Do Financial Frictions Help to Understand the Economy? Some Bayesian Results*

Virginia Queijo†

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Preliminary

Abstract

In this paper we focus on the financial accelerator mechanism developed by Bernanke, Gertler, and Gilchrist (1999) in which information asymmetries between lenders and entrepreneurs introduce inefficiencies in financial markets, affecting the supply of credit and amplifying business cycles. We develop and estimate a DSGE model with financial frictions using Bayesian methods. Our purpose is to estimate the parameters of the model and to identify the main shocks affecting the economy. Our preliminary results support the idea that exist a financial accelerator mechanism in the U.S. but not in the euro area.

Keywords: DSGE models; Bayesian estimation; financial accelerator

JEL: E3, E4, E5

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†Institute for International Economic Studies, Stockholm University. E-mail:que@iies.su.se


1 Introduction

The works of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Carlstrom and Fuerst (1998), in which endogenous procyclical movements in entrepreneurial net worth magnify investment and output fluctuations, are the cornerstone of most recent theoretical papers with financial frictions. In this paper we focus on the financial accelerator mechanism developed by Bernanke, Gertler, and Gilchrist (1996) in which information asymmetries between lenders and entrepreneurs introduce inefficiencies in financial markets, affecting the supply of credit and amplifying business cycles. The argument is that during booms (recessions), an increase (fall) in borrowers’ net worth decreases (increases) the borrowers’ cost of obtaining external funds and then stimulates (destimulates) investment amplifying the effects of the initial shock. This approach has become widely spread in the literature and many studies have introduced these type of frictions in DSGE models (Bernanke, Gertler, and Gilchrist (1999), henceforth BGG, Christiano, Motto, and Rostagno (2004)). Also the same idea has been used in growth models (Aghion, Bacchetta, and Banerjee (2004), Aghion, Howitt, and Mayer-Foulkes (2003)) and in open economies models (Gertler, Gilchrist, and Natalucci (2003), Gilchrist, Hairault, and Kempf (2002)).

However, not so much has been done when it comes to the econometric estimation of these models. We only account for two papers estimating models with a financial accelerator mechanism. Christiano, Motto, and Rostagno (2004) estimate a model with financial accelerator but they fix the parameters related to the financial frictions and use the same calibration as in BGG. Christensen and Dib (2004) estimate the standard BGG model for the U.S. using maximum likelihood and they find evidence in favour of the financial accelerator model.

In this paper, we extend the BGG model and estimate it using Bayesian methods for U.S. and European data. Our purpose is to estimate the parameters of the model and to identify the main shocks affecting the economy. Moreover, we want to determine if a model with financial frictions delivers a better description of the data than a model without such frictions.

The benefits of using Bayesian methods is that we can include prior information about the parameters, especially information about structural parameters from microeconomic studies. Another advantage is related to the fact that some parameters have a specific economic interpretation and a bounded domain, which can be incorporated in the priors.

Our preliminary results support the existence of a financial accelerator mechanism in the U.S. but not in the euro area. In particular, when we compare a model with monitoring cost and a model without, the Bayes factor favors the model with frictions in the case of the U.S.
In the case of the Euro area and using the standard BGG model, our estimation results support the financial accelerator model. However, once we incorporate other frictions into the model, such as sticky wages, variable capital utilization and consumption habits, financial frictions introduced through a financial accelerator mechanism loose relevance and do not help to explain the behavior of the economy.

The rest of the paper is organized as follows. In Section 2 we describe an alternative model to the standard BGG model which incorporate other frictions to the economy, maintaining the existence of financial frictions. This model is going to be our benchmark model. Section 3 presents the estimation methodology while Section 4 presents the results for the benchmark and the standard BGG model using US data. In Section 5 we compare these results against alternative model specifications and also using European data. In Section 6 we do our sensitivity analysis, while Section 7 concludes.

2 The Model

Our specification of the model follows the work of BGG which develops financial market frictions through a financial accelerator mechanism in a general equilibrium model. The main idea behind the financial accelerator mechanism is that exists a negative relationship between the external financial premium (the difference between the cost of funds raised externally and the opportunity cost of funds) and the net worth of potential borrowers. Since net worth is procyclical (because of the procyclicality of profits and asset prices), the external finance premium will be countercyclical amplifying business cycles through an accelerator effect on investment, production and spending.

Moreover, and following the recently literature in DSGE models, we modify the model to improve its performance. We allow for habit persistence in consumption, variable capital utilization and Calvo prices and wages with full indexation to past inflation. Christiano, Eichenbaum, and Evans (2005) show that variable capital utilization and wage stickiness are fundamental frictions in order to explain inflation inertia and persistent, hump-shaped responses in output after policy shocks. The other frictions in the model help to account for the response of other variables such as consumption and investment. We include these frictions because in order to test the relevance of the financial accelerator mechanism, it is important to see if financial market frictions become important as a consequence of other underlying frictions in the economy.

There are six types of agents in our model: households, retailers, wholesale sector, entrepreneurs, intermediaries and government.
2.1 Households

There is a continuum of monopolistically competitive individuals whose total is normalized to unity. At each period, each of these households maximize their expected lifetime utility choosing a final consumption good, $C_{j,t}$, nominal bonds, $NB_{j,t+1}$, and real deposits held at intermediates, $D_{j,t+1}$, which pay a real gross free risk rate $R_t$. Moreover, as in Erceg, Henderson, and Levin (2000), each household supplies differentiated labor services to the wholesale sector, $H_{j,t}$. Household discount the future at a rate $\beta$.

The representative household’s one period utility is:

$$U_t = \Upsilon_t \left[ \frac{1}{1-\sigma} (C_{j,t} - hC_{t-1})^{1-\sigma} - \frac{\Xi_t}{2} H_{j,t}^2 \right]$$

and the budget constraint

$$\frac{NB_{j,t+1}}{P_t} + D_{j,t+1} + C_{j,t} = \frac{W_{j,t}}{P_t} H_{j,t} + R_{t-1} D_{j,t} + R_{t-1}^n \frac{NB_{j,t}}{P_t} - T_t + \Delta_t$$

where $W_{j,t}$ is the nominal wage of household $j$, $P_t$ is the nominal level of prices, $T_t$ are lump sum taxes and $\Delta_t$ are dividends received from ownership of wholesale firms. $\Upsilon_t$ and $\Xi_t$ are shocks to consumer preferences and preference for leisure respectively which follow AR(1) processes with mean equal to one.

Households also supply differentiated labor services to the wholesale sector, where the labor aggregator has the Dixit-Stiglitz form:

$$L_t = \left[ \int_0^1 H_{j,t}^{1/(\tau_t+1)} dj \right]^{(\tau_t+1)}$$

and $\tau_t$ is a shock to the elasticity of substitution between labor with mean $\tau$ (the steady state wage mark up). Firms minimize the cost of hiring a fixed amount of total labor given the different price of labor. The optimal demand for labor is

$$H_{j,t} = \left( \frac{W_t}{W_{j,t}} \right)^{(\tau_t+1)/\tau_t} L_t$$

Integrating this equation and imposing the Dixit-Stiglitz aggregator for labor, we can express the aggregate wage index as

$$W_t = \left[ \int_0^1 W_{j,t}^{-1/\tau_t} dj \right]^{-\tau_t}$$
We assume that households can choose their wages with probability 
\((1 - \vartheta)\) at each period. Whenever the household is not allowed to reset his 
wage contract, wages are set \(W_{j,t} = \pi_{t-1}W_{j,t-1}\). According to Christiano, 
Eichenbaum, and Evans (2005), this kind of wage dynamics plays a crucial role 
in the performance of the model. The FOC with respect to wages is

\[
E_t \sum_{k=0}^{\infty} (\beta \vartheta)^k r_{t+k} (C_{j,t+k} - h_{t-1+k})^{-\sigma} \left( \frac{W_{j,t+k} H_{j,t+k}}{P_{t+k}} \left[ \frac{1}{\tau_{t+k}} \right] \right)
\]

\[
= E_t \sum_{k=0}^{\infty} (\beta \vartheta)^k r_{t+k} \Xi_{t+k} H_{j,t+k}^{\eta+1} \left[ \frac{(\tau_{t+k} + 1)}{\tau_{t+k}} \right]
\]

2.2 Final Good Sector

Firms in the final good sector produce a consumption good, \(Y_t\), in a 
perfectly competitive market, combining intermediate goods, \(Y_t(z)\). The pro-
duction function that transforms intermediate goods into final output is the 
usual Dixit-Stiglitz aggregator given by

\[
Y_t = \left[ \int_0^1 Y_t(z)^{1/ \{(\lambda_t + 1)\}} dz \right]^{\{(\lambda_t + 1)\}}
\]

where \(\lambda_t \geq 0\) is a mark up shock with mean \(\lambda\). Firms take prices as given 
and chose \(Y_t(z)\) to minimize costs

\[
\min_{Y_t(z)} \int_0^1 P_t(z)Y_t(z)dz
\]

subject to the Dixit Stiglitz aggregator. The first order condition of this 
problem imply

\[
Y_t(z) = \left( \frac{P_t}{P_t(z)} \right)^{\{(\lambda_t + 1)/\lambda_t\}} Y_t
\]

Integrating this equation and imposing the constraint, we can express the 
aggregate price index as

\[
P_t = \left[ \int_0^1 P_t(z)^{-1/\lambda_t} dz \right]^{-\lambda_t}
\]
2.3 Wholesale Sector

A variety of intermediate inputs are produced by a continuum of monopolistically competitive firms indexed by \( z \in [0, 1] \). Each firm hires the services of capital, \( K_t(z) \), and labor, \( L_t(z) \), to face the demand curve for its product. They rent capital from an entrepreneur sector which owns the capital stock. Firms produce according to the following technology function

\[
Y_t(z) = A_t K_t(z)^\alpha L_t(z)^{1-\alpha}
\]

where \( A_t \) is a productivity shock which follows a first order autoregressive process with mean one. Because constant returns to scale, firms choose capital and labor to minimize total costs, taken factor prices as given. Profits are distributed to households. The minimization problem can be written as

\[
\min_{L_t(z), K_t(z)} \frac{W_t}{P_t} L_t(z) + Z_t K_t(z)
\]

subject to

\[
A_t K_t(z)^\alpha L_t(z)^{1-\alpha} \geq Y_t(z)
\]

where \( Z_t \) is the real rental price of capital.

