Bundling and Complementarity

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January 2005

Abstract

We analyze bundling incentives in markets where products are composed of two complementary components (e.g. hardware and software). One of the components is monopolized and the other is sold by a duopoly. We assume quality differentiation between components, and we develop a model of quality competition à la Mussa and Rosen (1978). We demonstrate that the monopolist’s bundling decision depends on the quality of the component where he faces competition. We show that: (i) if the monopolist’s component has a higher quality than the competitor’s, then the monopolist prefers not to bundle (separate selling); and (ii) if the quality of monopolist’s component is lower than the competitor’s, then the monopolist prefers bundling. Thus, the monopolist’s incentive not to bundle is related to narrowing of market for its monopolized component with bundling. We also consider a dynamic game where firms compete in two-stages (quality and price) and determine the subgame perfect Nash equilibrium. Finally, we discuss antitrust implications of these findings.

Keywords: bundling, complementarity, quality, social welfare, antitrust.
JEL Classification: L14, L15, L40, L41, L42

1I am indebted to Abraham Hollander and Lars Ehlers for continued guidance and encouragements. All errors are of course my own. Correspondence: Thierno Diallo, département de sciences économiques, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal QC H3C 3J7, Canada, email : thierno.diallo@umontreal.ca
I. Introduction

I.1 Overview

Many products are consumed in combination with other products to form a complete system of products that cannot be consumed separately. E.g., a computer system can be decomposed into a basic unit and a monitor, a stereo system is an amplifier and a speaker, a computer is composed of hardware and software. But products can be sold and purchased either separately or as packages. Selling as packages can be achieved either by technical bundling, i.e. by making one component compatible only with components sold by the same firm or by engaging in tied selling.

A component is incompatible with components sold by other firms', if it cannot be assembled with them to form a usable system. The economic consequence of compatibility versus incompatibility have been examined by Chou and Shy (1989), Matutes and Regibeau (1989) for the case where each component is sold by an independent firm. Matutes and Regibeau (1988), Economides (1989, 1991), and Einhorn (1992) have looked at the case where each firm supplied all the components necessary to form the complete system. Economides (1991) proved that for firms supplying all components necessary to form a usable system, compatibility is profitable if the relative increase in the demand for the systems when making components compatible exceeds the number of complementary components supplied by other firms. A standard result of this literature is that compatibility increases industry demand and profits.

Tied selling or tying restrictions are of two types: requirement tie-ins and bundling. Under requirement tie-ins the seller of product \( A \) requires all purchasers of \( A \) also to purchase all their requirements of product \( B \) from him. Under a bundling arrangement or package tie-ins the seller offers two or more products in fixed combination as a single

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1 Formally, Economides (1991) considers a model where the market for one component (say market \( A \)) is a duopoly, and the other (say market \( B \)) is monopolistically competitive. He shows that when the addition of a new variety of \( B \) leads to little or no increase in the demand for the system \( AB \), the profits of a firm of type- \( A \) are higher under incompatibility and they will choose this regime. By contrast, when the addition of a new variety of \( B \) increases the demand for the system \( AB \), the profits of a firm of type- \( A \) are higher under compatibility. The profitability of compatibility depends on its capacity to increase the demand for \( A \).
package. The difference between these two types of tying is that, in the case of bundling the price of \( B \) is not separately stated but is bundled into the price of \( A \).

A producer who makes its components incompatible with components produced by others or engages in technical bundling imposes a \textit{de facto}\(^3\) tie. In this paper, we are interested in bundling. To rule out technical bundling, we assume a compatible world. Thus, selling as packages is achieved only by bundling. We analyze bundling incentives in markets where products are composed of two complementary components (e.g. hardware and software) in the context of vertical product differentiation. One of the components is monopolized and the other is sold by a duopoly. We ask what is the monopolist’s best selling strategy between separate selling and bundling? What are the consequences for welfare? To answer these questions, we review first the literature on bundling and then we solve a model of quality competition for systems composed of two complementary components.

### I.2 The Bundling Literature

Three reasons are given in the literature to explain bundling. The traditional explanation for bundling is that it achieves better price discrimination by a monopolist [Stigler (1968), Adams and Yellen (1976), Schmalensee (1984), and McAfee, McMillan and Whinston (1989)]. Usually, a firm has to charge one price to all consumers. In these cases heterogeneity in consumers’ valuations frustrates the seller’s ability to capture consumer surplus. Bundling reduces consumers’ heterogeneities in reservation prices and captures the maximum of their surplus. Stigler’s analysis and Adams and Yellen (1976) are based on stylized examples with a discrete number of customers. In these examples, the reservation values of the components of the bundle are negatively correlated. Bundling serves much the same purpose as price discrimination. Schmalensee (1984) considers the entire class of Gaussian demands and shows that bundling also helps to reduce the effective dispersion of reservation values and thereby makes it possible for the seller to

\[^{3}\text{See Tirole (1988)}\]
extract a greater fraction of the potential surplus even if demands are uncorrelated and possibly positively correlated.

The second reason for bundling is the leveraging of market power that exists in one market into another market, i.e. the use of monopoly power position in one market to exclude rivals, deter entry or foreclose competition in others markets [Burstein (1960), Blair (1978), Schmalensee (1982)]. As noted by Burstein (1960), firms can raise profits by bundling goods that are complementary. Burstein has argued that a firm with a monopoly over one good could monopolize another by selling them in conjunction with each other. If it did so, a firm seeking to compete in the market for the bundled good would be foreclosed from selling to all those who received the bundled good in conjunction with their purchase of the bundling good. This theory has come under heavy attack from the Chicago School [Director and Levi (1956), Posner (1976), Bork (1978)]. They argued that if a monopolist does employ tying or bundling, his motivation cannot be leverage. The reason is that there is only one monopoly profit that can be extracted. Also, they defended the fact that bundling could provide convenience for customer and lower transaction cost. Whinston (1990) and Nalebuff (2004) re-examined the role of bundling as an entry deterrent. Whinston (1990) presents a series of models in which he first makes the assumption in which bundling does not increase profits and then alters the assumption slightly so that it does. He shows that the Chicago School's criticism of leveraging monopoly power from one market (say market $A$) to another (say market $B$) applies only if market $B$ is perfectly competitive. With this latter assumption he proves that the use of leverage to affect the market $B$ is actually impossible. However, when the monopolized product $A$ is no longer essential for all uses of product $B$, he shows that in the presence of scale economies, bundling can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for bundled good rivals and eventually induce them to exit. Due to the scale economies, an entrant would need to attain sufficiently scale to survive in the market of $B$. Bundling can foreclose the entrant from sales to be the difference between making entry profitable an unprofitable. In these cases, bundling can increase the profits of the monopolist of $A$ and harm consumers.
The third reason is that bundling leads to reduce costs by enabling economies of scope in production and distribution to be achieved. Salinger (1995) recognizes that cost synergies from bundling are most valuable when consumer valuations are positively correlated. Focusing on the case of pure bundling, Salinger introduces the role of cost savings to interact with demand effects and proposes a graphical analysis of the economic properties of bundling. Thus, if most consumers would buy both (or neither) $A$ and $B$ when sold separately, then any cost savings from selling them together will create an incentive for a monopolist to sell bundled products when valuations are positively correlated.

