Measuring Distortion in Multitask Agencies*

Veikko Thiele†

School of Business and Economics
Humboldt-University at Berlin

February 28, 2005

Abstract

In this paper I consider a multitask-agency model to measure the induced distortion between the agent’s activities when the performance measure is not congruent to the principal’s objective. I distinguish between two cases. First, the agent’s marginal costs are equal across tasks. Then the measure for distortion is the same as for congruity emphasized by Baker [2002]. In the more general case where marginal costs differ, the measure depends on the outcome/cost sensitivity ratios and is different to the measure of congruity. Besides deriving conditions to rank performance measures in a multitask setting, I show that a more congruent performance measure can lead to a lower expected profit.

Keywords: Performance measurement, distortion, multi-task agencies, congruence, incentives.

JEL classification: D23, D82, M52

---

*Exceedingly preliminary. Please do not quote without permission. Financial support by the Konrad-Adenauer Foundation is gratefully acknowledged.

†Address: Spandauer Str. 1, D-10178 Berlin, Germany, phone: +49 30 2093-1344, fax: +49 30 2093-1343, e-mail: thiele@wiwi.hu-berlin.de
1 Introduction

Since the paper by Holmstrom and Milgrom [1991], a major research branch of agency theory focus on the possibility that the agent’s effort choice is multidimensional. Besides outweighing risk and insurance, the additional objective of an incentive scheme in a multitask setting is to ensure that the agent chooses the appropriate effort allocation across tasks. Feltham and Xie [1994] shows that incentive schemes based on verifiable performance measures can imply a further drawback: this performance measure is not necessarily congruent to the principal’s objective. Consequently, the agent may focus on less valuable activities and implement suboptimal intensities of other activities. This induced distortion between the multitask activities has a negative affect on the principal’s expected profit.

Obviously, the characterization of the performance measure congruity and its induced distortion between the agent’s activities is essential to explains incentive problems in multitask agency. Baker [2002] emphasized that the congruence of the performance measure can be measured by the angle between the vector of the outcome sensitivity and the vector of the performance measure sensitivity. So far, none of the paper dealing with incongruent performance measures derived a measure for the induced distortion.\(^1\) Accordingly, the main purpose of this paper is to derive a measure to quantify distortion in a multitask agency.

To provide an elementary measure for distortion I consider a multitask-agency model with a risk-neutral principal and a risk-averse agent. To do so, I distinguish between two cases. First, the agent’s marginal costs are equal across tasks. Then the measure for distortion is the same as for congruity. In contrast, in the more general case where marginal costs differ across tasks, the measure for distortion depends on the outcome/cost sensitivity ratios and is different to the measure of congruity. For each considered case I provide a criteria in order to rank the value of performance measures. As this analyzes shows, the signal-noise ratio as useful measure for the one-dimensional case cannot be transferred to the more complex multidimensional case. Interestingly, the analyzes show that a more congruent performance measure with the same noise does not necessarily lead to less distortion and hence, to a higher expected profit. In specific circumstances it can me beneficial for the principal to apply a less congruent performance measure.

\(^1\)Feltham and Xie [1994] is an exception. They derived a single measure for the two-dimensional case.
In his paper, Kerr [1975] discussed the consequences if a reward system overemphasizes some activities and underemphasize others than it would be optimal from a principal’s point of view. In his paper he provides many examples e.g. faculty at universities and their trade-off between research and teaching. Since teaching is harder to measure, teacher usually focus on research at the expense of teaching.\footnote{See Brickley and Zimmerman [2001] for an empirical study of this example.} The problem occurs because the principal has no access to a measure that perfectly coincides with his objective.

In the more recent literature, researcher focus on the induced consequences if performance measures are not congruent to the principal’s objective. Feltham and Xie [1994] developed a multitask agency model and find that the application of incongruent performances induces distortion between the agent’s effort allocation and hence, reduce the principal’s expected outcome. They show that this problem can be mitigated by the application of additional performance measures. The work by Feltham and Xie [1994] arouses further analysis such as Baker [2000], Datar, Kulp, and Lambert [2001], Baker [2002] and Gibbons [2005]. However, all models have one thing in common: In order to keep their analyzes simple, they assume that the agent’s marginal effort costs are constant across tasks. By considering agents in organizations, it is obvious that this assumption is rather restrictive. That is, differing marginal costs across tasks are more reasonable. For instance, a worker in the car manufacturing industry needs more effort to place the engine in the car instead of fixing the engine with screws. Furthermore, for the sales force in an insurance company it is easier to care for clients instead of the acquisition of new clients.

The main purpose of this paper is to consider an agency relationship with a risk-neutral principal and a risk-averse agent in order to derive a measure for the occurred distortion between the agent’s multitask activities induced by incongruent performance measures. This measure can be used in future multitasking settings to keep the analysis trackable. In order to shed light on the agent’s effort costs and their consequences to the induced distortion, I distinguish between to cases. First, the effort costs are constant across tasks, and second, they differ. This distinction leads to different observation regarding distortion and the criteria in order to rank performance measures. To address these issues I combine basically three elements in my analysis: (i) a moral hazard framework with a risk-averse agent, (ii) an incongruent and noisy performance measure about the agent’s multitask activities and (iii), constant or differing marginal
costs across tasks. To the best of my knowledge this paper is the first one that extends the approaches by Feltham and Xie [1994], Datar et al. [2001] and Baker [2002] with the opportunity for differing marginal costs in order to derive a measure for distortion induced by a congruity problem within an organization.

In general, this paper shows that the implication emphasized e.g. by Feltham and Xie [1994], Baker [2000], Datar et al. [2001], Baker [2002] and Gibbons [2005] derived for the case with constant marginal costs cannot be transferred to the more general case where marginal costs differ across tasks. In particular, I show that in a first-best setting the principal determines each single activity according to its respective outcome/cost sensitivity ratio. In this case the agent implements the first-best effort intensity and the first-best effort allocation, referred to be non-distorted. In the second-best case, the agent implements only the first-best effort intensity, if his performance measure is congruent to the principal’s objective. When the agent’s marginal costs are equal across tasks, the measure of congruity is as well the measure for the induced distortion. Therefore, a more congruent performance leads to less distortion and hence, to a higher expected profit. In contrast, if marginal costs differ across tasks, the measure for distortion differs considerable from the measure of congruity. In this case, a more congruent performance measure can yield a lower expected profit for the principal. Finally, I show that the signal-noise ratio cannot be used in a multitask setting to rank performance measures regarding their value to the principal. The new derived measure is a function of the measured distortion for each considered case.

The remainder of this paper is structured as follows. In section 12 I present the assumption of the model and derive the first-best solution in section 3. In section 5 I analyze the case where marginal costs are constant and in section 6 the more general case where costs differ across tasks. Section 7 concludes.

2 The Model

A risk-neutral principal owns an asset that is worthless without the productive effort of an agent. The agent is risk-averse and his preferences are represented by negative exponential utility function with

\[ U^A(w^A) = - \exp\{-\rho w^A\}, \]

where \( \rho \) denotes the Arrow-Pratt measure of absolute risk-aversion and \( w^A \) the agent’s wage payment. For parsimony, the agent’s reservation utility is normalized to zero.
The agent can implement $n$ non-verifiable multitask activities $e = (e_1, ..., e_n)'$, where $e_i \in \mathbb{R}^+$ for all $i = 1, ..., n$. These multitask activities induce additive separable and strictly convex increasing effort costs with

$$C(e) = \sum_{i=1}^{n} \psi_i e_i^2 / 2,$$

where $\psi = (\psi_1, \psi_2, ..., \psi_n)'$ is a $n$-dimensional vector of the agent’s effort costs sensitivity with $\psi_i \in \mathbb{R}^{++}$ for all $i = 1, ..., n$.

