( \frac{H_n}{H_x} - \frac{w_{ln}L_n}{w_{lX}L_x} \right)
\end{align*}
\]

which is the relation stated in the text.

7.3 Low-skilled Employment with Decentralised Bargaining

First of all we express the relative change of the low-skilled wage as a function of the relative change in bargaining power. Thus, we assume the rest of the mark-up determining variables as locally constant and unchanged. This gives the following equations:

\[
\begin{align*}
\hat{w}_{lX} &= a_2 \hat{\beta}_X + \hat{A}I \\
\hat{w}_{ln} &= a_3 \hat{\beta}_n + \hat{A}I
\end{align*}
\]

where the coefficients are given by:

\[
\begin{align*}
a_2 &\equiv \frac{(1 - \beta_X)s_{lX}}{(s_{lX} - \beta_X^1 - \eta_{lX} - 1)^2} > 0 \\
a_3 &\equiv \frac{(1 - \beta_n)s_{ln}}{(s_{ln} - \beta_n^1 + \eta_{ln} - 1)^2} > 0
\end{align*}
\]

With this, we get the relative labour demand relation as:

\[
a_2 \hat{\beta}_X - a_3 \hat{\beta}_n = -\eta_{lX} \hat{L}_X + \eta_{ln} \hat{L}_n.
\]

Expressing the union resource constraint in relative changes and assuming the overall low-skilled resource base \(\hat{L}\) to be unchanged, gives the following:

\[
0 = a_2 \frac{L_X}{\hat{L}} \hat{\beta}_X + \phi_X \frac{L_X}{\hat{L}} \hat{L}_X + a_3 \frac{L_n}{\hat{L}} \hat{\beta}_n + \phi_n \hat{L}_n
\]
where $\phi_X$ and $\phi_n$ are short for the mark-up in the intermediate and the research sector respectively. Writing the latter two equations in matrix notation gives:

$$
\begin{bmatrix}
-\eta_L X & \eta_L n \\
\phi_X \frac{L_X}{L} & \phi_n \frac{L_n}{L}
\end{bmatrix}
\begin{bmatrix} L_X \\ L_n \end{bmatrix} =
\begin{bmatrix} a_2 \beta_X - \beta_n \\ -a_2 \frac{L_X}{L} \beta_X - a_3 \frac{L_n}{L} \beta_n \end{bmatrix},
$$

where the determinant of the coefficient matrix is given by:

$$
-|B| = \eta_L X \phi_n \frac{L_n}{L} + \eta_L n \phi_X \frac{L_X}{L}.
$$

With this we get a solution to the relative change of employment in either sector of the economy:

$$
\hat{L}_X = \left( a_3 \frac{L_n}{L} (\phi_n - \eta_L n) \beta_n - a_2 \frac{L_n}{L} + \eta_L n \frac{L_X}{L} \beta_X \right) (-|B|)^{-1}
$$

$$
\hat{L}_n = \left( a_2 \frac{L_X}{L} (\phi_X - \eta_L X) \beta_X - a_3 (\eta_L X \frac{L_n}{L} + \phi_X \frac{L_X}{L}) \beta_n \right) (-|B|)^{-1}
$$

Sectoral low-skilled employment decreases with an increase in unionisation in that sector. This is just because the wage in that sector increases in this case and therewith employment declines. The impact of an increase in the other sectors unionisation has ambiguous effect. On the one hand, employment decreases, which dampens the alternative income. The other sector’s wage, however, increases and as such the alternative income. Form a theoretical point of view it is not clear which effect dominates. Using the definition $\hat{L} = \frac{L_X}{L} \hat{L}_X + \frac{L_n}{L} \hat{L}_n$ we get eventually the relative change in overall low-skilled employment:

$$
\hat{L} = c_X \hat{\beta}_X + c_n \hat{\beta}_n
$$

where the coefficients $c_X$ and $c_n$ are given by:

$$
c_X \equiv \frac{a_2}{-|B|} \left( \frac{L_n}{L} \frac{L_X}{L} (\phi_X - \phi_n) - \frac{L_X}{L} \eta_L X - \frac{L_n}{L} \eta_L n \right)
$$

$$
c_n \equiv \frac{a_3}{-|B|} \left( \frac{L_X}{L} \frac{L_n}{L} (\phi_n - \phi_X) - \frac{L_n}{L} \eta_L n - \frac{L_X}{L} \eta_L n \right)
$$