Finally, we conclude this review of literature with the specific cases of complementary and systems goods [Whinston (1990), Matutes and Regibeau (1992) and Economides (1993)]. Here, price discrimination plays a minor role because the goods are highly positively correlated in value and now there is more than one seller in the market (oligopoly markets). For these specific cases, a standard result is that bundling is dominated by separate selling. There is also no clear cut about welfare implications of bundling in these studies. Whinston (1990) demonstrates that separate selling is more profitable than pure bundling for a firm which is monopolist in $A$ and faces a competitor in $B$ when consumers’ demands are for the system made by $A$ and $B$. He shows that if a commitment to bundling causes the competitor in $B$ of the monopolist of $A$ to be inactive, the latter can do no worse and possibly better by committing to separate selling (Proposition 3). This is because if the competitor is inactive, this reduces the sales of $A$. Matutes and Regibeau (1992) and Economides (1993) extend the basic framework of monopoly bundling to a duopoly setting, with horizontally differentiated components. Here competition is between fully integrated firms, where each firm produces all components necessary to form a system. Matutes and Regibeau (1992), show that if two firms sell compatible components, then bundling is dominated by separate selling. This result comes from two effects. For example, if we consider that two components ($A$ and $B$) are available, the first effect is that under compatibility separate selling increases the number of systems that can be assembled from two ($AA$ and $BB$) to four ($AA, AB, BA, BB$). This enables some consumers to obtain a system that is closer to their ideal specification, and then industry demand is larger without bundling. The second
effect relates to the fact that separate selling softens price competition. The intuition is that in a compatible world with separate selling a decrease in the price of one firm’s component increases the demand for its own system, as well as for the mixed system (system composed by different firms' components). However, the greater demand for the mixed system reduces the demand for the firm’s second component. While with bundling, if a firm lowers the price of one component, it attracts additional consumers for its two components; cutting the price of one component increases equally the demand for both components in the system. It follows that a firms’ price-cutting incentives are lower with separate selling than with bundling.

I.3 Approach in this paper

This paper extends the literature on bundling in a number of ways. First, it considers quality differences in system components with consumers’ taste à la Mussa and Rosen (1978). Specifically, we study a market for systems composed of two complementary components where we assume the unit cost increasing in quality and variable in production. One of the components is sold by a firm that has a monopoly in that component; the other is sold by the same firm as well as by one other firm. Consumers derive utility only from using one unit of the monopolized component in combination with one unit of the other component.

Second, it answers the question how the introduction of quality affects the bundling decision of a firm that is a monopolist for product $A$ and faces a single competitor in market for product $B$. We prove that the monopolist’s bundling decision depends on the quality level of the component produced by two firms. We demonstrate that the monopolist of $A$ earns higher profits by selling $A$ and $B$ separately than selling the system $AB$ as bundle when its product $B$ is of higher quality than that of its rival. On the

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4 Quality differences in system components and consumers taste has been considered also by Einhorn (1992) and Economides and Lehr (1994). The former uses the model of quality differentiation of Gabszewicz and Thissé (1979) to analyze compatibility of systems’ components and find that profits are higher when components are fully compatible. The latter use the framework of Mussa and Rosen (1978) to study the quality choice of complete systems under many types of market structures with quadratic fixed cost of quality improvement. They do not find any equilibrium in quality in a similar market situation. In this paper by contrast, the unit cost is increasing in quality and variable in production. We find results that are qualitatively different and more general. As noted by Crampes and Hollander (1995), this specification of variable cost increasing in quality appears as the empirically more relevant case.
other hand if the monopolist’s product $B$ has the lower quality, then the bundling strategy yields more profits than separated selling. The intuitions for these results are quite simple and similar to those of Economides (1991). The profitability of separate selling depends on the relative increase in the demand for $A$ compared to the losses incurred due to competition faced in $B$. Separate selling leads to a considerable increase in the demand for $A$ through sales with $B$ when the monopolist produces the higher quality of $B$. Conversely, separate selling brings no increase in the demand for $A$ when the monopolist produces the lower quality of $B$. These results extend Whinston (1990) and Matutes and Regibeau (1992), when firms’ components are vertically differentiated. Also, in contrast to these studies, we demonstrate that bundling reduces social welfare.

Third, it considers the question how the decision to sell components separately or to bundle affects quality choice. We model a two stages game. In the first stage, firms choose qualities and determine whether they sell separately or bundle. In the second stage they set prices. We find the set of equilibrium quality and pricing decisions for each firm that are outcomes of the subgame perfect Nash equilibria. We show that a firm who is monopolist in product $A$ produces always the highest quality of product $B$ and makes more profits by selling $A$ and $B$ separately.

The plan of the paper is as follows. In section II, we present the model. In section III, we determine the possible relevant selling strategies. In section IV, we study and compare the separated selling strategy to the bundling strategy under different assumptions about qualities of firms’ products. Also in this section, we perform a welfare analysis and we investigate the endogenous case of quality choices with a quadratic cost function. Finally, we give antitrust implications of findings and conclude in section V.

II. The Model

There are two firms, denoted 1 and 2 and two products denoted $A$ and $B$. Firm 1 sells the products $A$ and $B$, whereas firm 2 sells only product $B$. The product $B$ comes in two distinct levels of quality, high ($H$) and low ($L$). Quality means reliability. It is the
probability of not breaking down. Let $\alpha_A$ denote the probability that product $A$ does not breakdown. Similarly, let $\alpha_{B_i}$ denote the probability that product $B$ of quality $i$ not breakdown, where $i \in \{H, L\}$. By assumption $\alpha_{B_H} > \alpha_{B_L}$.

We simplify by assuming that $A$ is produced at zero marginal cost with no fixed cost. While $B$ is produced at constant unit cost that depends on the probability of breakdown. We assume the cost function to be of the form:

$$C(q, \alpha) = qc(\alpha),$$

where $q$ and $\alpha$ denote respectively the quantity of output and the probability that a component does not breakdown. Also, we assume that $c(\alpha) \geq 0, c'(\alpha) \geq 0, c''(\alpha) \geq 0$ for all $\alpha$. Convexity of the unit cost function implies for $\alpha_{B_H} > \alpha_{B_L}$, that:

$$\alpha_{B_L} c(\alpha_{B_H}) - \alpha_{B_H} c(\alpha_{B_L}) > 0. \quad (1)$$

Consumers derive utility only from using one unit of $A$ in combination with one unit of $B$. They can, however, combine $A$ with any quality of $B$. Consumers are indexed by $\theta$, where $\theta$ is uniformly distributed on $[0,1]$. When none of the component breakdown, consumer $\theta$ obtains utility $\theta$; if one or both components breakdown his utility is zero. As shown in figure 1, under separate selling, consumers can potentially choose between two systems: $AB_H$ and $AB_L$. $AB_H$ includes $A$ and the higher quality of $B$ and $AB_L$ includes $A$ and the lower quality of $B$.

\[\begin{array}{ccc}
0 & \bar{\theta} & \tilde{\theta} & 1 \\
\hline
\text{Buy:} & \text{nothing} & AB_L & AB_H \\
\end{array}\]

$\theta \sim U[0,1]$

Figure 1

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5 We have this form of cost structure when the main part of quality improvement comes from more expensive material and input.
We note by $P_{AB_i}$, the price of the complete system $AB_i$, $i \in \{H, L\}$. If the components of the complete system are purchased individually, then the price of the complete system is the sum of the individual components’ price $(P_A + P_B)$. Else the price of the complete system is the bundle price $P_{G_i}$, where $G_i$ represents the bundle $AB_i$, $i \in \{H, L\}$.