By exerting the multitask activities, the agent influence the non-verifiable outcome $V(e) = \mu e + \varepsilon_V$, where $\mu = (\mu_1, ..., \mu_n)'$ is vector of the outcome sensitivity with $\mu_i \in \mathbb{R}$ for all $i = 1, ..., n$. In addition, the noise term $\varepsilon_V$ is assumed to be normally distributed with $\varepsilon_V \sim N(0, \sigma_V^2)$. Since the outcome is non-verifiable it cannot be part of an explicit one-period contract. The only verifiable information about the agent’s action choice is the performance measure $P(e) = \omega e + \varepsilon_P$, where $\omega = (\omega_1, ..., \omega_n)'$ denotes the vector of the performance measure sensitivity with $\omega_i \in \mathbb{R}$ for all $i = 1, ..., n$. Moreover, $\varepsilon_P$ is a random noise variable assumed to be normally distributed with $\varepsilon_P \sim N(0, \sigma_P^2)$ and represents effects outside the agent’s control.

Obviously, as long as there exists a multiplier $\vartheta > 0$ such that $\mu = \vartheta \omega$, the performance measure is congruent to the principal’s objective. If not, the performance measure is referred to be incongruent to the principal’s objective. Then, the application of this performance measure in an incentive contract leads to an inefficient effort allocation across all task (Feltham and Xie [1994], Baker [2002]). Baker [2002] provided a simple measure for congruity. Since his result is important throughout this paper I summarize his observation in the following definition.

**Lemma 1** The congruence between the vector of the outcome sensitivity $\mu$ and the vector of the performance measure sensitivity $\omega$ is measured by $\tilde{\Upsilon}(\xi) = \cos \xi$, where $\xi$ is the angle between both vectors.

According to this lemma, as long as vector $\mu$ and vector $\omega$ are not linear dependent such that $\mu = \lambda \omega$ with $\lambda > 0$, $\xi > 0$ and there exists a congruency problem.

---

3 All used vectors are column vectors where "'" denotes the transpose of an vector.

4 Nevertheless, the principal can use the outcome in an implicit contract in a multi-period setting under certain conditions, see e.g. Telser [1980], Bull [1987] and Levin [2003] for the theoretical foundation and Pearce and Stacchetti [1998] and Baker, Gibbons, and Murphy [2002] for an application.
Due to moral hazard, the principal provides the agent a linear compensation scheme $w^A$ contingent on the only available contractible information, the performance measure $P(e)$. The structure of this payment is

$$w^A(e) = \alpha + \beta P(e),$$

(3)

where $\alpha$ denotes the fixed payment and $\beta$ denotes the incentive parameter. Since the agent’s utility is exponential, maximizing his expected utility is equal to maximize his certainty equivalent. Given the linear compensation $w^A(e)$ and the assumptions explained above, the agent’s certainty equivalent takes the form

$$CE(e) = \alpha + \beta P(e) - C(e) - \frac{\rho}{2} \beta^2 \sigma_P^2,$$

(4)

where $0.5\rho\beta^2\sigma_P^2$ is the required risk premium in order to compensate the agent for his risk bearing due to $P(e)$.

The timing of this problem is as follows. First, the principal offers the agent a contract $(\alpha^*, \beta^*, e^*)$. Given that this contract provides the agent an expected utility that is at least his reservation utility, he accepts the contract and enters into this relationship. After implementing the multitask activities, all random variables are realized. Finally, the payments take place.

3 The First-Best Contract

Before turning to the second-best contract it is useful to derive the first-best solution as benchmark of this problem. The first-best effort allocation and first-best effort intensity can then be compared to the second-best case where the agent’s effort is not contractible and hence, his incentive contract depends on the stochastic performance measure.

First it is vitally to explain the distinction between effort intensity and effort allocation in more detail. Formally, let two arbitrary multitask activities $e_k$ and $e_j$ alter to $\hat{e}_k$ and $\hat{e}_j$. Provided that the ratio between these activities remains equal such that

---

3Despite real contracts are usually non-linear they are widely used in economic analyzes (see e.g. Holmstrom and Milgrom [1987], Spremann [1987], Blickle [1987] and Baker [1992]). The advantage of linear compensation schemes is that they provide less complex analysis than non-linear compensation schemes. Nevertheless, for a continuous time model with Brownian motion, Holmstrom and Milgrom [1987] have shown that in this setting the optimal incentive scheme is linear in output.
$e_k/e_j = \hat{e}_k/\hat{e}_j$ for all $k, j = 1, \ldots, n$ and $k \neq j$, the effort intensity changed whereas the effort allocation remains constant. In contrast, if $e_k/e_j \neq \hat{e}_k/\hat{e}_j$ for at least one pair with $k \neq j$, the relative effort between tasks changes and therefore, the effort allocation across tasks.

Suppose now that the principal can verify the agent’s multitask activities and can therefore write an enforceable contract. He chooses the effort intensity and allocation in order to maximize the expected outcome $V(e)$ minus the production costs $C(e)$,

$$\max_e \Pi(e) = \mu e - \frac{1}{2} \sum_{i=1}^{n} \psi_i e_i^2. \quad (5)$$

By using the first derivative with respect to $e$, the first-best effort is

$$e_{i}^{fb} = \max \left\{ \frac{\mu_i}{\psi_i}, 0 \right\}. \quad i = 1, \ldots, n. \quad (6)$$

Obviously, the principal maximizes his expected profit by determining each single activity according to its outcome/cost sensitivity ratio. It is optimal to implement activities with high ratios more intensively in contrast to activities with low ratios. Therefore, the marginal affect on the outcome (measured by $\mu_i$) and the costs sensitivity (measured by $\psi_i$) determine the optimal intensity for each activity and therefore, the optimal effort allocation across tasks. If $\mu_j < 0$, i.e. one activity has a negative effect on outcome, this activity will be excluded.\(^6\)

Keep in mind that the vector $e^{fb}$ characterizes the first-best effort intensity and the first-best effort allocation across task. Accordingly, there are two possible inefficiencies which can occur in a moral hazard environment: (i) a suboptimal effort intensity and (ii), an inefficient effort allocation. For clarification, the first-best effort allocation is referred to be non-distorted throughout this paper. Consequently, if for any implemented effort vector $e$ there exists no constant $\lambda > 0$ such that $e = \lambda e^{fb}$, the effort allocation is distorted.

By substituting the vector of the first-best effort in (5), the expected first-best profit becomes

$$\Pi^{fb} = \frac{1}{2} \sum_{i=1}^{n} \frac{\mu_i^2}{\psi_i}, \quad (7)$$

\(^6\)Note, this statement is in general true if costs are additive separable as assumed in this model. However, given that two activities $e_k$ and $e_k$ have cost complementarities such that $\partial^2 C(e)/(\partial e_k \partial e_j) < 0$, it can be optimal to implement both activities even $e_j$ has a negative effect on outcome ($\mu_j < 0$). The benefit of implementing $e_j$ is then to reduce the marginal costs for $e_k$.\(\)
where $\bar{\mu}_i = \max\{\mu_i, 0\}$ for all $i = 1, \ldots, n$ since $e_i^{fb} = \max\{\mu_i/\psi, 0\}$.

