The Trader’s Dilemma: Trading Strategies and Endogenous Pricing in an Illiquid Market

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Abstract

We investigate a large trader’s trading strategies in a search-based security market, in which all traders are subject to type switching. The large trader has pressure to liquidate her position by the end of the horizon to avoid extra holding costs and faces a tradeoff: if she trades quickly, she moves the price; if she trades slowly, she may not be able to find counterparties in the market. We show that there exists a subgame perfect equilibrium in which she chooses the optimal trading strategies taking into account both the price impact effect and liquidity uncertainty. Asset prices are generated endogenously through a dynamic search and bargaining process and reflect the impact of the large trader’s trades. Small traders, who are price takers and possess no manipulating power, cannot be ignored because their reactions to the big trader’s trading strategy jointly determine the market liquidity. We show that when market liquidity uncertainty is high, the large trader would rather put off trading because it is very costly to induce the small trader to undertake the liquidity risk. When the market liquidity uncertainty is very low, however, she unloads her position quickly despite the price impact incurred. Furthermore, we examine the two effects of liquidity uncertainty and imperfect competition on prices by studying three cases: the benchmark case, the case of a liquid market with a monopolist and the case of a competitive market with liquidity uncertainty. We show that the two effects on prices are nonlinear. When the market is perfectly liquid, there is no price impact from the large trader’s quick trading; however, when the market is illiquid, and the large trader cannot discriminate across small traders, the large trader has to pay a discount for the liquidity uncertainty. Lastly, we explore limiting pricing results by extending the number of small traders and the number of trading periods and show that competitive intuition applies in the model.
1. Introduction

Large traders’ trading behaviour is always of special interest to both academics and industry practitioners. The fact that their orderflow can be large enough to move prices is a significant concern to large institutional investors. This price impact of trading has been verified by many empirical studies to exist in almost all kinds of markets. Disasters can occur when investors find themselves in desperate need to liquidate their long positions but market liquidity suddenly dries up. The most recent well-known occurrence was the LTCM crisis in 1998. Studies of this crisis show that, in addition to poor risk management, it is suspected that LTCM became a victim of predatory trading. Studying the trading behaviour of market makers during the crisis using a unique dataset of audit trail transactions, Cai (2003) infers that market makers exploited their informational advantage on customers’ orderflows (LTCM needed to cover its short position in the treasury bond future market) and front run their customers’ trades.


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1 For example, Holthausen, Leftwich and Mayers (1990) examine price effects associated with block trades by investigating the largest 50 trades for 109 firms traded on the NYSE in 1983 and find that most of the price effects are permanent and related to block size. They report a price impact of around 1 percent. Keim and Madhavan (1996) report an even larger price impact (8 percent) in an up-stairs market. Harris and Piwowar (2004) study transaction costs and trading volumes in the U.S. municipal bond market and find that municipal bond trades are substantially more expensive than similar sized equity trades due to the lack of price transparency.
Market liquidity anomalies have aroused a lot of interest but have not been given convincing explanations. For example, Longstaff (2001) defines an illiquid market as one in which traders are unable to initiate or unwind a position and studies a trader’s optimal portfolio selection problem in such a market. The illiquid market can be regarded as an exogenous trading constraint faced by market participants. However, his model doesn’t provide an explanation how such an extreme situation comes into being, or in other words, why market participants retreat from trading under such circumstances.

Other questions also arise from studying the performance of “large traders”, such as hedge and mutual funds. If large traders have supreme analytical technological skills and information, how could they not consistently “beat the market”? Contrary to popular belief, analyses by Braas and Bralver (2003) on the trading profits of more than 40 large trading rooms throughout the world conclude that speculative positioning cannot be the major source of trading revenues. More often than not, this practice loses money rather than makes money.

We may then infer that a large trader, whose trades impact prices, can either benefit or suffer from her own market power conditioning on market conditions. To better understand interactive effects of large traders’ trading activities on market illiquidity, we need to put a large trader in a model and examine how her behaviour reacts to market conditions.

In this paper, we study a large trader’s trading strategy in a scenario of distressed sale, and show how her trading strategy affects the equilibrium price using a dynamic search-based model of asset trading following Duffie, Garleanu and Pedersen (2004;
henceforth DGP). Trades on an illiquid security with no dividend risk, occur through searching for counterparties and pairwise bargaining over the transaction price. A good example of such a market is the over-the-counter market for derivatives. Investors have heterogeneous initial endowments (i.e., one large trader with two shares vs. two small traders who have a maximum capacity to hold one share each) and bargaining power. They also differ by their intrinsic type, high-type and low-type, in the sense of the valuation of assets they hold. In addition to different valuations on assets, a low-type asset owner, who places a low valuation on the asset she holds, also incurs a holding cost. Therefore, in an economy with symmetric information, only asset owners with low valuation want to sell and non-owners with high valuation want to buy. We assume that their intrinsic types are subject to change over time, which generates uncertainty on their own future types and therefore on the future number of liquidity providers. In this sense there is uncertainty in the future market liquidity. We then are able to study a dilemma often faced by large traders: trade fast and you move the market; wait to trade and the market moves around you.

We show that under symmetric information, the price impact of trades by the large trader arise from a combination of the large trader’s monopolistic market power and the uncertainty of market liquidity in the future periods. We find that the large trader’s trading in the first period incurs a price impact, i.e., she gets a lower price if she sells two shares in one period as opposed to spreading the sale over two periods. Another finding is that when the probability of type switching is small, i.e., the risk of liquidity shock is low, the large agent would like to sell two shares in the first period despite the price impact; when there is high liquidity uncertainty, the large agent chooses to leave all trades to the last period. The reason for this effect is that when liquidity uncertainty is low, small traders are more willing to hold the risky asset for a
longer time and therefore their surplus from trading early is higher. With constant bargaining power, the large trader can claim more from small traders. On the other hand, when there is a high expectation of a liquidity shock across the market, large traders are forced to postpone trading, because it becomes very costly to induce small traders to buy. In the extreme case that small traders stop trading, the large trader’s nightmare turns into reality: nobody is on the other side of the market and liquidity suddenly disappears.

Having obtained the equilibrium strategies and prices, we further examine the two effects of liquidity uncertainty and imperfect competition on prices. We find that these two effects are nonlinear. In order to compare their partial effects on prices, we study three cases: the benchmark case, the case of a liquid market with a monopolist, and the case of a competitive market with liquidity uncertainty. We find that with a perfectly liquid market there is no price impact from the large trader’s quick trading. However, if the market liquidity is limited, even when the large trader cannot discriminate across small traders, the large trader still has to sell at a discount, accounting for the liquidity uncertainty.

The introduction of a large trader into the search model distinguishes our model from the search model with a competitive market in DGP, extended by Vayanos and Wang (2003) and Weill (2003), which all assume identical small traders and focus on steady-state equilibria, without considering the effect of time-varying liquidity risk. In contrast, we introduce a large trader into the model. We solve the model by characterizing the subgame perfect equilibrium of a dynamic search and bargaining game, finding the large trader’s optimal trading strategy and associated prices. The model generates a number of different results. First, the impact of a large trader’s type-switching is different from a small trader’s type-switching in that upon the large
trader’s type-shifting, a significant change occurs to the security’s demand or supply. Second, a large trader is able to choose trading strategies to maximize the liquidation value, which in turn influences future market liquidity. Third, when choosing trading strategies the large trader takes into consideration both the price impact and liquidity uncertainty, which endogenizes illiquidity cost and price impact.

In reality, large traders sometimes hold dominant market power relative to their dealers or other traders and extract more value from the bargaining. Braas and Bralver’s (2003) analyses on trading profits of large intermediations demonstrate that trading profits from market making and from customer business are a function of the relative power of the two trading parties. Green, Hollifield and Schurhoff (2004), estimating a structural bargaining model using transaction data of the U.S. municipal bond market, attribute their finding of decreasing profits on trade sizes to the dealers’ relative market power.

Our results contribute to the market microstructure literature in several ways. For example, we show that even without asymmetric information [Kyle (1985)] or the need to share risk [Vayanos (1999, 2001)] large traders still trade strategically when the market is incomplete and illiquid. Moreover, our study shows that the price impact effect could be magnified by market illiquidity, whilst monopoly power has less of an effect in a more liquid market.

Our model and method also contribute to an increasing literature trying to incorporate liquidity risk into asset pricing by endogenizing illiquidity cost into asset prices. For example, Pritsker (2004) studies a general equilibrium model in which the competitive fringe takes price as given whereas large investors face prices as a function of their own orderflows. Illiquidity in this model stems from imperfect competition only. He is able to derive a multi-factor asset pricing formula, capturing
the imperfect risk sharing by other temporary factors\(^2\) than market risk factor. Acharya and Pederson (2004), on the other hand, explicitly assume a stochastic illiquidity cost and develop a liquidity adjusted CAPM model. Since the stochastic transaction cost is still exogenous, the net-of-transaction-cost returns should satisfy the CAPM in a frictionless economy. Using this insight they are able to derive asset prices in an overlapping generation model. They show that in the liquidity-adjusted CAPM, the expected return of an asset has a four-factor structure with a non-zero constant term representing the expected illiquidity cost. Vayanos (2004) complements Acharya and Pederson (2004) by introducing a link between the liquidity and the volatility. Instead of a time-varying transaction cost assumed by Acharya and Pederson, he assumes a constant transaction cost but time-varying horizon which depends on the volatility of market return. By modeling investors as fund managers subject to performance-based withdrawals, he shows that assets in equilibrium can be priced by a conditional two-factor CAPM adjusted for the transaction cost, with two factors being the market risk and the volatility. Without assuming an exogenous transaction cost, either deterministic or stochastic, illiquidity cost, which arises from both imperfect competition and liquidity uncertainty, is endogenized into prices in our model in the process of search and bargaining. In addition, the existence of a large trader alters the bargaining situation from bilateral bargaining to multilateral bargaining, which brings more complexity to the model but also allows us to better examine how market competition influences prices.

Lastly, our model provides a theoretical base to Longstaff’s (2001) interpretation of an illiquid market. Our results show that there is some probability that there may be no counterparty on the other side of the market, either because of the sudden co-

\(^2\) These risk factors are temporary in that it is the deviations from Pareto optimal asset holdings by large investors that affect asset prices and these deviations will eventually disappear when the investors’ risky asset holdings converge to the competitive levels as time goes to infinity.
switching of traders or because traders are not willing to trade due to the high uncertainty of liquidity. In either case, markets disappear temporarily.

Our work is also related to the literature on market manipulation. For example, Jarrow (1992) investigates market manipulation trading strategies by large traders when their trades move prices. He studies the conditions on the price process, under which large traders generate profits at no risk. Subramanian and Jarrow (2001) study the liquidity cost when a trader’s trades have a price impact and there are execution lags in trading. The differences between their models and ours are as follows. First in their model the price process and price impact function are explicitly assumed, while in our model prices are produced endogenously and price impact exists as a result of imperfect competition. Second, they study the large trader’s trading behaviour in a partial equilibrium model while we study the large trader’s trading behaviour in a search and bargaining game. Finally, there is no liquidity uncertainty in their model.

The rest of the paper is organized as follows. Section 2 describes the basic model. The security market resembles an over-the-counter market, in which traders search to trade. Section 3 analyses the model and describes the optimal trading strategy for the large trader. In this section, we also try to isolate and compare the effects of imperfect competition and liquidity uncertainty on prices by studying extreme cases. In Section 3.5, we briefly describe the case of a monopolistic buyer and show that symmetric results apply based on the same method. Section 4 extends our model to $n$ small traders and $t$ periods and shows how an infinite number of small traders and time periods affect the market liquidity and pricing. Conclusions and further implications are discussed in Section 5. Calculations and proofs can be found in Appendices A and B.
2. The Basic Model

This is a three-date model. People trade at \( t_1 \) and \( t_2 \). No trade takes place at the last date \( t_3 \). Investors can either invest in a perfectly liquid risk-free money market account with a return of \( r \) or an illiquid security in an over-the-counter market, paying a dividend \( D > r \) at date \( t_3 \). The security can only be traded upon the encounter of two traders. Thus it is “risky” not because of any uncertainty on its underlying cash flow but the uncertainty on the availability of trading counterparty on the market.

There are three traders in the market: one big trader with an initial endowment of two shares of the illiquid security; two small traders with either one share of this security or \( M \) dollars, \( D < M < 2D \), as initial endowment. Borrowing or short sale is not allowed. Investors are risk neutral. They are heterogeneous in their intrinsic types: high (\( h \)) or low (\( l \)). We assume that when a low-type investor owns a share of the illiquid asset, she incurs a cost of \( \varepsilon \) such that \( D - \varepsilon < r \); while a high-type owner does not incur this cost. This captures the incentive of liquidation for low-type owner.

