Competition and Innovation:
An Alternative Explanation for the Inverted-U
Relationship and some Policy Implications*

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Abstract

We model the relationship between competition and innovation using a continuous time version of the dynamic industry model of Ericson-Pakes (Review of Economic Studies, 1995). We show that if there are increasing returns to innovation when innovation stock of a firm is low and decreasing returns when it is high, there is an inverted-U relationship between competition and innovation intensity as observed in the data for US and UK manufacturing firms. We also study the impact on welfare of the policies designed to affect competition or innovation. Our policy experiments show that R&D subsidies, though good for innovation and welfare, increase degree of monopoly in the industry. On the other hand, higher barriers to entry reduce innovation, increase concentration and degree of monopoly, hurt consumers and reduce overall welfare.

JEL Classification Codes: L10; O30

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1 Introduction

The question of the relationship between market structure and innovation is an old one and has been the subject of active research since 1950s. Kamien and Schwartz (1982) and Cohen and Levin (1989) are two good surveys of the earlier literature on the subject. More recently, the Schumpeterian models of economic growth have rekindled interest in this classic question.\(^1\) Since growth in these models is driven by innovation, the question becomes of a wider interest.

The motivation of this study comes from the work of Aghion et al (2005) who, in a general equilibrium setting, show that the relationship between competition and innovation is inverted-U shaped. They test this prediction by using data for manufacturing firms in the UK and find empirical support for it.\(^2\) In order to keep the general equilibrium model tractable, Aghion et al (2005) abstract from many important features of the real life industries. For example, they assume a duopoly structure for the market and do not allow for entry and exit. Moreover, they model the relationship between competition and innovation as being one-sided i.e. they focus on how competition affects innovation and do not allow for innovation to have any impact on competition. Their model is also not suitable for policy analysis since they measure competition by the substitutability parameter in consumers’ CES utility function and it is hard to think of a competition policy that can affect this parameter in a predictable way.

We propose an alternative explanation for the inverted-U relationship between competition and innovation in a partial equilibrium setting. This allows us to extend the market structure to an oligopoly, introduce entry/exit, model competition and innovation jointly and do some policy experiments.\(^3\)

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\(^2\)We observe a similar inverted-U relationship for manufacturing firms in the US. Figure 1 scatter plots innovation (measured by citation-weighted patents and R&D expenditures) against competition (measured by the inverse of markup) for the manufacturing firms in the US. As the competition increases, there is an increase in average innovation. When the competition gets too intense, the average innovation declines. Another notable feature of Figure 1 is that the dispersion of innovation also first increases and then declines.

\(^3\)Dasgupta and Stiglitz (1980) is an early example of the model in which industrial
Our framework draws heavily on a continuous time version (proposed by Doraszelski and Judd (2004)) of the model in Ericson-Pakes (1995). As Doraszelski and Judd (2004) show, writing the model in continuous time makes its computation a lot easier. The key feature of our framework that drives the inverted-U relationship is that there are increasing returns to innovation when innovation stock of a firm is low and decreasing returns when it is high i.e. first few innovations decrease the cost of production at an increasing rate but later innovations do so at a decreasing rate.

This framework also allows for policy analysis. Since in this environment competition and innovation are jointly determined, it is possible to see how policies designed to affect innovation impact on competition and vice versa. In other words, in this setting we can study competition policy as a tool to promote innovation and hence growth. However, we do not model growth and limit our attention to the welfare effects of these policies.

We define innovation as a new idea that reduces the cost of production. Hence our focus is on process innovations. We use two definitions of competition. The first is the inverse of the degree of monopoly and defined as \( mc/p \). The second is the inverse of the degree of concentration and defined as \( 1 - H \), where \( H \) is the Herfindhal’s index. The first measures competition at the firm level though it can be averaged for the industry. The second measures competition at the industry level only. As we shall see below, the relationship between competition and innovation changes when the definition of competition is changed.

The rest of the paper is organized as follows. In Section 2 we present the model and characterize the equilibrium. Section 3 discusses numerical computation of the equilibrium and the results for the simple case of a monopolist. Section 4 contains the main simulation results of this paper. In section 5 we discuss some policy experiments and section 6 concludes.
2 The Model

There are $n$ firms in the industry of which $n_1$ are active in production and $n_2 = n - n_1$ are potential entrants. Firms produce a homogeneous product but differ in productivity $\omega \in \{1, 2, \ldots, \bar{\omega}\}$. The potential entrants do not produce but have a token productivity of 0 to distinguish them from active firms. When a firm exits, regardless of its state at the time of exit, its individual state changes to 0. When a potential entrant enters, its state changes from 0 to $\omega_e \in [1, \bar{\omega}]$, which is exogenously given. Market structure $s$ is defined as a descending $n$-tuple describing the states of incumbents and potential entrants. For example, if $s = [8 \ 3 \ 1 \ 0 \ 0]$, there are five firms in the industry. Three of them are active with productivities equal to 8, 3 and 1 and remaining two are potential entrants.

Firms engage in a Cournot game in the product market. Differences in productivities translate into differences in marginal costs. More specifically, $mc_i = g(\omega_i)$ with $g'(\omega_i) < 0$. Market demand is given by $P(Q) = D - Q$, where $Q = \sum q_i$. It is straightforward to compute profit function $\pi(\omega_i, s)$ from the solution to this Cournot game (see Appendix A).

We write the model in continuous time as suggested by Doraszelski and Judd (2004). The great advantage of doing so is that even though there are $n$ firms in the industry, in a very small instant in time, only one of them can experience a change in its state. This simplifies the computation enormously compared to the discrete time version of the model in which in each period a firm has to form expectations about all possible combinations of changes in its own and in its rivals’ productivities.