Obviously, the higher outcome/cost sensitivity ratios for the implemented tasks, the higher is the expected first-best profit. This observation reveals the importance of the outcome sensitivity vector and the cost sensitivity vector for the specification of the expected first-best profit. To illustrate this finding define a new vector $\phi = (\mu_i/\psi_i, \ldots, \mu_n/\psi_n)'$ as $n$-dimensional vector of the outcome/cost sensitivity ratios. By using the relation $\sum \bar{\mu}_i \phi_i = \|\bar{\mu}\|\|\phi\|\cos \kappa$, the first-best profit can be rewritten as

$$\Pi^{fb} = \frac{\|\bar{\mu}\|\|\phi\|\cos \kappa}{2}, \quad (8)$$

where $\kappa$ denotes the angle between vector $\bar{\mu}$ and $\phi$ and $\|\bar{\mu}\|$ the length of vector $\bar{\mu}$ and $\|\phi\|$ the length of vector $\phi$, respectively. Accordingly, the relation between these two vectors plays an important role for the magnitude of the expected first-best profit.

Now suppose the outcome sensitivities are exogenous given whereas the agent’s effort cost structure can be changed up to a certain degree.\(^7\) For instance, by lowering the cost sensitivities of activities with a high marginal effect on outcome and enhancing the cost sensitivities for activities with a low marginal outcome such that $\|\phi\|$ remains constant, the effort allocation across tasks changes. Then, the angle $\kappa$ decreases and obviously, the principal obtains a higher expected first-best profit. However, this observation is only true if vector $\phi$ is linear independent of vector $\bar{\mu}$. Note, $\phi$ is linear dependent of $\bar{\mu}$ if $\psi_i = \hat{\psi} > 0$ for all $i = 1, \ldots, n$, i.e. the effort cost sensitivity is the same across all tasks.\(^8\) In this case the expected first-best profit simplifies to

$$\hat{\Pi}^{fb} = \frac{\|\bar{\mu}\|^2}{2\hat{\psi}}. \quad (9)$$

Already here it is to see that it is important to distinguish whether cost sensitivities are equal or differ across tasks. Usually in the previous multitask agency literature with incongruent performance measures, marginal costs are assumed to be constant across tasks.\(^9\) However, the proceeding analysis in this paper shows that the results regarding distortion induced by incongruent performance measures depends on whether costs are constant across task or not.

\(^7\)Possible mechanisms are considered in section ??.

\(^8\)Throughout this paper I refer to equal cost sensitivities as constant marginal costs.

4 The Second-Best Contract

In the previous section the agent’s effort intensity and allocation was assumed to be contractible. In this case the principal can write a contract about the so-called first-best effort. However, the principal is usually not able to verify the agent’s effort choice so that moral hazard occurs. Then the principal uses the performance measure $P(e)$ as single contractible information about the agent’s effort to provide him an incentive contract. Since the principal is not able to directly induce the agent to implement a required effort level, the application of incentive contracts contingent on the performance measure induces inefficiencies. First, the performance measure is risky so that the principal must pay the agent a risk premium. Second, the applied performance measure does not necessarily coincide with the principal’s objective. These effects are pointed up for a multitask setting after the so-called second-best solution is derived.

The principal’s problem is now to find a contract $(\alpha^*, \beta^*, e^*)$ that maximizes his expected profit. Accordingly, he maximizes his outcome minus compensation for the agent with

$$\max_{\alpha, \beta, e} \Pi \equiv \mu' e - \alpha - \beta \omega' e$$  \hspace{2cm} (10)

s.t.

$$\alpha + \beta \omega' e - \frac{1}{2} \sum_{i=1}^{n} \psi_i e_i^2 - \frac{\rho}{2} \beta^2 \sigma^2_P \geq 0$$  \hspace{2cm} (11)

$$e = \arg \max_{\tilde{e}} \alpha + \beta \omega' \tilde{e} - \frac{1}{2} \sum_{i=1}^{n} \psi_i \tilde{e}_i^2 - \frac{\rho}{2} \beta^2 \sigma^2_P,$$  \hspace{2cm} (12)

where (11) is the participation and (12) the incentive constraint.

First consider the agent’s incentive constraint. By defining the new vector $\Psi = (\omega_1/\psi_1, ..., \omega_n/\psi_n)'$ as $n$-dimensional vector of the measure/cost sensitivity ratios,
the agent chooses\textsuperscript{10}

\[ e_i = \max \left\{ \frac{\omega_i}{\psi_i}, 0 \right\} \beta = \bar{\Psi}_i \beta \quad i = 1, \ldots, n. \]  

(15)

where \( \bar{\Psi}_i = \max \{ \Psi_i, 0 \} \) for all \( i = 1, \ldots, n \). In contrast to the first-best case, the agent’s single effort choice depends now on magnitude of the incentive parameter \( \beta \) and on the single measure/cost sensitivity ratio. Provided that \( \omega_j \leq 0 \), i.e. the marginal effect of activity \( j \) on the performance measure is negative, the agent maximizes his certainty equivalent by implementing \( e_j = 0 \). This is in opposite to the first-best case where the principal wants to exclude activities that have a negative effect on his outcome. The consequences of these different preferences are explained later on in this section.

In order to maximize his expected utility the principal chooses the fixed transfer \( \alpha \) so that the agent’s participation constraint binds. Thus, solving (11) to \( \alpha \) and substitute this expression together with (15) in (10), the principal’s maximization problem can be rewritten without side constraints to

\[ \max_{\beta} \Pi \equiv \mu' \bar{\Psi} \beta - \frac{\beta^2}{2} \omega' \bar{\Psi} - \frac{\rho}{2} \beta^2 \sigma_P^2. \]  

(16)

The first-derivative of \( \Pi \) with respect to \( \beta \) yields

\[ \beta^* = \frac{\mu' \bar{\Psi}}{\omega' \bar{\Psi} + \rho \sigma_P^2}. \]  

(17)

Besides the precision \( \sigma_P^2 \) of the performance measure together with the agent’s risk aversion \( \rho \), the optimal incentive parameter is a function of the outcome sensitivity \( \mu \), the performance measure sensitivity \( \omega \) and the non-negative measure/cost sensitivity ratios \( \bar{\Psi} \).

\textsuperscript{10}Alternatively, it is as well possible to define the agent’s cost function as \( C(e) = e' \Gamma e \), where

\[ \Gamma = \begin{pmatrix} \psi_1 & 0 & \ldots & 0 \\ 0 & \psi_2 & \ldots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ldots & \psi_n \end{pmatrix} \]  

(13)

is the diagonal \( n \times n \) matrix of the effort cost sensitivities. Since \( \psi_i > 0 \) for all \( i = 1, \ldots, n \) it follows,

\[ \Psi = \omega \Gamma^{-1} = (\omega_1/\psi_1, \ldots, \omega_n/\psi_n). \]  

(14)
Finally, by substituting the optimal incentive parameter \( \beta^* \) in \( \Psi \) and applying the relations \( \sqrt{\sum \mu_i^2} = \| \mu \| \) and \( \sum \mu_i \Psi_i = \| \mu \| \| \Psi \| \cos \theta \), the principal expected profit becomes

\[
\Pi = \frac{\| \mu \|^2 \| \Psi \|^2 \cos^2 \theta}{2(\| \omega \| \| \Psi \| \cos \varphi + \rho \sigma_P^2)},
\]

(18)

where \( \theta \) denotes the angle between the vector of the outcome sensitivity \( \mu \) and the vector of the non-negative measure/cost sensitivity ratios \( \Psi \). Additionally, \( \| \mu \| \) is the length of vector \( \mu \) and \( \| \Psi \| \) the length of vector \( \Psi \), respectively. Due to the definition of the cosine, the principal obtains always a strictly positive expected profit if \( \theta \in (-90^\circ, 90^\circ) \) and \( \| \omega \| \| \Psi \| \cos \varphi + \rho \sigma_P^2 > 0 \). If this is not the case, it is optimal to provide the agent a negative incentive parameter. Then the agent maximizes his expected utility by implementing these activities that have a negative effect on the performance measure \( (\omega_i < 0) \) and neglect activities with a non-negative effect \( (\omega_i \geq 0) \). Then, if the implemented activities with a negative effect on the performance measure have as well a negative effect on outcome, it might be optimal for the principal to not provide the agent an incentive contract in order to avoid a strictly negative expected profit. In the remainder of this paper I focus on the case where it is optimal for the principal to provide the agent a strictly positive incentive parameter.