Moreover, wholesale firms have market power and can chose prices to maximize expected profits with probability \( 1 - \theta \) in each period (Calvo, 1983). As in the case of wages, firms that not maximize index their prices according to last period’s inflation rate

\[
P_{t+1}(z) = \pi_t P_t(z)
\]

For those firms that can choose prices, \( \hat{P}_t \), the optimal FOC is

\[
E_t \sum_{k=0}^{\infty} (\beta \theta)^k Q_{t,t+k} Y_{t+k}(1/\lambda_{t+k}) \left[ \frac{\hat{P}_t}{P_{t-1} \pi_{t+k}} \right]^{-1/\lambda_{t+k}}
\]

\[
= E_t \sum_{k=0}^{\infty} (\beta \theta)^k Q_{t,t+k} Y_{t+k}(\lambda_{t+k} + 1)/\lambda_{t+k} S_t^{\pi} \left[ \frac{\hat{P}_t}{P_{t-1} \pi_{t+k}} \right]^{-(\lambda_{t+k} + 1)/\lambda_{t+k}}
\]

where \( \beta^k Q_{t,t+k} \) is the stochastic discount factor between periods \( t \) and \( t + k \) and \( S_t^{\pi} \) is the nominal marginal cost.
2.4 Entrepreneurs

Entrepreneurs own the physical stock of capital, \( \tilde{K}_t \) (where the subscript indicates when capital is actually used) and provide capital services. They finance their capital both with own net worth and debt. Capital services are related to the physical stock of capital by

\[ K_t = U_t \tilde{K}_t \]

where \( U_t \) is the rate of capital utilization\(^1\).

Entrepreneurs are risk neutral and have finite horizons, being \( \gamma \) the probability of survival to next period. This assumption rules out the possibility that entrepreneurs accumulate enough wealth to be fully self-financed: part of their capital must be financed through bank loans with a standard debt contract.

At the end of period \( t \), entrepreneurs decide how much to invest in \( t + 1 \). Then, at the beginning of period \( t + 1 \), after observing all the shocks, they choose how intensely to use their capital.

2.4.1 Optimal Investment Decision

We follow BGG assuming the investment decisions are determined one period in advance and that there exist increasing marginal adjustment costs in the production of capital. Entrepreneurs investment expenditures, \( I_{i,t} \), deliver \( \Phi \left( \frac{I_{i,t}}{\tilde{K}_{i,t}} \right) \tilde{K}_{i,t} \) new capitals goods. Entrepreneurs choose investment in the next period to maximize

\[
\max_{I_{i,t+1}} E_t \left[ Q_{t+1} \Phi \left( \frac{I_{i,t+1}}{\tilde{K}_{i,t+1}} \right) \tilde{K}_{i,t+1} - I_{i,t+1} \right]
\]

where \( Q_{t+1} \) is the relative price of capital, \( \Phi(0) = 0, \Phi'(.) > 0, \Phi''(.) < 0 \).

We also assume that in steady state the relative price of capital is one.

The law of motion of the aggregate capital stock is

\[
\tilde{K}_{t+1} = \Phi \left( \frac{I_t}{\tilde{K}_t} \right) \tilde{K}_t + (1 - \delta(U_t)) \tilde{K}_t
\]

where \( \delta(U_{t+1}) \) is a convex depreciation function described below.

\(^1\)\( U_t \) can take any value \( \geq 0 \), where values greater than one mean there exist over utilization of capital.
2.4.2 Optimal Capital Utilization Decision

After observing the shocks at the beginning of period \( t + 1 \), entrepreneurs decide how intensively to use their capital. Higher capital utilization is costly because of higher depreciation rates.\(^2\) This is an important assumption because it allows for variable capital utilization which is a relevant feature in the data. They choose capital utilization, \( U_{t+1} \) to solve

\[
\max_{U_{t+1}} \left[ U_{t+1}Z_{t+1} + \left( 1 - \delta(U_{t+1}) \right)Q_{t+1} \right]
\]

with \( \delta(U_{t+1}) \in (0, 1) \), \( \delta'(U_{t+1}) > 0 \), and \( \delta''(U_{t+1}) > 0 \) around the steady state. We choose the function \( \delta(U) \) such that in steady state \( U_{ss} = 1 \).\(^3\)

2.5 Optimal Contract

The return on capital depends both in aggregate and idiosyncratic shocks as in BGG. The ex post return on capital for each entrepreneur is \( \omega^i_{t+1}R^k_{t+1} \), where \( \omega^i \) is an \( i.i.d. \) lognormal random variable with pdf \( F(\omega) \) and mean one. \( R^k_{t+1} \) is the average return of capital in the economy

\[
R^k_{t+1} = \frac{U_{t+1}Z_{t+1} + \left( 1 - \delta(U_{t+1}) \right)Q_{t+1}}{Q_t}
\]

Entrepreneurs finance their capital stock at the end of period \( t \) with their own net worth at the end of the period, \( N_{t+1,i} \), and banks loans, \( B_{t+1,i} \).

\[
Q_tK_{t+1,i} = N_{t+1,i} + B_{t+1,i}
\]

The entrepreneur borrows from a financial intermediary that obtains its funds from households, with an opportunity cost equal to the riskless gross rate of return, \( R_t \). In equilibrium, the intermediary holds a perfectly safe portfolio and the entrepreneurs absorb any aggregate risk.

Following a "costly state verification" problem of the type analyzed by Townsend (1979), in which lenders must pay a fixed "auditing cost" in order to observe an individual borrower’s realized return, BGG assume that the monitoring cost is a proportion \( \mu \) of the realized gross payoff to the firms’ capital, i.e., the monitoring cost equals \( \mu\omega^i_{t+1}R^k_{t+1}Q_tK_{t+1,i} \).

\(^2\)This approach has been used among others, by Baxter and Farr (2001).

\(^3\)One example of this kind of function can be

\[
\delta(U_t) = 1 - \frac{1 + p}{p + \exp^{\varepsilon U_t}} \quad \text{with} \quad p, \varepsilon > 0
\]

In this case, \( \delta(0) = 0 \), \( \delta(\infty) = 1 \), \( \delta(1) = 1 - \frac{1 + p}{p + \exp^\varepsilon} = \delta \). However we focus in a more general case of functional forms and we also estimate \( \delta_{ss}' / \delta_{ss} \).
The optimal contract will be characterized by a schedule of state contingent threshold values of the idiosyncratic shock $\omega_{t+1}^i$, such that for values of the idiosyncratic shock greater than the threshold, the entrepreneur is able to repay to the lender, and for values below the entrepreneur declares default and the lender gets $(1 - \omega_{t+1}^i) R_{t+1}^k Q_t K_{t+1}$. Only one-period contracts between borrowers and entrepreneurs are feasible.

Under these assumptions, the optimal contract is chosen to maximize expected entrepreneurial utility conditional on the return of the lender, for each possible realization of $R_{t+1}^k$, equals in expectations the riskless rate, $R_t$. In the appendix we show that the following two first order conditions must hold in the optimal contract between entrepreneurs and banks.

\[ E_t \left\{ (1 - \Gamma(\omega_{t+1}^i)) \frac{R_{t+1}^k}{R_t} + \lambda(\omega_{t+1}^i) \left[ (\Gamma(\omega_{t+1}^i) - \mu G(\omega_{t+1}^i)) \frac{R_{t+1}^k}{R_t} - 1 \right] \right\} = 0 \]

\[ \left[ \Gamma(\omega_{t+1}^i) - \mu G(\omega_{t+1}^i) \right] R_{t+1}^k Q_t K_{t+1} = R_t \left[ Q_t K_{t+1} - N_{t+1} \right] \]

where $\mu G(\omega_{t+1}^i) = \mu \int_0^{\omega_{t+1}^i} \omega dF(\omega)$ is the expected monitoring costs, $\Gamma(\omega_{t+1}^i) = (1 - F(\omega_{t+1}^i)) \omega_{t+1}^i + G(\omega_{t+1}^i)$ is the expected gross share of profits going to the lender, and $\lambda(\omega_{t+1}^i) = \frac{\Gamma(\omega_{t+1}^i) - \mu G(\omega_{t+1}^i)}{\Gamma(\omega_{t+1}^i)}$.