Consumers are assumed to be risk neutral. They maximize expected surplus. This means that they choose the combination of reliabilities of $A$ and $B$ that maximizes:

$$\alpha_A \alpha_B \theta - P_{AB_i},$$

$i \in \{H, L\}$,

where $\alpha_A \alpha_B \theta$, is the expected utility derived by consumers indexed $\theta$ from the quality $i$ of product $B$ consumed with product $A$. We assume that the reliability of $A$ and $B_i$, $i \in \{H, L\}$ is common knowledge.

We indicate by $\tilde{\theta}$ the preference parameter of a consumer who is indifferent between purchasing the higher quality system ($AB_H$) and the lower quality system ($AB_L$). The consumer indexed $\tilde{\theta}$ is indifferent between purchasing the lower quality system ($AB_L$) and not purchasing at all.

The demand function for each complete system depends on prices and breakdown probabilities. To obtain the demands, we use the individual-rationality ($IR$) constraints and the incentive constraints ($IC$) of each demand consumer. For the consumer who purchases the higher quality system the following constraints are used:

$$\alpha_A \alpha_{B_H} \theta - P_{AB_H} \geq 0,$$  \hspace{2em} ($IR_{H}$)

$$\alpha_A \alpha_{B_H} \theta - P_{AB_H} \geq \alpha_A \alpha_{B_L} \theta - P_{AB_L}.$$ \hspace{2em} ($IC$)
For the one who purchases the lower quality system the individual rationality constraint is:
\[ \alpha_A \alpha_B \theta - P_{AB} \geq 0. \quad (IR_L) \]

This implies that \( \bar{\theta} = \frac{P_{AB_H} - P_{AB_L}}{\alpha_A (\alpha_B - \alpha_{B_L})} \). From the \( (IR_L) \) constraint we derive: \( \bar{\theta} = \frac{P_{AB_L}}{\alpha_A \alpha_B} \).

If we denote by \( D_H^S \) and \( D_L^S \) the respective demands for \( AB_H \) and \( AB_L \) under separate selling, we can write that:
\[ D_H^S = 1 - \bar{\theta}, \quad \text{and} \quad D_L^S = \bar{\theta} - \bar{\theta}. \]

We consider in turn two cases. Case 1 is where firm 1 produces \( A \) and \( B_H \) and firm 2 produces \( B_L \); Case 2 is where firm 1 produces \( A \) and \( B_L \) and firm 2 produces \( B_H \). Without any further loss of generality, we set \( \alpha_A = 1 \), i.e. we assume that \( A \) is produced with perfect reliability.

### III. Relevant Possible Selling Strategies

We assume that bundling is achieved costlessly. Since firm 1 produces the two complementary components that form the complete system, it has five marketing strategies. In case 1 (firm 1 produces \( A \) and \( B_H \) and firm 2 produces \( B_L \)) these are:

(i) sell \( A \) and \( B_H \) separately;
(ii) sell the bundle \( AB_H \) only;
(iii) sell \( A \) and \( B_H \) separately and sell the bundle \( AB_H \) at a price lower than the sum of individual prices (mixed bundling);\(^6\)

\(^6\) According to Adam and Yellen (1976) the first three marketing strategies refer respectively to pure components strategy or separate selling strategy (S), pure bundling strategy (PB) and mixed bundling strategy (MB).
(iv) sell $B_H$ and the bundle $AB_H$;
(v) sell $A$ and the bundle $AB_H$.

The strategies (iv) and (v) are not relevant. Indeed, strategy (iv) is equivalent to strategy (ii). The reason is that without $A$, $B_H$ alone has no value. Thus (iv) and (ii) yield only a demand for $AB_H$. This analysis applies equally to strategy (v), which is equivalent to strategy (iii). Both strategies yield demands for $A$ and $AB_H$.

The remaining strategies to consider are: (i), (ii), and (iii). We show that strategy (iii) is equivalent to strategy (i).

**Proposition 1:** Mixed bundling yields the same profits as separate selling.

**Proof**

Case 1 (firm 1 produces $A$ and $B_H$ and firm 2 produces $B_L$): the profits of firm 1 under separate selling are:

$$\pi_1^S = [P_A^S + P_{B_H}^S - c(\alpha_{B_H})]D_H^S + P_A^S D_L^S.$$  \(\text{(2)}\)

Let $D_{G_{H}}^{MB}$ and $P_{G_{H}}^{MB}$ respectively denote the demand and the price for the bundle $AB_H$.

Under mixed bundling profits are:

$$\pi_1^{MB} = [P_{G_{H}}^{MB} - c(\alpha_{B_H})]D_{G_{H}}^{MB} + [P_A^{MB} + P_{B_H}^{MB} - c(\alpha_{B_H})]D_H^{MB} + P_A^{MB} D_L^{MB}.$$  \(\text{(3)}\)

If $P_{G_{H}}^{MB} \geq P_A^S + P_{B_H}^S$, clearly $D_{G_{H}}^{MB} = 0$. Therefore:

$$\pi_1^{MB} = [P_A^{MB} + P_{B_H}^{MB} - c(\alpha_{B_H})]D_H^{MB} + P_A^{MB} D_L^{MB} \text{ and } \pi_1^{MB} = \pi_1^S.$$  \(\text{(3)*}\)

If $P_{G_{H}}^{MB} \leq P_A^S + P_{B_H}^S$, also $D_{H}^{MB} = 0$. Therefore:

$$\pi_1^{MB} = [P_{G_{H}}^{MB} - c(\alpha_{B_H})]D_{G_{H}}^{MB} + P_A^{MB} D_L^{MB}.$$  \(\text{(3)*}\)
This shows that separate selling (2) yields the same profits as mixed bundling (3*), when $P_{G_H}^{MB}$ is defined as the sum of individual components’ prices ($P_A^S + P_{B_L}^S$). Therefore, $\pi_1^{MB} = \pi_1^S$. This proves that mixed bundling strategy can’t do better than separate selling.