Now it is necessary to focus more intensively on the concepts of congruity and distortion. According to Feltham and Xie [1994], congruence refers to the degree the agent can influence his performance measure, vector \( \omega \), compared to his contribution to the principal’s expected outcome, vector \( \mu \). Baker [2002] shows that congruence can be measured by the angle between these two vectors, i.e. by the angle \( \xi \). Recall the first-best effort allocation \( e_i^{fb} = \max \{ \mu_i/\psi_i, 0 \} \). Throughout this paper every implemented effort allocation as the same as the first-best allocation is referred to be non-distorted. The application of an performance measure in an incentive contract leads to the effort allocation \( e_i = \max \{ \omega_i/\psi_i \beta, 0 \} \) for all \( i = 1, ..., n \). Now it is easy to see that the effort allocation between tasks differs from the first-best effort allocation as long as \( \mu \neq \lambda \omega, \lambda > 0 \). In this case the performance measure is incongruent and consequently, the agent’s effort allocation is distorted. The next corollary gives an implication derived from the previous observation.

\[\text{This is true because the optimal incentive parameter can be rewritten as}\]

\[
\beta^* = \frac{\| \mu \|^2 \| \Psi \|^2 \cos^2 \theta}{(\| \omega \| \| \Psi \| \cos \varphi + \rho \sigma_P^2)}.
\]

(19)

Accordingly, \( \beta \leq 0 \) if \( \theta \notin (-90^\circ, 90^\circ) \) and \( \| \omega \| \| \Psi \| \cos \varphi + \rho \sigma_P^2 > 0 \).
**Corollary 1** Only a congruent performance measure with \( \mu = \lambda \omega \) and \( \lambda > 0 \) leads to a first-best and therefore non-distorted effort allocation.

This corollary implies that the application of a performance measure that is not congruent to the principal’s objective yields a distorted effort allocation across tasks. This observation is independent of whether the agent’s effort cost sensitivities are equal or differ across tasks. However, from this corollary it cannot inferred that a more congruent performance measure induces a higher expected profit. I show in section 5 that this conclusion is in general only true if effort cost sensitivities are constant across task. However, given that cost sensitivities differ, I show in section 6 that a more congruent performance measure is not a sufficient condition for a higher expected profit. Then there are cases where a more congruent performance measure leads to a lower expected profit, given the same precision of the performance measures. Before considering these cases it useful to derive a necessary condition in order that a new performance measure is more congruent.

**Lemma 2** A performance measure with sensitivity \( \dot{\omega} \) is more congruent than a performance measure with sensitivity \( \omega \), if and only if,

\[
\left( \frac{\sum_{i=1}^{n} \mu_i \dot{\omega}_i}{\sum_{i=1}^{n} \mu_i \omega_i} \right)^2 > \frac{\sum_{i=1}^{n} \dot{\omega}_i^2}{\sum_{i=1}^{n} \omega_i^2}.
\]  

(20)

**Proof** Due to lemma 1 it follows that \( \dot{\xi} < \xi \). Therefore, \( \Delta \cos \xi = \cos \xi - \cos \dot{\xi} < 0 \). Since \( \mu \omega = \|\mu\|\|\omega\| \cos \xi \), it follows

\[
\Delta \cos \xi = \frac{\mu \omega}{\|\mu\|\|\omega\|} - \frac{\mu \dot{\omega}}{\|\mu\|\|\dot{\omega}\|} < 0.
\]  

(21)

Accordingly,

\[
\frac{\mu \dot{\omega}}{\mu \omega} > \frac{\|\dot{\omega}\|}{\|\omega\|},
\]  

(22)

which can be rewritten to the relation stated in the lemma.

Q.E.D.

Recall again the agent’s optimal effort choice \( e_i^* = \max \left\{ \frac{\phi_i}{\psi_i} \beta^*, 0 \right\} \) for all \( i = 1, ..., n \). As already mentioned, it is important to distinguish between effort intensity and allocation. Next, I identify factors that affect the effort intensity without changing the effort allocation and vice versa.
Proposition 1  The agent’s effort intensity is affected by $\rho \sigma_P^2$, $\|\Psi\|$ and $\|\omega\|$ whereas his effort allocation across tasks is influenced by the vector $\Psi$.

Proof  See appendix.

It is common knowledge that the precision of the performance measure and the agent’s risk aversion determines the agent’s effort intensity in the single-action case. Proposition 1 emphasizes that this observation holds as well for multitask settings. According to Baker [2002] and Gibbons [2005], the length $\|\omega\|$ measures the scaling of the performance measure sensitivity, i.e. how sensitive the performance measure is against changes in the agent’s effort.$^{12}$ Furthermore, $\|\Psi\|$ determine additionally how sensitive the agent’s effort costs are regarding to a change in $e$. At the first glance this result is surprising since every single activity responds in a different way if $\bar{\phi}_i \neq \bar{\phi}_j$. However, changing $\beta$ by varying the factors given in the proposition by a certain percentage leads to in change in each activity by the same percentage. The relations between these activities remain unchanged. Accordingly, the factors described in the proposition affect only the overall effort intensity and not the relative effort allocation across tasks.

The allowance for single negative outcome sensitivities and performance measure sensitivities requires further explanation. Since $e_i = \max\{\omega_i/\psi_i, 0\}$, the agent neglects activities that reduces his performance measure, given the optimal incentive parameter is strictly positive as considered on throughout this paper. Suppose an arbitrary activity $e_j$ has a negative effect on the performance measure $P(e)$ with $\omega_j < 0$. Accordingly, the agent chooses $e_j = 0$. Despite a negative effect on the measure, this activity is valuable for the principal if it has a strictly positive contribution to the firm outcome with $\mu_i > 0$. Or vice versa, an arbitrary activity $e_k$ with a negative contribution on outcome $\mu_k$ should be avoided. However, if this activity has a strictly positive effect on the performance measure with $\omega_k > 0$, the agent implements $e_k$ and thereby reduces the principal’s expected profit. The general problem for this case is that the principal is not able to exclude single activities since the only available and contractible information about the agent’s effort choice is an aggregation and not separable in order to provide incentives.

Given the previous explanations it is worthwhile to take a closer look on the agent’s effort allocation provided the principal uses a more congruent performance measure.

$^{12}$See e.g. Baker [2002] and Gibbons [2005] for a more detailed explanation.
Proposition 2 A more congruent performance measure with sensitivity $\hat{\omega}$ changes the agent’s effort allocation if and only if for at least one $i \in \{1, \ldots, n\}$,

$$\max\{\omega_i, 0\} \neq \max\{\hat{\omega}_i, 0\}. \quad (23)$$

Proof See appendix.