From this first FOC, we see that when $\mu = 0$, $\lambda(\omega_{t+1}^i) = 1$ and $E_t R_{t+1}^k = R_t$, meaning that the ex ante return on capital equals the risk free rate when there are no monitoring costs. The second FOC has to do with the fact that the financial intermediary receives an expected return equal to the opportunity cost of its funds. In this case, we can express the lender’s expected return simply as a function of the average cutoff value of the firm’s idiosyncratic shock, $\omega_{t+1}^i$.

Because the entrepreneur is risk neutral, he cares only about the mean return on his wealth. He guarantees the lender a return that is free of any systematic risk: conditional on $R_{t+1}^k$, he offers a state-contingent contract that guarantees the lender a return equal in expected value to the riskless rate.

From these two equations aggregation is straightforward and it can be shown that capital expenditures by each entrepreneur are proportional to the net worth. Aggregate entrepreneurial net worth (in consumption units) at the end of period $t$, $N_{t+1}$ is given by

\[ N_{t+1} = \gamma \left\{ R_t^k Q_t K_t - \left[ R_t \left( Q_{t-1} K_t - N_t \right) + \mu \int_0^{\omega_{t+1}^i} \omega dF(\omega) R_t^k Q_{t-1} K_t \right] \right\} + W^c \]

For more details see BGG (1999).
where $\gamma$ is the fraction that survives to next period, and $W^r$ are net transfers to entrepreneurs. At each period, a fraction $(1 - \gamma)$ enters the market receiving some transfers and the wealth of the fraction that did not survive is given to the government.

2.6 Competitive Equilibrium

The competitive equilibrium is obtained when all the optimality conditions above are satisfied and markets clear. This involve an aggregate resource constraint, the government budget constraint and a monetary policy. The aggregate resource constraint is

$$Y_t = C_t + I_t + Gov_t + \mu \int_0^\infty wdF(w)R^k_t Q_{t-1}\tilde{K}_t$$

where the government expenditure, $Gov_t$, follows a first order autoregressive process. Final goods are allocated in consumption, investment, government expenditure and monitoring cost. In equilibrium, the credit market clears, $B_t = D_t$, and the government budget constraint is satisfied.

The monetary authority conducts monetary policy by controlling the gross nominal interest rate, $R^n_t$. For convenience, we assume a cashless economy, but the monetary authority can set the interest rate directly in the interbank market. The Central Bank policy rule is a Taylor type rule of the form

$$R^n_t = f(R^n_{t-1}; E_t (\pi_{t+1}); Y_t; g^*_{t})$$

where $g^*_{t}$ is a monetary policy shock and $\pi_{t+1}$ is inflation in $t + 1$.

2.7 The log-linearized model

In order to solve the model we loglinearize the equilibrium conditions around the steady state values. We then can write the model in terms of three blocks of linear equations where small letters represent log deviations from the steady state and capital letters without subscript represent the steady state values of the variables.

2.7.1 Equilibrium conditions

$$y_t = \frac{C}{Y} a_t + \delta \bar{K} t_t + \frac{Gov}{Y} g_t + \mu \frac{G(\bar{x}) R^k \bar{K}}{Y} (r^k_t + g_{t-1} + \bar{k}_t) + \frac{\mu R^k G'(\bar{x}) \bar{K} \bar{m}_t}{Y} \bar{w}_t$$ (2.1)

$$y_t = a_t + \alpha k_t + (1 - \alpha)l_t$$ (2.2)
Equations 2.1 and 2.2 are the loglinearized version of the aggregate demand and supply. $\delta$ is the steady state capital depreciation.

$$c_t = \frac{(1-h)}{\sigma(1+h)} (v_t - E_t v_{t+1}) + \frac{h}{(1+h)} c_{t-1} - \frac{(1-h)}{\sigma(1+h)} r_t + \frac{E_t c_{t+1}}{(1+h)} \quad (2.3)$$

$$E_t \left\{ \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 \hat{\pi}_{t-1} + \eta_4 \hat{\pi}_t + \eta_5 \hat{\pi}_{t+1} + \eta_6 \hat{L}_t + \eta_7 \hat{\psi}_{z,t} + \eta_8 \hat{\xi}_t + \eta_9 \hat{\lambda}_t \right\} = 0 \quad (2.4)$$

where $b_w = [(\tau + 1) \sigma_L + \tau] / [(1 - \xi_w) (1 - \beta \xi_w)]$

$$\eta = \begin{pmatrix} b_w \xi_w \\ -b_w (1 + \beta \xi_w^2) + (\tau + 1) \\ \beta \xi_w b_w \\ b_w \xi_w \\ -\xi_w b_w (1 + \beta) \\ b_w \beta \xi_w \\ \tau \\ \tau \sigma (1 - h)^{-1} (c_t - h c_{t-1}) \\ \tau \\ \tau \tau \tau \end{pmatrix} = \begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \end{pmatrix}$$

$$r^n_t = r_t + E_t \pi_{t+1} \quad (2.5)$$

Equation 2.3 is the consumption Euler equation. Equation 2.4 is the law of motion of real wages\(^5\), and Equation 2.5 is the arbitrage condition for nominal bonds. The three equations are derived from the households FOC. $\tau$ is the steady state wage mark up, $v_t$ is the preferences shock, and $\xi_t$ is the labor supply shock.

$$y_t - l_t + s_t = w_t^r \quad (2.6)$$

$$s_t + y_t - k_t = z_t \quad (2.7)$$

$$\hat{\pi}_t = \frac{\pi_{t-1}}{(1 + \beta)} + \frac{\beta}{(1 + \beta)} E_t \hat{\pi}_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{(1 + \beta) \theta} s_t + \frac{(1 - \theta)(1 - \beta \theta)}{(1 + \beta) \theta} \frac{\lambda}{(\lambda + 1)} \hat{\lambda}_t \quad (2.8)$$

Equations 2.6 and 2.7 are the demand for labor and capital of the wholesale sector, where factor prices are equal to marginal productivity plus real

\(^5\)This is the same notation as in Christiano, Eichenbaum, and Evans (2005) but where we have introduced a wage mark up shock and the mark up is in net terms.
marginal cost, $s_t$. Equation 2.8 is the Phillips curve derived from the whole-
sale sector optimization problem for prices, where $(1 - \theta)$ is the probability of
adjusting prices and $\lambda$ is the steady state price mark up.

Next we present the equilibrium conditions of the entrepreneurs.

$$E_t q_{t+1} + \varphi \left[ i_{t+1} - \bar{k}_{t+1} \right] = 0$$

(2.9)

$$E_t r^k_{t+1} - r_t = E_t \bar{\omega}_{t+1} R^k \left( 1 - \Gamma(\varpi) \right) \left[ \Gamma''(\varpi) \frac{\Gamma'(\varpi)}{\lambda(\varpi)} - \frac{\Gamma''(\varpi)}{\lambda(\varpi)} + \mu G''(\varpi) \right]$$

(2.10)

$$[(1 - F(\varpi)) - \mu G'(\varpi)] \frac{\bar{K}}{N} R^k \bar{\omega}_{t+1} + \left[ \frac{\bar{K} - N}{N} \right] (r^k_{t+1} - r_t) = \bar{k}_{t+1} + q_t - n_{t+1}$$

(2.11)

$$k_t = u_t + \bar{k}_t$$

(2.12)

$$z_{t+1} = \frac{\delta''(UK)}{\delta'(UK)} u_{t+1} + q_{t+1}$$

(2.13)

$$r^k_{t+1} = \frac{Z}{R^k} z_{t+1} + \left( 1 - \delta \right) \frac{R^k}{R^k} q_{t+1} - q_t$$

(2.14)

Equation 2.9 links asset prices and investment, where $\varphi = \Phi'' \left( \frac{L}{K} \right) \left( \frac{L}{K} \right)$ is the elasticity of the price of capital with respect to the investment-capital ratio. Equations 2.10 and 2.11 are the FOC of the optimal lending contract which are derived in the Appendix. In steady state, the probability of default, $F(\varpi) = \Theta(zz)$, where $\Theta$ is the cumulative distribution of a standard normal. In what follows we will estimate $zz$ and then pin down the other parameters. Equation 2.12 relates capital services to the stock of capital, while Equation