2.1 The Problem of an incumbent

The state of an incumbent $i$ changes with a Poisson hazard rate $p_i$. This hazard rate has three components: $x_i$, $\delta$ and $y_i$. $x_i$ is the hazard rate of firm’s productivity going up by one and can be changed by investing in R&D. We assume that in order to have a hazard rate of $x_i$, a firm needs to

\[\bar{\omega}\] can also be interpreted as the innovation stock of a firm. In the text we use ‘productivity’ and ‘innovation stock’ interchangeably.
invest $\frac{\beta}{2} x_i^2$ in R&D. $\delta$ is the hazard rate of a firm’s productivity declining by one and is given exogenously. $y_i$ is the hazard rate of exit. Exit in this model is not once for all. Instead, it is moving from an active to a dormant state with a chance of reentry at a later stage.\(^{5}\) We assume that a firm receives an exogenously fixed scrap value $\phi$ upon exit. A firm can invest to change the hazard rate of exit. We assume that in order to have a hazard rate $y_i$, the firm needs to invest $\frac{\beta}{2} y_i^2$. We shall see below that in equilibrium this hazard rate depends on the difference between the scrap value and the value of the firm. If the scrap value is higher than the value of the firm, the firm would be willing to invest to hasten its exit. Intuitively this investment to change $y_i$ can be thought of as a loss in the scrap value: if a firm wants to quit early, it will have to accept a lower net scrap value.\(^{6}\) To sum up, an incumbent firm’s productivity can change with a Poisson hazard rate $p_i = x_i + \delta + y_i$. And when a change occurs, the transition probabilities are:

\[
\begin{align*}
\Pr(\omega' = \omega_i + 1|\omega = \omega_i) & = \frac{x_i}{p_i}, \\
\Pr(\omega' = \omega_i - 1|\omega = \omega_i) & = \frac{\delta}{p_i}, \\
\Pr(\omega' = 0|\omega = \omega_i) & = \frac{y_i}{p_i}.
\end{align*}
\]

The problem of a potential entrant will be described in some detail in the next sub-section. For the time being, we note that each potential entrant has a Poisson hazard rate of entry equal to $p_e = y_e$ that depends on the variable cost of entry. Let $e_j$ be a $1 \times n$ vector with 1 at $j^{th}$ location and zeros everywhere else. The individual state variable for firm $i$ is $\omega_i$ and aggregate state variable is $s$. Having set the necessary notation in place, we

\(^{5}\)We do not require the potential entrants to be the same firms that were either the potential entrants last period or that exited during the last period. All we require is that the number of incumbents and potential entrants adds up to $n$. So it is possible that an exiting firm may exit forever. However, to keep the total fixed we require that one potential entrant be added. In other words, if $n_1$ falls by one, $n_2$ must increase by one to keep $n$ constant. This formulation makes the computation much simpler and quicker.

\(^{6}\)In this environment, it need not be the case that the firm with lowest productivity is always the first one to exit. Instead, it is possible that a higher productivity firm may exit first because of the randomness of exit. However, a lower productivity firm will always have a higher probability of exit.
can write an incumbent’s problem as

\[
V(\omega_i, s) = \max_{x_i, y_i} \left\{ \left( \pi(\omega_i, s) - \frac{\beta y^2}{2} y^2_i \right) dt + e^{-\rho dt} \right. \\
\left. \left[ x_i dtV(\omega_i + 1, s + e_i) + \delta dtV(\omega_i - 1, s - e_i) + y_i dt\phi \\
+ \sum x_j dtV(\omega_i, s + e_j) + \sum \delta dtV(\omega_i, s - e_j) + \sum y_j dtV(\omega_i, s - \omega_j e_j) \\
\left. \right) + n_2 y_e dtV(\omega_i, s + \omega e_{n_1+1}) + \left( 1 - p_i dt - \sum p_j dt - n_2 p_i dt \right) V(\omega_i, s) \right\}.
\]

(1)

The value of a firm with productivity \(\omega_i\) and facing market structure \(s\) is given by the right-hand side of (1). The firm chooses its Poisson hazard rates of success \((x_i)\) and exit \((y_i)\) taking the actions of its active and potential competitors as given.\(^7\) The return function consists of three terms. The first term \(\pi(\omega_i, s)\) is the instantaneous profit from production (see appendix A). The second term \((\frac{\beta x_i^2}{2} x_i^2)\) is the cost of R&D. For simplicity it is assumed to be quadratic in \(x_i\). \(\beta_x\) is the inverse of the efficiency of R&D investment: with a fixed R&D investment, a lower \(\beta_x\) implies a higher rate of success. The third term \((\frac{\beta y_i^2}{2} y_i^2)\) is the investment made by the firm to expedite its exit. This will be positive if the value of a firm is less than the scrap value at exit. We can think of it as the loss in scrap value that the firm is willing to accept in order to increase its hazard rate of exit. \(\beta_y\) is the inverse of the efficiency of this investment. A low \(\beta_y\) implies an institutional set up in which it is very easy to exit.

The rate of time preference is \(\rho\) and hence \(e^{-\rho dt}\) is the continuous time discount factor. The continuation value is simply a weighted average of values under different scenarios. The weights are the hazard rates of success, depreciation, exit and entry. Now we explain the continuation value terms one by one.

\(^7\)The hazard rate of exit, as we shall see below, is zero in most of the cases except when the value of the firm falls below the fixed scrap value it can get upon exit.
\(x_i dt V(\omega_i + 1, s + e_i)\): \(x_i\) is the hazard rate of firm \(i\)'s success in its quest for innovation and if the firm succeeds its productivity will increase from \(\omega_i\) to \(\omega_i + 1\) and the value of the firm will change to \(V(\omega_i + 1, s + e_i)\).

\(\delta dt V(\omega_i - 1, s - e_i)\): The productivity of a firm can also depreciate. We assume a fixed hazard rate of depreciation for all active firms and denote it by \(\delta\). If this depreciation takes place the value of the firm will be \(V(\omega_i - 1, s - e_i)\).

\(y_i dt \phi\): The hazard rate of exit is \(y_i\) and the firm would receive the scrap value \(\phi\) in the event of exit.