In addition, investors’ intrinsic types are subject to changes. The switching rate from high-type to low-type is \( \rho \) per unit of time, and the opposite switching rate from low to high is \( \rho \) per unit of time. The investor types and changes capture the effects of several situations. For instance, a) a liquidity shock, i.e., the need for cash; b) a risk management requirement, eg., to meet the VaR restriction or hedging needs; c) low utilities for an asset, eg., a low expectation of future dividend flow.

Therefore an investor’s type is drawn from the set \{high-type owner, high-type non-owner, low-type owner, low-type non-owner\}, which is denoted as \( I = \{ho, hn, lo, ln\} \).

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3 Note that the number of shares held by an agent doesn’t necessarily mean the exact number of shares. It can represent any number of shares. What the numbers try to capture here is that a large trader owns significantly more shares than a small trader.
When the intrinsic type of an investor switches from *high* to *low*, the investor’s valuation of the asset becomes lower and he wants to liquidate the asset. Similarly when an investor’s type switches from *low* to *high*, he may want to buy the risky asset and consume the dividend at the end. Therefore the asset transfers from low-type owners, i.e., sellers, to high-type non-owners, i.e., buyers. In equilibrium, we shall see only the low-type asset owners sell to the high-type non-owners. Low-type non-owners or high-type owners would have no incentive to trade.

The market, however, is decentralized in the sense that buyers and sellers are separated. Although any two agents are free to trade the security whenever they meet, they have to search for a counterparty and bargain over the price. The timing of the model is as follows.

At the beginning of each date, each trader recognizes her type and endowment and decides whether to trade in this period or not. The agent who decides to trade contacts some other agents at random. The contact and match processes of agents are pairwise and independent. Once she is matched with another trader, they bargain over the transaction price. If they reach an agreement, the transaction occurs. From the time after transaction to next date they can trade, their intrinsic types are subject to changes. By the next date, they learn their new types and trade if necessary. The timing is clearly demonstrated in Figure 1.

**Figure 1. Timing of type switching and trade**

![Figure 1](image-url)
Firstly, there are several things about investors that should be emphasized. The investors are heterogeneous in two ways. First, they are heterogeneous in their initial endowments. The large trader is endowed with two shares while the small traders with only one share. Second, they are heterogeneous in their bargaining power. We assume small traders’ bargaining power is \( q, \ 0 \leq q \leq 1 \) and the large trader’s bargaining power is \( 1-q \). The bargaining power partly captures the idea of the “market power”, which translates the market power of an agent into the control of bargains. In this model, the “market power” is also reflected in how many shares a trader owns. We assume the large trader can search and bargain with more than one trader in the market at the same time. For example, the large trader can announce a price to several or all potential counterparties and wait for counter offers. They bargain until an agreement is reached. In this case, the large trader acts as a market-maker. Since the large trader has more than one share and is assumed to have the advantage to move first, she can choose the trading strategy to her best interests by spreading the sales, for example in this simple case, selling one share in each period, or concentrating the sales, i.e., selling two shares in one period, \( t_1 \) or \( t_2 \). In order to study a particular case of depressed sale by the large trader, we assume that the large owner’s type, once it jumps down to low type, cannot switch back to high-type unless she unwinds her long position.\(^4\) With this assumption, the distressed large trader, who suffers a liquidity shock suddenly, cannot take her chances by doing nothing and hoping the situation will improve itself.

Secondly, we need to specify the matching and bargaining process in more detail. Buyers and sellers are randomly matched and bargain over prices. They don’t know

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\(^4\) This can be justified by situations such as a) a large trader facing margin calls from her broker; b) a fund manager facing sudden withdrawals from fund holders; and c) a risk manager facing the binding constraint, eg., VaR constraint. In all these cases she has to liquidate at least part of her position to meet the cash need. She cannot wait for the situation to improve by itself.
with whom they will meet in the market place but they do have a whole picture of the
distribution of all traders. The traders are anonymous in the sense that their identities
are not revealed until they are matched. However, they are also aware of the
existence of a large trader, her type and the numbers of traders in all four categories.
The assumption is plausible because large traders always attract more attention and
they can be identified from their trading activities, regulatory revelations and even
from rumors. If someone intends to trade with some specific trader, she can take her
chances by rejecting all traders before that specific one appears in the market. But it
is not worth doing because she may neither be matched with the one she wants to
trade with nor be able to trade with her favorite one if they cannot reach an agreement.
This is, basically, a non-cooperative game.

If in period $t$ a seller and a buyer fail to reach an agreement, they remain in their
own categories until period $t + 1$, when they participate in the market again given that
their types keep unchanged. We assume that investors have short memories about
with whom they’ve been matched and negotiated before. In this sense, they cannot
predict who they will meet next so that they cannot discriminate between different
traders.

When they are matched, price is determined through bargaining. We use the Nash
bargaining solution to model the outcome of the negotiation. In any pairwise
negotiation the buyer and the seller split the joint surplus of trading according to their
bargaining power. The price is given by

$$P(t) = q_{lo}\Delta V_i(t) + q_{hn}\Delta V_h(t)$$  \hspace{1cm} (1)

where $q_{lo}$ and $q_{hn}$ are the bargaining powers of low-type owner and high-type non-
owner respectively, and $\Delta V_i$ and $\Delta V_h$ refer to the reservation values of $lo$ and $hn$
traders respectively.
Lastly, we assume the information is symmetric in that no one can hide her identity when she enters a negotiation. This distinguishes our model from many market microstructure models, such as Kyle (1985) and others, in which the information asymmetry is the major incentive for some market participants to trade strategically.

Next we give the definition for a subgame perfect equilibrium of the model and solve the model by finding the optimal trading strategy and associated prices.
3. Analyses of the Model

To focus our attention on the distressed large trader’s trading behavior, we only study in detail the case that at $t_1$ the large trader is the only low-type owner with two shares to sell while the other two small traders are both high-type non-owners, who each would like to buy one share.\(^5\) Figure 2 describes the dynamics of population structure, which evolves according to trades and type switching.

Before we define the subgame perfect equilibrium of this dynamic game, we would like to explain the structure of Figures 2 and 3 in more detail. In these two figures, a cluster of ovals represents a mass configuration at some specific time. The following illustration is the leftmost cluster of ovals taken from Figure 2, providing the information regarding how many traders in each category and how many shares held by each of them.

\[
\begin{array}{c}
\text{ho} \\
0 \\
\hline
\text{lo} \\
1*2 \\
\hline
\text{hn} \\
2*1 \\
\hline
\text{ln} \\
0
\end{array}
\]

In each oval there are letters and numbers describing the type and the number of traders of this type and shares they hold respectively. Take the above oval cluster for example, before trading at $t_1$ there are one lo (low-type owner) type trader with two shares, i.e., 1*2, and two hn (high-type non-owner) traders with one share each (2*1). There is no trader in other two categories: ho (high-type owner) and ln (low-type non-owner). The number to the left of the cluster denotes the subgame (I). Since the only

\(^5\) Two similar cases can be studies in the same way. One case is the large trader is the only seller, whilst two small traders are a high-type non-owner and a low-type non-owner respectively. The other case starts with the large trader being the only agent in low-type-owner category and the two small traders being low-type non-owners. In this case, there is no trade in the first period but there could be trades in the second period. To avoid the repetition, we do not study these two cases explicitly in the paper.
lo trader has two shares, making her the large trader in our model, she can choose to trade one share, two shares or not at all. This is shown in Figure 2 as three branches leading to three mass distributions after trading at \( t_1 \). Before the next trading date arrives, traders’ types are subject to change. This is represented in the figure as dotted lines leading to possible mass distributions at \( t_2 \). On arriving \( t_2 \) they find themselves in one cluster of ovals. In some cases there will be no chance to trade any more. For those with trading opportunities we number them (i to vi) and discuss them in detail below. Each one of these cases develops to a subgame starting from \( t_2 \). For instance, the mass configuration highlighted in the rectangle evolves to the subgame demonstrated in Figure 3. We see that even with two periods, the structure of the game can become very complicated.

At the beginning of each period the large trader chooses the optimal strategy to maximize her expected payoff at the last date. Since she has two shares to sell and she can sell only one share to a buyer upon one encounter, her major concern is “when” to sell and “how many” at each period. Selling quickly, for example, two shares in the first period, she is afraid that she has to sell them at a low price, but then the pressure of liquidation is gone and the payoff is guaranteed. Employing the strategy of smoothing the sales across two periods may be conducive to a higher transaction price by exploiting the large trader’s monopolistic power, but the uncertainty of not being able to find a buyer in a later period increases (due to the type switching of \( hn \) traders). Portfolio managers have always faced the question of how to balance the price impact of trades and the possibility of market deterioration.

Next we define the equilibrium outcome to a dynamic matching and bargaining process.
**Definition:** An outcome profile consists of a trading strategy profile and associated transaction prices \((\psi(t), P(t))\). An *equilibrium outcome profile* \((\psi(t)^*, P(t)^*)\) is an outcome profile such that for a trader configuration at each time, given split-the-difference negotiations, the large trader cannot improve her expected payoff by adopting any other strategy profile \((\psi(t)', P(t)')\), and no small trader can improve his expected payoff in a pairwise negotiation with the large trader.

Note that there are several differences between this model and the search models in DGP and in Vayanos and Wang (2003). Firstly, this model studies a dynamic process of the liquidation and associated asset prices while their models study a search market in the steady state; Secondly, agents in our model are heterogeneous not only in their intrinsic types but in their initial endowments, while in DGP model agents are identical except their types and in Vayanos and Wang (2003) agents are also heterogeneous in their horizons, i.e., different preferences to liquidity; Thirdly, they assume a continuum of traders but we do not assume that because otherwise the large trader can always find counterparties and liquidate her position. We study in Section 4 extensions of the model to \(n\) small traders and \(t\) periods such that \(n\) and \(t\) are allowed to go to infinity. The results verify our claims here that the market is always liquid in the sense that the large trader can liquidate her position at any speed or at any time she wants. This fact is also critical in that since a single trader’s activity cannot be ignored the market is not perfectly competitive, i.e., the large trader can discriminate across small traders so that she is not a price taker, and the price is therefore affected by traders’ strategic activities.

Without the convenient properties of the steady state, we have to start by analyzing payoffs to different strategies in the last period and work backward to find an equilibrium path. In addition, the existence of a big trader alters the bargaining
situation from the bilateral bargaining framework to the multilateral bargaining framework. In addition to our model employing different technology relative to other models, we expect that the implications derived from our model will be quite different too. For example, the liquidity premium reflects not only a trader’s ease of searching and matching with a counterparty and his bargaining power (DGP) or exogenous transaction cost [Amihud and Mendelson (1986), Vayanos (1998) and others], or asymmetric information on initial endowments or endowment shocks [Vayanos (1999, 2001)], it also reflects the price effect of strategic considerations.

Below we obtain the equilibrium profile through backward induction. We briefly summarize the approach as follows.

The second period: Starting from a possible node at time $t_2$ (e.g., Figure 3), we first calculate the large trader’s expected payoffs corresponding to different trading strategies. Comparing her expected utilities across different trading strategies, the optimal trading strategy can be identified for this subgame. Note that we need to apply the requirement for equilibrium at each step to make sure it is the equilibrium strategy in each subgame such that no one wants to deviate. In our case, in equilibrium, the large trader chooses the strategy with which small traders are willing to play.

The first period (Figure 2): Repeat the above steps with expected payoffs at $t_3$ replaced by value functions for all traders at each possible node that reaches $t_2$. Note that the whole branch demonstrated in Figure 3 now collapses to the right uppermost “node” in the rectangle in Figure 2. The combination of the optimal trading strategies employed in the first and the second period and the associated transaction prices

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6 The bargaining situation discussed in this model, i.e., one large trader vs. several small traders, is similar to the multilateral bargaining game of employment by Stole and Zwiebel (1996).
constitute the equilibrium outcome profile for this dynamic match and bargaining game.

In order to demonstrate how the large trader chooses one trading strategy over another and how the price is determined in a multilateral bargaining game, we analyze in detail the subgame illustrated in Figure 3. Other subgames can be analyzed in the same way and the calculations are provided in Appendix A.

3.1 The Analysis of Subgame (i)

Upon the arrival at $t_2$ the type of each trader is revealed, as shown in Figure 3. The large trader is the only seller who wants to liquidate two shares before date $t_3$. She observes there are two traders in the market seeking to buy one share each. Before she is matched with another trader, she considers all possible strategies she could use to maximize her expected payoff at date $t_3$. The fact that she incurs an extra holding cost $\epsilon$ for holding one share at the end motivates her to sell it as long as the price is acceptable. For the two small buyers, they are eager to buy a share since this is the last chance to earn a higher expected return than $r$. The definition of the equilibrium outcome requires that for any small buyer in an $n$-buyer-one-seller negotiation game the net surplus is split between the large seller and that small buyer according to their bargaining power, thereby satisfying

$$q_{hn}(V_{lo}(n,t) - V_{lo}(n-1,t)) = q_{lo}(V_{hn}(1,t) - P(n,t))$$

(2)

where $V_{lo}(n,t)$ denotes the value function of a low-type owner who sells $n$ shares at time $t$ and $V_{hn}(1,t)$ denotes the value function of a high-type non-owner who buys one share at time $t$. $P(n,t)$ is the price paid by all buyers for acquiring one share.
Equation (2) states that the large seller will keep selling until the portion of the marginal profit from selling the $n$-th share given up to a buyer is equal to the portion of the gain that can be claimed from the marginal buyer.