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6 In the model without financial frictions these equations and the law of motion of net worth become

$$E_t r^k_{t+1} = r_t$$

$$[(1 - F(\varpi))] \frac{\bar{K}}{N} \bar{\omega}_{t+1} + \left[ \frac{\bar{K} - N}{N} \right] (r^k_{t+1} - r_t) = \bar{k}_{t+1} + q_t - n_{t+1}$$

$$n_{t+1} = \gamma R \left \{ \left( \frac{\bar{K}}{N} \right) r^k_t - \left( \frac{\bar{K} - N}{N} \right) r_{t-1} + n_t \right \}$$

so ex-ante the risk premium is cero.
2.13 is the optimality condition for capital utilization. Equation 2.14 specifies the ex post return on capital.

\[ n_{t+1} = \gamma \left\{ \left( \frac{\tilde{K}-\mu G\tilde{K}}{N} \right) R_k + \left( \frac{R^k - \tilde{R} - \mu GR^k}{N} \right) q_{t-1} + \left( \frac{R^k - R - \mu GR^k}{N} \right) \tilde{R} \right\} \]

Equations 2.15 and 2.16 are the law of motion of net worth and capital respectively.

\[ \bar{k}_{t+1} = \delta i_t + (1 - \delta) \bar{k}_t - \delta' u_t \] (2.16)

2.7.2 Monetary policy rule

The loglinearized monetary policy rule is

\[ r^m_t = \rho^r r^m_{t-1} + (1 - \rho^r) (\gamma^r E\pi_{t+1}) + (1 - \rho^r) (\gamma^r y_t) / 4 + \bar{\varphi}_t \] (2.17)

2.7.3 Shock Process

There exists seven shocks in our model:

\[ \bar{\varphi}_t = \varepsilon^r_t \] (2.18)

\[ \lambda_t = \varepsilon^\lambda_t \] (2.19)

\[ \tau_t = \varepsilon^\tau_t \] (2.20)

\[ \xi_t = \rho^\xi \xi_{t-1} + \varepsilon^\xi_t \] (2.21)

\[ u_t = \rho^u u_{t-1} + \varepsilon^u_t \] (2.22)

\[ g_t = \rho^g g_{t-1} + \varepsilon^g_t \] (2.23)

\[ a_t = \rho^a a_{t-1} + \varepsilon^a_t \] (2.24)

where \( \varepsilon^i_t \) are white noise shocks affecting the economy.
Equations 2.18-2.20 are the monetary policy, price mark up and wage mark up shocks. We specify these shocks as white noise shocks. The rest of the shocks in the model, labor supply, preferences, government spending and technology follow first order autoregressive process. We choose this specification for the shocks in order to avoid identification problems.

2.7.4 Solution Method

To solve the model we use the method described in Sims (2000) and his matlab code gensys.m. Our loglinearized model can be written as

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi Z_t + \Pi \eta_t$$

where $Z_t$ is a vector of exogenous random disturbances, and $\eta_t$ is a vector of expectational errors with mean zero.

3 Estimation Methodology

There is a total of 30 free parameters in our model. We first fix three steady state parameters, while the rest of the parameters are determined by estimating the model. In particular we set the steady state rate of depreciation of capital ($\delta$) equal to 0.025 which corresponds to a rate of depreciation equal to 10% annual, the discount factor $\beta$ equal to 0.99, which corresponds to an annual real rate in steady state of 4%, and the steady state share of government spending equal to 0.195.

The other 27 parameters are estimated using Bayesian procedures. The advantage of Bayesian estimation relative to maximum likelihood (the only realistic alternative), is that the solution of the model imposes many restrictions and boundary values for the parameters which is difficult to implement using maximum likelihood. Besides, using Bayesian methods it is possible to formally incorporate our beliefs about the parameters.

We start by solving the model for an initial set of parameters. Then, we use the Kalman Filter to calculate the likelihood function of the data (for given parameters). The observed variables used in the Kalman Filter correspond to seven variables of the model: real output, real consumption, real investment, hours worked, nominal interest rate, inflation and real wages.\(^7\) We used U.S.

\(^7\) We do not include financial variables because in order to compare the model with and without financial frictions, the first one will present a natural advantage in the case you include this variable.
quarterly detrended data for the period 1980:1-2004:1.8

Combining prior distributions with the likelihood of the data, gives as result the posterior density function of the parameters. Since the posterior distribution is unknown, we use Monte Carlo Markov Chain (MCMC) simulation methods to conduct inference about the parameters. A description of some of these aspects is discussed in what follows in the remaining of the section.

3.1 Prior Distribution

All the prior distributions were selected from the normal, beta, gamma and uniform distribution, depending on the different supports and characteristics of the parameters. Table 1 shows the prior distributions.

Many of the priors are standard and follow the literature (Smets and Wouters (2004), Christiano, Motto, and Rostagno (2004), Altig, Christiano, Eichenbaum, and Linde (2003), Adolfo, Lasén, Lindé, and Villani (2004)). The relative risk aversion coefficient, $\sigma$, has a normal distribution with mode 1; the habit persistence parameter, $h$, has a beta distribution with mode 0.70; $\alpha$, the parameter of the Cobb-Douglas function has also a beta distribution with mode 0.35.

The parameters determining prices and wages follow a beta distribution. The modes of the Calvo parameters $\theta$ and $\phi$, the probability of not adjusting prices and wages, were set equal to 0.70, while the modes of the price and wage mark ups, $\lambda$ and $\tau$, were set equal to 0.20.

Some of the parameters are particular to this model and the way we modelled some frictions. This is the case of the elasticity of the price of capital with respect to the investment-capital ratio, $\varphi$. There is not consensus about this parameter: BGG set this parameter equal to -0.25 while King and Wolman (1996) use a value of -2 based on estimations of Chirinko (1993). Since we do not have enough information about this parameter we use a uniform prior distribution between -1 and 0. The prior for $\delta''/\delta'$ is a gamma distribution with mode equal to 1, following the calibrations of Baxter and Farr (2001).

---

8U.S. data was taken from the Bureau of Economic Analysis of the U.S. Department of Commerce (BEA), the IMF database and the Bureau of Labor Statistics (BLS). Real output is measured by real GDP converted into per capita terms dividing by the population over 16 year old (P16). Real consumption is Real personal consumption expenditures divided by P16. Real investment is Real gross private domestic investment also in per capita terms. Hours worked is measured by the product of average weekly hours in the private sector times the population over 20. The nominal interest rate is the Federal Funds Rate, and inflation is calculated as the difference of the GDP deflator. Real wages are measured by the average hourly earnings of production workers in real terms. All series were detrended with a linear trend and in the case of the interest rate we used the same trend as inflation.
Other non standard parameter in our model are the ones related to the financial frictions. \(zz\) is the parameter determining the probability of having a idiosyncratic shock below the threshold value in steady state (probability of default). Our prior for this parameter is a normal pdf and we follow BGG and set the mode equal to -2.43, which correspond to a probability of default in steady state equal to 3% per year\(^9\). We set the prior for the ratio of capital stock to net worth in steady state, \(K/N\), gamma distributed with mode equal to 2, which correspond to a leverage ratio of 0.50 per cent. The fraction of entrepreneurs who survive to next period, \(\gamma\), has a beta distribution with mode .975 which implies that in average entrepreneurs live 10 years. Last, we set the prior for the risk premium, \(R - \bar{R}\), gamma distributed with a mode equal to 0.005, which correspond to a 2% annual risk premium as in BGG.

The priors for the parameters of the monetary policy rule are based in the estimations of Clarida, Gali, and Gertler (2000) for the period post-82. The long run coefficient on inflation and output, \(\gamma_x\) and \(\gamma_y\), are normal distributed with mode 1.5 and 0.5 respectively. The interest rate smoothing parameter, \(\rho_r\), follows a beta distribution with mode 0.85.

Regarding the shocks affecting the economy, all the autoregressive coefficients have a beta distribution with mode 0.85, while the standard deviations for the shocks follow a gamma distribution with mode 0.10.

### 3.2 Posterior Distribution

We first estimate the mode of the posterior distribution maximizing the posterior density \(p(\kappa | Y)\) with respect to the parameters, given the data \(Y\).

The objective is to maximize

\[
\log p(\kappa | Y) = \log p(Y | \kappa) + \log p(\kappa) - \log p(Y)
\]

where \(p(Y | \kappa)\) is the sample density or likelihood function, \(p(\kappa)\) is the prior density of the parameters and \(p(Y)\) is the marginal likelihood.

However, since \(p(Y)\) does not depend on \(\kappa\), the posterior mode can be obtained maximizing (Hamilton (1994))\(^10\)

\[
\log p(\kappa, Y) = \log p(Y | \kappa) + \log p(\kappa)
\]

We use MCMC to obtain the posterior distributions. This is necessary when it is not possible to sample the parameters directly from the posterior.

---

\(^9\)Since the probability of default in one quarter is \(F(zz) = \Theta(zz)\), where \(\Theta\) is the cumulative distribution of a standard normal.

\(^10\)We maximize the RHS using Sims’ code csminwel.
distribution. The idea behind MCMC is to draw values of the parameters from an approximate distribution and then correct these draws to better approximate the posterior distribution. Starting from an initial arbitrary value of the parameters, the samples are drawn sequentially, and each draw will depend on the last value drawn. The approximate distribution is improved at each step of the simulation until it converges to the posterior.