Due to the interaction effect between wage setting and the alternative income, overall employment effect of higher unionisation is ambiguous. If e.g. unionisation is very strong in one sector and the increase in unionisation leads to a reallocation of low-skilled labour into the other sector overall employment might improve. Since we want to analyse the ”pure” effects of union
wage bargaining, we assume an initial situation in which there is no union wage setting. Thus, we can set \( \phi_n = \phi_X = 1 \) in the above equations. This implies that \( c_X \) and \( c_n \) are negative. As such, unionisation in the research and in the intermediate sector will decrease overall-low-skilled employment. This decrease, however, will not be symmetric and will be a function of the strength of unionisation and of the sectoral labour demand elasticities.
7.4 Rybczynski Effect

Figure 2: The Rybczynski Effect with Centralised Low-skilled Bargaining

Assumption: 
\( \theta_m - \theta_s < 0 \)

no-arbitrage Relation

\( \gamma \)

\( \gamma_1 \)

\( \gamma_0 \)

\( \gamma_2 \)

\( X \)
References


8 Referee’s Appendix (not to be published)

8.1 Supply Function in the Intermediate Sector

The profit function in the intermediate sector reads:

\[ \Pi_x = x p_x - [l_x w_l + h_x w_h] \]  
\[ \Pi_x = x p_x - x \left[ l_x w_l + \frac{h_x}{x} w_h \right] \]  
\[ \Pi_x = x p_x - x \left[ a_x l w_l + a_x h w_h \right] \]  

(65)  
(66)  
(67)

Firms choose \( x \) to maximise profits. They take into account that they face a downward sloping demand function with inverse price elasticity of \( \alpha - 1 \).

\[ \frac{\partial \Pi_x}{\partial x} = p_x + x \frac{\partial p_x}{\partial x} - [a_x l w_l + a_x h w_h] = 0 \]  
\[ = p_x \left( 1 + \frac{\partial p_x}{\partial x} \frac{x}{p_x} \right) = [a_x l w_l + a_x h w_h] \]  
\[ = p_x (1 + \alpha - 1) = [a_x l w_l + a_x h w_h] \]  
\[ = p_x = \frac{1}{\alpha} [a_x l w_l + a_x h w_h] \]  

(68)  
(69)  
(70)  
(71)

8.2 Supply Function in the Research Sector

Profits in this sector reads:

\[ \Pi_n = n p_n - [l_n w_l + h_n w_h] \]  
\[ \Pi_n = n p_n - \left[ \frac{l_n}{n} w_l + \frac{h_n}{n} w_h \right] n \]  
\[ \Pi_n = n p_n - \frac{1}{n} \left[ \frac{l_n}{\gamma} w_l + \frac{h_n}{\gamma} w_h \right] \hat{n} \]  
\[ \Pi_n = n p_n - \frac{1}{n} [a_n l w_l + a_n h w_h] \hat{n} \]  

(73)  
(74)  
(75)  
(76)

Profit maximisation yields:

\[ \frac{\partial \Pi_n}{\partial \hat{n}} = p_n - \frac{1}{n} [a_n l w_l + a_n h w_h] = 0 \]  
\[ \Leftrightarrow p_n = \frac{1}{n} [a_n l w_l + a_n h w_h] \]  

(77)  
(78)

which is equation (6).
8.3 Deriving the Log-Linear System

1. Intermediate Supply:

\[ p_x = \frac{1}{\alpha} [a_{xl}w_l + a_{xh}w_h] \]  
\[ \frac{dp_x}{p_x} = \frac{1}{\alpha} [a_{xl}dwl + w_l/da_{xl} + a_{xh}dw_h + w_h/da_{xh}] \]  
\[ \frac{dp_x}{p_x} = 1 \left[ a_{xl}\frac{dwl}{wl} + a_{xl}\frac{da_{xl}}{a_{xl}} + a_{xh}\frac{dw_h}{wh} + a_{xh}\frac{dw_h}{wh} \right] \]  
\[ \hat{p}_x = \frac{a_{xl}w_l}{\alpha p_x} (\hat{w}_l + \hat{a}_{xl}) + \frac{a_{xh}w_h}{\alpha p_x} (\hat{w}_h + \hat{a}_{xh}) \]  
\[ \hat{p}_x = \frac{l_{xw_l}}{\alpha xl} (\hat{w}_l + \hat{a}_{xl}) + \frac{h_{xw_h}}{\alpha xh} (\hat{w}_h + \hat{a}_{xh}) \]  