According to this proposition, a more congruent performance measure does not yield an improved effort allocation and hence, expected profit for the principal as long as only negative sensitivities are increased and remain negative. Due to this observation one must be careful to argue that a more congruent performance measure induces less distortion and consequently, enhance the principal’s expected profit. That is why it is straightforward to use $\bar{\Psi} = \max\{\omega_i/\psi_i, 0\}$ for each partial solution in order to avoid misleading observations.\(^{13}\)

Since the effect of the risk in performance measures in connection with the agent’s risk aversion is intensively discussed in the previous literature, the next sections focus on the question how distortion induced by incongruent performance measures can be measured. Moreover, I emphasize whether a more congruent performance measure leads to a higher expected profit or not. Since the results differ considerably, I first consider the special case where the agent’s effort cost sensitivities are constant across tasks and then, the more general case where they differ.

5 Distortion with Constant Marginal Costs

In this section I consider distortion between the agent’s multitask activities induced by incongruent performance measures given that marginal costs are constant across tasks. This separate consideration is useful since in the previous multitask literature assuming incongruent performance measures it is usually assumed that cost sensitivities are constant across tasks in order to provide a less complex analysis.\(^{14}\) The main objective of this section is to provide an appropriate measure for distortion and derive a criteria in order to rank a set of available measures.

\(^{13}\)This distinction seems to be straightforward. However, in the previous literature dealing with congruence this distinction is neglected even negative outcome sensitivities are not explicitly excluded, see e.g. Feltham and Xie [1994], Datar et al. [2001] and Baker [2002].

\(^{14}\)See e.g. Feltham and Xie [1994], Datar et al. [2001] and Baker [2002] for the risk-averse case and for the risk-neutral case where the agent is protected by a liability limit.
Suppose the agent’s effort cost sensitivities are constant across tasks such that \( \psi_i = \bar{\psi} > 0 \) for all \( i = 1, \ldots, n \). Accordingly, the agents cost function becomes \( C(e) = 1/\psi \sum_{i=1}^{n} e_i^2 \). Using this observation, the principal’s expected profit can be rewritten as (the derivation is relegated to the appendix)

\[
\Pi = \frac{\|\mu\|^2 \|\omega\|^2 \cos^2 \xi}{2\psi(\|\omega\|^2 + \psi \rho \sigma^2)},
\]

where \( \xi \) is the angle between vector \( \mu \) and vector \( \omega \). According to Baker [2002] and Gibbons [2005], the length \( \|\omega\| \) measures the scaling of the performance measure, i.e. how sensitive the performance measure reacts when the agent changes his effort intensity.

The next proposition emphasizes a measure for distortion, given that an applied performance measure is incongruent and marginal costs are constant across tasks.

**Proposition 3** Provided that marginal costs are constant across tasks, the measure for congruity \( \bar{\Upsilon}(\xi) = \cos \xi \) is as well the measure for distortion. Moreover, the expected profit is increasing in \( \cos \xi \).

**Proof** See appendix.

The key factor driving this result is that vector \( \bar{\Psi} \) is linear dependent of vector \( \omega \) due to constant marginal costs. Only in this special case the measure for congruity is as well the measure for the induced distortion. The more the performance measure sensitivity is aligned to the outcome sensitivity, the less distorted become the agent effort allocation. Formally, the angle \( \xi \) decreases and thus, \( \cos \xi \) increases.\(^{15}\) The consequence is that lower distortion induces a higher expected profit as stated in the proposition. However, as I show in the next section this is not necessarily true in the more general case with differing marginal costs.

Finally, the question arises how different performance measures can be compared in a multitask setting with an risk-averse agent. A performance measure \( P^k(e) = \omega^k e + \varepsilon_{Pk} \) is referred to be strictly superior, if it provides the principal a strictly higher expected profit compared to all other available performance measures \( P^j(e) = \omega^j e + \varepsilon_{Pj} \) for all \( j \in \mathbb{P} \) and \( k \neq j \). By comparing the respective expected profits and rearranging leads immediately to the next proposition.

\(^{15}\)Note, this is true since I consider the case \( \xi \in (-90^{\circ}, 90^{\circ}) \) such that \( \beta^* > 0 \). Otherwise, the incentive parameter becomes negative.
Proposition 4 Given that marginal effort costs are constant across tasks, a performance measure \( P^k(e) \) is strictly superior to any other performance measure \( P^j(e) \) for all \( j \in P \) and \( k \neq j \), if and only if
\[
\tilde{\Upsilon}^{-2}(\xi^k) - \tilde{\Upsilon}^{-2}(\xi^j) < \psi\rho \left( \frac{\sigma^{2j}}{||\omega^j||^2} \tilde{\Upsilon}^{-1}(\xi^j) - \frac{\sigma^{2k}}{||\omega^k||^2} \tilde{\Upsilon}^{-1}(\xi^k) \right),
\]
where \( \tilde{\Upsilon}_i(\xi_i) = \cos \xi_i, \ i \in P \), as the measure distortion induced by performance measure \( i \).

The condition stated in the proposition is a sufficient condition in order that one performance measure \( P^k(e) \) is superior to all other performance measures. Obviously, the characterization depends on the induced distortion measured by \( \tilde{\Upsilon}(\xi^i) \) and by the incorporated risk normalized by the length of vector \( \omega^i \). Again, the reason is that the higher the length, the more sensitive is the performance measure due to changes in the effort intensity. Given the agent’s risk aversion \( \rho \), the superior performance measure outweighs the induced distortion and the risk incorporated in the measure. If the agent is risk-neutral the condition simplifies to \( \tilde{\Upsilon}(\xi^k) > \tilde{\Upsilon}(\xi^j) \) which is satisfied for all \( \xi^k < \xi^j \). Accordingly, the performance measure that induces the less distortion is the superior one. In contrast, the more risk-averse the agent becomes, the more important becomes the risk incorporates in a performance measure.

For the one-dimensional action case performance measures with the same signal-to-noise-ratio lead to the same expected profit.\(^{16}\) In the multitask setting this signal-to-noise-ratio can be adjusted by \( ||\omega^k||^2/\sigma^{2k} \). The two performance measures \( P^k(e) \) and \( P^j(e) \) lead to the same expected profit if both sides of (25) in proposition ?? are equal. Obviously, the same signal-to-noise-ratio does not necessarily yield the same expected profit. If and only if both performance measures induce the same distortion, i.e. \( \tilde{\Upsilon}(\xi^k) = \tilde{\Upsilon}(\xi^j) \), the principal receives the same expected profit. In order to be able to assess the value of multiple performance measures in a multitask setting it is required to know besides the adjusted signal-noise ratio, the induced distortion and consequently, the characteristics of the vector of the outcome sensitivity \( \mu \).

6 Distortion with Differing Marginal Costs

The assumption of constant marginal costs across tasks considered in the previous section is a very restrictive assumption. It is more straightforward to assume that at least

\(^{16}\)See e.g. Banker and Datar [1989] and Kim and Suh [1991] for a theoretical foundation.
two activities have different marginal costs. For example, a manager may find it easier to maintain an existing customer relationship than to search for new customers. Or for worker in production facilities it is easier fixing screws than to move heavy items. This list can be easily extended. That is why it is more reasonable to consider multi-task agency-relationships where the agent chooses his effort allocation not only due to his performance measure but also due to the properties of his effort costs. However, assuming constant marginal costs is one way to keep analysis more trackable. Nevertheless, in this section I show that the results derived under the assumption of constant marginal costs cannot be transferred to the more reasonable case where marginal costs differ across tasks.

In the previous section I have shown that the vector $\Psi$ was linear dependent of vector $\omega$, given marginal costs are constant across task. Then, the angle $\xi$ between vector $\mu$ and vector $\omega$ measures besides the congruity of the performance measure as well as the distortion between the agent’S multitask activities. Suppose for at least two arbitrary activities $e_k$ and $e_j$ that the respective marginal costs are not identical such that $\psi_k \neq \psi_j$. Then, vector $\Psi$ is linear independent of $\omega$ and hence, the results from the previous section cannot be transferred to this more general case.