The posterior output can be used to compute any posterior function of the parameters: impulse responses, moments, etc.

In order to perform the simulations, we used the Metropolis-Hastings algorithm. This algorithm uses an acceptance/rejection rule to converge to the posterior distribution. The algorithm samples a proposal parameter from a jumping distribution $q(\kappa^{l+1}/\kappa^l)$ and accepts the draw with probability

$$
\alpha = \min\left\{\frac{p(\kappa^{l+1}/Y)q(\kappa^{l+1}/\kappa^l)}{p(\kappa^l/Y)q(\kappa^l/\kappa^{l+1})}, 1\right\}.
$$

If the new value of the parameters is rejected, then $\kappa^{l+1} = \kappa^l$. We use as the jumping function a random walk around the parameter space. In particular, we set $q(\kappa^{l+1}/\kappa^l) = N(\kappa^l, c^2 \Sigma)$ where $\Sigma$ is the inverse of the Hessian computed at the joint posterior mode, and $c$ is a scale factor set to obtain efficient algorithms.11. Our purpose when choosing the scale factor was to tune the acceptance rate around 20%.

To check convergence we run different simulations starting from dispersed points, as suggested by Gelman, Carlin, Stern, and Rubin (2004). We monitor the convergence by comparing variation between and within simulated sequences until ‘within’ variation approximates ‘between’ variation. Only when the distribution of each sequence is close to that of all the sequences mixed together, they can all be approximating the posterior distribution. We then can considered all the draws as coming from the same posterior distribution.

To be more specific, consider the between and within sequence variance of each parameter given respectively by

$$
B = \frac{n}{m-1} \sum_{j=1}^{m} (\hat{\kappa}_{j} - \hat{\kappa}_{..})^2 , \text{ where } \hat{\kappa}_{..} = \frac{1}{n} \sum_{i=1}^{n} \kappa_{ij} \text{ and } \hat{\kappa}_{j} = \frac{1}{m} \sum_{j=1}^{m} \kappa_{ij}
$$

$$
W = \frac{1}{m} \sum_{j=1}^{m} s_j^2 , \text{ where } s_j^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\kappa_{ij} - \hat{\kappa}_{..})^2
$$

$m$ is the number of sequences and $n$ the number of draws in each sequence. The marginal posterior variance of each parameter will be a weighted average of $W$ and $B$.

---

11Gelman, Carlin, Stern, and Rubin (2004) consider that among this kind of jumping rules, the most efficient has scale $c \approx 2.4\sqrt{d}$, where $d$ is the number of parameters to estimate,
\[ \text{var}(\kappa/Y) = \frac{n - 1}{m} W + \frac{1}{n} B \]

One way to check convergence is to calculate the potential scale reduction

\[ \hat{R} = \sqrt{\frac{\text{var}(\kappa/Y)}{W}} \] (3.1)

which declines to 1 as \( n \to \infty \). If the potential scale reduction is high, we should proceed with further simulations to improve our inference. We compute this ratio for all the parameters.

Moreover, to avoid the effect of the starting points and considering the assumption that for large simulations the distribution converges to the posterior, we ignored the first half of each sequence.

4 Results

For each model we run two different simulations starting from the mode of the posterior plus-minus one standard deviation, with a total of 100,000 draws in each simulation. In all cases, the acceptance rate was around 20 per cent. Table 1 reports the mean, median and the 5th and 95th percentile of the posterior distribution of the benchmark model for US data. When we plot the path of the different parameters along the chain, as well as the value of the posterior likelihood function, we see there is convergence to a stationary distribution. Moreover, when we calculate the potential scale reduction as in equation 3.1 the results confirm this idea. However, it can be seen from Table 1 that for some of the parameters \( \hat{R} \) is considerably larger than one. In particular, there exists some convergence problems for \( \sigma \), the risk aversion coefficient.

Analyzing the different parameters we observe that some of them substantially differ from the priors. In particular the mean value of \( \alpha \) is 0.30, while the steady state mark up is 0.44. These values imply a capital output ratio of 4.9 which is too low, and consequently a too low investment output ratio.

The mean of the posterior likelihood for \( \theta \) implies that prices adjust in average once every 13 months. This result implies more flexible prices than the estimations of Smets and Wouters (2004). The same occurs with wages, with an average duration of contracts of 3 quarters.

When we look at the risk premium in steady state \( (Rk - R) \), the mean of the posterior distribution implies an annual premium of 3.2\% which is higher than the value used by BGG and Christiano, Motto, and Rostagno (2004). Moreover, the mean of \( zz \) implies a probability of default of 3\% yearly while the mean for the fraction of entrepreneur who survive, \( \gamma \), is 0.99, implying an
average duration of entrepreneurs of 36 years. From these values we pin down that monitoring costs, $\mu$, are 18% and the fraction of GDP used in bankruptcy cost is 0.3%.

Concerning the coefficients in the instrument rule, all the coefficients differ from the estimations of Clarida, Gali, and Gertler (2000). The coefficient on future inflation, $\gamma_\pi$, is higher than their estimations while the coefficient on output, $\gamma_y$, and the interest rate smoothing parameter, $\rho_r$, are lower.

### 4.1 Volatility

Table 2 shows the standard deviation of the data and the simulated model with and without financial frictions evaluated at the mean of the posterior. The model with financial frictions represents slightly better the volatility in the data. However both models are a good approximation of the volatility in the data.

When we look at the historical variance decomposition of the model, Table 3 shows that for one quarter horizons the relevant shocks affecting output are government spending, consumer preferences and labor supply. After three years however, labor supply and technology shocks explain more than 90% of output volatility. In the case of inflation, mark up shocks are the main cause of one quarter volatility, but in the long run also preferences, technology and wage mark ups play an important role explaining inflation volatility. For real wages, wage mark ups are important in the short run but in the long run productivity shocks explain 50% of the variation. Variation in nominal interest rates are explained mainly by monetary shocks in the short run, but after three years preferences shocks are the most important.

### 5 Robustness and Model Comparison

To check the model robustness and the relevance of the financial accelerator mechanism for different models, we cancel one by one the frictions of the model and reestimate the parameters. We also estimate the standard BGG model which does not include consumption habits, sticky wages or variable capital utilization.

In order to compare the performance of the different models, we need to calculate the marginal data density for the models. Let’s call $M_{fa}$ the model with financial frictions and $M_{alt}$ an alternative specification of the model. The marginal data density for each model will be

$$p(Y \mid M_i) = \int p(Y \mid \kappa_i, M_i)p(\kappa_i \mid M_i)d\kappa_i$$
where $\kappa_i$ are the parameters of model $i$ and $p(\kappa_i \mid M_i)$ is the prior density of the parameters for model $i$. The posterior probability for each model will be

$$p(M_i \mid Y) = \frac{p(Y \mid M_i)p(M_i)}{\sum_{i=1}^{f_a,alt} p(Y \mid M_i)p(M_i)}$$

Bayesian model selection is done comparing the models through the posterior odds ratio

$$PO_{i,j} = \frac{p(M_i \mid Y)}{p(M_j \mid Y)} = \frac{p(Y \mid M_i)p(M_i)}{p(Y \mid M_j)p(M_j)}$$

where the prior odds $\frac{p(M_i)}{p(M_j)}$ are updated by the Bayes factor $\frac{p(Y \mid M_i)}{p(Y \mid M_j)}$.

One problem that arise is to obtain the marginal likelihood. We follow Geweke (1999) and use the modified harmonic mean method to approximate the marginal likelihood. In particular, Gelfand and Dey (1994) show that for any pdf $f(\kappa)$ whose support $\Theta_m$ is contained in the parameter space,

$$E\left[\frac{f(\kappa)}{p(Y \mid \kappa_i, M_i)p(\kappa_i \mid M_i)} \mid Y, M_i\right] = \int_{\Theta_m} \frac{f(\kappa_i)}{p(Y \mid \kappa_i, M_i)p(\kappa_i \mid M_i)} p(\kappa_i \mid Y, M_i) d\kappa_i = p(Y \mid M_i)^{-1}$$

We then can use the sample posterior mean of $\left[\frac{f(\kappa_i)}{p(Y \mid \kappa_i, M_i)p(\kappa_i \mid M_i)}\right]$ as an approximation for the inverse of the marginal density.

5.1 Empirical Results

In Table 4 and 5 we report the mean of the parameters for alternative models and also using European data. When comparing the marginal data densities, we observe that a model with a financial accelerator mechanism delivers a better representation of the data in the U.S. but not in the euro area. In Table 4 we observe that the Bayes Factor favors a model with monitoring costs in the U.S. However, when we use European data (see Table 5), the data favors other frictions in our model but not the financial accelerator mechanism.

5.1.1 Benchmark Model

In the benchmark model, which is discussed in Section 2, the parameters of the model with and without financial frictions are robust in both specifications. However, the elasticity of capital price with respect to the investment-capital ratio is lower in the model without frictions. Also we can see some
differences concerning the steady state capital to net worth ratio and the entre-
preneurs rate of survival.