\(\sum x_j dt V(\omega_i, s + e_j)\): The sum is over \(j\) that denotes one of the \(n_1 - 1\) active competitors of the firm \(i\). The firm \(i\) takes the hazards rates of success and exit of its competitors as given but in equilibrium they will be the same as anticipated by the firm.\(^8\) The hazard rate of a competitor \(j\) innovating is \(x_j\). If this happens, the productivity of the firm will remain \(\omega_i\) but it will face a new structure \(s + e_j\). Hence the value of the firm will be \(V(\omega_i, s + e_j)\).

\(\delta dt V(\omega_i, s - e_j)\): If the productivity of a competitor falls, and the hazard rate of this happening is \(\delta\), the value of the firm will be \(V(\omega_i, s - e_j)\). Again we sum over all active competitors.

\(\sum y_j dt V(\omega_i, s - \omega_j e_j)\): In the event of exit of a competitor, the value of the firm will be \(V(\omega_i, s - \omega_j e_j)\) and the hazard rate of this happening is \(y_j\). Once again the sum is over \(n_1 - 1\) active competitors.

\(n_2 y_e dt V(\omega_i, s + \omega_e e_{n_1 + 1})\): There are \(n_2\) identical potential entrants. The hazard rate of each of them entering is \(y_e\). In the event of entry, the structure changes to \(s + \omega_e e_{n_1 + 1}\) and the value of the firm to \(V(\omega_i, s + \omega_e e_{n_1 + 1})\). Notice that rather than taking the sum we simply multiply with \(n_2\). This highlights the fact that all potential entrant are identical.

\((1 - p_i dt - \sum p_j dt - n_2 p_e dt) V(\omega_i, s)\): Finally we have to take into account the possibility that during the infinitesimally small interval of time nothing happens. The hazard rate of this non-event is one minus the hazard rates of all above events combined. And if this happens (i.e. nothing happens) the value of the firm will remain at \(V(\omega_i, s)\).

By using the approximation \(e^{-\rho dt} \approx 1 - \rho dt\), ignoring the second order

\(^8\)In other words, we have a rational expectation equilibrium.
terms in \( dt \) and simplifying, the Bellman equation in (1) can be written as

\[
\rho V(\omega_i, s) = \pi(\omega_i, s) - \frac{\beta_x}{2} x_i^2 - \frac{\beta_y}{2} y_i^2 \\
+ x_i \left[ V(\omega_i + 1, s + e_i) - V(\omega_i, s) \right] \\
+ \delta \left[ V(\omega_i - 1, s - e_i) - V(\omega_i, s) \right] \\
+ y_i \left[ \phi - V(\omega_i, s) \right] \\
+ \sum x_j \left[ V(\omega_i, s + e_j) - V(\omega_i, s) \right] \\
+ \sum \delta \left[ V(\omega_i, s - e_j) - V(\omega_i, s) \right] \\
+ \sum y_j \left[ V(\omega_i, s - \omega_j e_j) - V(\omega_i, s) \right] \\
+ n y_e \left[ V(\omega_i, s + \omega_e e_{n+1}) - V(\omega_i, s) \right]
\]  

(2)

This equation has an intuitive interpretation: The flow value of firm’s assets \( \rho V(\omega_i, s) \) is the sum of the following:

\[
\pi(\omega_i, s) - \frac{\beta_x}{2} x_i^2 - \frac{\beta_y}{2} y_i^2, \text{ instantaneous profits from product market operations, net of R&D investment and investment to change the hazard rate of exit;}
\]

\[
x_i \left[ V(\omega_i + 1, s + e_i) - V(\omega_i, s) \right], \text{ probability of firm’s productivity going up by one multiplied by net change in the value of firm’s assets as a result of this change.}^{9}
\]

\[
\delta \left[ V(\omega_i - 1, s - e_i) - V(\omega_i, s) \right], \text{ expected loss from depreciation;}
\]

\[
y_i \left[ \phi - V(\omega_i, s) \right], \text{ net expected value of exit;}
\]

\[
\sum x_j \left[ V(\omega_i, s + e_j) - V(\omega_i, s) \right], \text{ sum of expected loss from competitors’ innovations;}
\]

\[
\sum \delta \left[ V(\omega_i, s - e_j) - V(\omega_i, s) \right], \text{ sum of expected gain from depreciation of competitors’ productivities;}
\]

\[
^{9}\text{In deriving (2), we have assumed that } dt \text{ approaches zero. When this happens, the Poisson hazard rates can be interpreted as instantaneous probabilities. Hence } x_i \text{ in (2) is the instantaneous probability of firm } i \text{'s productivity going up by one. The other hazard rates can be interpreted similarly.}
\]
\[ \sum y_j \left[ V(\omega_i, s - \omega_j e_j) - V(\omega_i, s) \right], \] sum of expected gain from exit by the competitors;

\[ n_2 y_e \left[ V(\omega_i, s + \omega_e e_{n_1+1}) - V(\omega_i, s) \right], \] sum of expected loss from entry.

The Bellman equation (2) is much simpler to solve numerically than its discrete time counterpart. In discrete time, a firm has to form expectations about the joint distribution of its competitors’ (both active and potential) productivities. Let \( n_1 = 10 \) and \( n_2 = 5 \) and suppose that only one firm can enter in a single time period. Then in discrete time a firm has to form expectations over \( 2 \cdot 3^{n_1} = 118,098 \) possibilities.\(^{10}\) In continuous time, during a very small interval of time only one change can occur and hence a firm has much fewer possibilities to keep track of. In the above example, under continuous time, the number of possibilities reduces to \( 3n_1 + n_2 = 35 \).\(^{11}\)

### 2.2 The Problem of an Entrant

We assume that each potential entrant has a Poisson hazard rate of entry equal to \( y_e \).\(^{12}\) This hazard rate depends on the investment \( \varphi(y_e) \) that each potential entrant makes to enter. \( \varphi(y_e) \) is a strictly increasing and convex function. It can be thought of as the variable cost of entry that varies with the hazard rate of entry chosen by the potential entrant according to the following rule:

\[
y_e = \begin{cases} 
\arg\max V_e & \text{if } V_e > 0 \\
0 & \text{otherwise},
\end{cases}
\]

\(^{10}\)For each of the \( n_1 \) incumbents, there are 3 possibilities i.e. its productivity might go up by one, might fall by one or it may exit (we ignore the possibility of the productivity remaining the same as it does not change the state). These make \( 3^{n_1} \) possibilities. We multiply by 2 to take into account the fact that there may or may not be entry during the period under consideration.