Recall the general expected profit derived in section 4. The expected profit depends on two angles. First, the angle $\theta$ between the vector of the marginal outcome sensitivity $\mu$ and the vector of the measure/cost sensitivity ratios $\Psi$. Second, on the angle $\varphi$ between the vector of the performance measure sensitivity $\omega$ and the vector of the measure/cost sensitivity ratios $\Psi$. Keep in mind that the congruence of the performance measure is still measured by the angle $\xi$. As the next proposition emphasizes, the measure for congruity is different to the measure of the induced distortion.

**Proposition 5** Given that marginal costs differ across tasks, the measure for the induced distortion is $\Upsilon(\varphi, \theta) = \cos \varphi / \cos^2 \theta$.

**Proof** See appendix.

The key difference between this proposition and proposition 3 is that here vector $\Psi$ is linear independent of $\omega$ in contrast to the case with constant marginal costs. Obviously, the observation from the previous section where the measure of congruity is as well a measure of distortion is a special case and cannot be transferred to the more general case with differing marginal costs. Therefore, it is important to distinguish precisely between congruence and distortion given that the effort cost sensitivities dif-
fer. Finally, as for the measure with constant marginal costs, the higher the measure, the less distortion is induced. Accordingly, the higher is the principal’s expected profit.

This observation is illustrated in figure 1 for the three-dimensional case. In this figure the vector of the outcome sensitivity $\mu$ and vector of the performance measure sensitivity $\omega$ is presented where the angle $\xi$ describes the relation between them. Note, the vector $\Psi$ of the measure/cost sensitivity ratios is not necessarily on the plane built by $\mu$ and $\omega$. Finally, the relation of $\Psi$ to $\mu$ and to $\omega$ are characterized by the angles $\theta$ and $\varphi$ respectively.

First suppose that marginal costs are constant across tasks and thus, vector $\Psi$ is linear dependent on $\omega$. In this case vector $\Psi$ has the same direction as vector $\omega$ and is therefore perfectly congruent. Only the lengths differ according to the magnitude of $\bar{\psi}$. Then it is easy to see that in this case the angle $\xi$ measures the relation between these two vectors and hence, the congruity and the induced distortion. However, given that $\Psi$ is linear independent of $\omega$, it is necessary to distinguish between two cases, i.e. whether if $\Psi$ is on the plane built by $\mu$ and $\omega$ or outside the plane. Provided that $\Psi$ is on the plane, this vector is a linear combination of $\mu$ and $\omega$ and obviously, $\xi = \theta + \varphi$. Only in this case it is possible to use as well the measure of congruity to characterize distortion. Note that this is always true for the two-dimensional case, i.e. the agent...
can only chose between two activities.\footnote{It is important to note that there are two possibilities for the two-dimensional case. In the first case the vector $\psi$ is on the plane in between $\mu$ and $\omega$. Accordingly, $\varphi_1(\theta, \xi) = \xi - \theta$. In the second case, $\psi$ is outside the plane and therefore $\varphi_2(\theta, \xi) = \theta - \xi = -\varphi_1(\theta, \xi)$. Due to the definition of the cosine, both results in the same measure for distortion since the absolute value is identical.} The second case occurs if $\Psi$ lies outside the plane between $\mu$ and $\omega$. Then, $\xi \neq \theta + \varphi$. That is, the measure for distortion $\Upsilon(\varphi, \theta)$ cannot be inferred by the measure of congruity $\cos \xi$. In order to do this it is necessary to know the specific properties of the agent’s effort costs, and hence the direction of vector $\Psi$.

The analyzes in the previous section revealed that the principal applies a more congruent performance measure in an incentive contract, everything else equal. However, the question arises if this is as well true for the case with differing marginal costs. By considering figure 1 it is easy to see that by reducing the angle $\xi$ the respective angles $\theta$ and $\varphi$ change as well. However, as long as $\Psi$ is not on the plane built by $\mu$ and $\omega$, the change is not linear in $\theta$ and $\varphi$. Then it is not clear cut whether a more congruent performance measure with a smaller angle $\xi' < \xi$ provides the principal a higher expected profit. The next proposition gives a sufficient condition in order that a more congruent performance measure is more beneficial for the principal.

**Proposition 6** Suppose marginal costs differ across tasks and a subset $l \leq n$ of elements in $\omega$ changes such that the new vector $\dot{\omega}$ is more congruent. Then, the new expected profit is strictly higher, if and only if,

$$
\left( \frac{\sum_{i=1}^{l} \frac{\mu_i \omega_i}{\psi_i} + B}{\sum_{i=1}^{l} \frac{\mu_i \omega_i}{\psi_i} + B} \right) > \frac{\sum_{i=1}^{l} \frac{\dot{\omega}_i^2}{\psi_i} + D + \rho \sigma^2_P}{\sum_{i=1}^{l} \frac{\dot{\omega}_i^2}{\psi_i} + D + \rho \sigma^2_P},
$$

(26)

where $B \equiv \sum_{i=l+1}^{n} \frac{\mu_i \omega_i}{\psi_i}$ and $D \equiv \sum_{i=l+1}^{n} \frac{\omega_i^2}{\psi_i}$ are the relations for the unchanged elements in $\dot{\omega}$.

**Proof** See appendix.

The previous proposition provides a sufficient condition in order that a more congruent performance measure leads to a higher expected profit. Note, that this is in general true if factor affecting the effort intensity such that $\rho \sigma^2_P$ remain constant. Obviously, as long as $\rho \sigma^2_P = 0$, the sufficient condition is always satisfied. Moreover, it can easily checked that the same is true if marginal costs are constant across tasks.\footnote{This provides an additional theoretical foundation of the statement that a more congruent performance measure is beneficial for the principal as long as marginal costs are constant across tasks.}
Given that the condition holds, reducing the congruity problem reduces the distortion and hence, improves the principal’s expected profit.

However, it is important to note that this condition is not always satisfied. Thus, there might exists environments where a more congruent performance measure reduces the expected profit. Consider for instance the two-dimensional example with \( \mu = (2, 4), \omega = (1, 6), \dot{\omega} = (1.5, 5), \psi = (2, 4) \) and \( \rho \sigma_P^2 = 10 \). It can be checked by using lemma 2 that \( \dot{\omega} \) is more congruent than \( \omega \). However, substituting the specific value in the sufficient condition stated in proposition 6, one obtains \( 0.8623 > 0.8910 \) which is a contradiction. Therefore, in this specific example the expected profit decreases if the performance measure becomes more congruent.

**Proposition 7** Suppose marginal costs differ across tasks and a subset \( l \leq n \) of elements in \( \omega \) changes such that the new vector \( \dot{\omega} \) is more congruent. Then, it is more (less) likely that the expected profit decreases the higher \( \rho \sigma_P^2 \) is and if \( \sum_{i=1}^l \dot{\omega}_i^2 / \psi_i > (\leq) \sum_{i=1}^l \omega_i^2 / \psi_i \).