Case 2 (firm 1 produces $A$ and $B_L$ and firm 2 produces $B_H$): firm 1’s profits under separate selling are:

$$\pi_1^S = [P_A^S + P_{B_L}^S - c(\alpha_{B_L})]D_L^S + P_A^S D_H^S.$$  

Whereas with mixed bundling they are:

$$\pi_1^{MB} = [P_{G_L}^{MB} - c(\alpha_{B_L})]D_{G_L}^{MB} + [P_A^{MB} + P_{B_L}^{MB} - c(\alpha_{B_L})]D_L^{MB} + P_A^{MB} D_H^{MB}.$$  

if $P_{G_L}^{MB} \geq P_A^{MB} + P_{B_L}^{MB}$, clearly $D_{G_L}^{MB} = 0$. Therefore:

$$\pi_1^{MB} = [P_A^{MB} + P_{B_L}^{MB} - c(\alpha_{B_L})]D_L^{MB} + P_A^{MB} D_H^{MB}.$$  

if $P_{G_L}^{MB} \leq P_A^{MB} + P_{B_L}^{MB}$, also $D_L^{MB} = 0$, therefore:

$$\pi_1^{MB} = [P_{G_L}^{MB} - c(\alpha_{B_L})]D_{G_L}^{MB} + P_A^{MB} D_H^{MB}.$$  

It is obvious that $\pi_1^{MB} = \pi_1^S$ when $P_{G_L}^{MB}$ is the sum of prices of individual components’ prices ($P_A^S + P_{B_L}^S$).

From proposition 1, it follows that a complete discussion of strategies only requires a discussion of the choice between separate selling and pure bundling. To gain some intuitions for this result, note that mixed bundling induces a consumer who prefers to purchase only one component of the system, to purchase the whole system of components. Here products are composed of different components that cannot be used individually. Therefore, the incentive to buy the whole system is related to the functional dependence of components. Thus, all forms of mixed bundling are irrelevant in this model.
IV. Choosing a Selling Strategy

We now characterize, for case 1 and case 2, the model’s equilibrium with separate selling, and develop the comparison with that arising with bundling.

IV.1 Case 1: the producer of $A$ also produces $B_H$

Separate selling

When firm 1 sells $A$ and $B_H$ separately, there are two available systems in the market, $AB_H$ and $AB_L$. Since the demands for $AB_H$ and $AB_L$ are $D_H$, and $D_L$ respectively, the profits under separate pricing are given for firm 1 by:

$$\pi_1^S = [P_A + P_{B_H} - c(\alpha_{B_H})]D_H + P_A D_L,$$

(2)

And for firm 2 by:

$$\pi_2^S = [P_{B_L} - c(\alpha_{B_L})]D_L.$$

(4)

Firms choose prices to maximize their profits (2) and (4). We have the following first-order conditions for firm 1:

$$\frac{\partial \pi_1^S}{\partial P_A} = D_H + [P_A + P_{B_H} - c(\alpha_{B_H})] \frac{\partial D_H}{\partial P_A} + P_A \frac{\partial D_L}{\partial P_A} = 0,$$

$$\frac{\partial \pi_1^S}{\partial P_{B_H}} = D_H + [P_A + P_{B_H} - c(\alpha_{B_H})] \frac{\partial D_H}{\partial P_{B_H}} + P_A \frac{\partial D_L}{\partial P_{B_H}} = 0.$$

And for firm 2:

$$\frac{\partial \pi_2^S}{\partial P_{B_L}} = D_L + [P_{B_L} - c(\alpha_{B_L})] \frac{\partial D_L}{\partial P_{B_L}} = 0.$$

From these conditions, we obtain:

$$P_A^S = \frac{3\alpha_{B_H} \alpha_{B_L} - 2\alpha_{B_H} c(\alpha_{B_L}) - \alpha_{B_L} c(\alpha_{B_H})}{6\alpha_{B_H}},$$

(5)
By substituting these equilibrium prices back into profits functions, we obtain for each firm the following level of profits:

\[
\begin{align*}
\pi_1^s &= \frac{\alpha_{bh} - c(\alpha_{bh})}{2} D_{h}^S + \left[ \frac{\alpha_{bl} - c(\alpha_{bl})}{2} - \frac{\alpha_{bl} c(\alpha_{bh})}{6\alpha_{bh}} \right] D_{l}^S, \\
\pi_2^s &= \frac{\alpha_{bl} - c(\alpha_{bl})}{2} D_{l}^S.
\end{align*}
\]

These results differ from the standard model of vertically differentiated duopoly. In our model, firm 1 sells \(A\) to all consumers and sells \(B_h\) to those with high reservation prices for \(B\). While firm 2 sells \(B_L\) to consumers with low reservation prices for \(B\). These consumers also purchase \(A\). Therefore the willingness of firm 1 to capture its rival’s consumers is limited since it loses component \(A\)’s consumers. Firm 1 prices its components such that rival in market \(B\) cannot capture enough surplus of component \(B\)’s consumers, then it extracts all remaining surplus in the market \(A\), where it is monopoly.

Also, in contrast to a single integrated firm providing both components, firm 1 raises the price of \(A\) and lowers the price of \(B_h\) to induce firm 2 to lower the price of \(B_L\). Firm 1 does not engage in such practice to sell more of \(B\), but instead to increase the demand of \(A\).

**Pure bundling**

Now assume that firm 1 bundles \(A\) and \(B_h\). Under this strategy no one purchases \(B_L\). Therefore, this is a way to exclude firm 2 from the market. Only the system \(AB_h\) is available at price \(P_{G_H}\). Thus firm 1’s profits are:
\[ \pi_{1}^{PB} = \left[ P_{G_{H}} - c(\alpha_{B_{H}}) \right] \hat{D}_{G_{H}}, \]  
(10)

where \( \hat{D}_{G_{H}} = 1 - \hat{\theta}_{H} \), with \( \hat{\theta}_{H} = \frac{P_{G_{H}}}{\alpha_{B_{H}}} \).

\( \hat{\theta}_{H} \) represents the preference parameter of a consumer who is indifferent between purchasing the bundle \( AB_{H} \) and not purchasing.

The first-order condition is:

\[ \frac{\partial \pi_{1}^{PB}}{\partial P_{G_{H}}} = \hat{D}_{G_{H}} + \left[ P_{G_{H}} - c(\alpha_{B_{H}}) \right] \frac{\partial \hat{D}_{G_{H}}}{\partial P_{G_{H}}} = 0. \]

The equilibrium price and profit are:

\[ P_{G_{H}}^{PB} = \frac{\alpha_{B_{H}} + c(\alpha_{B_{H}})}{2}, \]  
(11)

\[ \pi_{1}^{PB} = \frac{\left[ \alpha_{B_{H}} - c(\alpha_{B_{H}}) \right]^{2}}{4\alpha_{B_{H}}}. \]  
(12)

Observe that the price of the bundle (11) is the sum of (5) and (6), which are the prices of the individual components of the bundle when firm 1 sells its components separately. In fact, the decision to bundle does not affect the systems’ prices given the qualities. The intuition is that, there is no need to offer a discount to the consumers who purchase the whole system from firm 1. The reasons are: (i) components are perfectly positively correlated in value; and (ii) components are differentiated in quality.

**Separate selling vs. pure bundling**

To see whether separate selling or bundling is more profitable for firm 1, we must compare (8) to (12).