2. Research Sector Supply

\[ p_n = \frac{1}{n} [a_{nl}w_l + a_{nh}w_h] \]  
\[ \frac{dp_n}{p_n} = \frac{1}{n} [a_{nl}dwl + w_l/da_{nl} + a_{nh}dw_h + w_h/da_{nh}] \]  
\[ \frac{dp_n}{p_n} = \frac{1}{n} \left[ a_{nl}\frac{dwl}{wl} + a_{nl}\frac{da_{nl}}{a_{nl}} + a_{nh}\frac{dw_h}{wh} + a_{nh}\frac{dw_h}{wh} \right] \]  
\[ \hat{p}_n = \frac{a_{nl}w_l}{np_n} (\hat{w}_l + \hat{a}_{nl}) + \frac{a_{nh}w_h}{np_n} (\hat{w}_h + \hat{a}_{nh}) \]  
\[ \hat{p}_n = \frac{l_{nw_l}}{n\gamma p_n} (\hat{w}_l + \hat{a}_{nl}) + \frac{h_{nw_h}}{n\gamma p_n} (\hat{w}_h + \hat{a}_{nh}) \]  
\[ \hat{p}_n = \frac{l_{nw_l}}{n\gamma p_n} (\hat{w}_l + \hat{a}_{nl}) + \frac{h_{nw_h}}{n\gamma p_n} (\hat{w}_h + \hat{a}_{nh}) \]
3. Blueprints Demand

\[
\frac{p_n}{p_x} = \frac{(1 - \alpha)x}{\rho + \frac{1 - \alpha}{\alpha} \gamma}
\]

(90)

\[
d \left[ \frac{p_n}{p_x} \right] = \frac{(1 - \alpha)}{\rho + \frac{1 - \alpha}{\alpha} \gamma} \left( dx - \frac{(1 - \alpha)x}{(\rho + \frac{1 - \alpha}{\alpha} \gamma)^2} \frac{1 - \alpha}{\alpha} d\gamma \right)
\]

(91)

\[
d \left[ \frac{p_n}{p_x} \right] = \frac{(1 - \alpha)x \ dx}{\rho + \frac{1 - \alpha}{\alpha} \gamma} - \frac{(1 - \alpha)x}{(\rho + \frac{1 - \alpha}{\alpha} \gamma)^2} \frac{1 - \alpha}{\alpha} \gamma \ d\gamma
\]

(92)

\[
d \left[ \frac{p_n}{p_x} \right] = \frac{p_n \ dx}{p_x} - p_n \ \frac{1 - \alpha}{\alpha} \gamma \ d\gamma
\]

(93)

\[
\frac{p_x dp_n - p_n dp_x}{p_x^2} = \frac{p_n}{p_x} \left( \dot{x} - \frac{1 - \alpha}{\rho + \frac{1 - \alpha}{\alpha} \gamma} \dot{\gamma} \right)
\]

(94)

\[
\frac{p_x dp_n - p_n dp_x}{p_x^2} \frac{p_x}{p_n} = \dot{x} - \frac{1 - \alpha}{\rho + \frac{1 - \alpha}{\alpha} \gamma} \dot{\gamma}
\]

(95)

\[
\dot{p}_n - \dot{p}_x = \dot{x} - \frac{1 - \alpha}{\rho + \frac{1 - \alpha}{\alpha} \gamma} \dot{\gamma}
\]

(96)

Remember that \( r = \rho + \frac{1 - \alpha}{\alpha} \gamma \). With this equation (11’) in the text follows.

4. Resource Constraints-Low-skilled Employment

\[
L = a_{xl} X + a_{nl} \gamma
\]

(97)

\[
dL = a_{xl} dX + X da_{xl} + a_{nl} d\gamma + \gamma da_{nl}
\]

(98)

\[
\frac{dL}{L} = a_{xl} X \frac{dX}{X} + a_{xl} X \frac{da_{xl}}{a_{xl}} + a_{nl} \gamma \frac{d\gamma}{\gamma} + a_{nl} \gamma \frac{da_{nl}}{a_{nl}}
\]

(99)

\[
\frac{dL}{L} = a_{xl} X \left( \frac{dX}{X} + \frac{da_{xl}}{a_{xl}} \right) + a_{nl} \gamma \left( \frac{d\gamma}{\gamma} + \frac{da_{nl}}{a_{nl}} \right)
\]

(100)

\[
\dot{L} = \frac{a_{xl} X}{L} \left( \dot{X} + \dot{a}_{xl} \right) + \frac{a_{nl} \gamma}{L} (\dot{\gamma} + \dot{a}_{nl})
\]

(101)

\[
\dot{L} = \frac{a_{xl} X}{L_{\lambda_{xl}}} \left( \dot{X} + \dot{a}_{xl} \right) + \frac{a_{nl} \gamma}{L_{\lambda_{nl}}} (\dot{\gamma} + \dot{a}_{nl})
\]