\(^{11}\)There are 3 possible outcomes for each of \( n_1 \) incumbents and one possible outcome for each of the potential entrants. But now we just have to add them up because no two changes can take place simultaneously.

\(^{12}\)Since all potential entrants are identical, they have the same hazard rate.
where \( V_e(s) \) is the net present value of entering the industry defined as

\[
V_e(s) = \max_{y_e} \left\{ e^{-\rho dt} \left[ y_e dt V(e, s + \omega e^{n_1 + 1}) \right] - \left[ c_e + \varphi(y_e) \right] dt \right\}. \tag{3}
\]

Once again using the approximation \( e^{-\rho dt} \simeq 1 - \rho dt \) and ignoring the second order terms in \( dt \), we can write (3) as

\[
V_e(s) = \max_{y_e} \left\{ y_e V(e, s + \omega e^{n_1 + 1}) - c_e - \varphi(y_e) \right\}, \tag{4}
\]

where \( \omega_e \) is the productivity level of an entrant and \( c_e \) is the fixed cost of entry. Both \( \omega_e \) and \( c_e \) are given exogenously.

2.3 The Equilibrium

Let \( \Omega \) be the space of \( \omega \) and \( S \) be the space of \( s \). The Equilibrium consists of \( V(\omega, s), x(\omega, s), y(\omega, s), y_e(s), Q(s'|s) \) and \( s_0 \) such that:

1. \( \forall (\omega, s) \in (\Omega, S), V(\omega, s) \) satisfies (1).

2. \( \forall (\omega, s) \in (\Omega, S), x(\omega, s), y(\omega, s) \) and \( y_e(s) \) solve (1).

3. \( Q(s'|s) \) is the Markov transition matrix that gives the probability of moving from structure \( s \) today to structure \( s' \) in the next period.

4. The initial structure \( s_0 \) is given.

2.4 Characterization of Equilibrium

An incumbent chooses \( x_i \) and \( y_i \) to maximize the right hand side of (2), taking \( x_j, y_j \) (\( \forall j \neq i \)) and \( y_e \) as given. The first order conditions are:

\[
x_i = \frac{1}{\beta_e} \left[ V(\omega_i + 1, s + e_i) - V(\omega_i, s) \right]
\]

\[
y_i = \max \left\{ 0, \frac{1}{\beta_y} \left[ \phi - V(\omega_i, s) \right] \right\}. \tag{5}
\]

The interpretation of the first order conditions is straight forward. An incumbent firm will invest more in R&D if the difference between the value
of its assets now and the value of its assets if it moves one step ahead is high. It will also invest more if the efficiency of R&D investment is high (i.e. $\beta_x$ is low). The interpretation of the second condition is the following. If the scrap value upon exit is greater than the current value of a firm’s assets, the firm would like to invest money to hasten its exit. Otherwise it will set $y_i = 0$.

A potential entrant makes its decision in two steps. First, it chooses $y_e$ to maximize $V_e(s)$. The first order condition for this problem is

$$\varphi'(y_e) = V(\omega_e, s + \omega_e e_{n_1 + 1}).$$

For the rest of the discussion we assume $\varphi(y_e) = y_e^\alpha/\alpha$ with $\alpha > 1$. Then the optimal $y_e$ is given by: $y_e^* = [V(\omega_e, s + \omega_e e_{n_1 + 1})]^{1/(\alpha-1)}$. By substituting $y_e^*$ back into (4) and simplifying we get the optimized net present value of entry.

$$V_e^* = \left(\frac{\alpha - 1}{\alpha} V(\omega_e, s + \omega_e e_{n_1 + 1})^{\alpha-1} - c_e\right) dt.$$  

In the second step, the potential entrant compares this value with zero. If $V_e^* > 0$, it will invest $\varphi(y_e)$ and wait for its chance to enter. Else, it will not invest anything. Let $I_n$ be a generic indicator function such that $I_n(x) = 1$ if $x > 0$, then an entrant’s choice of $y_e$ is

$$y_e(s) = I_n(V_e^*)y_e^*$$
We can substitute (8) and (5) into (2) and simplify to get

\[ \rho V(\omega_i, s) = \pi(\omega_i, s) \]

\[ + \frac{1}{2\beta x} \left[ V(\omega_i + 1, s + e_i) - V(\omega_i, s) \right]^2 \]

\[ + I_n(\phi - V(\omega_i, s)) \frac{1}{2\beta y} \left[ \phi - V(\omega_i, s) \right]^2 \]

\[ + \delta \left[ V(\omega_i - 1, s - e_i) - V(\omega_i, s) \right] \]

\[ + \frac{1}{\beta x} \sum_j \left[ V(\omega_j + 1, s + e_j) - V(\omega_j, s) \right] \left[ V(\omega_i, s + e_j) - V(\omega_i, s) \right] \]

\[ + \frac{1}{\beta y} \sum_j I_n(\phi - V(\omega_j, s)) \left[ \phi - V(\omega_j, s) \right] \left[ V(\omega_i, s - \omega_j e_j) - V(\omega_i, s) \right] \]

\[ + n_2 I_n(V_e^s)[V(\omega_e, s + \omega_e e_{n_1+1})]^{1/(\alpha-1)} \left[ V(\omega_i, s + \omega_e e_{n_1+1}) - V(\omega_i, s) \right] \]

\[ (9) \]

We assume that when an incumbent firm is already at the maximum level of productivity, it cannot increase its productivity any more and when it is at the lowest level of productivity, its productivity does not decline any further. With these assumptions, (9) is a system of \( N \times n \) non-linear equations with as many unknowns.\(^{13}\) In the next section we solve this system numerically.

3 Numerical Computation of Equilibrium

Our benchmark set of parameters is the following.