**Proposition 2:** If the monopolist produces the higher quality version of \( B \), then he earns higher profits from separate selling than from bundling.
**Proof**

Under separate selling firm 1’s profits are:

$$\pi_1^s = [P_A^s + P_B^s - c(\alpha_{B_H})]D_H^s + P_A^s D_A^s = [P_A^s + P_B^s - c(\alpha_{B_H})](1 - \theta) + P_A^s(\theta - \theta), \quad (2)$$

While under bundling its profits are:

$$\pi_1^{pb} = [P_{G_H}^p - c(\alpha_{B_H})]\hat{D}_{G_H}^{pb} = [P_{G_H}^p - c(\alpha_{B_H})](1 - \hat{\theta}_H). \quad (10)$$

We can rewrite this latter function as:

$$\pi_1^{pb} = [P_{G_H}^p - c(\alpha_{B_H})](1 - \hat{\theta}_H) = [P_{G_H}^p - c(\alpha_{B_H})](1 - \tilde{\theta}) + [P_{G_H}^p - c(\alpha_{B_H})](\tilde{\theta} - \hat{\theta}_H) \cdot (10^*)$$

Since $$P_A^s + P_B^s = P_{G_H}^p$$, the first terms of (2) and (10*) are equal. Then to compare $$\pi_1^s$$ to $$\pi_1^{pb}$$, we have just to compare the last term of (2), $$P_A^s (\tilde{\theta} - \tilde{\theta})$$ to the one of (10*), $$[P_{G_H}^p - c(\alpha_{B_H})](\tilde{\theta} - \hat{\theta}_H)$$. When we compute these two terms, we find that the former can be written as:

$$P_A^s (\tilde{\theta} - \tilde{\theta}) = P_A^s \left( \frac{\alpha_{B_L} c(\alpha_{B_H}) - \alpha_{B_H} c(\alpha_{B_L})}{3\alpha_{B_L} (\alpha_{B_H} - \alpha_{B_L})} \right).$$

While the latter is:

$$[P_{G_H}^p - c(\alpha_{B_H})](\tilde{\theta} - \hat{\theta}_H) = [P_{G_H}^p - c(\alpha_{B_H})]\left( \frac{\alpha_{B_L} c(\alpha_{B_H}) - \alpha_{B_H} c(\alpha_{B_L})}{3\alpha_{B_L} (\alpha_{B_H} - \alpha_{B_L})} \right).$$

Therefore separate selling is better than bundling if and only if:

$$P_A^s (\tilde{\theta} - \tilde{\theta}) > [P_{G_H}^p - c(\alpha_{B_H})](\tilde{\theta} - \hat{\theta}_H) \iff P_A^s > [P_{G_H}^p - c(\alpha_{B_H})] \frac{\alpha_{B_L}}{\alpha_{B_H}}.$$  

As $$P_A^s + P_B^s = P_{G_H}^p$$, we have:

$$P_A^s (\tilde{\theta} - \tilde{\theta}) > [P_{G_H}^p - c(\alpha_{B_H})](\tilde{\theta} - \hat{\theta}_H) \iff \frac{\alpha_{B_L} c(\alpha_{B_H}) - \alpha_{B_H} c(\alpha_{B_L})}{3\alpha_{B_L}} > 0.$$  

Since $$\alpha_{B_L} c(\alpha_{B_H}) - \alpha_{B_H} c(\alpha_{B_L}) > 0$$, we have $$\pi_1^s > \pi_1^{pb}.$$
The intuition behind the result of proposition 2 is the following (see figure 2): when firm 1 sells its products separately, it sells $A$ and $B_H$ to consumers $(1 - \bar{\theta})$ and $A$ to consumers $(\bar{\theta} - \bar{\theta})$. When it decides to bundle, then it sells the bundle $AB_H$ to consumers $(1 - \hat{\theta}_H)$ and does not sell any more $A$ to consumers $(\bar{\theta} - \bar{\theta})$. Then to see which strategy is better, we must compare the additional benefits from bundling that result from selling $AB_H$ to additional consumers $(\bar{\theta} - \hat{\theta}_H)$, to losses from bundling that result from not selling $A$ to former consumers $(\bar{\theta} - \bar{\theta})$. The additional benefits from bundling are: $[P_{c_{H}}^{PB} - c(\alpha_{B_{H}})](\bar{\theta} - \hat{\theta}_H)$, the benefits coming from new consumers demanding $AB_H$. While the losses from bundling are: $P_{A}^{S}(\bar{\theta} - \bar{\theta})$, the resulting losses from consumers who do not buy $A$ any more. Therefore separate selling is better than bundling if and only if:

$$P_{A}^{S}(\bar{\theta} - \bar{\theta}) > [P_{c_{H}}^{PB} - c(\alpha_{B_{H}})](\bar{\theta} - \hat{\theta}_H).$$
This is the case when the production cost is convex: \( \alpha_{B_h} c(\alpha_{B_h}) - \alpha_{B_l} c(\alpha_{B_l}) > 0 \). Here the result of bundling is a narrowing of market since consumers \( (\hat{\theta}_H - \theta) \) who bought the system of lower quality do not buy under bundling.

IV.2 Case 2: the producer of \( A \) also produces \( B_L \).

Separate selling

When firm 1 sells \( A \) and \( B_L \) separately, the profits of firm 1 and firm 2 are respectively given by:

\[
\begin{align*}
\pi_1^S &= [P_A + P_{B_L} - c(\alpha_{B_L})]D_L + P_A D_H, \\
\pi_2^S &= [P_{B_H} - c(\alpha_{B_H})]D_H.
\end{align*}
\]

And equilibrium prices are:

\[
\begin{align*}
P_A^S &= \frac{2\alpha_{B_H} + \alpha_{B_L} - 2c(\alpha_{B_H}) - c(\alpha_{B_L})}{6}, \\
P_{B_H}^S &= \frac{2c(\alpha_{B_H}) + c(\alpha_{B_L}) - \alpha_{B_L} + \alpha_{B_H}}{3}, \\
P_{B_L}^S &= \frac{c(\alpha_{B_H}) + 2c(\alpha_{B_L}) - \alpha_{B_H} + \alpha_{B_L}}{3}.
\end{align*}
\]

We observe that the price of \( A \) is higher and the prices of \( B_H \) and \( B_L \) are lower compared to the case where the monopolist produces \( B_H \). This comes to the fact that firm 1 increases the price of \( A \) to capture more surplus of consumers with high reservation prices for system \( AB_H \). Therefore the prices of strategic substitute, \( B_H \) and \( B_L \), decrease. Here there is much more surplus to capture in the market of \( B \).

By substituting (15), (16) and (17) into the profit functions (13) and (14), we obtain:
\[
\pi_1^S = \left[\frac{2\alpha_{B_H} + \alpha_{B_L} - 2c(\alpha_{B_L}) - c(\alpha_{B_L})}{6}\right]D_H^S + \left[\frac{\alpha_{B_L} - c(\alpha_{B_L})}{2}\right]D_L^S, \quad (18)
\]
\[
\pi_2^S = \left[\frac{\alpha_{B_L} - c(\alpha_{B_L})}{2}\right]D_H^S. \quad (19)
\]

In comparison to case 1, we find that firm 1’s profits are lower while firm 2’s profits are higher. This is because now firm 2 serves the high reservation price consumers of \(B\), which is the most profitable segment of the market.