Consider the outcome if the large seller only sells one share. The net surplus the buyer receives from buying one share is $\frac{1}{r} \left[ \rho_d \Delta t (\bar{D} - \varepsilon) + (1 - \rho_d \Delta t) \bar{D} \right] - P(1, t_2)$; the net surplus the seller obtains from selling one share is $P(1, t_2) - \frac{1}{r} \left( D - \varepsilon \right)$. They split the total net surplus according to their bargaining power. According to equation (2), the price is determined by

$$q \left[ P(1, t_2) - \frac{1}{r} (\bar{D} - \varepsilon) \right] = (1 - q) \left[ \frac{1}{r} (\bar{D} - \rho_d \Delta t \varepsilon) - P(1, t_2) \right]$$

$$\Rightarrow P(1, t_2) = q \left[ \frac{1}{r} (\bar{D} - \varepsilon) \right] + (1 - q) \left[ \frac{1}{r} (\bar{D} - \rho_d \Delta t \varepsilon) \right]$$

We must check to ensure that the outcome to this bargaining process does not violate the traders’ outside option constraint. Intuitively the joint net surplus from the buyer and the seller engaging in a pairwise negotiation should be positive; otherwise no agreement will be reached. The total net surplus is $\varepsilon (1 - \rho_d \Delta t) / r$, which is greater than zero because the probability of type switching down during a short time interval $\Delta t$ is less than one. We also want to check whether the value function of each trader after trading is larger than her value function of adopting an alternative strategy.

$$V_{lo}(1, t_2) = P(1, t_2) + \frac{\bar{D} - \varepsilon}{r} = (1 + q) \frac{\bar{D} - \varepsilon}{r} + (1 - q) \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r}$$

which is greater than the value function of “no trade”, i.e., $V_{lo}(0, t_2) = \frac{2(\bar{D} - \varepsilon)}{r}$.

And $V_{hn}(1, t_2) - V_{hn}(0, t_2) = \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} - P(1, t_2) = q \frac{r}{r} (1 - \rho_d \Delta t) \varepsilon > 0$.  

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Now consider the outcome that the large seller plans to sell two shares. She searches and contacts with two traders. The price results from

\[ q \left( 2P(2,t_2) - P(1,t_2) + \frac{\bar{D} - \varepsilon}{r} \right) = (1-q) \left( \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} - P(2,t_2) \right) \]  

\[ \Rightarrow P(2,t_2) = q \left( \frac{1}{r}(\bar{D} - \varepsilon) \right) + (1-q) \left( \frac{1}{r}(\bar{D} - \rho_d \Delta t \varepsilon) \right) = P(1,t_2) \]

The value function of the seller selling two shares is

\[ V_{lo}(2,t_2) = 2P(2,t_2) = 2q \left( \frac{1}{r}(\bar{D} - \varepsilon) \right) + 2(1-q) \left( \frac{1}{r}(\bar{D} - \rho_d \Delta t \varepsilon) \right) \]  

\[ \text{(4)} \]

Comparing \( V_{lo}(1,t_2) \) and \( V_{lo}(2,t_2) \), we get

\[ V_{lo}(1,t_2) - V_{lo}(2,t_2) = (1-q) \left[ \frac{\bar{D} - \varepsilon}{r} - \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} \right] < 0 \quad \text{iff} \quad \rho_d \Delta t < 1 \]  

\[ \text{(5)} \]

Therefore in this subgame the optimal strategy for the large trader is to sell two shares at \( P(2,t_2) \) and each \( hn \) trader buys one share at this price. The small \( hn \)-type traders are always willing to buy as long as the probability of their types switching from high-type to low-type is strictly less than one.

Several points are worth further discussion.

First, of particular interest is equation (3), which can be rewritten as

\[ q \left[ P(2,t_2) - \frac{\bar{D} - \varepsilon}{r} \right] + \left[ P(2,t_2) - P(1,t_2) \right] = (1-q) \left( \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} - P(2,t_2) \right) \]  

\[ \text{(6)} \]

The LHS shows us that by selling an additional share the seller not only gains \( P(2,t_2) - (\bar{D} - \varepsilon)/r \) on the margin, but also incurs the price impact captured by \( P(2,t_2) - P(1,t_2) \). This secondary price effect is crucial in our analysis that when the large trader sells an additional share she has to take into consideration the effect on the price of her own trading. Note that in this very subgame in the last period, the price of selling one share is the same as the price of selling two shares. This is so...
because of the-last-period effect. Traders in the last period know this is the last chance to trade so that the motivation to trade is stronger. For the large seller the marginal costs of selling one share and selling two shares are the same because the payoff of holding one share at time $t_3$ is fixed at $\bar{D} - \varepsilon$. As we will show shortly, this is not the case in subgames starting at time $t_1$. The marginal costs of selling one share and of selling two shares are different due to the additional uncertainty of being matched and trade in a later period. More specifically, the trading strategy at $t_1$ involves consideration of such dilemma: selling too fast you may move the price; selling slowly, you may not be able to trade later. Such dilemma is not presented in the last period since there is no further chance to trade and every trader has taken it into account when choosing trading strategy at $t_2$.

Secondly, we restrict our attention to Markov perfect equilibrium, that is, the subgame perfect equilibrium in which the strategies are independent of the history of the game. In other words, traders cannot condition their behavior in a bargaining encounter on their previous bargaining experience. We have assumed that at the beginning of each date, all that the buyers and sellers know is the numbers of agents remaining in each category. Note that the assumption of anonymity does not conflict with the small finite number of traders in our model. What is implied is that for the large seller she cannot tell the two small traders apart. What she cares about is how much she can extract from a marginal buyer’s expected utility, which in turn depends on the configuration of market participants. For a small buyer, if he is matched, which cannot be predicted, he becomes the marginal buyer. If he is not matched, he remains in the pool of buyers. The value function of a marginal buyer $V_{ho}(1)$ is the same for any small buyers. Therefore the payoff to any trader depends on her own

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7 This is also referred to as the “anonymity” assumption. It describes the agents’ information. Further discussion can be found in Osborne and Rubinstein (1990) and Gale (2000).
strategy and the distribution of other traders’ strategies. This assumption allows us to analyze the impact of a single agent’s trading strategies on the equilibrium price profile in a relatively simple framework as the number of traders becomes large. In addition, this assumption is not implausible. The assumption that small traders are anonymous in a financial market is a plausible assumption.

3.2 The Analysis of Subgame at $t_1$

The analyses of other subgames beginning at $t_2$ are similar to the above subgame we discuss in detail. We leave the calculation in Appendix A. After obtaining the value function of every trader at $t_2$, we calculate the expected payoff to each trader at $t_1$ corresponding to each strategy that the large trader may choose. This is the standard backward induction approach used in finite dynamic games.

The extensive-form game described in Figure 2 gives a hint how many possible situations investors may end up in even though there is only two trading periods. Basically when the large trader decides a trading strategy at time $t_1$, she has to consider all possible payoffs. But once she chooses one trading strategy at $t_1$, the game is greatly reduced. For example, if she chooses to sell two shares in the first period, she sells two shares at

$$P(2,t_1) = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2(1+q)} \left[ q^2(1+q) + \rho_d \Delta t \left( 2 + 3q^2 - 4q^2 - q^3 \right) + \rho_o^2 \Delta t^2 \left( 1 - q \right)(q-1)(2-q) - \rho_o^2 \Delta t^3 \left( 1 - q^2 \right) + \rho_d \rho_o \Delta t^2 (q-1) - 2q(1-q) \left( \rho_d^2 \rho_o^2 \Delta t^4 + \rho_o^3 \rho_o \Delta t^4 + \rho_d \rho_o^2 \Delta t^3 \right) \right]$$

and her expected payoff is
\[ V_{b}(2,t) = \frac{2D}{r^2} - \frac{2\varepsilon}{r^2} q^2 + \frac{2+3q-4q^2-q^3}{1+q} \rho_d\Delta t - \frac{(1-q)^2(2-q)}{1+q} \rho^2\Delta t^2 - \frac{q(1-q^2)}{1+q} \rho^3\Delta t^3 \\
+ \frac{(q-1)(2+q)}{1+q} \rho_d\rho_u\Delta t^2 + \frac{(1-q)^2}{1+q} \left( \rho^2_d\rho^2_u\Delta t^2 + \rho^2_d\rho_u\Delta t^4 + \rho_d\rho^3_u\Delta t^4 \right) \]

(8)

If she decides to sell two shares in two periods, she sells one share at \( t_1 \) at

\[ P(1,t) = \frac{D}{r^2} - \frac{\varepsilon}{r^2} \left[ q^2 + (2-q-q^2)\rho_d\Delta t + (2q-q^2-1)\rho^2_d\Delta t^2 - (q-q^2)\rho^3_d\Delta t^3 + (2q-1)\rho_d\rho_u\Delta t^2 \right] \]

(9)

and sells the other share at \( t_2 \) at a price depending on the specific type configuration evolved till that time. Her payoff becomes

\[ V_{b}(1,t) = \frac{2D}{r^2} - \frac{\varepsilon}{r^2} \left[ q(1+q) + (5-4q-q^2)\rho_d\Delta t + (1-q)(2-q)\rho^2_d\Delta t^2 - (1-q^2)\rho^3_d\Delta t^3 - 2(1-q)\rho_d\rho_u\Delta t^2 \right] \]

(10)

If she decides to leave two shares to the last period, her expected payoff at \( t_1 \) would be

\[ V_{b}(0,t) = \frac{2D}{r^2} - \frac{2\varepsilon}{r^2} \left[ q + (1-q)(2\rho_d\Delta t - \rho^3_u\Delta t) \right] \]

(11)

Since the probability of type switching during a short period \( \Delta t \) is pretty small, e.g., \( \rho_d \) and \( \rho_u \) are both less than 1 and \( \Delta t \) can be set to less than one day, the higher order effects of \( \rho_d\Delta t \) or \( \rho_u\Delta t \) on prices are negligible. In order to facilitate the comparison of prices and value functions, we ignore higher order terms containing second and higher orders of \( \rho_i\Delta t, i = u, d \). \( P(1,t) \) and \( P(2,t) \) then can be rewritten as

\[ P(1,t) \approx \frac{D}{r^2} - \frac{q^2\varepsilon}{r^2} - \frac{\rho_d\Delta t\varepsilon}{r^2} \left( 2-q-q^2 \right) \]

(12)

and

\[ P(2,t) \approx \frac{D}{r^2} - \frac{q^2\varepsilon}{r^2} - \frac{\rho_d\Delta t\varepsilon}{r^2} \left( 2+3q-4q^2-q^3 \right) \]

(13)
The value functions of the large trader engaging in different trading strategies become

\[
V_{ls}(1, t_i) = \frac{2D}{r^2} - \frac{e}{r^2} \left[ q(1+q) + (5-4q-q^2)\rho_d\Delta t \right]
\]

(14)

\[
V_{ls}(2, t_i) = \frac{2D}{r^2} - \frac{2q^2e}{r^2} - \frac{2\rho_d\Delta t\varepsilon}{r^2} \frac{2+3q-4q^2-q^3}{1+q}
\]

(15)

\[
V_{ls}(0, t_i) = \frac{2D}{r^2} - \frac{2\varepsilon}{r^2} \left[ q + 2(1-q)\rho_d\Delta t \right]
\]

(16)

Comparing the payoffs to three trading strategies, she chooses the optimal strategy with the highest expected payoff. The following theorem states the optimal strategy of the large trader.

**Theorem 1.** There exists a unique subgame perfect equilibrium in this game, in which the large trader’s trading strategy depends on the probability of type switching. When \( \rho_d\Delta t \) is greater than some value \( f(q) = \frac{q+1}{q+3} \), the large trader leaves the entire trading to the last period; otherwise she chooses to sell two shares in the first period.