\(^{13}\)\( N \) is the cardinality of \( S \), i.e. it is the total number of possible structures \( s \).
Table 1: Parameter Values

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<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\omega}$</td>
<td>15</td>
<td>$\omega_e$</td>
<td>7</td>
</tr>
<tr>
<td>$n$</td>
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<td>$\rho$ (annual)</td>
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<tr>
<td>$c_e$</td>
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<td>$\beta_x$</td>
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<tr>
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<td>$\alpha$</td>
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We start off by computing the static profit function $\pi(\omega, s)$ as explained in Appendix A. Next, we guess initial matrix of values $V_0$ to be equal to $\pi/\rho$. Then we update each element of $V_0$ by substituting the relevant elements of $V_0$ on the right hand side of (9). Once all the elements have been updated, the updated matrix $V_1$ is used as $V_0$ in the next iteration and this process continues until the difference between $V_1$ and $V_0$ is less than a prespecified tolerance level. Two comments on our computation procedure are in order. First, each element of $V_0$ is updated separately and updated elements are not used in the current iteration. Second, we do not compute policies at each iteration. Instead, we numerically solve (9) for equilibrium and then use (5) to compute optimal policies. This saves a lot of computation time.

3.1 Insights from a Monopolist’s Problem

The easiest way to get the intuition behind the main idea of this paper is to look at the solution to a monopolist’s problem. Since there is only one firm, the equilibrium values and policies are vectors instead of matrices and can easily be analysed with the help of simple graphs.

Figure 2 plots the value function of a monopolist. The state variable $\omega$ is on x-axis. The slope of the value function first increases and then decreases. The first order condition for $x$ in (5) tells us that this slope is exactly what is driving the innovation intensity of the monopolist. Since monopolist is the only firm in the market, its productivity defines the market structure and

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14The choice of continuous time drastically reduces the time needed to update each element. However, the state space remains the same as with discrete time and hence the monster of dimensionality is only partially contained.
it does not have to worry about the actions of its rivals. Hence, its research intensity at $\omega$ is solely determined by the difference between its value at $\omega$ and at $\omega + 1$. As this difference increases, the monopolist innovates more and when this difference declines, it innovates less.

But the degree of monopoly power (measured by markup) is directly related to productivity: higher productivity leads to a lower marginal cost, which in turn leads to a higher markup. Hence if we measure competition in the market by the inverse of markup (i.e. $mc/p$), we should expect an inverted-U relationship between competition and innovation. The argument is the following: when competition increases (i.e. the markup falls), it is because the firm is becoming less productive (we are moving down the value function from right to left) and the expected benefit from innovation increases initially as the value function is becoming steeper. However, as the competition further increases, we move to the convex portion of the value function and expected benefit from innovation falls, resulting in lower intensity of innovation. This is seen graphically in Figure 3. The variable on $x$-axis is competition defined as the inverse of markup and on $y$-axis is the intensity of innovation $x$ as defined in (5). We see an inverted-U relationship between the two as described above.

The value function in Figure 2 takes its shape from the specific function $g(\omega)$ that translates productivity into marginal cost (see Appendix A). If we only require $g'(\omega) < 0$, a whole range of functions can satisfy this condition. However, it turns out that in order to have the inverted-U relationship that we observe in the data, the cost function must be such that marginal cost first decreases at an increasing rate and then at a decreasing rate.\textsuperscript{15} Hence the key assumption is that $g'(\omega)$ should not only be negative but also U-shaped. What if this condition is not satisfied? The answer is, we would not get the inverted-U relationship shown in Figure 3. For example, if we use the cost function in Ericson and Pakes (1995) i.e. $g(\omega) = \gamma e^{-\omega}$, then the $g'(\omega)$ would be negative but an increasing function of $\omega$ and the

\textsuperscript{15}This is the same as saying that there are increasing returns to innovation when $\omega$ is low and decreasing returns when $\omega$ is high.
resulting value function would be strictly concave.\textsuperscript{16} This will generate a positive relationship between competition and innovation. Hence choice of the specific cost function is the key to generate the relationship we observe in the data.

Although the example of a monopolist is useful to understand the intuition behind our main result, it presents an overly simplified picture of the model because there is no entry/exit and no interaction between firms. Things get a bit complicated when we increase the number of firms and allow for entry/exit, though the basic intuition remains the same.

\section{Simulation Results}

Having computed the equilibrium as defined in section 2.3 by using the computation strategy outlined in section 3, we simulate the model for 10,000 periods starting from an arbitrary structure with only one firm at the productivity level equal to $\omega_e$. In the following we sketch how the simulation procedure works. Let $n = 5$ then the initial structure is $s_0 = [\omega_e \ 0 \ 0 \ 0 \ 0]$.

From the equilibrium solution we know the Poisson hazard rates of productivity increase for active firms ($x(\omega, s)$), productivity depreciation (which is fixed at $\delta$), exit ($y(\omega, s)$) and entry $y_e(s)$.$^17$ We normalize the sum of these hazard rates to one and then draw a random number from uniform $[0,1]$ distribution. The draw determines which of the $3n_1 + n_2$ events takes place. Suppose that our random draw indicates that the event is that of entry, then we update the structure to $s_1 = [\omega_e \ \omega_e \ 0 \ 0 \ 0]$ i.e. add a second firm to the market with productivity equal to $\omega_e$. Again from our equilibrium solution we know all possible transition probabilities for this new structure. We normalize their sum to one and draw from uniform $[0,1]$ dis-

\textsuperscript{16}$g'(\omega)$ would be an increasing function of $\omega$ because $\omega = \{1, 2, \ldots, \overline{\omega}\}$.

\textsuperscript{17}One of the simplifying assumptions that needs to be made for the continuous time model is that, in a small interval of time, at most one firm can experience a productivity decline. Hence, although the probability that an incumbent’s productivity will decline is the same for all incumbents and given by $\delta$, in any period at most one firm can experience a productivity decline. This is in contrast to the original model of Ericson-Pakes in which if productivity declines, it declines for all the firms simultaneously.
Suppose this time the draw indicates that the event is an increase in the productivity of the second firm. We add one to the productivity of the second firm and reorder (since the market structure has to be a weakly descending n-tuple). This gives \( s_2 = [\omega_e + 1 \ \omega_e \ 0 \ 0 \ 0] \). We allow this process to continue for 10,000 periods. For each period we record the values of variables of interest. This completes one round of simulations. We conduct 1,000 such rounds and report the average results below.