**Proof** See appendix.

According to this proposition, a higher risk-aversion or more risk incorporated in the performance measures make it more likely that the expected profit decreases in the more congruent the applied performance measure is. Note that despite this observation, a congruent performance measure is always efficient since it ensures that the agent implements a non-distorted effort allocation. To explain this finding in more detail rewrite first the condition given in this proposition as

\[
\sum_{i=1}^l \dot{\omega}_i^2 / \psi_i + \sum_{i=l+1}^n \omega_i^2 / \psi_i > \sum_{i=1}^l \omega_i^2 / \psi_i + \sum_{i=l+1}^n \omega_i^2 / \psi_i. \tag{27}
\]

I follows,

\[
\frac{\| \dot{\Psi} \|}{\| \Psi \|} > \frac{\cos \dot{\varphi}}{\cos \varphi}. \tag{28}
\]

First it is important to note that a change of \( \omega \) directly influence the length of vector \( \Psi \). Then, the relation above suggests that if the ratio of the lengths is strictly greater than the ratio of the change in the relations of \( \Psi \) to \( \omega \) measured by \( \cos \varphi \), the more likely is that the expected profit decreases for small changes in the congruence. Note that \( \cos \dot{\varphi} \) is a part of the distortion measurement. Accordingly, the lower \( \cos \dot{\varphi} \), the higher is the measure \( \Upsilon(\theta, \varphi) \) and hence, the less is the induced distortion. Recall that \( \| \Psi \| \) only affects the effort intensity by reducing the optimal incentive parameter \( \beta \). Thus,
a higher length of vector $\Psi$ implies that the performance measure is more sensitive in changes of the agent’s effort. Moreover, the higher $\rho \sigma_j^2$ is, the more important is this affect on the principal’s expected profit. In order to restrict the required risk-premium, it is optimal for the principal to reduce the incentives. Accordingly, if the condition above is satisfied, the change in the congruence outweighs the change in the length of vector $\Psi$ and hence, the effect on the agent’s risk premium.

The key for this observation is the vector $\psi$ of the marginal effort costs and hence, differences in the costs structure for different activities. To illustrate this observation suppose two activities 1 and 2. Assume activity 1 is undervalued and 2 is overvalued by the performance measure. Then, enhancing the sensitivity of activity 1 and reducing the sensitivity for 2 in the performance measure improves the congruence. However, if the length of vector $\Psi$ is as well increased, it is optimal for the principal to reduce the incentive parameter in order to counteract the increased effort intensity and hence, the higher required risk-premium. Whether $\|\Psi\|$ increases or decreases, depends on the marginal effort costs. If the marginal effort costs for 1 is relatively higher than for 2, $\|\Psi\|$ increases.

Therefore, given that marginal costs differ across tasks, a more congruent performance measure does not necessarily leads to a higher expected profit, *ceteris paribus*, like in the case with constant costs. Accordingly, if a principal can choose one performance measure from a set of performance measures, it is not in general true that the most congruent one leads to the highest expected profit. To assess the usefulness the principal needs more precise knowledge about the agent’S costs structure.

As for the special case considered in the previous section, it is important to know which properties the more valuable performance measure $P_k(e)$ from a set $\mathcal{P}$ has if marginal costs are not constant across tasks. Again, a performance measure $P_k(e) = \omega_k e + \sigma_k^2$ is referred to be superior, if it provides the principal a strictly higher expected profit compared to all other available performance measures $P_j(e) = \omega_j e + \sigma_j^2$ for all $j \in \mathcal{P}$ and $k \neq j$. By comparing the resulting expected profits and rearranging leads to the final next proposition.

**Proposition 8** Suppose marginal costs differs across tasks. Then, a performance measure $P_k(e)$ is strictly superior to any other performance measure $P_j(e)$ for all $j \in \mathcal{P}$ and $k \neq j$, if and only if

$$
\frac{\|\omega_k\|}{\|\Psi_k\|} \Gamma_k^{-1}(\varphi_k, \theta_k) - \frac{\|\omega_j\|}{\|\Psi_j\|} \Gamma_j^{-1}(\varphi_j, \theta_j) < \rho \left( \frac{\sigma_j^2}{\|\Psi_j\|^2 \cos^2 \theta_j} - \frac{\sigma_k^2}{\|\Psi_k\|^2 \cos^2 \theta_k} \right), \quad (29)
$$

20
where $\Upsilon_i = \cos \varphi_i / \cos^2 \theta_i$, $i \in P$ is the measure of the induced distortion by performance measure $i$.

The condition emphasized by this proposition is again a sufficient condition in order that one performance measure $P^k(e)$ is superior to all other performance measures. In contrast to the case with constant marginal costs, the characterization depends on the induced distortion measured by $\Upsilon(\theta, \varphi)$ and on the incorporated risk normalized by the length of vector $\omega_i$. The most valuable performance measure outweighs the induced distortion and the risk incorporated in the measure given the agent’s risk aversion $\rho$ better than any other available performance measure. It is easy to see that under the assumption of a risk-neutral agent the principal chooses the performance measure that induce the least distortion, normalized by the lengths of the vector $\omega$ and $\Psi$ respectively. Contrary, the more risk-averse the agent becomes, the more important becomes the risk incorporates in a performance measure.

As for the case with constant marginal costs, the same signal-noise-ratio does not necessarily yield the same expected profit. If and only if both performance measures induce the same distortion such that $\bar{\Upsilon}(\theta^k, \varphi^k) = \bar{\Upsilon}(\theta^j, \varphi^j)$, the principal receives the same expected profit. Accordingly, the value of performance measures cannot be assessed by using the signal-noise ratio. It is merely important to know the induced distortion and therefore, the vector of the outcome sensitivity $\mu$.

## 7 Conclusion

In this paper I considered a multitask agency-relationship with a risk-neutral principal and a risk-averse agent. The primary purpose of this paper was to derive a measure for distortion between the agent’s multitask activities. Thereby I distinguished between two cases. First, if the agent’s marginal costs are constant across tasks that is a common assumption in the existing literature. Second, the more realistic case where his marginal costs differs between task.

First I have shown that in a first-best setting the principal determines each single activity according to its respective outcome/cost sensitivity ratio. This optimal effort allocation is referred to be non-distorted. Furthermore, the first-best effort allocation and therefore the expected first-best profit can be characterized by the vector of the outcome/cost sensitivity ratios in the $n$-dimensional space.
In the second-best case, where the principal uses a verifiable performance in order to provide the agent incentives, two inefficiencies arise: the agent implements a suboptimal effort intensity and allocation. The second-best effort allocation is a result of the trade-off between incentives and insurance due to the agent’s risk bearing. In contrast, the distortion occurs because the performance measure is not congruent to the principal’s objective. Only if a performance is congruent to the organization’s outcome, the first-best and therefore non-distorted effort allocation is implemented even the intensities are suboptimal due to the risk incorporate in that measure. Consequently, the performance measure sensitivities and the agents marginal costs affect the effort allocation, whereas the precision of the performance measure in connection with the agent’s risk-aversion affect the overall effort intensity.

When the agent’s marginal costs are equal across tasks, the measure of congruity emphasized by Baker [2002] is as well the measure for the induced distortion. Consequently, the more congruent a performance measure is, the less distorted becomes the agent’s effort allocation and hence, the higher is the expected profit. This is in contrast to the more general case where marginal costs differ across tasks. Here the measure of distortion is different to the measure of congruity. More precisely, the measure of distortion depends then on the ratio of the relationships between the vector of the measure/cost sensitivity ratio first with the vector of the outcome sensitivity and second, with the vector of the measure sensitivity. Then, a more congruent performance measure can reduce the principal’s expected profit. This observation depends on the properties of the agent’s risk aversion and the precision of the performance measure.

In general, the optimal linear contract depends on three factors. First, the dimension of the congruity problem. Second, the induced distortion between the agent’s multitask activities. And finally, the incorporated risk. Furthermore, the first and the second factor have the same measure if costs are constant across tasks whereas they have difference measures otherwise.