This is directly from the comparison of the expected payoffs to three trading strategies. We first compare \( V_{ls}(1, t_i) \) and \( V_{ls}(2, t_i) \). When \( 0 \leq q \leq \sqrt{5} - 2 \), \( V_{ls}(1, t_i) \leq V_{ls}(2, t_i) \); when \( \sqrt{5} - 2 < q \leq 1 \), \( V_{ls}(1, t_i) > V_{ls}(2, t_i) \) if and only if \( \rho_d\Delta t \leq \frac{q(1+q)}{q^2+4q-1} \). Comparing \( V_{ls}(1, t_i) \) and \( V_{ls}(0, t_i) \), we find that \( V_{ls}(1, t_i) \geq V_{ls}(0, t_i) \) if and only if \( \rho_d\Delta t \leq \frac{q}{1+q} \). Similarly, \( V_{ls}(2, t_i) \geq V_{ls}(0, t_i) \) if and only if \( \rho_d\Delta t \leq \frac{q+1}{q+3} \). Denote \( x = \frac{q(1+q)}{q^2+4q-1} \), \( y = \frac{q}{1+q} \), \( z = \frac{q+1}{q+3} \). It is easy to see that \( x \geq z \geq y \) for \( q \in [0,1] \).
Figure 4 draws the $x$, $y$, $z$ as functions of $q$. When $1 \geq \rho_d \Delta t \geq x$, $V_{lo}(0,t_1) > V_{lo}(1,t_1) \geq V_{lo}(2,t_1)$; when $z \leq \rho_d \Delta t < x$, $V_{lo}(0,t_1) \geq V_{lo}(2,t_1) > V_{lo}(1,t_1)$; when $y \leq \rho_d \Delta t < z$, $V_{lo}(2,t_1) > V_{lo}(0,t_1) \geq V_{lo}(1,t_1)$; when $\rho_d \Delta t < y$, $V_{lo}(2,t_1) > V_{lo}(1,t_1) > V_{lo}(0,t_1)$. Therefore, when $\rho_d \Delta t \geq z$, i.e., the value of $\rho_d \Delta t$ locates above the dashed line, leaving all sales in the second period is the optimal strategy to the large trader. When $\rho_d \Delta t < z$, the large trader prefers a quick sale in the first period.

Figure 4. Trading strategy and probability of type switching

![Figure 4](image.png)

**Proposition 1:** (i) The equilibrium price $P$ is decreasing in $\varepsilon$ and $\rho_d \Delta t$. Further,$$
\frac{\partial P(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial P(1,t_1)}{\partial (\rho_d \Delta t)}, \text{ with equality as } q = \{0,1\};$

(ii) The expected payoffs to both large and small traders are decreasing in $\rho_d \Delta t$, i.e.,$\frac{\partial V_{lo}(\cdot,t_1)}{\partial (\rho_d \Delta t)} \leq 0, \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} \leq 0$. Further,$\frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} \leq 0$ when $\sqrt{5} - 2 \leq q \leq 1$; $\frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} \leq 0$ when $0 \leq q < \sqrt{5} - 2$. 

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The above results are quite intuitive (a proof is provided in Appendix B). The higher the holding cost to a low-type owner, the less attractive the asset becomes, therefore the lower the price. The equilibrium prices are also decreasing in the type-switching probability. The price has to be lower when the probability of type switching is high. On the one hand, the likelihood of finding a trading counterpart in the later period diminishes; on the other hand, the high-type buyer hesitates because of the high possibility of falling into the category of low-type owner. Both worries push up the price.

Not surprisingly, the value functions of both the large trader and small traders are sensitive to the change of $\rho_d \Delta t$. The sensitivities of different strategies depend on the relative bargaining power $q$. When $q \in \left[\sqrt{5} - 2, 1\right]$, or $1-q$ is less than $1 - \left(\sqrt{5} - 2\right) \approx 0.764$, the strategy of selling two shares in the first period is more sensitive to the type-switching probability than the strategy of selling one share in each period because the impact of $\rho_d \Delta t$ on prices is more prominent when employing the sell-two-in-the-first-period strategy. As the large trader’s bargaining power grows beyond some certain level, i.e., 0.764, the increase of $\rho_d \Delta t$ has less impact on $V_{lo}(2,t_1)$ than on $V_{lo}(1,t_1)$ because the large trader has greater compensation from bargaining. Anyway, the total effects of higher probability of type-switching tend to lower the trader’s value function, which is plausible if we interpret $\rho_d \Delta t$ as one cause of the future liquidity uncertainty.

3.3 Equilibrium Prices and Liquidity Discounts

One interesting feature of this model is that the effects of liquidity uncertainty and imperfect competition on prices are both nonlinear. Although it is impossible to isolate each effect completely, it is still worthwhile to examine and compare their
partial effects on prices. We first study the benchmark case where traders behave competitively in a market with no liquidity uncertainty. We then study two cases: a liquid market with a monopolist and a competitive market with liquidity uncertainty. Comparisons of the prices obtained in these two cases with the prices produced in our original model are also performed.

**Benchmark Case**

The benchmark market is defined as a perfectly competitive and liquid market. This can be approximately achieved by assuming a zero switching rate and an equal number of low-type owners and high-type non-owners with equal endowments. Although prices are still determined in pairwise bargaining, competitiveness drives the price to such a level that traders are indifferent to trading today or tomorrow. Therefore, the transaction prices in the two periods are given by:

\[
P^e(t_1) = \frac{\bar{D} - q\epsilon}{r^2} \quad (17)
\]

\[
P^e(t_2) = \frac{\bar{D} - q\epsilon}{r} \quad (18)
\]

The following two propositions state how liquidity uncertainty and lack of competition alone affect prices. Both propositions are proven in Appendix B.

**Proposition 2 (no liquidity uncertainty):** As the type switching rate \( \rho_{d/u} \) goes to zero, or the time interval \( \Delta t \) goes to zero, price impact goes to zero such that \( P^l(1,t_1) = P^c(2,t_2) = \frac{\bar{D} - q^2\epsilon}{r^2} \). In this case the large trader sells two shares in the first period.

It is not surprising that \( P^l(t_1) \), the transaction price at \( t_1 \) in a liquid market, is greater than \( P^e(t_1) \), the transaction price in the benchmark case, which reflects the large trader’s monopoly status in the market. When traders do not need to worry
about their type switching, or they have very frequent trading opportunities such that the probability of type switching between two trading opportunities is ignorable, the prices are totally decided by traders’ relative market power, which is accordingly decided by their relative bargaining power in a pairwise bargaining. This result is somewhat consistent with the Coase conjecture in the study of durable goods monopoly. In the presence of liquidity uncertainty, the monopolist is unable to credibly commit to the second-period price (her trading strategy in the second period depends on how many counterparties she can find in the market, which is uncontrollable and unpredictable in the first period), thus her trading strategy profile is not deterministic in the first period and her sale in the second period becomes the competitor to her sale in the first period. In the case of no liquidity uncertainty, the monopolist can then fully execute her monopoly power by credibly committing to the second-period price. As one may expect, the monopolist sells very quickly when she can set the price path.

The case of zero switching rates can also be interpreted as a perfectly liquid market in the sense that there are always sufficient competitive small traders on the other side of the market, willing to buy at the price set up by the large trader. Although the price in the second period, \( \frac{(D - q\varepsilon)}{r} \), is lower than the price in the first period, \( \frac{(D - q^2\varepsilon)}{r} \), small traders are still willing to buy in the first period due to the competition.

**Proposition 3 (competitive market):** If the market is competitive, in the sense that the large trader cannot discriminate across small traders conditional on her trading with high-type small traders, employing different trading strategies results in different prices. The large trader’s trading strategy depends on the relationship between the
dividend payoff, the holding cost, the relative bargaining power and the rate of type switching.

Since the large trader is unable to discriminate across small traders who are high-type non-owners, i.e., she offers a price at which small traders are indifferent between trading and not trading. Recall that in an imperfectly competitive market the large trader can discriminate across small traders and negotiate as if she can move along their supply curves. That’s why a small trader, when entering a pairwise bargaining, only compares her value function of trading to her reservation value, but not to her peer’s value function.

Note that the large trader still can choose her trading strategy in a competitive market. If she decides to sell one share, she contacts with a small trader randomly and negotiates the transaction price. The price must be such that the small trader is indifferent between trading with her and staying as a high-type non-owner. If trade occurs, the small trader’s value function is \( V_{ho}(t_i) - P^C(1, t_i) \); if there is no trade, her value function will be the same as the other small trader who has not been contacted, i.e., \( V_{hn}(t_1) \). The price is hence determined by

\[
V_{ho}(t_i) - P^C(1, t_i) = V_{hn}(t_1)
\]

(19)

\[
P^C(1, t_i) = \frac{\overline{D}}{r^2} (1 - \rho_d \Delta t) - \frac{\varepsilon}{r^2} \left[ \rho_d \Delta t - \rho_d^2 \Delta t^2 - \rho_d \rho_u \Delta t^2 + q \left( 1 - 3 \rho_u \Delta t + \rho_d^2 \Delta t^2 + \rho_u^2 \Delta t^3 \right) \right]
\]

(20)

If the large trader tries to sell two shares, she contacts both of the small traders. Again the price is set as such that both the small traders are indifferent between trading and not trading. To determine small traders’ value function of no trade, we need the following lemma.
Lemma 1: In a competitive market, small traders’ response functions to the large trader’s trading strategy are the same, i.e., they would either trade or not trade with the large trader at the same time.

For the large trader, the identical responses from small traders imply that she cannot sell the marginal share at a price different to the infra-marginal ones. The large trader’s value function, conditional on both small traders trading, is \( V_{lo}^C(2,t_1) \) and her value function, conditional on neither small traders trading, is \( V_{lo}^C(0,t_1) \). For a small trader the gain from trading is \( V_{ho}(1,t_1) - P^C(2,t_1) \). Bargaining with both gives us the price

\[
q \left[ V_{lo}^C(2,t_1) - V_{lo}^C(0,t_1) \right] = 2(1-q) \left[ V_{ho}(1,t_1) - P^C(2,t_1) \right]
\]  

(21)

where \( V_{lo}^C(2,t_1) \) and \( V_{lo}^C(0,t_1) \) are value functions of the large trader engaging in selling two shares and zero share in a competitive market, and \( P^C(2,t_1) \) is the first-period transaction price in a competitive market. \( V_{lo}^C(2,t_1) \) and \( V_{lo}^C(0,t_1) \) are given by equations (A.7) and (A.5) respectively as in the non-competitive case we discuss above. Equation (21) tells us in a competitive market the two sides of a bargain are both representatives of their types and bargain in an aggregate form.

Substituting \( V_{lo}^C(2,t_1) \) and \( V_{lo}^C(0,t_1) \) into equation (21), we get the price \( P^C(2,t_1) \).

\[
P^C(2,t_1) = \frac{D}{r^2} - \frac{q^2 \epsilon}{r^2} - \frac{\epsilon}{r^2} \left[ 2(1-q^2) \rho_u \Delta t - (1-q) \rho_u^2 \Delta t^2 - q(1-q) \rho_d^3 \Delta t^3 
\right. 
\left. -(1-q)^2 \rho_u \rho_d \Delta t^2 - q(1-q) \left( \rho_u^2 \rho_d^2 \Delta t^3 + \rho_u \rho_d^3 \Delta t^4 + \rho_d^3 \rho_u \Delta t^4 \right) \right]
\]  

(22)

In order to compare \( P^C(2,t_1) \) and \( P(2,t_1) \), we ignore the second and higher order terms of \( \rho_i \Delta t, i = u, d \). Then \( P^C(2,t_1) \) becomes
\[ P^c(2,t_1) \approx \frac{\bar{D}}{r^2} - q^2 \frac{\epsilon}{r^2} - \frac{2(1-q^2)\rho_d \Delta t \epsilon}{r^2} \]  

(23)

It is easy to show that \( P^c(2,t_1) > P(2,t_1) \). The relationship between \( P^c(1,t_1) \) and \( P(1,t_1) \) is unclear because it depends on values of \( \bar{D} \), \( \epsilon \), \( q \) and \( \rho_d \Delta t \). The difference between \( P^c(2,t_1) \) and \( P(2,t_1) \) can be attributed to the price impact effect in a non-competitive market.

**Equilibrium Discounts**

Let’s take a further look at equilibrium prices derived above. We can see that all equilibrium prices in the first period take similar forms such as the discounted dividends, \( \bar{D}/r^2 \), minus some values, which represent pricing discounts. These pricing discounts can be further broken up to components accounting for different factors. In order to see this, we list all first-period-sell-two prices below.

**Benchmark case:**

\[ P^*(t_1) = \frac{\bar{D}}{r^2} - q^2 \frac{\epsilon}{r^2} \]  

(22)

**Liquid but non-competitive market:**

\[ P^{L_{111}}(2,t_1) = \frac{\bar{D}}{r^2} - q^2 \frac{\epsilon}{r^2} \]  

(23)

**Competitive but illiquid market:**

\[ P^c(2,t_1) \approx \frac{\bar{D}}{r^2} - q^2 \frac{\epsilon}{r^2} - \frac{2(1-q^2)\rho_d \Delta t \epsilon}{r^2} \]  

(24)

**Illiquid and non-competitive market:**

\[ P(2,t_1) \approx \frac{\bar{D}}{r^2} - q^2 \frac{\epsilon}{r^2} - \frac{\rho_d \Delta t \epsilon}{r^2} \frac{2+3q-4q^2-q^3}{1+q} \]  

(25)

\( P^c(2,t_1) \) contains only one discount term \( q^2 \frac{\epsilon}{r^2} \), which we would like to call the “monopoly discount” since this accounts for market non-competitiveness alone. Similarly, we call the second and the following terms on the right hand side of (24) the “liquidity uncertainty discount”. Interestingly, the liquidity-uncertainty-discount
term contains the monopoly-discount term, which implies that the existence of market uncertainty of liquidity could worsen other market frictions. Lastly we call all terms except the first term on the right hand side of (25) the “liquidity discount” for simplicity. The liquidity discount accounts for both liquidity uncertainty and market non-competitiveness. Comparing all three discounts, we find the liquidity uncertainty effect contributes more to the liquidity discount than imperfect market competition. The following proposition states the relation of these two effects on prices.