Figure 4 scatter plots competition and innovation intensity. Competition in this picture is measured by the inverse of markups. We see that when competition is low, firms innovate less. This is because their markups are already so high that they expect little net benefit from further innovation. As we move towards higher competition, we see that the equilibrium outcome for the firms is to innovate more. In Figure 4 when competition is around 0.6, innovation is at its maximum level. Intuitively, in this region the value function of a firm is very steep. However, as competition increases further, we observe that firms innovate less. This is because at low levels of productivity the additional gain from innovation is small.

The most important thing about Figure 4 is that it does not imply a one-sided causal relationship between competition and innovation. Instead, it portrays the equilibrium outcome of a dynamic system in which both competition and innovation are endogenously determined.

We started out by showing some data in Figure 1. How the results in Figure 4 explain the data in Figure 1? Two comments are in order here. First, Figure 4 shows in an exaggerated way what we observe in Figure 1: at both low and high levels of competition, firms innovate less while in the intermediate range, they innovate more. Hence, our model provides an explanation for the inverted-U relationship that we observe in the data. Second, in Figure 1, we not only observe higher innovation at the intermediate level of competition, we also observe more dispersion. However, we do not see this dispersion in Figure 4 because we have averaged the simulation results to avoid clutter.\(^{18}\) If the results were not averaged, we

\(^{18}\)Figure 4 has 10,000 points and each point is the average of 1,000 rounds of simulations. If we don’t average, the figure will have 10 million points and will put unnecessary storage
would see an increase in dispersion at intermediate levels of competition.

To make the last point clear, we scatter plot the innovation intensity \( (x) \) against competition \( (mc/p) \) for the entire state-space in Figure 7. It is clear that when competition increases the innovation intensity gets more dispersed. However, when the competition gets too high, this dispersion also declines. Why the dispersion first increases and then declines? The simple answer is that firms facing different market structures can have very similar markups. Hence these firms will share the same point on \( x \)-axis. However, because of the different structures they face, their innovation intensity will vary.

To sum up, our model provides an explanation for why the average innovation is high at intermediate levels of competition and it can also explain why it is more widely dispersed in this range.

Figure 5 shows the same relationship as in Figure 4 but at the industry level. The average level of competition in the industry is on \( x \)-axis. It is simply the average of \( mc/p \) for all active firms in the industry. The sum of R&D intensity of firms in the industry is on \( y \)-axis. The overall relationship remains inverted-U shaped. As the average industry competition increases, the aggregate R&D intensity in the industry also increases. However, as competition increases further, the aggregate R&D intensity tends to decrease. These results are in line with what we observe in the data (not reported here) at the industry level.

5 Policy Analysis

In this section we analyze some policy tools that can affect the market structure and innovation. Because the market structure and innovation intensity are jointly determined in our model, it is interesting to see how a policy designed to affect one of these impacts on the other. We focus on three policies: R&D subsidies, bankruptcy laws and barriers to entry. First of these is a policy to affect R&D. We are interested to see how it might affect the market structure. The last two policies affect market structure and our burden.
interest is in their effect on innovation. We also study welfare implications of these policies. There are four policy parameters in our model that can be used to analyze these policies. These are: $\beta_x$, $\beta_y$, $c_e$ and $\alpha$. In the following we briefly comment on each.

$\beta_x$ is the inverse of the efficiency of R&D investment. A higher $\beta_x$ implies that in order to achieve a certain hazard rate, the firm has to invest more in R&D. This parameter has another interpretation too. Let $\beta_x = 1$, then in order to have a hazard rate of $x$ a firm needs to invest $x^2/2$ in R&D. Now suppose that the government gives a 20% subsidy on all R&D investments. We can model this subsidy simply by changing $\beta_x$ to 0.8. Now, in order to have a hazard rate of $x$, the firm needs to invest only $0.8x^2/2$ i.e. 20% less. A $\beta_x$ greater than 1 would mean an R&D tax. So we can vary $\beta_x$ to study the effects of R&D subsidies on market structure, innovation and welfare.

$\beta_y$ measures the ease of exit. If $\beta_y$ is close to zero, the optimal $y$ would be very high whenever $\phi > V(\omega_i, s)$ (see (5)) and hence the firm could exit the market very quickly. However, as $\beta_y$ increases, it becomes harder to exit. We interpret this difficulty of exit as a change in bankruptcy laws that makes exit harder.\(^\text{19}\) Our intuition is that as exit becomes harder, less efficient firms will stay in the market longer. This will hamper innovation and overall welfare will decline.

Both $c_e$ and $\alpha$ affect the ease of entry. To understand their role in the entry process, note that the total cost of entry ($T_{C e}$) paid by a successful entrant is\(^\text{20}\)

$$T_{C e} = c_e + \frac{y_e^\alpha}{\alpha}. \quad \text{(10)}$$

The parameter $\alpha$ is the elasticity of $y_e$ (the hazard rate of entry) with

\(^{19}\)Strict bankruptcy laws may also discourage entry as the riskier ventures may not get underway in the first place. However, it is irrelevant to our model as entrants do not take the difficulty of exit into account when making their entry decisions.

\(^{20}\)Each of the potential entrants pays $\frac{y_e^\alpha}{\alpha}$ to buy a Poisson hazard rate of entry equal to $y_e$. However, at most one potential entrant can be successful in a small instant of time (because time is continuous). The successful entrant then pays the fixed cost of entry and becomes an incumbent in the next period with efficiency level $\omega_e$. 

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respect to the variable cost of entry. An increase in either \( c_e \) or \( \alpha \) will make entry harder. Hence \( c_e \) and \( \alpha \) represent barriers to entry. We shall focus on \( c_e \) and see how it affects the market structure, innovation and welfare.