5.1.2 Flexible Prices

In a model with flexible prices real marginal cost are equal to the inverse of price mark ups. Loglinearizing, we obtain

\[ s_t = -\frac{\lambda}{\lambda + 1} \lambda t \]

which replaces equation 2.8 in our model.

[Remains to be done]

5.1.3 Flexible Wages

When wages are flexible equation 2.4 becomes the standard consumer first order condition respect to labor

\[ w_t^r - \sigma (1 - h)^{-1} (c_t - h c_{t-1}) = \xi_t + \eta l_t + \frac{\tau}{\tau + 1} \tau_t \]

where we have added the existence of wage mark up shocks.

[Remains to be done]

5.1.4 No Variable Capital Utilization

In a model without variable capital utilization \( K_t = \tilde{K}_t \forall t \). In our model this is the case when the depreciation rate is constant, and equation 2.13 is replaced by \( u_t = 0 \).

Table 4 shows that variable capital utilization is an important feature in the data. Departing from the benchmark model, variable capital utilization is more important than financial frictions, and not including this feature lowers the marginal density in greater extent than when we remove the financial frictions.

5.1.5 No Consumption Habits

When we do not consider the existence of external consumption habits, \( h = 0 \) and equation 2.3 becomes the standard Euler equation plus a preference shock

\[ c_t = \frac{1}{\sigma} (v_t - E_t v_{t+1}) - \frac{1}{\sigma} \tau_t + E_t c_{t+1} \]

[Remains to be done]
5.1.6 The Standard BGG Model

The standard BGG model does not include consumption habits or variable capital utilization. Moreover, wages are flexible and prices do not adjust according to past inflation. Equation 2.8 becomes

\[ \tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{(1 - \theta) (1 - \beta \theta)}{(1 + \beta \theta)} s_t + \frac{(1 - \theta) (1 - \beta \theta)}{(1 + \beta \theta)} \frac{\lambda}{(\lambda + 1)} \lambda_t \]

where inflation does not depend on past inflation as in our benchmark model.

It is important to notice that when we restrict our estimation to the standard BGG model, the marginal data densities favors the model with frictions. In particular, for the US we obtain a Bayes factor of $1e + 15$ which clearly prefers the model with the financial accelerator. This result is in line with the one in Christensen and Dib (2004), where they find that in the standard BGG model the estimation results provide evidence in favour of the financial accelerator model. However, according to our results, the data prefers a model which include other frictions instead of monitoring costs.

5.1.7 European Data

For the euro area we use six variables for the period 1980:1-2002:4: real output, real consumption, real investment, nominal interest rate, inflation and real wages\(^{12}\). In this case, the data favors a model without financial frictions.

Comparing the estimations of the model with and without monitoring costs, both average lifetime of entrepreneurs and capital net worth ratios are lower in the model without financial frictions.

When we compare the results between the U.S. and the euro area we notice that steady state wage mark ups, \(\tau\), and wage mark ups shocks are larger in the U.S., while the coefficient in the production function, \(\alpha\), and the elasticity of the price of capital with respect to the investment capital ratio, \(\varphi\), are larger in Europe. Also prices and wages seems to be more sticky in the euro area. Moreover, the response of monetary policy to output is higher in the euro area.

\(^{12}\)European data was taken from the AWM database of the ECB. Real output is measured by real GDP converted into per capita terms dividing by the labor force. Real consumption is Real consumption divided by the labor force. Real investment is Real gross investment also in per capita terms. The nominal interest rate is the quarterly short-term interest rate, and inflation is calculated as the difference of the GDP deflator. Real wages are measured by the wage rate deflated by the GDP deflator. All series were detrended with a linear trend and in the case of the interest rate we used the same trend as inflation.
6 Sensitivity Analysis

[Remains to be done]

7 Conclusions

[Remains to be written]

8 Bibliography


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A Optimal Contract

The return on capital depends both in aggregate and idiosyncratic shocks as in BGG. The ex post return on capital in state $s$ of the economy is $\omega'_{t+1} R_{s,t+1}^k$, where $\omega'$ is an i.i.d. lognormal random variable with pdf $F(\omega)$ and mean one.

Entrepreneurs finance their capital stock at the end of period $t$ with their own net worth at the end of the period and banks loans.

$$Q_t K_{t+1,i} = N_{t+1,i} + B_{t+1,i}$$

where $Q_t$ is the relative price of capital at the end of the period. As in BGG, the entrepreneur borrows from a financial intermediary that obtains its funds from households, with an opportunity cost equal to the riskless gross rate of return, $R_t$. Following a "costly state verification" problem of the type analyzed by Townsend (1979), lenders must pay a fixed "auditing cost" in order to observe an individual borrower's realized return. BGG assume that the monitoring cost is a proportion $\mu$ of the realized gross payoff to the firms' capital, i.e., the monitoring cost equals $\mu \omega'_{t+1} R_{s,t+1}^k Q_t K_{t+1,i}$.

The optimal contract will be characterized by a schedule of state contingent threshold values of the idiosyncratic shock $\omega'_{s,t+1}$, such that for values of the idiosyncratic shock greater than the threshold, the entrepreneur is able to repay to the lender, and for values below, the entrepreneur declares default and the lender gets $(1 - \mu) \omega'_{t+1} R_{s,t+1}^k Q_t K_{t+1,i}$. Because the entrepreneur is risk neutral, he is willing to guarantee the lender a return that is free of any aggregate risk.

Under these assumptions, the optimal contract is chosen to maximize expected entrepreneurial utility conditional on that the return of the lender, for each possible realization of $R_{t+1}$, equal in expected value the riskless rate, $R_t$. The problem to solve is

$$\max_{\{\omega'_{s,t+1}\}_{s,s_{t+1}}} \sum_s \Pi_s (1 - \Gamma(\omega'_{s,t+1})) R_{s,t+1}^k Q_t K_{t+1,i}$$

subject to

$$[\Gamma(\omega'_{s,t+1}) - \mu G(\omega'_{s,t+1})] R_{s,t+1}^k Q_t K_{t+1,i} = R_t \left[Q_t K_{t+1,i} - N_{t+1,i}\right] \quad \forall s$$

where $\Pi_s$ is the probability to reach state $s$, $\mu G(\omega'_{s,t+1}) = \mu \int_0^{\omega'_{s,t+1}} \omega dF(\omega)$ is the expected monitoring costs and $\Gamma(\omega'_{s,t+1}) = (1 - F(\omega'_{s,t+1})) \omega'_{s,t+1} + G(\omega'_{s,t+1})$ is the expected gross share of profits going to the lender given state
s of the economy. Associating for each constraint a multiplier \( \Pi_s \lambda_s \), the FOC are

\[
\Gamma' (w_{s,t+1}^i) R^i_{s,t+1} Q_t \tilde{K}_{t+1,i} + \lambda_s \left[ \Gamma' (w_{s,t+1}^i) - \mu G' (w_{s,t+1}^i) \right] R^i_{s,t+1} Q_t \tilde{K}_{t+1,i} = 0
\]

\[
\sum_s \Pi_s \left( 1 - \Gamma (w_{s,t+1}^i) \right) R^i_{s,t+1} Q_t + \sum_s \Pi_s \lambda_s \left[ (\Gamma (w_{s,t+1}^i) - \mu G (w_{s,t+1}^i)) R^i_{s,t+1} - R_t \right] = 0
\]

\[
\left[ \Gamma (w_{s,t+1}^i) - \mu G (w_{s,t+1}^i) \right] R^i_{s,t+1} Q_t \tilde{K}_{t+1,i} = R_t \left[ Q_t \tilde{K}_{t+1,i} - N_{t+1,i} \right] \quad \forall s
\]

Rearranging, we get

\[
\lambda_s (w_{s,t+1}^i) = \frac{\Gamma' (w_{s,t+1}^i)}{\Gamma' (w_{s,t+1}^i) - \mu G' (w_{s,t+1}^i)} \quad \forall s
\]

\[
E_t \left\{ \left( 1 - \Gamma (w_{t+1}^i) \right) R^i_{t+1} + \lambda (w_{t+1}^i) \left[ (\Gamma (w_{t+1}^i) - \mu G (w_{t+1}^i)) R^i_{t+1} - R_t \right] \right\} = 0
\]

\[
\left[ \Gamma (w_{s,t+1}^i) - \mu G (w_{s,t+1}^i) \right] R^i_{s,t+1} Q_t \tilde{K}_{t+1,i} = R_t \left[ Q_t \tilde{K}_{t+1,i} - N_{t+1,i} \right] \quad \forall s
\]

Since all entrepreneurs have the same distribution of the idiosyncratic risk, \( w_{s,t+1}^i = w_{s,t+1} \) and \( \lambda_s (w_{s,t+1}^i) = \lambda_s (w_{s,t+1}) \). This implies from the third FOC, that \( N_{t+1,i} \) will also be the same across entrepreneurs.

From the second FOC, we see that when \( \mu = 0 \), \( \lambda (w_{t+1}^i) = 1 \) and \( E_t R^i_{t+1} = R_t \). The third FOC has to do with the fact that bank profits are zero ex post. In this case, we can express the lender’s expected return simply as a function of the average cutoff value of the firm’s idiosyncratic shock, \( w_{t+1} \).