**Proposition 4:** If the market is perfectly liquid, in the sense that there are always enough liquidity providers in the market, then large traders do not need to worry about price impacts when they need to trade quickly. However, if the market liquidity is limited, even the market is competitive, traders still have to sell at a discount, accounting for the liquidity uncertainty.

Last but not least, we would like to mention that the liquidity discount in our model is unlike the liquidity risk premium derived in Pritsker (2004), which is driven by large investors’ different risk aversions, or the liquidity discount in Subramanian and Jarrow (2001), which only reflects the price impact of a large trader’s trading. The risk neutrality of the large trader isolates the liquidity effect from her risk preference or the need to share risk: the need to trade quickly or slowly is purely a consideration of future availability of liquidity provider.

### 3.4 The Equilibrium Trading Strategy

Nevertheless, the large trader’s trading strategy seems counterintuitive. The optimal trading strategy is to sell all shares in the first period when the type switching probability is low and to postpone the trading to the second period when the type switching probability is high. Intuitively we would conjecture that when \( \rho_{d} \Delta t \) is low (high), which implies that the type shifting is unlikely (likely), the large trader should
prefer to spread the sale in two periods to lessen the price impact, i.e.,
\[ P(1,t_i) > P(2,t_i) \] (or to hasten the sale to avoid being unmatched in the second period). However this is true if we only consider the price impact of trading. This is not true when both traders, large and small, are rational and maximize their expected payoffs. There are two effects that affect the trading behavior: the price impact effect and the liquidity uncertainty effect. The price impact is reflected in the fact that \( P(1,t_i) > P(2,t_i) \). It largely stems from the large trader’s monopoly position. On the other hand, the liquidity uncertainty exists due to the uncertainty of type switching, and in consequence, the existence of liquidity providers. As stated in Proposition 1, this uncertainty affects not only the large trader but the small trader as well. Furthermore the large trader’s expected payoff is affected by the small trader’s expected payoff through their bargaining over the transaction price. These two effects are intertwined. The optimal strategy depends on which effect dominates. Next we analyze how these two effects affect the equilibrium trading strategy, considering the cases of very small value \( \rho_s \Delta t \) and very large \( \rho_c \Delta t \).

Case A: small \( \rho_s \Delta t \)

Firstly recall that in the last period the large trader’s trading strategy does not incur a price impact effect. This is so because there is no incentive for a large trader to delay trading, thus she loses the manipulating power. Trading in the last period, given that she can find a counterparty, is beneficial to the large trader since the impact on prices of her trades doesn’t exist. More importantly, however, the \( hn \)’s are more willing to buy in the first period because the chance of switching from \( ho \) to \( lo \) is small, which leads to higher expected utility for a small \( hn \) trader. From equation (1) we can see since the price is determined by the Nash bargaining solution, with constant bargaining power, the large trader can claim more from the negotiation, i.e.,
by receiving a higher price. Therefore the trading decision is a tradeoff between the two effects.

Case B: large $\rho_{d}\Delta t$

When $\rho_{d}\Delta t$ becomes larger, for example, $\rho_{d}\Delta t \geq z$, the uncertainty of the availability of liquidity provider becomes a greater concern to the large trader. At the same time higher probability of type switching also becomes more important to small traders because they are afraid of being shifted to low-type owners after purchase and end up with the low return, $\bar{D} - \epsilon$. Thus the higher $\rho_{d}\Delta t$, the more hesitant they are to trade, and the price has to be lower to induce them to trade. In addition proposition 1 tells us $P(2, t_1)$ is more sensitive than $P(1, t_1)$ to the increase of $\rho_{d}\Delta t$, which means when $\rho_{d}\Delta t$ increases the price is depressed more if the large trader wants to liquidate her position quickly. When the value of $\rho_{d}\Delta t$ increases beyond a certain level, the large trader would rather try her luck in the second period than sell them in the first period at a very low price. Thus this “gambling” behavior is the optimal reaction to high liquidity uncertainty.

3.5 A Symmetric Case: A Large Buyer vs. Two Small Sellers

It is natural to ask whether these arguments apply in a symmetric case, where a large buyer ($hn$-type) seeks to buy two shares in two periods and two small traders ($lo$-type) hold one share each and hence are eager to sell before $t_3$. Similar to the scenario of distressed sale, the large buyer has to buy two shares at the end of the second period to avoid a penalty of $\delta$ per share. This assumption gives a stronger incentive to the large trader than just being a high-type non-owner. She will then aggressively seek to trade but not just trade in an indifferent manner. This can be observed for situations such as when a large trading desk faces margin calls and is
forced to cover up its short position reluctantly, or when a fund manager faces unexpected withdrawals and has to liquidate her short positions. Basically, what happens to a large seller can happen to a large buyer, which forces her to liquidate a “short” position instead of a “long” position.

The large buyer faces the same dilemma: buying aggressively, she may push up the price; waiting to buy at a better price, she could miss the last opportunity to purchase. On the other side of the market, small sellers are balancing between the payoff of selling it today and the expected payoff of keeping it until the next period. Given the same searching and bargaining process, we can find the subgame perfect equilibrium for this dynamic game. As in the above game, we expect that prices in the first period are functions of the type switching rates, $\rho_u$ and $\rho_d$, the holding cost $\epsilon$ for the low-type owner and the penalty cost $\delta$ for the large high-type non-owner. The large trader’s trading strategies should depend on the current market liquidity and the expected future market liquidity. Therefore, conditional on the current market condition, there should exist some condition under which the large trader would rather bargain harder now than wait and vice versa.

In summary, given the same model structure and assumptions, we can conclude that the large-buyer-vs.-small-sellers case is symmetrical to the previous case we studied and all of the same arguments apply in this case.
4. Extensions and Further Interpretations of the Model

When talking about a liquid (or illiquid) market, there is no unanimous definition or measurement. Among efforts to achieve a definition, Kyle (1985) provides a thorough characterization of “market liquidity”, which is widely accepted by academics and practitioners. He describes market liquidity in three aspects: tightness, depth and resiliency.

In this paper we also try to provide some insights into the definition of market liquidity. “Market liquidity” has two levels of meaning in our model. It refers to current and future liquidity levels, i.e., the liquidity providers available in our model. To model these two aspects of market liquidity, we assume a limited number of small traders and type switching rates which brings uncertainty to the future liquidity level. We have shown that such a market, from the large trader’s perspective, is neither infinitely tight, i.e., she cannot turn over a position costlessly in two periods even if she can perfectly discriminate across small traders, nor deep enough to avoid a price impact.

This, however, is not the case if there are a large number of small traders in the market and longer periods to trade. To show this, we now extend our model to \( n \) small traders, each being either a high-type non-owner or a low-type non-owner with a probability of \( 
\frac{1}{2} \). In any period, the probability of there being at least two high-type non-owners is \( 1 - (1/2)^{n-2} \), which is asymptotically equal to one for large \( n \). This implies that the large trader is pretty sure she can always find somebody on the market even when type switching probabilities are significantly greater than zero. Therefore the market is perfectly liquid in the sense that she can sell whatever number of shares whenever she wants.
Next we would like to ask what the market would be like if there are a limited number of small traders but an extended horizon is used, for example, \( t \) periods. Suppose that the large trader only knows that at time \( \tau \), the probability of a small trader being a high-type or a low-type is \( p \). Then after \( \tau \) periods the probability of a small trader being a high-type is

\[
\left(1 - \frac{(1 - \rho_u \Delta t - \rho_d \Delta t)^\tau}{\rho_u \Delta t + \rho_d \Delta t}\right),
\]

which is \( \frac{\rho_u}{\rho_u + \rho_d} \) in the limit when \( \tau \) goes to infinity.\(^8\) It is easy to see that when two switching rates are equivalent the probability that the large trader will find at least one small trader to trade with is approximately \( \frac{1}{2} \). The probability will be greater than \( \frac{1}{2} \) when the downward switching rate \( \rho_d \) is greater than the upward switching rate \( \rho_u \), and be less than \( \frac{1}{2} \) when \( \rho_d \) is less than \( \rho_u \). But whatever rate dominates, the probability of the availability of at least one high-type small trader is a constant in the limit and is significantly greater than zero. Therefore if the large trader is allowed to

\(^8\) We provide a brief derivation here. At time \( \tau \), the probability that a small trader being either a high-type or a low-type is \( p \), i.e., \( p_s(\tau) = p \cdot p_l(\tau) = 1 - p \).

After one period, \( p_s(\tau + \Delta t) = p \cdot (1 - \rho_u \Delta t + (1 - p) \rho_d \Delta t) = p - \gamma \) and \( p_l(\tau + \Delta t) = 1 - p + \gamma \), where \( \gamma = p \rho_u \Delta t - (1 - p) \rho_d \Delta t \).

After two periods,

\[
p_s(\tau + 2\Delta t) = p_s(\tau + \Delta t)(1 - \rho_u \Delta t) + p_l(\tau + \Delta t) \rho_d \Delta t
= p - \gamma (1 + 1 - \rho_u \Delta t - \rho_d \Delta t)
= p - \gamma (1 + x)
\]

where \( x = 1 - \rho_u \Delta t - \rho_d \Delta t \). \( p_l(\tau + 2\Delta t) = 1 - p + \gamma (1 + x) \).

Following the same method, we have \( p_s(\tau + 3\Delta t) = p - \gamma \left(1 + x + x^2\right) \).

We can show by induction that after \( \tau \) periods,

\[
p_s(\tau + \tau \Delta t) = p - \gamma \left(1 + x + x^2 + \ldots + x^{\tau - 1}\right)
= p - \gamma \left(1 - x^\tau \right) \frac{1}{1 - x}
\]

because \( x \in (0, 1) \). Therefore, \( \lim_{\tau \to \infty} p_s(\tau + \tau \Delta t) = p - \frac{p \rho_u \Delta t - (1 - p) \rho_d \Delta t}{\rho_u \Delta t + \rho_d \Delta t} = \frac{\rho_u}{\rho_u + \rho_d} \).
trade in a long enough horizon, she can always liquidate her position without disturbing the price or worrying about illiquidity.

If the horizon is not infinite, then the probability that there is no high-type small trader is non-zero in some period. This could even last for several periods, which to the large trader would seem as if the market had disappeared.

The above two extensions show that limited traders and limited trading opportunities are both crucial to market illiquidity. Our model also provides some theoretical base to the definition of illiquidity in Longstaff (2001), in which a trader is unable to trade because the market has just disappeared. And our model shows that it is indeed possible for this effect to occur.
5. Conclusions and Future Research

The purpose of this simplified three-date model is to demonstrate the price effect of trading strategies. We demonstrate that with three traders, one large trader and two small traders, the transaction price of the security is determined by the future dividend flow, the trader’s type-switching rate and bargaining power. Unlike other papers studying the price impact of strategic trading assuming the exogenous price impact function, transaction price is endogenously determined in the dynamic bargaining game. We show that the asset price is indeed a decreasing function of shares traded and liquidity uncertainty.

By studying the large trader’s trading strategy, we learn how asset prices are jointly affected by the market condition of trading and by the large trader’s own trading strategy. The risk neutrality of all market participants ensures that the liquidity effect is purely a consideration of future market liquidity. We would conjecture that drastic price changes during a large trader’s depressed sale would dramatically increase the asset’s volatility, which may also drive risk-averse small traders out of the market. In addition, similar to the propagation mechanism of financial contagion described in Allen and Gale (2000), liquidity risk may be contagious across assets and markets through portfolio adjustment or other constraints. Then two questions may be interesting to ask. Will liquidity risk affect market risk and how? Furthermore, is liquidity risk systemic and how should it be hedged?

Keeping these questions in mind, we would like to extend this simple multi-period model in several ways. (1) Extend the game to multiple large traders and study if the existence of other large traders would affect individual large trader’s trading strategy, how the oligopolistic competition would affect asset prices and the predatory
activities when one large trader is in financial distress;\(^9\) (2) Generalize the model by letting bargaining power be a function of the shares held and study how improvements of distressed large trader influence her trading behaviours and asset prices; (3) Add into the large trader’s portfolio a derivative and study how hedging strategies change due to imperfect competition and liquidity risks on the underlying asset market.