**Pure bundling**

If firm 1 bundles, its profits are:
\[
\pi_1^{PB} = [P_{G_L} - c(\alpha_{B_L})]\hat{D}_{G_L}, \quad (20)
\]
where \(\hat{D}_{G_L} = 1 - \hat{\theta}_L\) with \(\hat{\theta}_L = \frac{P_{G_L}}{\alpha_{B_L}}\),

where \(\hat{\theta}_L\) is the preference parameter of a consumer indifferent between purchasing bundle \(AB_L\) and not purchasing.

Since there is compatibility, the consumers who desire to purchase the higher quality system \(AB_H\), must purchase \(B_H\) in addition to bundle \(AB_L\). They will do so if:
\[
\alpha_{B_H} \theta - P_{G_L} - P_{B_H} \geq \alpha_{B_L} \theta - P_{G_L}. \quad (21)
\]

If we denote by \(\theta_H^*\), the preference parameter of an indifferent consumer between purchasing \(B_H\) in addition to the bundle \(AB_L\) to form the higher quality bundle \(AB_H\) and the lower bundle \(AB_L\) alone, we obtain from (21):
\[
\theta_H^* = \frac{P_{B_H}}{\alpha_{B_H} - \alpha_{B_L}}. \quad (22)
\]

Consequently the demand for the higher quality bundle \(AB_H\) is:
\[
D_H^* = 1 - \theta_H^*. \quad (23)
\]

This gives firm 2’s profits as:
\[
\pi_2^{PB} = [P_{B_H} - c(\alpha_{B_H})]D_H^*. \quad (23)
\]
By maximizing (20) and (23) with respect to prices, one obtains:

\[ P_{B_h}^{PB} = \frac{\alpha_{B_h} - \alpha_{B_L} + c(\alpha_{B_h})}{2} \]

\[ P_{B_L}^{PB} = \frac{\alpha_{B_h} + c(\alpha_{B_L})}{2} \]  

(24)  

From (25), we can rewrite \( \theta_h^* \) as:

\[ \theta_h^* = \frac{P_{B_h}^{PB}}{\alpha_{B_h} - \alpha_{B_L}} = \frac{1}{2} + \frac{c(\alpha_{B_h})}{2(\alpha_{B_h} - \alpha_{B_L})} \]  

(26)

We have \( 0 \leq \theta_h^* \leq 1 \), i.e.:

\[ \frac{1}{2} + \frac{c(\alpha_{B_h})}{2(\alpha_{B_h} - \alpha_{B_L})} \leq 1 \iff \alpha_{B_h} - \alpha_{B_L} \geq c(\alpha_{B_h}) \]  

(27)

Condition (27) is the constraint for firm 2 to remain in the market. Firm 2 sells \( B_h \) if and only if the difference of reliability between \( B_h \) and \( B_L \) is greater than the unit production cost of \( B_h \). Otherwise all consumers prefer to purchase the bundle of lower quality \( AB_L \) (i.e. nobody will purchase \( B_h \) in addition). Equilibrium profits for firm 1 and firm 2 are respectively:

\[ \pi_1^{PB} = \frac{[\alpha_{B_h} - c(\alpha_{B_h})]^2}{4\alpha_{B_L}} \]  

(28)

\[ \begin{cases} \pi_2^{PB} = \frac{[\alpha_{B_h} - \alpha_{B_L} - c(\alpha_{B_h})]^2}{4(\alpha_{B_h} - \alpha_{B_L})} & \text{if } \alpha_{B_h} - \alpha_{B_L} \geq c(\alpha_{B_h}) \\ \pi_2^{PB} = 0 & \text{else} \end{cases} \]

Separate selling vs. pure bundling

The ranking of profits for the monopolist (firm 1) is summarized in the following proposition.
**Proposition 3:** If the monopolist produces the lower quality version of B, then he earns lower profits from separate selling than from bundling.

**Proof**

Since the price of the bundle $AB_L$ is the same under both strategies and since we have $\hat{\theta}_L = \hat{\theta}$; i.e. the consumer who is indifferent between purchasing $AB_L$ and not purchasing at all, is the same under both strategies, we have:

$$\hat{\theta}_L = \frac{P_{g_{l}}^{p_{g_{l}}}}{\alpha_{B_{l}}} = \frac{P_S + P_{B_{l}}^S}{\alpha_{B_{l}}} = \hat{\theta}.$$  

Therefore the demand for $A$ under separate selling equals the demand for $A$ under pure bundling, i.e. $1 - \hat{\theta} = 1 - \hat{\theta}_L$. On the other hand the demand for $B_L$ under separate selling is smaller than the demand for $B_L$ under pure bundling. Because: $\hat{\theta} - \hat{\theta} \leq 1 - \hat{\theta}$. Consequently the demand for the bundle $AB_L$ is weakly higher under pure bundling for the same bundle prices. Thus, firm 1’s profits are higher under pure bundling than those under separate selling.

---

**Figure 3**

<table>
<thead>
<tr>
<th>Bundling</th>
<th>Buy:</th>
<th>nothing</th>
<th>$AB_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[\hat{\theta}_L]</td>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>Separate Selling</td>
<td>Buy:</td>
<td>nothing</td>
<td>$AB_L$</td>
</tr>
<tr>
<td></td>
<td>[\theta \sim U[0,1]]</td>
<td>---------</td>
<td>--------</td>
</tr>
</tbody>
</table>
For proposition 3, the intuition is simple (see figure 3). The price of the bundle is the same under both regimes and demands under pure bundling are greater than demands under separate selling. Therefore for firm 1, bundling is better than separate selling. We found a case where bundling is a profitable strategy and can lead to foreclosure and exclusion of the rival from the market.

To sum up, firm 1 who is monopolist on $A$ never finds it worthwhile to bundle in order to reduce the level of competition in market for $B$ when the result of bundling is a narrowing of market for its monopolized component. The reason lies in the fact that since the monopolized product $A$ is essential for all uses of products $B$ ($B_{H}$ and $B_{L}$), the monopolist can benefit from competition in $B$ through sales of its monopolized product $A$ with separate selling. This is the case when the monopolist of $A$ produces $B_{H}$. Therefore he prefers separate selling over bundling. On the other hand when the monopolist of $A$ produces $B_{L}$, separate selling brings no increase in the demand for its monopolized product $A$ and even reduces the demand for its product $B_{L}$ through competition with $B_{H}$. Therefore, separate selling is dominated by bundling.