(102)
5. Resource Constraints-High-skilled Employment

\[ H = a_{zh}X + a_{nh}\gamma \]  
(103)

\[ \frac{dH}{H} = a_{zh}dX + Xd\gamma + a_{nh}\gamma \frac{d\gamma}{\gamma} + a_{nh}\gamma \frac{d\gamma}{a_{nh}} \]  
(104)

\[ \frac{dH}{H} = a_{zh}X \left( \frac{dX}{X} + a_{zh}X \frac{d\gamma}{a_{zh}} \right) + a_{nh}\gamma \left( \frac{d\gamma}{\gamma} + a_{nh}\gamma \frac{d\gamma}{a_{nh}} \right) \]  
(105)

\[ \frac{dH}{H} = a_{zh}X \left( \frac{\dot{X} + \dot{\gamma}}{\lambda_{zh}} \right) + a_{nh}\gamma \left( \frac{\dot{\gamma} + \dot{\gamma}}{\lambda_{nh}} \right) \]  
(106)

\[ \frac{dH}{H} = a_{zh}X \left( \frac{\dot{X} + \dot{\gamma}}{\lambda_{zh}} \right) + a_{nh}\gamma \left( \frac{\dot{\gamma} + \dot{\gamma}}{\lambda_{nh}} \right) \]  
(107)

\[ \frac{dH}{H} = a_{zh}X \left( \frac{\dot{X} + \dot{\gamma}}{\lambda_{zh}} \right) + a_{nh}\gamma \left( \frac{\dot{\gamma} + \dot{\gamma}}{\lambda_{nh}} \right) \]  
(108)

8.4 Cost Minimization

Firms choose input coefficients such that unit costs will be minimized. Input prices will be assumed exogenous by the firm. Moreover, since the choice concerning the optimal amount of production has already taken place, the firm also takes the output price as given.

An intermediate sector firm thus will choose \( a_{xl} \) and \( a_{xh} \) such that the following holds:

\[ \alpha p_x = a_{xl}w_l + a_{xh}w_h \]  
(109)

\[ w_l d_{axl} + w_h d_{axh} = 0 \]  
(110)

\[ w_l a_{xl} \frac{d_{axl}}{a_{xl}} + w_h a_{xh} \frac{d_{axh}}{a_{xh}} = 0 \]  
(111)

\[ \frac{w_l l_x}{x} \dot{\alpha}_{xl} + \frac{w_h h_x}{x} \dot{\alpha}_{xh} = 0 \]  
(112)

\[ \frac{w_l l_x}{x \alpha p_x} \dot{\alpha}_{xl} + \frac{w_h h_x}{x \alpha p_x} \dot{\alpha}_{xh} = 0 \]  
(113)

\[ \frac{w_l l_x}{x \alpha p_x} \dot{\alpha}_{xl} + \frac{w_h h_x}{x \alpha p_x} \dot{\alpha}_{xh} = 0 \]  
(114)
For the research sector firm, the cost minimization problem reads:

\[ np_n = a_{nl}w_l + a_{nh}w_h \quad (115) \]
\[ w_l^na_{nl} + w_h^na_{nh} = 0 \quad (116) \]
\[ w_l^na_{nl} \frac{da_{nl}}{a_{nl}} + w_h^na_{nh} \frac{da_{nh}}{a_{nh}} = 0 \quad (117) \]
\[ w_l^na_{nl} \frac{\hat{a}_{nl}}{\gamma} + w_h^na_{nh} \frac{\hat{a}_{nh}}{\gamma} = 0 \quad (118) \]
\[ w_l^na_{nl} \frac{\hat{a}_{nl}}{\hat{a}_{nl}} + w_h^na_{nh} \frac{\hat{a}_{nh}}{\hat{a}_{nh}} = 0 \quad (119) \]
\[ w_l^na_{nl} \frac{\hat{a}_{nl}}{\theta_{nl}} + w_h^na_{nh} \frac{\hat{a}_{nh}}{\theta_{nh}} = 0 \quad (120) \]

8.5 The Elasticities of Substitution

By definition the elasticity of substitution in the intermediate sector reads:

\[ \sigma_x = \frac{d[h_x/l_x]}{h_x/l_x} \quad (121) \]

Simplifying the numerator gives:

\[ \frac{d[h_x/l_x]}{h_x/l_x} = d[h_x/l_x] \frac{l_x}{h_x} = \frac{l_x dh_x - h_x dl_x}{h_x} = \frac{dh_x}{h_x} - \frac{dl_x}{l_x} = \frac{d[h_x/x]}{h_x/x} - \frac{d[l_x/x]}{l_x/x} \]
\[ = \frac{da_{xl}}{a_{xl}} - \frac{\hat{a}_{xl}}{\hat{a}_{xl}} = \hat{a}_{xl} - \hat{a}_{xl} \quad (122) \]