\(^9\) Predatory trading or front-run like behaviour are also studied in Brunnermeier and Pederson (2004), Attari, Mello and Ruckes (2002) and Pritsker (2004).
Appendix A

In the dynamic game described in Figure 2, there are 6 possible subgames (i to vi) starting at time $t_2$. The game begins at $t_1$ can also be considered as subgame (I). We have demonstrated in the text for subgame (i) how traders choose their trading strategies and how price is determined through negotiation. In this appendix we are going to (1) calculate the value function of each trader in all subgames begins at time $t_2$; (2) determine the payoff to each trading strategy considered at $t_1$; (3) compare the payoffs to all trading strategies and determine the optimal trading strategy for each trader in subgame (I).

(1) Subgames beginning at $t_2$

If the large trader delays the selling to the second period, she would end up in one of three possible subgames at $t_2$. In subgame (i) (Figure 3), the large trader chooses to sell two shares at price $P(t_2;i) = q\left[\frac{1}{r} (\bar{D} - \varepsilon)\right] + (1-q)\left[\frac{1}{r} (\bar{D} - \rho \Delta t \varepsilon)\right]$. The value function of the large trader is

$$V_{l2} (2,t_2) = 2q\left[\frac{1}{r} (\bar{D} - \varepsilon)\right] + 2(1-q)\left[\frac{1}{r} (\bar{D} - \rho \Delta t \varepsilon)\right]$$

(A.1)

In subgame (ii), there is only one small buyer. The other small trader occurs a type switching from high type to low type and therefore exits the pool of buyers. In this case the large trader still has three choices: sell one share, sell two shares or no trade. The choice of “no trade” is always available as her outside option, i.e., delay trading until next date. If the seller wants to sell two shares, she has to sell it at extremely low price since the buyer will drive the price so low that seller’s value function is equal to her outside option $V(0,t_2) = \frac{2}{r} [\bar{D} - \varepsilon]$. If she sells one share, then they bargain over price such that
\[ q \left[ P(t_2; ii) - \frac{1}{r} (\bar{D} - \varepsilon) \right] = (1-q) \left[ \frac{1}{r} (\bar{D} - \rho_d \Delta t \varepsilon) - P(t_2; ii) \right] \]

and again \( P(t_2; ii) = q \left[ \frac{1}{r} (\bar{D} - \varepsilon) \right] + (1-q) \left[ \frac{1}{r} (\bar{D} - \rho_d \Delta t \varepsilon) \right] \)  \hspace{1cm} (A.2)

\[ V_{lo}(1, t_2) = P(t_2; ii) + \frac{\bar{D} - \varepsilon}{r} = (1+q) \frac{\bar{D} - \varepsilon}{r} + (1-q) \frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} \]  \hspace{1cm} (A.3)

\( V_{lo}(1, t_2) \) is greater than \( V(0, t_2) \) as long as \( q + (1-q) \rho_d \Delta t < 1 \), which is the case since the probability of type switching during the short period, \( \rho_d \Delta t \), is strictly less than 1. Thus if the traders end up in subgame (ii) the large trader will only sell one share to the only buyer in the market.

Now we can calculate the value function of the large trader (the only \( lo \) agent) at \( t_1 \) if she takes the strategy of “no trade”. Thus,

\[ V_{lo}(0, t_1) = \frac{1}{r} \left[ (\rho_d \Delta t)^2 \right] 2 \left( \frac{\bar{D} - \varepsilon}{r} \right) + (2 \rho_d \Delta t) V_{lo}(1, t_2) + \left( 1 - 2 \rho_d \Delta t - (\rho_d \Delta t)^2 \right) V_{lo}(2, t_2) \]  \hspace{1cm} (A.4)

The value function of the large trader at \( t_1 \) is the discounted expected payoff at \( t_2 \), which not only depends on her own strategy but also depends on the availability of a trading counterpart. With probability \( \rho_d^2 \Delta t^2 \) both \( hn \)-type small traders switch to \( ln \).

In such case the large trader cannot find anyone to trade with and has to hold shares till the end. With probability \( 2 \rho_d \Delta t \), only one \( hn \) trader switches type, so at time \( t_2 \) the large trader finds herself in the subgame (ii) and her payoff would be \( V_{lo}(1, t_2) \). With probability \( 1 - 2 \rho_d \Delta t - (\rho_d \Delta t)^2 \) there is no type switching and the large trader still has a pretty good chance to sell her two shares. In this case she would sell two shares and the expected payoff at \( t_2 \) would be \( V_{lo}(2, t_2) \). Substituting \( V_{lo}(1, t_2) \) and \( V_{lo}(2, t_2) \) into equation (A.4), we have
\[ V_{lo}(0, t_1) = \frac{2D}{r^2} - \frac{2\epsilon}{r^2} \left[ q + (1-q) \left(2\rho_s\Delta t - \rho_s^2\Delta t^2\right) \right] \]  

(A.5)

Next we consider the possible subgames begin at \( t_2 \) if the large trader sells only one share at \( t_1 \). After buying one share at time \( t_1 \), the \( hn \) agent becomes the \( ho \). The other \( hn \)-agent stays in the economy but subjects to change. There are four possibilities: no type-switching leads to subgame (iii); the \( hn \) switches to \( ln \) or at the same time the \( ho \) agent also switches to \( lo \) and becomes a seller. In both cases no trade is taken place in the last period; the \( hn \) doesn’t switch type but the \( ho \) agent switches to \( lo \). In this case there is trade in the last period but there will be two agents competing for one share.

Consider subgame (iii). In this case the large trader sells one share at

\[ P(t_2; iii) = q \left[ \frac{1}{r} (D - \epsilon) \right] + (1-q) \left[ \frac{1}{r} (D - \rho_s\Delta t\epsilon) \right] \]

and the value function is

\[ V_{lo}(t_2) = rP(1, t_1) + P(1, t_2) = rP(1, t_1) + \frac{D}{r} - \frac{\epsilon}{r} \left[ q + (1-q)\rho_s\Delta t \right], \]

where \( P(1, t_1) \) is the price at which the large trader sells one share in the first period.

If a high-type owner, who just bought one share in the first period, incurs type switching, the competition situation can then be described as subgame (iv). This resembles the monopoly buyer case. Note here the reservation values for a large seller and a small seller are different. A large trader is willing to sell as long as the price is greater than \( (D - \epsilon)/r \); while for a small trader, she won’t trade unless the price exceeds her reservation value \( (D - (1-\rho_s\Delta t)\epsilon)/r \), which is her expected payoff if she waits. It is easy to see that the small seller’s reservation value is higher than the large seller’s reservation value. If there were no small seller, the small buyer and the large seller would enter the negotiation and end up with a price somewhere between the large trader’s reservation value \( (D - \epsilon)/r \) and the small buyer’s expected payoff.
of holding one share at $t_3, \left(\bar{D} - \rho_d \Delta t \epsilon \right)/r$. However the existence of a small seller changes the negotiation situation and brings the price down to her reservation value. If the price is below this level, this small seller is driven out, i.e., she would rather wait than trade because there is still some chance, $\rho_u \Delta t$, that she switches back to the high type in the last period and avoids the holding cost. Without the competition from this small seller, the large seller will bargain hard and drive the price up to $\left[\bar{D} - \epsilon \left( q + \left( 1 - q \right) \rho_d \Delta t \right) \right]/r$, which, to make the argument simple, is assumed to be greater than $\left[\bar{D} - (1 - \rho_u \Delta t) \epsilon \right]/r$, or equivalently, $\rho_u \Delta t < (1 - q)(1 - \rho_d \Delta t)$. In order to keep the small trader in the game, the small buyer won’t drive the price below $\left[\bar{D} - (1 - \rho_u \Delta t) \epsilon \right]/r$. On the other hand, the price will not be higher than $\left[\bar{D} - (1 - \rho_u \Delta t) \epsilon \right]/r$ due to the limited competition between the two sellers. Since the large trader has the right to move first, she trades with the small buyer at $\left[\bar{D} - (1 - \rho_u \Delta t) \epsilon \right]/r$ and her value function is $rP(1, t_f) + \left[\bar{D} - (1 - \rho_u \Delta t) \epsilon \right]/r$.

In the cases that there is no trade taken place, i.e., the other $hn$-agent shifts down to low-type, the value function of the large trader is the same as her value function in subgame (iv).

Therefore the expected payoff if the large trader employs strategy of selling one share would be

$$V_{io}(1, t_i) = \frac{1}{r} \left\{ \left( \rho_d \Delta t + \rho_d \Delta t^2 \right) \left[ rP(1, t_i) + \left( \bar{D} - \epsilon \right)/r \right] ight. $$

$$+ \rho_d \Delta t \left[ rP(1, t_i) + \left( \bar{D} - (1 - \rho_u \Delta t) \epsilon \right)/r \right] $$

$$+ \left( 1 - 2 \rho_d \Delta t - \rho_d \Delta t^2 \right) \left[ rP(1, t_i) + \left( \bar{D} - \epsilon \left[ q + (1 - q) \rho_d \Delta t \right] \right)/r \right] \right\} $$

$$= P(1, t_i) + \frac{\bar{D}}{r^2} - \frac{\epsilon}{r^2} \left[ q + (1 - q) \left( 3 \rho_d \Delta t - \rho_d \Delta t^2 - \rho_d \Delta t^3 \right) - \rho_d \rho_u \Delta t^2 \right] \quad \text{(A.6)}$$
If at $t_1$ the large trader decides to sell two shares, she then is released from the pressure of liquidation but still faces the uncertainty of type switching. With positive possibility she could end up in one of five subgames at $t_2$, two of which imply trading opportunities. If after trading at $t_1$ there is no type shifting between $t_1$ and $t_2$, or either or both $ho$ agents switch to $lo$, or only the $ln$ switches to $hn$, then neither $lo$ nor $hn$ can find a trading counterparty. But if the $ln$ trader and an $ho$ trader change their types at the same time, trading is possible at $t_2$. For the large trader who sells two shares in the first period, she’s afraid that she’s having her day at $t_2$ but cannot find any sellers on the market. She has to let go the last chance to earn a higher return by investing in the risky asset. For those $hn$ traders who buy one share in the first period, the least situation they want to see is that they have to sell the share acquired a period before (due to type shifting) but cannot find any buyers. Therefore both types of agents have to take account the possibility of type switching when negotiating the transaction price in the first period.

We now analyze two subgames (v) and (vi) starting at $t_2$. In subgame (v), the $hn$ agent with $2rP(2,t_j)$ in money market account seeks to buy shares, while the $lo$ agent can only provide one share. They negotiate over the transaction price. The price cannot fall outside the interval $[\frac{D -(1-\rho_u\Delta t)e}{r},2rP(2,t_j)]$, or the negotiation breaks down. The price is given by Nash Bargaining Solution

$$(1-q)\left[ P(1,t_2) - \frac{D -(1-\rho_u\Delta t)e}{r} \right] = q \left[ \frac{D - \rho_u\Delta t e}{r} - P(1,t_2) \right]$$

$$\Rightarrow P(1,t_2) = \frac{D}{r} - \frac{e}{r} \left[ (1-q)(1-\rho_u\Delta t) + q\rho_u\Delta t \right]$$
The big buyer’s value function becomes
\[ V_{hu}(t_2) = \frac{D - \rho_d \Delta t \varepsilon}{r} + 2rP(2, t_1) - P(1, t_2), \]
and the small seller’s value function is
\[ V_{lo}(t_2) = P(1, t_2) - rP(2, t_1). \]

The subgame (vi) resembles the subgame (i) but with a monopoly buyer. The large trader is now the large buyer who has three choices: buy one share, buy two shares or zero share. She is also subject to the budget constraint such that the maximal price she can pay for one share is \( rP(2, t_1) \). Similar to the monopoly seller case, the monopoly buyer considers the marginal profit of buying one more share. Suppose it is always the big trader bids first. She offers to buy one share such that
\[
(1-q) \left( P(1, t_2) - \frac{D - (1-\rho_d \Delta t) \varepsilon}{r} \right) = q \left( \frac{D - \rho_d \Delta t \varepsilon}{r} - P(1, t_2) \right)
\]
\[ \Rightarrow \quad P(1, t_2) = \frac{D}{r} - \frac{\varepsilon}{r} \left[ (1-q)(1-\rho_d \Delta t) + q \rho_d \Delta t \right] \]

Since \( P(1, t_2) \) is greater than \( \frac{D - (1-\rho_d \Delta t) \varepsilon}{r} \), the small lo’s reservation value, the small seller must be willing to sell. The big buyer offers to buy 2 shares such that
\[
(1-q) \left( P(2, t_2) - \frac{D - (1-\rho_d \Delta t) \varepsilon}{r} \right) = q \left\{ 2 \left[ \frac{D - \rho_d \Delta t \varepsilon}{r} - P(2, t_2) \right] - \left[ \frac{D - \rho_d \Delta t \varepsilon}{r} - P(1, t_2) \right] \right\}
\]
\[ \Rightarrow \quad P(2, t_2) = \frac{D}{r} - \frac{\varepsilon}{r} \left[ (1-q)(1-\rho_d \Delta t) + q \rho_d \Delta t \right] = P(1, t_2) \]

The big buyer is willing to buy two shares as long as \( P(2, t_2) < rP(2, t_1) \) (need to check). Her value function of buying two shares is
\[ V_{hu}(t_2) = 2 \left[ \frac{D - \rho_d \Delta t \varepsilon}{r} + rP(2, t_1) - P(2, t_2) \right] \]

Comparing the payoffs of selling one share and selling two shares, \( V_{hu}(2, t_2) > V_{hu}(1, t_2) \), the big buyer must buy two shares.