The desirability (or otherwise) of a policy ought to be judged by its effect on welfare. For the following analysis we define welfare as:

\[
W(s|\beta_x, \beta_y, c_e, \alpha, \chi, \theta) = CS + PS - \frac{\beta_x}{2} \sum_{i=1}^{n_1} x_i^2 - \frac{\beta_y}{2} \sum_{i=1}^{n_1} y_i^2 - n_2 \frac{y_0^x}{\alpha} - c_e \chi, \tag{11}
\]

where \( \chi = 1 \) if there is entry and 0 otherwise. \( CS \) and \( PS \) are consumers’ and producers’ surpluses and \( \theta \) is the vector of all other model parameters. In the following we report the results of our policy experiments.

### 5.1 R&D Subsidies

Suppose the government gives a subsidy of \( \tau \) on R&D investment. Hence a firm has to pay only \( 1 - \tau \) of its R&D cost. In other words, \( \beta_x \) with the subsidy will be \( 1 - \tau \) fraction of the \( \beta_x \) without the subsidy: \( \beta^s_x = (1 - \tau) \beta_x \). How will this policy affect market structure and innovative activity? Induced by the lower cost of R&D, the incumbents invest more in R&D and hence their hazard rate of successful innovation \((x)\) increases. This results in their moving up the productivity ladder since \( \delta \) (the hazard rate of their productivity decline) is unchanged. This discourages entry and increases incumbents’ markups. As a result, the degree of monopoly in the industry increases.

However, since all the incumbents benefit from these subsidies, their productivities relative to one another do not change much. Hence these subsidies have no significant effect on the degree of concentration in the industry. To sum up, R&D subsidies increase innovation and monopoly power of incumbents and do not affect the degree of concentration in the industry.

What is the net effect of these changes on welfare? The answer is: the welfare increases. For example, if we set the welfare in the benchmark case equal to 100 then a \( \tau = 0.2 \) increases the welfare to 102.4 and a \( \tau = 0.5 \)
increases it further to 105.1. The next question is how this gain in welfare is distributed between producers and consumers? Apparently, the producers enjoy higher markups after the subsidy so it could be the case that they benefit at the expense of consumers. However, this is not the case and it turns out that both consumers and producers are the net beneficiaries of this policy. The subsidy encourages innovation that reduces costs. Lower costs of production result in lower prices though prices do not fall by as much as the cost. The net result is that producers benefit because of higher markups and consumers benefit because of lower prices. In fact, with the above parameter values, the consumers gain a little more than the producers from this subsidy.

5.2 Bankruptcy Laws

We interpret $\beta_e$ as a measure of how strict the bankruptcy laws are. A low value of $\beta_e$ corresponds to easy bankruptcy laws and a high value to strict laws. This need not be the only interpretation of this parameter. Since it basically measures the ease of exit, any institution that affects ease of exit would affect $\beta_e$. The reason for relating it to bankruptcy laws is that it gives this parameter under the government control and hence makes it a policy parameter.

We set $\beta_y = 0.1$ in our benchmark model. To see what happens to market structure and innovation when it becomes difficult for firms to exit, we increase $\beta_y$ to 1. The first effect of this policy is to make the market more competitive because even if a firm has low productivity, it is not easy for it to exit and hence number of firms in the market is higher than the benchmark case. However, this effect is small. The effect on innovation is also small though positive. The overall welfare remains almost unchanged. However, when we decompose the welfare into consumer and producer components, consumers appear to gain a little (0.34%) and producers lose a little (0.49%). The offsetting gain and loss in welfare implies that increase in competition reduces the prices more than reduction in cost that results from increase in R&D. Hence, consumers gain and producers lose. However, we reiterate that these gains and losses are really small (though persistent) despite the
fact that $\beta_y$ was increased by an order of magnitude.

5.3 Barriers to Entry

In the benchmark model the fixed cost of entry was set equal to 0.06. In this policy experiment, we increase it to 0.6. This gives the incumbents a clear advantage. Market becomes more concentrated and the degree of monopoly increases. Since incumbents face less competition, they innovate less and the average productivity in the economy is lower than the benchmark case. On the welfare front, consumers lose and producers gain. Specifically, compared to our benchmark case, consumers’ welfare is down by 20% and and producers’ welfare is up by 21%. The overall welfare is down by 4%.

To sum up, barriers to entry reduce innovation, increase the degree of monopoly and market concentration, hurt consumers and help producers. There is an overall welfare loss associated with them which results because incumbents do not innovate as much as they would in the absence of the barriers and hence production costs are higher. Producers can offset these higher costs by charging even higher prices. They can do so because there are fewer firms in the market. Hence their markups are higher than the benchmark case. Consumers lose because they have to pay higher prices.

6 Related Literature

It is generally said that most of the recent empirical studies point to a positive relationship between competition and innovation. However, we observe in Figure 1 for US manufacturing that the relationship is inverted-U shaped. This finding is also confirmed by Aghion et al (2005) for UK manufacturing firms. Can we reconcile these apparently contradictory results?

The answer is, yes. In fact it is not correct to say that the relationship is unambiguously positive, negative or inverted-U. There are many dimensions of competition and emphasizing one dimension may lead one to con-

clude that the relationship is positive but another may point to a negative or inverted-U relationship. Hence it is important to be clear about which dimension we are talking about. For example, in our model the inverted-U relationship is claimed when competition is measured by the inverse of mark ups and innovation is measured by the intensity of R&D as reflected in R&D investment of firms. However, if we change the definition of competition to ‘one minus the Herfindhal’s Index (H)’, we observe a positive relationship (see Figure 6).

A good example to support our last point is a recent paper by Carlin et al (2004). They use various measures of competition in the same regression equation in which innovation is the left-hand side variable. They distinguish between old and new firms in their sample. Their findings point to all sort of relationships between competition and innovation. For example, if competition is measured by the number of competitors then the relationship between competition and innovation is negative for new firms and insignificant for old firms. I.e. the new firms facing more competitors innovate less. A similar conclusion is reached for old firms when competition is measured by the inverse of the degree of market power: the old firms with more market power innovate more. However, in case of new firms the relationship becomes inverted-U i.e. as market power increases, firms innovate more but when market power increases further, they innovate less.