BGG show that the capital to wealth ratio is an increasing function of the ex ante premium on external funds.
## B Tables and Figures

### Table 1-A: Prior and Posterior Distribution of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Mode</th>
<th>St. Error</th>
<th>5%</th>
<th>Median</th>
<th>Mean</th>
<th>95%</th>
<th>𝑅̅</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜎_r Std. dev. monetary shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td>1.000</td>
</tr>
<tr>
<td>𝜎_a Std. dev. technology shock</td>
<td>Gamma</td>
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<td>0.05</td>
<td>0.006</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
<td>1.001</td>
</tr>
<tr>
<td>𝜎_g Std. dev. Gov. spending shock</td>
<td>Gamma</td>
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<td>0.05</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
<td>0.017</td>
<td>1.003</td>
</tr>
<tr>
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<td>Gamma</td>
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<td>0.05</td>
<td>0.096</td>
<td>0.120</td>
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<td>0.148</td>
<td>1.107</td>
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<tr>
<td>𝜎_x Std. dev. labor supply shock</td>
<td>Gamma</td>
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<td>0.05</td>
<td>0.029</td>
<td>0.036</td>
<td>0.036</td>
<td>0.044</td>
<td>1.048</td>
</tr>
<tr>
<td>𝜎_y Std. dev. price mark up shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.142</td>
<td>0.188</td>
<td>0.189</td>
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<td>𝜎_c Std. dev. wage mark up shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.567</td>
<td>0.740</td>
<td>0.744</td>
<td>0.932</td>
<td>1.000</td>
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<tr>
<td>𝜌_r Smooth coef. in instrument rule</td>
<td>Beta</td>
<td>0.85</td>
<td>0.10</td>
<td>0.460</td>
<td>0.532</td>
<td>0.531</td>
<td>0.594</td>
<td>1.002</td>
</tr>
<tr>
<td>𝜌_a Autocor. coef. technology</td>
<td>Beta</td>
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<td>0.10</td>
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<td>0.982</td>
<td>0.981</td>
<td>0.995</td>
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<tr>
<td>𝜌_g Autocor. coef. Gov. spending</td>
<td>Beta</td>
<td>0.85</td>
<td>0.10</td>
<td>0.913</td>
<td>0.951</td>
<td>0.949</td>
<td>0.980</td>
<td>1.000</td>
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<tr>
<td>𝜌_v Autocor. coef. preferences shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.10</td>
<td>0.986</td>
<td>0.990</td>
<td>0.989</td>
<td>0.992</td>
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<tr>
<td>𝜌_x Autocor. coef. labor supply shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.10</td>
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<td>0.988</td>
<td>0.987</td>
<td>0.996</td>
<td>1.016</td>
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</table>

Note: Benchmark FA model using US data.
Table 1-B: Prior and Posterior Distribution of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Mode</th>
<th>St. Error</th>
<th>5%</th>
<th>Median</th>
<th>Mean</th>
<th>95%</th>
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<tbody>
<tr>
<td>$\gamma_x$ Coef. inflation in instrument rule</td>
<td>Normal</td>
<td>1.50</td>
<td>0.05</td>
<td>1.563</td>
<td>1.638</td>
<td>1.637</td>
<td>1.709</td>
</tr>
<tr>
<td>$\gamma_y$ Coef. output in instrument rule</td>
<td>Normal</td>
<td>0.50</td>
<td>0.05</td>
<td>0.117</td>
<td>0.19</td>
<td>0.19</td>
<td>0.264</td>
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<td>$\sigma$ risk aversion</td>
<td>Normal</td>
<td>1.00</td>
<td>0.20</td>
<td>0.776</td>
<td>1.092</td>
<td>1.095</td>
<td>1.335</td>
</tr>
<tr>
<td>$\alpha$ Cobb-Douglas</td>
<td>Beta</td>
<td>0.35</td>
<td>0.02</td>
<td>0.278</td>
<td>0.303</td>
<td>0.303</td>
<td>0.328</td>
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<tr>
<td>$\lambda$ SS price mark up</td>
<td>Beta</td>
<td>0.20</td>
<td>0.05</td>
<td>0.318</td>
<td>0.435</td>
<td>0.439</td>
<td>0.574</td>
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<td>$\tau$ SS wage mark up</td>
<td>Beta</td>
<td>0.20</td>
<td>0.05</td>
<td>0.531</td>
<td>0.649</td>
<td>0.647</td>
<td>0.757</td>
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<td>$\theta$ prob. of not adj. prices</td>
<td>Beta</td>
<td>0.70</td>
<td>0.05</td>
<td>0.760</td>
<td>0.786</td>
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<td>$\phi$ prob. of not adj. wages</td>
<td>Beta</td>
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<td>0.607</td>
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<td>0.661</td>
<td>0.71</td>
</tr>
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<td>$\phi$ elasticity of capital price wrt I/K</td>
<td>Uniform</td>
<td>-0.5</td>
<td>0.29</td>
<td>-0.601</td>
<td>-0.457</td>
<td>-0.459</td>
<td>-0.33</td>
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<tr>
<td>$\delta''/\delta'$ Variable dep. parameter</td>
<td>Gamma</td>
<td>1.00</td>
<td>0.05</td>
<td>0.98</td>
<td>1.029</td>
<td>1.03</td>
<td>1.081</td>
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<tr>
<td>$h$ Habit formation</td>
<td>Beta</td>
<td>0.70</td>
<td>0.05</td>
<td>0.593</td>
<td>0.656</td>
<td>0.655</td>
<td>0.715</td>
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<td>$\varepsilon$ Std normal parameter</td>
<td>Normal</td>
<td>-2.43</td>
<td>0.10</td>
<td>-2.604</td>
<td>-2.439</td>
<td>-2.438</td>
<td>-2.273</td>
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<tr>
<td>$K/N$ Capital to net worth ratio</td>
<td>Gamma</td>
<td>2.00</td>
<td>0.10</td>
<td>2.127</td>
<td>2.28</td>
<td>2.281</td>
<td>2.444</td>
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<tr>
<td>$\gamma$ Entrepreneurs rate of survival</td>
<td>Beta</td>
<td>.975</td>
<td>0.01</td>
<td>0.989</td>
<td>0.994</td>
<td>0.993</td>
<td>0.997</td>
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<tr>
<td>$Rk - R$ Risk premium</td>
<td>Gamma</td>
<td>0.005</td>
<td>0.002</td>
<td>0.006</td>
<td>0.008</td>
<td>0.008</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Note: Benchmark FA model using US data.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model with Financial Frictions</th>
<th>Model with NO Financial Frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.041</td>
<td>0.032</td>
<td>0.050</td>
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<tr>
<td>Consumption</td>
<td>0.041</td>
<td>0.029</td>
<td>0.050</td>
</tr>
<tr>
<td>Investment</td>
<td>0.126</td>
<td>0.133</td>
<td>0.234</td>
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<tr>
<td>Inflation</td>
<td>0.004</td>
<td>0.010</td>
<td>0.008</td>
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<tr>
<td>Nominal interest rate</td>
<td>0.007</td>
<td>0.015</td>
<td>0.011</td>
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<tr>
<td>Real wages</td>
<td>0.036</td>
<td>0.051</td>
<td>0.049</td>
</tr>
<tr>
<td>Hours</td>
<td>0.024</td>
<td>0.054</td>
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</table>

Note: Actual and small sample standard deviation of US data evaluated at the mean of the posterior distribution.
Table 3: Historical Variance Decomposition

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<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Inflation</th>
<th>Nominal interest rate</th>
<th>Real wages</th>
<th>Hours</th>
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<td><strong>1 Quarter</strong></td>
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<td></td>
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<tr>
<td>Monetary shock</td>
<td>0.01</td>
<td>0.04</td>
<td>0.09</td>
<td>0.01</td>
<td>0.71</td>
<td>0</td>
<td>0.07</td>
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<tr>
<td>Price mark up shock</td>
<td>0</td>
<td>0.01</td>
<td>0.06</td>
<td>0.87</td>
<td>0.18</td>
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<td>0.01</td>
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<tr>
<td>Wage mark up shock</td>
<td>0.01</td>
<td>0.03</td>
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<td>0.08</td>
<td>0.07</td>
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<td>0.42</td>
<td>0.21</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
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<tr>
<td>Preferences shock</td>
<td>0.35</td>
<td>0.27</td>
<td>0.42</td>
<td>0</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>Gov. spending shock</td>
<td>0.26</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.11</td>
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<td>Technology shock</td>
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<td><strong>4 Quarters</strong></td>
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<td>Monetary shock</td>
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<td>0.04</td>
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<td>0.01</td>
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<td>Gov. spending shock</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>Technology shock</td>
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<td>0.03</td>
<td>0.12</td>
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<td>0.26</td>
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<tr>
<td>Wage mark up shock</td>
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<td>0.02</td>
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<td>0.13</td>
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<tr>
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<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
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<tr>
<td>Technology shock</td>
<td>0.29</td>
<td>0.25</td>
<td>0.15</td>
<td>0.17</td>
<td>0.14</td>
<td>0.55</td>
<td>0.01</td>
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</table>