Now we can calculate the large trader (lo)’s value function at \( t_1 \) if she sells 2 shares.
\[ V_{lo}(2,t_1) = \frac{1}{r} \left[ 2P(2,t_1) + \frac{\varepsilon}{r} (1 - q) (1 - \rho_u \Delta t - \rho_d \Delta t) \right] \left( 2\rho_d \rho_u \Delta t^2 \right) + \frac{2}{r} \left[ rP(2,t_1) + \frac{\varepsilon}{r} (1 - q) (1 - \rho_u \Delta t - \rho_d \Delta t) \right] \left( \rho_u^2 \Delta t^2 \rho_u \Delta t \right) \]

\[ = 2P(2,t_1) + \frac{2\varepsilon}{r} (1 - q) \rho_d \rho_u \Delta t^2 \left( 1 + \rho_d \Delta t \right) \left( 1 - \rho_u \Delta t - \rho_d \Delta t \right) \]  

(A.7)

(2) Subgame beginning at \( t_1 \)

Having obtained payoffs of three trading strategies in first period, we now can determine the optimal strategy and transaction price at \( t_1 \) using the same method as in the second period. Given that the initial demographic structure, one \( lo \) agent with two shares to sell and two \( hn \) agents want to purchase one share each, the game is similar to the subgame (i). If the large seller decides to sell one share, then

\[
q \left[ V_{lo}(1,t_1) - V_{lo}(0,t_1) \right] = (1 - q) \left[ V_{lo}(1,t_1) - P(1,t_1) \right] 
\]

(A.8)

This equation states the marginal profit that the large seller gives up to a marginal buyer must be equal to the fraction that she obtains from the marginal buyer. \( V_{lo}(1,t_1) \) and \( V_{lo}(0,t_1) \) are the expected payoffs to strategies “sell one” and “no trade” employed in the first period.

Note that the outside option for a buyer to buy one share is the riskless return \( r \), but not the expected payoff to remain as a potential buyer. The reason is that small traders are treated as fully competitive and therefore price takers. They cannot manipulate the market by strategic trading. A marginal buyer is a buyer who is matched and is willing to make a deal as long as the price is above his outside option, being an unmatched \( hn \) agent. At the next trading date the matching opportunity is the same for all \( hn \) agents, unmatched at the last date or newly entered. Therefore, unlike the large trader, small traders are short-sighted in the sense that they don’t take
into account the future opportunity of trading. Thus for an anonymous marginal small
buyer, his expected value function at \( t_1 \) is

\[
V_{ho}(t_1) = \frac{1}{r} \left[ (1 - \rho_d \Delta t) \frac{\bar{D} - \rho_e \Delta t \epsilon}{r} + \rho_d \Delta t \left( \frac{\bar{D} - (1 - \rho_e \Delta t) \epsilon}{r} \text{prob(no trade)} + P(1, t_2) \text{prob(trade)} \right) \right]
\]

\[
= \frac{1}{r} \left[ (1 - \rho_d \Delta t) \frac{\bar{D} - \rho_e \Delta t \epsilon}{r} + \rho_d \Delta t \left( \frac{\bar{D} - (1 - \rho_e \Delta t) \epsilon}{r} \right) \right]
\]

\[
= \frac{1}{r^2} \left[ \bar{D} - (2 - \rho_d \Delta t - \rho_e \Delta t) \rho_d \Delta t \epsilon \right]
\]

(A.9)

Substituting \( V_{ho}(0, t_1) \), \( V_{lo}(1, t_1) \) and \( V_{ho}(1, t_1) \) into (A.8), we get the price of trading
one share at \( t_1 \).

\[
P(1, t_1) = \frac{\bar{D}}{r^2} - \frac{\epsilon}{r^2} \left[ q^2 + (2 - q - q^2) \rho_d \Delta t + (2q - q^2 - 1) \rho_d^2 \Delta t^2 - (q - q^2) \rho_d^3 \Delta t^3 + (2q - 1) \rho_d \rho_e \Delta t^2 \right]
\]

(A.10)

The price is equal to the present value of expected dividend, deducted by an illiquidity
discount. The illiquidity is due to the uncertainty of being able to trade in the future
period. It depends on the relative bargaining power, the rate of switching and the
frequency of trading. Substituting \( P(1, t_1) \) back into \( V_{lo}(1, t_1) \), we have

\[
V_{lo}(1, t_1) = \frac{2\bar{D}}{r^2} - \frac{\epsilon}{r^2} \left[ q(1 + q) + (5 - 4q - q^2) \rho_d \Delta t + (1 - q)(2 - q) \rho_d^2 \Delta t^2 \right. \\
- (1 - q^2) \rho_d^3 \Delta t^3 - 2(1 - q) \rho_d \rho_e \Delta t^2 \]
\]

(A.11)

If the big seller decides to sell two shares at \( t_1 \), the transaction price is determined by

\[
q \left[ V_{lo}(2, t_1) - V_{lo}(1, t_1) \right] = (1 - q) \left[ V_{ho}(1, t_1) - P(2, t_1) \right]
\]

\[
P(2, t_1) = \frac{\bar{D}}{r^2} - \frac{\epsilon}{r^2 (1 + q)} \left[ q^2 (1 + q) + \rho_d \Delta t (2 + 3q - 4q^2 - q^3) + \rho_d^2 \Delta t^2 (1 - q)(q - 1)(2 - q) \\
- \rho_d^3 \Delta t^2 (q - 1) - 2q (1 - q)(\rho_d^2 \rho_e \Delta t^4 + \rho_d^3 \rho_e \Delta t^3 + \rho_d \rho_e \Delta t^3) \right]
\]

(A.12)
Substituting \( P(2,t_i) \) in the (A.7), the big seller’s value function of selling 2 shares is given by

\[
V_{bs}(2,t_i) = \frac{2D}{r^2} - \frac{2e}{r^2} q^2 + \frac{2+3q-4q^2-q^3}{1+q} \rho_d \Delta t - \frac{(1-q)^2}{1+q} \rho_d^3 \Delta t^3 - \frac{q(1-q)^2}{1+q} \rho_d^3 \Delta t^3
\]

\[
+ \frac{(q-1)(2+q)}{1+q} \rho_d^2 \rho_u \Delta t^2 + \frac{(1-q)^2}{1+q} \left( \rho_d^2 \rho_u^2 \Delta t^4 + \rho_d^3 \rho_u \Delta t^4 + \rho_d \rho_u^2 \Delta t^3 \right)
\]

(A.13)

Since the probability of type switching during a short period \( \Delta t \) is pretty small, e.g., \( \rho_d \) and \( \rho_u \) are both less than 1 and \( \Delta t \) is set to be less than one day, the higher order effect of \( \rho_d \Delta t \) on prices is negligible. In order to facilitate the comparison of prices and value functions, we ignored higher order terms containing second order and beyond of \( \rho_i \Delta t, i = u, d \). \( P(1,t_i) \) and \( P(2,t_i) \) then can be written as

\[
P(1,t_i) \approx \frac{2D}{r^2} - \frac{q^2 e}{r^2} \frac{\rho_d \Delta t e}{r^2} \left( 2 - q - q^2 \right)
\]

(A.14)

and

\[
P(2,t_i) \approx \frac{2D}{r^2} - \frac{q^2 e}{r^2} \frac{\rho_d \Delta t e}{r^2} \left( 2 + 3q - 4q^2 - q^3 \right)
\]

(A.15)

\[
P(1,t_i) - P(2,t_i) = \rho_d \Delta t e \frac{2q(1-q)}{1+q} > 0
\]

We can see that the equilibrium price of selling one share at \( t_i \) is always higher than the price of selling two shares at \( t_i \). We call this result as price impact.

Applying the approximation to value functions, the large trader’s value functions becomes

\[
V_{ls}(1,t_i) \approx \frac{2D}{r^2} - \frac{e}{r^2} \left[ q(1+q) + (5 - 4q - q^2) \rho_d \Delta t \right]
\]

(A.16)

\[
V_{ls}(2,t_i) \approx \frac{2D}{r^2} - \frac{2q^2 e}{r^2} - \frac{2 \rho_d \Delta t e}{r^2} \left( 2 + 3q - 4q^2 - q^3 \right)
\]

(A.17)
and \( V_{lo}(0, t_1) = \frac{2D}{\rho^2} - \frac{2\epsilon}{\rho^2} q + 2(1 - q) \rho_d \Delta t \) \quad (A.18)

To determine the optimal strategy at \( t_1 \), we compare the value functions of all three trading strategies.

\[
V_{lo}(1, t_1) - V_{lo}(2, t_1) = \frac{\epsilon}{\rho^2} \left[ -q(1 - q) + \rho_d \Delta t(1 - q) \frac{q^2 + 4q - 1}{1 + q} \right]
\]

When \( 0 \leq q \leq \sqrt{5} - 2 \), \( V_{lo}(1, t_1) \leq V_{lo}(2, t_1) \); when \( \sqrt{5} - 2 < q \leq 1 \), \( V_{lo}(1, t_1) > V_{lo}(2, t_1) \)

if and only if \( \rho_d \Delta t \geq \frac{q(1 + q)}{q^2 + 4q - 1} \).

Comparing \( V_{lo}(1, t_1) \) and \( V_{lo}(0, t_1) \), we find that \( V_{lo}(1, t_1) \geq V_{lo}(0, t_1) \) iff \( \rho_d \Delta t \leq \frac{q}{1 + q} \).

Similarly, \( V_{lo}(2, t_1) \geq V_{lo}(0, t_1) \) iff \( \rho_d \Delta t \leq \frac{q + 1}{q + 3} \). Denote \( x = \frac{q(1 + q)}{q^2 + 4q - 1} \), \( y = \frac{q}{q + 1} \), \( z = \frac{q + 1}{q + 3} \). It is easy to see that \( x \geq z \geq y \) for \( q \in [0, 1] \).

For \( \rho_d \Delta t \geq x \), \( V_{lo}(0, t_1) > V_{lo}(1, t_1) \geq V_{lo}(2, t_1) \); for \( z \leq \rho_d \Delta t < x \), \( V_{lo}(0, t_1) \geq V_{lo}(2, t_1) > V_{lo}(1, t_1) \); for \( y \leq \rho_d \Delta t < z \), \( V_{lo}(2, t_1) > V_{lo}(0, t_1) \geq V_{lo}(1, t_1) \); for \( \rho_d \Delta t < y \), \( V_{lo}(2, t_1) > V_{lo}(1, t_1) > V_{lo}(0, t_1) \). Therefore, when \( \rho_d \Delta t \geq z \), large trader prefers the strategy of no trade in the first period and sell two in the second period and when \( \rho_d \Delta t < z \), she will do the opposite, selling two shares in the first period.

**Appendix B**

**Proof to Proposition 1:**
(i) According to equations (12) and (13), the equilibrium prices at $t_1$ are functions of $\epsilon$, $q$ and $\rho_d \Delta t$. It’s easy to see that the derivatives of $P(1,t_1)$ and $P(2,t_1)$ with respect to $\epsilon$ are negative when $0 < q < 1$, and zero when $q$ equals 0 or 1. That is,

$$\frac{\partial P(1,t_1)}{\partial \epsilon} = -\frac{q^2}{r^2} \frac{\rho_d \Delta t}{r^2} (2 - q - q^2) \leq 0$$

$$\frac{\partial P(2,t_1)}{\partial \epsilon} = -\frac{q^2}{r^2} \frac{\rho_d \Delta t}{r^2} \frac{(1-q)(q^2 + 5q + 2)}{1+q} \leq 0$$

To see if prices are decreasing in the probability of type-switching, $\rho_d \Delta t$, we differentiate prices with respect to $\rho_d \Delta t$ and we get

$$\frac{\partial P(1,t_1)}{\partial (\rho_d \Delta t)} = -\frac{\epsilon}{r^2} (q+2)(1-q) \leq 0,$$

$$\frac{\partial P(2,t_1)}{\partial (\rho_d \Delta t)} = -\frac{\epsilon}{r^2} \frac{(1-q)(q^2 + 5q + 2)}{1+q} \leq 0.$$  

Additionally, $\frac{\partial P(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial P(1,t_1)}{\partial (\rho_d \Delta t)}$ because $\frac{\partial P(2,t_1)}{\partial (\rho_d \Delta t)} - \frac{\partial P(1,t_1)}{\partial (\rho_d \Delta t)} = \frac{-2\epsilon(1-q)q}{r^2(1+q)}$ is less than or equal to zero, with equality when the seller or the buyer owns the entire power in the bargaining, that is, $q = 1$ or 0.