Another important aspect of the relationship is the direction of causality. Does competition influence innovation or innovation affects competition? Our answer is both affect each other and are also affected by a host of other factors. Hence it is misleading to focus on just one-directional causal relationship. This view is also supported by Cohen and Levin (1989) who survey a whole bunch of studies and conclude that the evidence of a relationship between market structure and innovation is fragile and it is mainly due to the failure of studies to take into account the fundamental conditions that affect this relationship. In our model, we do not model the relationship as a causal one. Instead we model competition and innovation as being simultaneously determined and being affected by a host of other factors.

On the theoretical front, our work is related to Boone (2000). Boone
(2000) identifies four type of firms: faint, struggling, eager and complacent. This classification depends on the cost of production: faint firms have the highest marginal cost of production and complacent firms have the lowest. The part of his theory that deals with process innovations predicts that when competitive pressure increases, the struggling and eager firms (the ones with marginal costs in the intermediate range) innovate more whereas the faint and complacent firms innovate less. In our model, when cost of entry is decreased (i.e. competition is increased exogenously), the firms at very low and very high rungs of productivity ladder innovate less and those in the intermediate range of productivity innovate more. This happens because the expected benefit from innovation for the firms in the intermediate productivity range increases.

Our work is also related to Laincz (2005). The main difference is that Laincz uses a discrete-time version of Ericson and Pakes (1995) and focuses on how R&D subsidies affect market structure. Our focus instead is on explaining the inverted-U relationship between competition and innovation and we employ a continuous-time version. Another related paper is Peretto (1999). However, his focus is on the interaction between market structure and growth.

7 Summary and Concluding Remarks

We model the relationship between market structure and innovation using a continuous time version of the model in Ericson and Pakes (1995). The idea of writing this model in continuous time is due to Doraszilski and Judd (2004) and makes the computation of the equilibrium very easy. To our knowledge, our paper is the first application of the continuous time version of the model.

We find that if innovations exhibit increasing returns when the innovation stock of a firm is low and decreasing returns when the stock is high, there exists an inverted-U relationship between competition and innovation intensity. This is in line with the finding of Aghion et al (2005) and with the data from US manufacturing.
Another contribution of our paper is to show that the inverted-U relationship observed in the data is an equilibrium outcome of the interaction between market structure and innovation. Hence it is not a one-sided causal relationship. In fact, competition and innovation are not only interdependent but also depend on a host of other factors.

We also see that the inverted-U relationship per se cannot help in policy making. In other words, it is not possible to manipulate the market structure in such a way that all the firms find themselves sitting at the peak of the inverted-U and hence achieve the highest level of innovation in the industry. However, there do exist policies that affect competition and innovation and can be used to achieve certain ends.

The policy options we analyze do not affect the overall inverted-U relationship. However, due to dynamic interaction between market structure and innovation, a policy intended to affect one will automatically impact the other. We analyze three particular policy options: R&D subsidies, change in bankruptcy laws and barriers to entry.

R&D subsidies have a positive effect on the welfare of consumers as well as producers. They also result in more innovation. However, they increase the degree of monopoly in the industry.

A change in bankruptcy laws (or any other policy that makes it harder for incumbents to exit) does not appear to have any significant effect on innovation, market structure and welfare.

Barriers to entry are certainly a harmful policy. In the present context, they reduce innovation, increase the monopoly power and concentration, hurt consumers and reduce overall welfare. The only beneficiaries of this policy are the incumbent producers.

In the light of our analysis, a desirable policy mix would be to combine R&D subsidies with a reduction in the barriers to entry. This mix is expected to increase innovation, reduce market concentration and monopoly power, benefit consumers and increase overall welfare. However, if R&D subsidies are small and entry barriers are reduced drastically, the producers will lose.

The future work could extend this paper in the following dimension. Although writing the model in continuous time makes the computation of
equilibrium a lot easier and each element of the value function is updated very quickly, it does nothing to reduce the size of the statespace. It would be worthwhile to explore some innovative assumptions that free the statespace from the curse of dimensionality as well. If it is achieved, we could construct more complex models and explore richer interactions between market structure and innovations. One such extension could be modelling expanding varieties in this environment.
References


A Appendix: Cournot Equilibrium

Firms differ in their productivities that are inversely related to their marginal costs.

\[ mc_i = g(\omega_i), \]

(12)

with \( g'(\omega_i) < 0 \). However, for numerical computations we use a multiple of Gaussian survival function. Specifically, we use \( mc_i = a \left[ 1 - F(\omega_i, \mu, \sigma) \right] \), where \( F(\cdot, \mu, \sigma) \) is the cdf of a Gaussian distribution with mean \( \mu \) and standard deviation \( \sigma \). We set \( a = 4 \), \( \mu = 5 \) and \( \sigma = 4 \). Demand is given by

\[ p(Q) = D - Q, \]

(13)

where \( Q = \sum_{i=1}^{n} q_i \). Given demand and the vector of marginal costs, equilibrium quantity, price and profits are given by

\[ q_i^* = \max\{0, p^* - mc_i\}, \]

(14)

\[ p^* = \frac{1}{n^* + 1} \left[ D + \sum_{i=1}^{n^*} mc_i \right], \]

(15)

\[ \pi(\omega_i) = \max \left\{ -f, \left[ p^* - mc_i \right]^2 - f \right\}. \]

(16)

Markup is simply given by \( \mu_i = p^*/mc_i \). Let \( L \) denote the Lerner’s index defined by \( L = 1/\mu \). \( L \) is our main measure of endogenous competition in the industry.
Figure 1: Competition and Innovation (for a description of the data, see Hashmi (2005))
Figure 2: Value Function of the Monopolist
Figure 3: Competition and Innovation Intensity
Figure 4: Competition and Innovation (Firm-level)
Figure 5: Competition and Innovation 1 (Industry-level)
Figure 6: Competition and Innovation 2 (Industry-level)
Figure 7: Competition and Innovation (Entire State-Space at the Firm-level)