**Corollary 1:** The monopolist will sell its products separately if and only if its product $B$ has the higher quality.

Whinston (1990) and Matutes and Regibeau (1992) stated that if firms sell compatible components, then bundling is dominated by separate selling. The set up of Whinston (1990) is same as here except that: (i) he considers the case of technical bundling; and (ii) he does not consider products differentiated in quality. In the model of Matutes and Regibeau (1992), no firm has a monopoly over either component and products are horizontally differentiated. Therefore our results cannot be directly compared to their results. However, our results extend theirs to the case where components are vertically differentiated.\textsuperscript{7,8}
IV.3 Welfare Analysis

Now let us see how consumers’ surplus and social welfare are affected by bundling decision. When firm 1 sells its components separately, consumers’ surplus denoted by $\text{CS}^S$ is:

$$\text{CS}^S = \int_0^1 (\alpha_{B_H} \theta - P_{AB_H}^S) d\theta + \int_0^{\bar{\theta}} (\alpha_{B_L} \theta - P_{AB_L}^S) d\theta .$$

While the social welfare denote by $\text{SW}^S$ is:

$$\text{SW}^S = \text{CS}^S + \pi_1^S + \pi_2^S .$$

When bundling is allowed consumers’ surplus ($\text{CS}^{PB}$) is:

$$\text{CS}_i^{PB} = \int_0^{\tilde{\theta}_i} (\alpha_{B_i} \theta - P_{G_i}^{PB}) d\theta = \frac{(1-\tilde{\theta}_i^2)}{2} (\alpha_{B_i} - P_{G_i}^{PB} (1-\tilde{\theta}_i)) , i \in \{H, L\} .$$

And the social welfare ($\text{SW}^{PB}$) is:

$$\text{SW}_i^{PB} = \text{CS}_i^{PB} + \pi_1^{PB} + \pi_2^{PB} , i \in \{H, L\} .$$

**Proposition 4:** Consumers’ surplus and social welfare are always higher under separate selling than under pure bundling.

**Proof**

Intuitively for case 1 (firm 1 produces $A$ and $B_H$, whereas firm 2 produces $B_L$), pure bundling increases the number of consumers of $AB_H$ from $(1-\tilde{\theta})$ to $(1-\tilde{\theta}_H)$ and eliminates $(\tilde{\theta} - \tilde{\theta})$ of consumers of $AB_L$ (see figure 2). Therefore, consumers’ surplus under pure bundling is greater than under separate selling if and only if the additional surplus obtained by new consumers of $AB_H$, $(\tilde{\theta} - \tilde{\theta}_H)$ when consuming the higher quality under bundling (rather than the lower quality under separate selling) is lower than the

\footnote{Remark that when costs of quality improvement are fixed, the result is easily deductible. The monopolist chooses bundling such to foreclose its competitor in $B$. Indeed, as proved by Diallo (2004), for symmetrical distribution of consumers’ taste for quality, the monopolist prefers to produce only one variety of a system when costs of quality improvement are fixed. So he would like to eliminate the others varieties from the market.}
losses of surplus of consumers \((\bar{\theta} - \bar{\theta})\) who do not consume \(AB_L\) under pure bundling. Formally we have \(CS^S \geq CS^PB\) if:

\[
\int_{\theta_H}^{\bar{\theta}} [(\alpha_{B_H} - \alpha_{B_L})\theta - (P_{PB_L} - P_{PB_H})]d\theta \geq \int_{\theta_H}^{\bar{\theta}} (\alpha_{B_L} - P_{PB_L})d\theta.
\]

(29)

After computation we obtain the following condition for (29) to be true:

\[
(\alpha_{B_H} P_{PB_H} - \alpha_{B_L} P_{PB_L})^2 \geq 0.
\]

Thus, consumers’ surplus is higher under separate selling than under pure bundling. Since this is also true for firms’ profits, this is true for social welfare.

For case 2 (firm 1 produces \(A\) and \(B_L\), whereas firm 2 produces \(B_H\)), the result is straightforward. Indeed by bundling, \((1 - \bar{\theta})\) consumers purchase \(AB_L\) instead of \(AB_H\). This reduces the surplus of these consumers who had higher surplus with \(AB_H\). At the same time, the surplus of consumers \((\bar{\theta} - \bar{\theta})\) does not change since prices and qualities are identical (see figure 3). Consequently consumer surplus decreases under bundling. Also, under bundling firm 2’s losses (not to sell \(B_H\)) exceed firm 1’s additional benefits (sell more \(B_L\)). Since firms’ profits also decrease under bundling, immediately social welfare also decrease under bundling.

\[\bullet\]

**IV.4 Numerical Application with Endogenous Quality Choice**

We consider now a game where firms compete in two-stages. In the first stage, the monopolist chooses its marketing strategy (bundling or not) and the two firms simultaneously choose their quality levels. In the second they concurrently determine prices- given the qualities and strategy already chosen- and produce the output which
satisfies consumers’ demands. A solution of this game consists of a set of equilibrium quality and pricing decisions for each firm. We focus on subgame perfect Nash equilibria. We also assume that the marginal cost of production is quadratic, i.e.:

$$c(\alpha) = \frac{(\alpha)^2}{2}.$$

Since quality is endogenous both firms may choose the same quality for $B$. If firm 1 does not bundle then Bertrand competition leads to a unique equilibrium where prices equal unit cost. Hence, both firms make zero profits. Therefore, firm 2 produces a component $B$ differentiated in quality from firm 1’s under separate selling and bundling.

From the previous results we know that the only certain equilibrium with two active firms is where firm 1 produces the higher quality of $B$. In that situation it sells its components separately (otherwise it bundles its components and this can exclude firm 2). Then the profits are given by (2) for firm 1 and by (4) for firm 2. With a quadratic cost function, we rewrite these profits as:

$$\pi_1^S = [P_A + P_{B_H} - \frac{(\alpha_{B_H})^2}{2}]D_H + P_A D_L,$$  (30)

$$\pi_2^S = [P_{B_L} - \frac{(\alpha_{B_L})^2}{2}]D_L.$$  (31)

For any given pair of reliability, maximizing (30) and (31) with respect to prices, gives the following equilibrium prices:

$$P_A^S = \frac{6\alpha_{B_L} - 2(\alpha_{B_H})^2 - \alpha_{B_H} \alpha_{B_H}}{12},$$  (32)

$$P_{B_H}^S = \frac{6\alpha_{B_L} + 2(\alpha_{B_H})^2 - 6\alpha_{B_L} + \alpha_{B_L} \alpha_{B_H} + 3(\alpha_{B_H})^2}{12},$$  (33)

$$P_{B_L}^S = \frac{\alpha_{B_L} (2\alpha_{B_L} + \alpha_{B_H})}{6}.$$  (34)

By the first derivatives in quality, we observe that $P_A^S$ is decreasing in $\alpha_{B_H}$ and increasing in $\alpha_{B_L}$, $P_{B_H}^S$ is increasing in $\alpha_{B_H}$ and decreasing in $\alpha_{B_L}$ and $P_{B_L}^S$ is increasing in
α_{B_H} and α_{B_L}. These variations suggest that for firm 1 to induce firm 2 to lower the price of B_L, it must choose α_{B_H} as close as possible to α_{B_L}. Also, we know that P^S_{B_L} is increasing in α_{B_L}; this implies that the quality dispersion (difference of qualities) will be lower under this market structure compared to the traditional model of duopoly in a Mussa and Rosen (1978) framework.