Note: Benchmark FA model using US data evaluated at the mean of the posterior.
Table 4-A: Robustness and Different Models Performance - US Data

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<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Flexible</th>
<th>Flexible</th>
<th>No Var.</th>
<th>No Cons.</th>
<th>Standard FA Model</th>
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<tr>
<td></td>
<td>FA</td>
<td>No FA</td>
<td>Prices</td>
<td>Wages</td>
<td>Cap.Utiliz.</td>
<td>Habits</td>
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<tr>
<td>$\sigma_r$ Std. dev. monetary shock</td>
<td>0.003</td>
<td>0.03</td>
<td>0.003</td>
<td></td>
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<tr>
<td>$\sigma_a$ Std. dev. technology shock</td>
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<td>0.006</td>
<td>0.006</td>
<td></td>
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<td>0.006</td>
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<tr>
<td>$\sigma_g$ Std. dev. Gov. spending shock</td>
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<td>0.014</td>
<td>0.014</td>
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<tr>
<td>$\sigma_v$ Std. dev. preferences shock</td>
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<td>0.157</td>
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<tr>
<td>$\sigma_{\xi}$ Std. dev. labor supply shock</td>
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<td>0.043</td>
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<td>0.012</td>
</tr>
<tr>
<td>$\sigma_{\lambda}$ Std. dev. price mark up shock</td>
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<tr>
<td>$\sigma_{\tau}$ Std. dev. wage mark up shock</td>
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<td>0.703</td>
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<td>$\rho_r$ Smooth coef. in instrument rule</td>
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<tr>
<td>$\rho_g$ Autocor. coef. Gov. spending</td>
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</table>

Note: Benchmark FA model is the one in the paper. The Standard FA model is the one in Bernanke, Gertler, and Gilchrist (1999), without consumption habits, no variable capital utilization and flexible wages.
## Table 4-B: Robustness and Different Models Performance - US Data

<table>
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<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\gamma_x$ Coef. inflation in instrument rule</td>
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<td>1.622</td>
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<tr>
<td>$\gamma_y$ Coef. output in instrument rule</td>
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<tr>
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<tr>
<td>$\theta$ prob. of not adj. prices</td>
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<td>0.781</td>
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<td>$\theta$ prob. of not adj. wages</td>
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<td>-0.278</td>
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<tr>
<td>$\delta''/\delta'$ Variable dep. parameter</td>
<td>1.030</td>
<td>1.001</td>
<td>1.000</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$h$ Habit formation</td>
<td>0.655</td>
<td>0.635</td>
<td>0.630</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$K/N$ Capital to net worth ratio</td>
<td>2.281</td>
<td>2.070</td>
<td>2.234</td>
<td>2.198</td>
<td>2.005</td>
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<td></td>
</tr>
<tr>
<td>$\gamma$ Entrepreneurs rate of survival</td>
<td>0.993</td>
<td>0.972</td>
<td>0.992</td>
<td>0.997</td>
<td>0.971</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$Rk - R$ Risk premium</td>
<td>0.008</td>
<td>-</td>
<td>0.006</td>
<td>0.025</td>
<td>-</td>
<td></td>
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<tr>
<td>Log Marginal Data Density</td>
<td>2615.2</td>
<td>2608.3</td>
<td>2605.3</td>
<td>2472.1</td>
<td>2438.0</td>
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<td>Bayes Factor</td>
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<td>1026.1</td>
<td>19928</td>
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</tbody>
</table>

Note: Benchmark FA model is the one in the paper. The Standard FA model is the one in Bernanke, Gertler, and Gilchrist (1999), without consumption habits, no variable capital utilization and flexible wages.
Table 5-A: Robustness and Different Models Performance - European Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Standard FA Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FA</td>
<td>No FA</td>
</tr>
<tr>
<td>$\sigma_r$ Std. dev. monetary shock</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma_a$ Std. dev. technology shock</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>$\sigma_g$ Std. dev. Gov. spending shock</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td>$\sigma_\nu$ Std. dev. preferences shock</td>
<td>0.091</td>
<td>0.110</td>
</tr>
<tr>
<td>$\sigma_\xi$ Std. dev. labor supply shock</td>
<td>0.154</td>
<td>0.107</td>
</tr>
<tr>
<td>$\sigma_\lambda$ Std. dev. price mark up shock</td>
<td>0.402</td>
<td>0.425</td>
</tr>
<tr>
<td>$\sigma_\tau$ Std. dev. wage mark up shock</td>
<td>0.110</td>
<td>0.172</td>
</tr>
<tr>
<td>$\rho_r$ Smooth coef. in instrument rule</td>
<td>0.543</td>
<td>0.539</td>
</tr>
<tr>
<td>$\rho_\sigma$ Autocor. coef. technology</td>
<td>0.992</td>
<td>0.991</td>
</tr>
<tr>
<td>$\rho_\sigma$ Autocor. coef. Gov. spending</td>
<td>0.858</td>
<td>0.954</td>
</tr>
<tr>
<td>$\rho_\nu$ Autocor. coef. preferences shock</td>
<td>0.991</td>
<td>0.994</td>
</tr>
<tr>
<td>$\rho_\xi$ Autocor. coef. labor supply shock</td>
<td>0.621</td>
<td>0.715</td>
</tr>
</tbody>
</table>

Note: Benchmark FA model is the one in the paper. The Standard FA model is the one in Bernanke, Gertler, and Gilchrist (1999), without consumption habits, no variable capital utilization and flexible wages.
Table 5-B: Robustness and Different Models Performance - European Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark FA</th>
<th>No FA</th>
<th>Standard FA Model FA</th>
<th>no FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_\pi$ Coef. inflation in instrument rule</td>
<td>1.653</td>
<td>1.669</td>
<td>1.486</td>
<td>1.495</td>
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<tr>
<td>$\gamma_\theta$ Coef. output in instrument rule</td>
<td>0.325</td>
<td>0.272</td>
<td>0.561</td>
<td>0.258</td>
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<tr>
<td>$\sigma$ risk aversion</td>
<td>0.975</td>
<td>1.099</td>
<td>1.241</td>
<td>1.402</td>
</tr>
<tr>
<td>$\alpha$ Cobb-Douglas</td>
<td>0.37</td>
<td>0.361</td>
<td>0.352</td>
<td>0.341</td>
</tr>
<tr>
<td>$\lambda$ SS price mark up</td>
<td>0.458</td>
<td>0.518</td>
<td>0.266</td>
<td>0.394</td>
</tr>
<tr>
<td>$\tau$ SS wage mark up</td>
<td>0.226</td>
<td>0.257</td>
<td>0.231</td>
<td>0.220</td>
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<tr>
<td>$\theta$ prob. of not adj. prices</td>
<td>0.873</td>
<td>0.892</td>
<td>0.878</td>
<td>0.931</td>
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<tr>
<td>$\phi$ prob. of not adj. wages</td>
<td>0.711</td>
<td>0.650</td>
<td>-</td>
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<tr>
<td>$\varphi$ elasticity of capital price wrt I/K</td>
<td>-0.818</td>
<td>-0.407</td>
<td>-0.505</td>
<td>-0.064</td>
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<tr>
<td>$\delta''/\delta'$ Variable dep. parameter</td>
<td>1.017</td>
<td>0.990</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h$ Habit formation</td>
<td>0.548</td>
<td>0.576</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$zz$ Std normal parameter</td>
<td>-2.401</td>
<td>-2.389</td>
<td>-2.387</td>
<td>-2.420</td>
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<tr>
<td>$K/N$ Capital to net worth ratio</td>
<td>2.180</td>
<td>2.030</td>
<td>1.963</td>
<td>2.001</td>
</tr>
<tr>
<td>$\gamma$ Entrepreneurs rate of survival</td>
<td>0.994</td>
<td>0.972</td>
<td>0.987</td>
<td>0.971</td>
</tr>
<tr>
<td>$Rk - R$ Risk premium</td>
<td>0.010</td>
<td>-</td>
<td>0.022</td>
<td>-</td>
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<tr>
<td>Log Marginal Data Density</td>
<td>2200.5</td>
<td>2212.8</td>
<td>2092.7</td>
<td>2092.1</td>
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<td>Bayes Factor</td>
<td>1</td>
<td>1e-4</td>
<td>1</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Note: Benchmark FA model is the one in the paper. The Standard FA model is the one in Bernanke, Gertler, and Gilchrist (1999), without consumption habits, no variable capital utilization and flexible wages.
Figure 1: Prior and Posterior Distribution of the model with Financial Frictions
Figure 2: Prior and Posterior Distribution of the model with Financial Frictions
Figure 3: Prior and Posterior Distribution of the model with Financial Frictions.
Figure 4: Actual and Fitted data: Dotted line - One side Kalman Filter Fitted data. Solid line - Actual data.
Figure 5: Two side Kalman Filter Estimated Shocks