The denominator gives:

\[ \frac{d[w_l/w_h]}{w_l/w_h} = d[w_l/w_h] \frac{w_h}{w_l} = \frac{w_h dw_l - w_l dw_h}{w_l} = \frac{dw_l}{w_l} - \frac{dw_h}{w_h} = \hat{w}_l - \hat{w}_h \quad (124) \]

Plugging the simplified numerator and denominator into the definition of the elasticity of substitution gives:

\[ \sigma_x = \frac{\hat{a}_{xl} - \hat{a}_{xl}}{\hat{w}_l - \hat{w}_h} \quad (125) \]
In the research sector, this elasticity is given by:

\[
\sigma_n = \frac{d[h_n/l_n]}{h_n/l_n} \cdot \frac{d[w_l/w_h]}{w_l/w_h}
\]

By the same reasoning as above, the numerator simplifies to:

\[
\frac{d [h_n/l_n]}{h_n/l_n} = d[h_n/l_n] = \frac{l_n dh_n - h_n dl_n}{h_n} = \frac{dh_n}{h_n} - \frac{dl_n}{l_n} = \frac{d[h_n/n]}{h_n/n} - \frac{d[l_n/n]}{l_n/n}
\]

\[
= \frac{d\hat{a}_{nh}}{a_{nh}} - \frac{d\hat{a}_{nl}}{a_{nl}} = \hat{a}_{nh} - \hat{a}_{nl}
\]

The denominator is the same as in the intermediate sector case, thus the elasticity reads:

\[
\sigma_n = \frac{\hat{a}_{nh} - \hat{a}_{nl}}{\hat{w}_l - \hat{w}_h}
\]

8.6 Deriving the Production Coefficients

The (relative change of the) four production coefficients can be determined with the help of equations (114), (120), (125) and (129), which makes four equations in four variables. Rewriting (114) and (120) gives, e.g.:

\[
\theta_{xl} \cdot \hat{a}_{xl} + (1 - \theta_{xl}) \cdot \hat{a}_{xh} = 0
\]

\[
\Leftrightarrow \hat{a}_{xh} + \theta_{xl}(\hat{a}_{xl} - \hat{a}_{xh})
\]

\[
\Leftrightarrow \hat{a}_{xh} = -\theta_{xl}(\hat{a}_{xl} - \hat{a}_{xh})
\]

\[
\theta_{nl} \cdot \hat{a}_{nl} + (1 - \theta_{nl}) \cdot \hat{a}_{nh} = 0
\]

\[
\Leftrightarrow \hat{a}_{nh} + \theta_{nl}(\hat{a}_{nl} - \hat{a}_{nh})
\]

\[
\Leftrightarrow \hat{a}_{nh} = -\theta_{nl}(\hat{a}_{nl} - \hat{a}_{nh})
\]

From the definition of the elasticity of substitution we know that:

\[
\sigma_x = \frac{\hat{a}_{xh} - \hat{a}_{xl}}{\hat{w}_l - \hat{w}_h}
\]

\[
\Leftrightarrow \hat{a}_{xh} - \hat{a}_{xl} = \sigma_x(\hat{w}_l - \hat{w}_h)
\]

\[
\Leftrightarrow \hat{a}_{xl} - \hat{a}_{xh} = -\sigma_x(\hat{w}_l - \hat{w}_h)
\]

\[
\sigma_n = \frac{\hat{a}_{nh} - \hat{a}_{nl}}{\hat{w}_l - \hat{w}_h}
\]

\[
\Leftrightarrow \hat{a}_{nh} - \hat{a}_{nl} = \sigma_n(\hat{w}_l - \hat{w}_h)
\]

\[
\Leftrightarrow \hat{a}_{nl} - \hat{a}_{nh} = -\sigma_n(\hat{w}_l - \hat{w}_h)
\]
Using this, we get:

\[
\hat{a}_{xh} = \theta_{xl} \sigma_x (\hat{w}_l - \hat{w}_h) \\
\hat{a}_{nh} = \theta_{nl} \sigma_n (\hat{w}_l - \hat{w}_h)
\]

Moreover, this yields:

\[
\hat{a}_{xl} = -\theta_{xh} \sigma_x (\hat{w}_l - \hat{w}_h) \\
\hat{a}_{nl} = -\theta_{nh} \sigma_n (\hat{w}_l - \hat{w}_h)
\]

which then gives the equations in the text.