We look now for the solution of the quality game. Firms choose their quality specification to maximize their profits (30) and (31). By substituting the equilibrium prices (32), (33), and (34) back into profit functions, and maximizing these latter functions with respect to qualities, we get a nonlinear system of equations of (α_{B_H}, α_{B_L}) which is solved numerically to obtain the following α_{B} s:

\begin{align*}
\alpha^S_{B_H} &= 0.710102, \\
\alpha^S_{B_L} &= 0.35505.
\end{align*}

Equilibrium prices and profits for separated selling are therefore:

\begin{align*}
P^S_A &= 0.1355; \\
P^S_{B_H} &= 0.3456; \\
P^S_{B_L} &= 0.08404
\end{align*}

\begin{align*}
\pi^S_1 &= 0.076329; \\
\pi^S_2 &= 0.002486.
\end{align*}

Expressions (35) and (36) give the pair of candidate equilibrium qualities. The second derivatives with respect to qualities given the equilibrium qualities are negative:

\begin{align*}
\frac{\partial^2 \pi^S_1}{(\partial \alpha_{B_H})^2} &\bigg|_{\alpha_{B_H}=0.71010} = -0.2139 \leq 0 \quad \text{and} \quad \\
\frac{\partial^2 \pi^S_2}{(\partial \alpha_{B_L})^2} &\bigg|_{\alpha_{B_L}=0.35505} = -0.0394 \leq 0.
\end{align*}

However, that shows only that (35) and (36) represent a local maximum. This is not enough to ensure we have found a Nash equilibrium. To be sure that our candidate maximum is indeed an equilibrium we also have to check first that firm 2 has no incentive to “leapfrog” firm 1 and itself produce the highest quality and second that firm 1 makes more profits as a monopolist in A and a duopolist producing B_H than as a monopolist in both markets A and B.
If firm 2 produces its component $B$ at a quality level above the one of firm 1, we know that the best strategy of firm 1 would be bundling. We show that in this case firm 2 makes zero or less profits than before “leapfrogging”. After “leapfrogging” firm 1 profits are given by:

$$\pi^P_1 = [P_{G_i} - c(\alpha_{B_i})]\hat{D}_{G_i}. \tag{37}$$

After optimization of (37), we obtain easily the following results for equilibrium quality, price and profits:

$$\alpha^P_{BL} = 0.6666 ; P^P_{G_i} = 0.44444 ; \pi^P_1 = 0.07407 .$$

Since $\pi_1$ is concave in quality, by showing that $$\frac{\partial^2 \pi_1}{\partial \alpha_B^2} < 0,$$ is a sufficient condition for $\alpha^P_B = 0.6666$ being a global maximum. That is the case since $$\frac{\partial^2 \pi_1}{\partial \alpha_B^2} |_{\alpha_B = 0.6666} = -1 \leq 0 .$$

For firm 2, its profits are:

$$\begin{align*}
\pi^P_2 &= \begin{cases} 
\frac{[\alpha_{BH} - \alpha_{BL} - (\alpha_{BH})^2]}{2} & \text{if } \alpha_{BH} - \alpha_{BL} \geq \frac{(\alpha_{BH})^2}{2} \\
0 & \text{else}
\end{cases} \tag{38}
\end{align*}$$

Under the constraints: $1 \geq \alpha_{BH} \geq \alpha_{BL}$ and $\alpha^P_B = 0.6666$, there is no solution $\alpha^P_{BH}$ which satisfies $\alpha^P_{BH} - \alpha^P_{BL} \geq \frac{(\alpha^P_{BH})^2}{2}$ for the maximization of (38) with respect to qualities. Therefore, we have $\pi^P_2 = 0$.

Now we must check whether firm 1 makes more profits as a duopolist producing $B_H$ than as a monopolist in $B$. It is straightforward that firm 1’s profits under separate selling are higher than those under bundling since:

$$\pi^S_1 = 0.076329 > 0.07407 = \pi^P_1 .$$
**Corollary 2:** The monopolist will always produce the higher quality of B in a dynamic game.

Thus, the outcome of the subgame perfect Nash equilibrium is such that firm 1 selects $\alpha_{BH}^S$ equals to 0.710102, and firm 2 selects $\alpha_{BL}^S$ equals to 0.35505. $\alpha_{BH}^S$ is half of $\alpha_{BH}^S$ and their difference equals to 0.35505. When we determine quality choice in a standard model of duopoly in a Mussa and Rosen (1978) framework, we obtain that $\alpha_{BH}^D$ equals to 0.8195 and $\alpha_{BL}^D$ equals to 0.3987; their difference equals to 0.4208.

Since 0.4208 is greater than 0.35505, immediately we deduce that the spread of component B quality in our market structure outcome is too low relative to the standard profit maximization duopoly outcome. The reason is that the monopolist want to extract more surplus in market A from intensifying price competition in market B. He achieves that by not differentiate its component B from rival’s. Consequently, quality differentiation in market B between the firms is tighter and price competition is intensified.

**V. Antitrust Implications and Conclusion**

We analyzed bundling incentives in markets where products are composed of two complementary components. One of the components is monopolized and the other is sold by a duopoly. We assumed quality differentiation between components. We developed a model of quality competition and we demonstrated the monopolist’s bundling decision depends on the quality level of the component where he faces competition. We proved that if the monopolist’s component has a quality higher than the competitor’s then the monopolist prefers not to bundle. On the other hand if the quality of this component is

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9 When we compute consumers’ surplus and social welfare under both marketing strategies, we find that:

$$CS^S = 0.038165; \quad CS^D = 0.03703; \quad SW^S = 0.117; \quad SW^D = 0.111$$

As proved in the preceding section, consumers’ surplus and social welfare are higher under separate selling than pure bundling.
lower than the competitor’s then the monopolist prefers bundling. For the monopolist the incentive not to bundle is related to market extensions for its monopolized component. He chooses the marketing strategy which helps to achieve this goal. Consequently it may appear a possible exclusion or foreclosure of competitor. Thus, we showed that this is an exclusionary bundling. However, when we look for the subgame perfect Nash equilibrium, we found that the monopolist chooses to produce the higher quality of the complementary product and separate selling is its best strategy.

The results of this paper, which show that bundling could be anticompetitive can be used to asses a proper legal rule regarding bundling in market for complementary components when quality matters. For example in the Microsoft litigation courts have had to determine whether it was lawful for Microsoft to bundle its Internet Explorer browser with Windows operating system. Since Microsoft is almost a monopoly in the personal computer operating system, faces competition in the browser market essentially from Netscape and both products are complementary, this case fits appropriately our model if we consider the two web browsers to be quality differentiated. Absent the possibility that the complementary component, Netscape Navigator might later in the future become a substitute to Windows, the presence of network externality, and predatory pricing, our results suggest that the bundling strategy adopted by Microsoft implies that Internet Explorer is of lower quality than Netscape Navigator and that Microsoft leverage its market power from the operating system market to the web browser market.

In most of the studies on bundling there is no clear cut about welfare effects of bundling. We found that the welfare effects of bundling are clearly negative. This establishes that an efficiency presumption of bundling in markets where products are composed of two strictly complementary components, with market power in one component is unwarranted. This suggests a prohibition per se of bundling in these types of markets.

In an imperfect and asymmetric information world, bundling may also be a signal for bad quality of the monopolist’s complementary product. It is worthwhile to note that the results of this paper must be interpreted under the following simplifications: we ignored
positive demand externalities (or network externalities) and we assumed no cost savings result from bundling. Network externalities will increase the profitability of separate selling, while cost savings that result from economy of scope in distribution will increase the profitability of bundling. Finally, for future research, it will be interesting to look at the case where each firm supplied all the components necessary to form the complete system.

References