(ii) For the second half of the proposition, a simple exercise of differentiating value functions with respect to $\rho_d \Delta t$ gives us

$$\frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} = -\frac{\epsilon}{r^2} (q+5)(1-q) \leq 0$$

and

$$\frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} = -\frac{2\epsilon}{r^2} \frac{(1-q)(q^2 + 5q + 2)}{1+q} \leq 0.$$  

Differentiating $V_{lo}(0,t_1)$ with respect to $\rho_d \Delta t$ gives us
\[
\frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} = -\frac{4\varepsilon}{r^2}(1-q) \leq 0
\]

We now compare \( \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} \), \( \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \) and \( \frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} \).

\[
\frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} - \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} = \frac{\varepsilon}{r^2} (1-q)(q+1) \geq 0
\]

and

\[
\frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} - \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} = \frac{\varepsilon}{r^2} \left( 1-q \right) \left( q^2 + 4q - 1 \right)
\]

\[
\frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} > \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \quad \text{when} \quad \sqrt{5} - 2 \leq q \leq 1 ; \quad \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} < \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \quad \text{when} \quad 0 \leq q < \sqrt{5} - 2.
\]

Thus, \( \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} \leq 0 \) when \( \sqrt{5} - 2 \leq q \leq 1 \); \( \frac{\partial V_{lo}(1,t_1)}{\partial (\rho_d \Delta t)} < \frac{\partial V_{lo}(2,t_1)}{\partial (\rho_d \Delta t)} \leq \frac{\partial V_{lo}(0,t_1)}{\partial (\rho_d \Delta t)} \leq 0 \) when \( 0 \leq q < \sqrt{5} - 2 \).

We last consider the sensitivity of a small trader’s value function to the change of type-switching rate. The value function of a small trader buying one share is given by (A.9). We rewrite it here as

\[
V_{ho}(1,t_1) = \frac{1}{r^2} \left[ \frac{\hat{D} - 2 - \rho_d \Delta t - \rho_d \Delta t \rho_d \Delta \varepsilon}{(2 - \rho_d \Delta t - \rho_d \Delta t \rho_d \Delta \varepsilon)} \right]
\]

\[
\frac{\partial V_{ho}(1,t_1)}{\partial (\rho_d \Delta t)} = -\frac{\varepsilon}{r^2} \left[ 2(1-\rho_d \Delta t) - \rho_d \Delta t \right], \text{which is less than zero since we assume above that } \rho_d \Delta t < (1-q)(1-\rho_d \Delta t) \text{ and } \rho_d \Delta t \in [0,1].
\]

**Proof to Proposition 2:**

When \( \rho_{d,u} = 0 \), or \( \Delta t \) goes to zero, if there are still trades in the second period, the price is given by

\[
q \left[ P^l(t_2) - \frac{1}{r \hat{D} - \varepsilon} \right] = (1-q) \left[ \frac{1}{r \hat{D} - P^l(t_2)} \right] \quad \text{(B.1)}
\]
\[ P^L(t_2) = \frac{D - q\varepsilon}{r} \]  \hspace{1cm} (B.2)

According to price formulae (7) and (9), the transaction prices in the first period become to

\[ P^L(1, t_1) = P^L(2, t_1) = \frac{D - q^2\varepsilon}{r^2} \]  \hspace{1cm} (B.3)

Since the transaction price is higher in the first period than in the second period, the large trader must sell two shares in the first period. The return for a small trader is \( q^2\varepsilon / r^2 > 0 \), which ensures that the small trader must be willing to trade.

**Proof to Proposition 3:**

First we show that the second-period trading prices in a competitive market are uniform across the trading strategies. We then show that the first-period trading prices, however, still present the impact of trading strategies. This price impact, as we may conclude, stems from the liquidity uncertainty alone.

As we have showed in the text and Appendix A, there is no price impact in the second period in our original model because the marginal costs of selling one share and selling two shares are equivalent. Since market non-competitiveness has no effect on prices in the last period, there won’t be any price impact in the second period in a competitive market as well.

In order to determine prices in \( t_1 \), we need the following lemma.

**Lemma 1:** In a competitive market, small traders’ response functions to the large trader’s trading strategy are the same, i.e., they would either trade or not trade with the large trader at the same time.

**Proof:** First of all, we assume all small traders are identical in this paper so that they have exactly the same chance to be contacted by and trade with the large trader.
Secondly, by symmetry their value functions are the same. When the large trader only sells one share, the small trader who buys the share has an outside option valued exactly the same as the small trader who does not buy the share. When the large trader sells two shares, it cannot be the case that only one small trader trades with her. Without losing generality we assume that if small traders are indifferent between trading and not trading, they trade. If one small trader retreats from trading, this will send a strong signal to the other trader that trading with the large trader is not a good decision. The other trader will be better off by just mimicking the first small trader’s withdrawal. Therefore in a competitive market small traders’ trading strategies would be identical.

□

Since the large trader is unable to discriminate across small traders who are high-type non-owners, i.e., she offers a price at which small traders are indifferent between trading and not trading. Note that the large trader still can choose her trading strategy in a competitive market. If she decides to sell one share, she contacts with a small trader randomly and negotiate the transaction price. The price must be such that the small trader is indifferent between trading with her and staying as a high-type non-owner. If trade, the small trader’s value function is \( V_{ho} \left( t_1 \right) - P^C \left( 1, t_1 \right) \); if not trade, her value function will be the same as the other small trader who has not been contacted, i.e., \( V_{hn} \left( t_1 \right) \). The price is hence determined by

\[
V_{ho} \left( t_1 \right) - P^C \left( 1, t_1 \right) = V_{hn} \left( t_1 \right)
\]

(B.4)

where \( V_{ho} \left( t_1 \right) \) is given by (A9), i.e.,

\[
V_{ho} \left( t_1 \right) = \frac{1}{\rho^2} \left[ \bar{D} - \left( 2 - \rho^2 \Delta t - \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \rho^2 \Delta t \right] .
\]

\( V_{ho} \left( t_1 \right) \) is the value function of a small trader remaining as a high-type non-owner at \( t_1 \) after trading.
\[ V_{hn}(t_1) = \frac{1}{r} \left[ \text{prob (subgame iv)} V_{hn}(iv) + \text{prob (subgame iii)} V_{hn}(iii) \right] \]
\[ = \frac{1}{r} \left\{ \frac{\rho_d \Delta t}{r} \left( \frac{D - \varepsilon}{r} \right) + \left( 1 - 2 \rho_d \Delta t - \rho_d^2 \Delta t^2 \right) \left[ \frac{D - \rho_d \Delta t}{r} - P(t_2; iii) \right] \right\} \]  
\[ (B.5) \]

where \( P(t_2; iii) = \frac{D - \varepsilon\left[ q + (1-q) \rho_d \Delta t \right]}{r} \), is the transaction price in subgame (iii).

Substitute \( V_{ho}(t_1), V_{hn}(t_1) \) and \( P(t_2; iii) \) into (B.4), we get the price as
\[ P^C(1,t_1) = \frac{D}{r^2} \left( 1 - \rho_d \Delta t \right) - \frac{\varepsilon}{r^2} \left[ \rho_d \Delta t - \rho_d^2 \Delta t^2 - \rho_d \rho_d \Delta t^2 + q \left( 1 - 3 \rho_d \Delta t + \rho_d^2 \Delta t^2 + \rho_d^3 \Delta t^3 \right) \right] \]
\[ (B.6) \]

If the large trader tries to sell two shares, she contacts both of small traders. Again the price is set as such that both the small traders are indifferent between trading and not trading. According to Lemma 1, both small traders would either trade or not trade with the large trader. For the large trader, the identical responses from small traders imply that she cannot sell the marginal share at a price different to infra-marginal ones.

The large trader’s value function, conditional on both small traders trade, is \( V_{ho}^C(2,t_1) \) and her value function, conditional on neither small trader trades, is \( V_{ho}^C(0,t_1) \). For a small trader the gain by trading is \( V_{ho}(1,t_1) - P^C(2,t_1) \). Bargaining with both gives us the price
\[ q \left[ V_{ho}^C(2,t_1) - V_{ho}^C(0,t_1) \right] = 2(1-q) \left[ V_{ho}(1,t_1) - P^C(2,t_1) \right] \]
\[ (B.7) \]

where the superscript “C” denotes “competitive market”. \( V_{ho}^C(2,t_1) \) and \( V_{ho}^C(0,t_1) \) are given by equations (A.7) and (A.5) respectively. This is so because the prices and the trading strategies in the second period are the same as in our non-competitive model.

Straightforward computation gives \( P^C(2,t_1) \) as
\[ P^C_{i} (2, t_i) = \frac{D}{r^2} - \frac{q^2 \epsilon}{r^2} \left[ 2(1-q^2) \rho_d \Delta t - (1-q) \rho_d^2 \Delta t^2 - q(1-q) \rho_d \Delta t^3 \right. \]

\[ \left. - (1-q)^2 \rho_d \rho_s \Delta t^2 - q(1-q) \left( \rho_d \rho_s^2 \Delta t^3 + \rho_s^2 \rho_d \Delta t^4 + \rho_d \rho_s \Delta t^4 \right) \right] \]  

(B.8)

In order to compare prices in different markets, we again ignore the second and higher order terms of \( \rho_i \Delta t, i = u, d \).  \( P^C (2, t_i) \) and \( P^C (1, t_i) \) can be rewritten as

\[ P^C (2, t_i) \approx \frac{D}{r^2} - \frac{q^2 \epsilon}{r^2} \left[ 2(1-q^2) \rho_d \Delta t \right] \]  

(B.9)

\[ P^C (1, t_i) \approx \frac{D}{r^2} \left[ 1 - \rho_d \Delta t \right] - \frac{\epsilon^2}{r^2} \left[ q + (1-3q) \rho_d \Delta t \right] \]  

(B.10)

In order to compare \( P^C (t_i) \) and \( P(t_i) \), we rewrite \( P(1, t_i) \) and \( P(2, t_i) \) here.

\[ P(1, t_i) \approx \frac{D}{r^2} - \frac{q^2 \epsilon}{r^2} - \frac{\rho_d \Delta t \epsilon}{r^2} (2 - q - q^2) \]

\[ P(2, t_i) \approx \frac{D}{r^2} - \frac{q^2 \epsilon}{r^2} - \frac{\rho_d \Delta t \epsilon}{r^2} \left[ 2 + 3q - 4q^2 - q^3 \right] \]

\[ P^C (2, t_i) - P(2, t_i) = \frac{\rho_d \Delta t \epsilon q(1-q)^2}{1+q} > 0 \]  

(B.11)

\[ P(1, t_i) - P^C (1, t_i) = \frac{\rho_d \Delta t \epsilon}{r^2} + \frac{\epsilon}{r^2} \left[ q - q^2 + (q^2 - 2q - 1) \rho_d \Delta t \right] \]  

(B.12)

\[ P(1, t_i) > P^C (1, t_i) \text{ if and only if } \frac{D}{\rho_d \Delta t} \left[ q^2 - q + (1+2q-q^2) \rho_d \Delta t \right], \text{ which is nonlinear in } q \text{ and } \rho_d \Delta t. \]

The value functions of selling one share and two shares in the first period are

\[ V^{C}_{i} (1, t_i) \approx \frac{D}{r^2} (2 - \rho_d \Delta t) - \frac{\epsilon}{r^2} \left[ 2q + (4-6q) \rho_d \Delta t \right] \]  

(B.13)

\[ V^{C}_{i} (2, t_i) \approx \frac{2D}{r^2} - \frac{2q^2 \epsilon}{r^2} - \frac{4(1-q^3) \rho_d \Delta t \epsilon}{r^2} \]  

(B.14)

\[ V^{C}_{i} (2, t_i) - V^{C}_{i} (1, t_i) = \frac{D}{r^2} \rho_d \Delta t + \frac{2\epsilon}{r^2} \left[ q - q^2 + \rho_d \Delta t (2q^2 - 3q) \right] \]  

(B.15)
Therefore the large trader’s trading strategy depends on values of parameter $\bar{D}$, $\epsilon$, $q$ and $\rho_d\Delta t$. She would sell two shares in the first period if
\[
\bar{D} > \frac{2\epsilon}{\rho_d\Delta t} \left[ q^2 - q + \rho_d\Delta t \left( 3q - 2q^2 \right) \right]
\] and sell one share in each period in the opposite case.
Figure 2. The dynamics of the trading and type evolution.
Figure 3. The evolution of a subgame begins at $t_5$. 
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