One additional purpose of this paper was to provide conditions for that one performance measure is superior compared to all other performance measures in an available information system. The general and most important observation is that the signal-noise ratio that is useful to rank measures in the single-action case, cannot be transferred to the multi-action case. To assess the value of a performance measure it is in addition important to know the induced distortion.

This paper is part a larger research agenda. The previous literature focused pri-
marily on the properties of performance measures and the incorporated risk in order to identify and to characterize occurred inefficiencies. However, according to this results derived in this paper it is as well insightful to take the agent’s costs function into account. As shown, the results derived with constant marginal costs are not in general transferable to the more realistic case where the agent’s marginal costs differ across tasks. One important question is how cost complementarities affect the distortion and efficiencies in a multitask setting. Furthermore, the results in this paper have shown that the agent’s marginal costs plays an important role to value different performance measures. Accordingly, if one performance measure is more valuable to provide incentives to one specific agent, it is not necessarily optimal for another agent with a different costs structure. This provides the principal some room to reduce distortion for a given performance measure by using adverse selection mechanisms in order to choose the "most appropriate" agent. These emphasized specification may be subject for future research.
Appendix

Proof of Proposition 2.

Recall that \( e_i = \max \{ \omega_i / \psi_i \beta, 0 \} \) for all \( i = 1, \ldots, n \). Now consider one arbitrary element \( \omega_k < 0 \). Hence, \( e_k = 0 \). Provided that the vector \( \omega \) changes in such a way that only \( \omega_k \) changes to \( \hat{\omega}_k < 0 \), the agent still implements \( \hat{e}_k = 0 \). Hence, in order that a more congruent performance measure change the distortion between the agent’s multitask activities, at least one element should change, i.e. \( \max \{ \omega_i, 0 \} \neq \max \{ \hat{\omega}_i, 0 \} \).

Q.E.D.

Proof of Proposition 1.

First, rewrite \( \beta \) as

\[
\beta = \frac{\| \mu \| \| \hat{\Phi} \| \cos \theta}{\| \omega \| \| \hat{\Phi} \| \cos \varphi + \rho \sigma_P^2}.
\]

First note that a change of \( \beta \) in \( \rho \sigma_P^2 \) influences the effort intensity without the effort allocation if and only if \( \{ e_i \}_{i=1}^n = \lambda \{ \bar{e}_i \}_{i=1}^n \) for \( \lambda > 0 \), where \( \bar{e}_i \) is the new effort. Now choose two arbitrary elements \( e_j \) and \( e_k \) without loss of generality, where \( j, k \in N \) and \( k \neq j \). Accordingly, \( \lambda = e_j / e_k \). Now define the new effort after a change in \( \beta \) with \( \bar{e}_j \) and \( \bar{e}_k \). Then, the effort allocation remains if \( \lambda = \bar{e}_j / \bar{e}_k = e_j / e_k \). Since \( \{ e_i \}_{i=1}^n = \{ \bar{\Psi}_i \}_{i=1}^n \beta \) it follows \( \lambda = \phi_j / \phi_k \). Due to the fact that the relationship between \( e_j \) and \( \bar{e}_j \) is linear, it follows

\[
\bar{e}_l = e_l + d e_k \quad l = \{ j, k \},
\]

where \( d e_k = \partial e_l / \partial \rho \sigma_P^2 d \rho \sigma_P^2 \) is the partial derivative of \( e_k \) with respect to \( \rho \sigma_P^2 \). Therefore,

\[
e_l = \bar{\phi}_l \beta + \bar{\phi}_l \frac{\partial \beta}{\partial \rho \sigma_P^2} d \rho \sigma_P^2 \quad l = \{ j, k \}.
\]

Then it follows,

\[
\lambda = \frac{\bar{e}_j}{\bar{e}_k} = \frac{\phi_j \left( \beta + \frac{\partial \beta}{\partial \rho \sigma_P^2} d \rho \sigma_P^2 \right)}{\phi_k \left( \beta + \frac{\partial \beta}{\partial \rho \sigma_P^2} d \rho \sigma_P^2 \right)} = \frac{\phi_j}{\phi_k}.
\]

Thus, the effort allocation remains if \( \beta \) changes due to \( \rho \sigma_P^2 \). This proof is the same for \( \| \Psi \| \) and \( \| \omega \| \).

Finally, note that \( \lambda = e_j / e_k = \Psi_j / \Psi_k \) for all \( j, k \in N \) and \( j \neq k \). Obviously, if one at least one element of vector \( \Psi \) changes to \( \hat{\Psi}_j \) such that \( \hat{\lambda} = \hat{\Psi}_j / \hat{\Psi}_k \) and \( \lambda \neq \hat{\lambda} \), the effort allocation changes.
Derivation of the principal’s expected profit with constant marginal costs.

The expected profit can be rewritten as

$$\Pi = \frac{\left(\sum_{i=1}^{n} \mu_i \frac{\omega_i}{\psi_i}\right)^2}{2 \left(\sum_{i=1}^{n} \omega_i \frac{\omega_i}{\psi_i} + \rho \sigma^2\right)}.$$  (34)

If $\psi_i = \psi$, this can be rewritten as

$$\Pi = \frac{\left(\sum_{i=1}^{n} \mu_i \omega_i\right)^2}{2\psi \left(\sum_{i=1}^{n} \omega_i \omega_i + \psi \rho \sigma^2\right)},$$  (35)

and hence,

$$\Pi = \frac{\|\mu\|^2 \|\omega\|^2 \cos^2 \xi}{2\psi (\|\omega\|^2 + \psi \rho \sigma^2)}. $$  (36)

Proof of Proposition 3.

From proposition 1 it can be inferred that a change of $\rho \sigma_P^2$ does not change the distortion. Hence, it is possible to use $\rho \sigma_P^2 = 0$ and rewrite the principal expected profit as

$$\Pi = \frac{\|\mu\|^2 \|\omega\|^2 \cos^2 \xi}{2\psi}.$$  (37)

Now suppose $\mu = \omega$, i.e. the performance measure is congruent. Then, the principal’s expected profit without distortion becomes $\Pi^c = \|\mu\|^2/(2\psi)$. Accordingly, $\Delta \Pi \equiv \Pi^c - \Pi^*$ is the expected loss due to the congruity problem. Thus,

$$\Delta \Pi = \frac{\|\mu\|^2 (1 - \cos^2 \xi)}{2\psi}.$$  (38)

Finally, it can be easily checked that $\partial \Pi / \partial \xi < 0$.

Q.E.D.

Proof of Proposition 5.

From proposition 1 it can be inferred that a change of $\rho \sigma_P^2$ does not change the distortion. Hence, it is possible to use $\rho \sigma_P^2 = 0$ and rewrite the principal expected profit given that marginal costs differ across tasks as

$$\Pi = \frac{\|\mu\|^2 \|\bar{\Psi}\|^2 \cos^2 \theta}{2\|\omega\|^2 \cos \varphi}.$$  (39)
Now suppose $\mu = \omega$, i.e. the performance measure is congruent and the agent implements the first-best effort allocation (see proposition xx). Then, the principal’s expected profit without distortion becomes $\Pi_c = \|\mu\|\|\phi\|\cos \kappa/2$. Accordingly, $\Delta \Pi \equiv \Pi_c - \Pi^*$ is the expected loss due to the congruity problem. Thus,

$$\Delta \Pi = \frac{\|\mu\|}{2} \left( \|\phi\| \cos \kappa - \frac{\|\mu\|\|\bar{\Psi}\|}{\|\omega\|} \cos^2 \theta \right).$$

(40)

Since $\|\Psi\|$ and $\|\omega\|$ influence the effort intensity and not the distortion, it follows that $\Upsilon(\varphi, \theta) = \cos \varphi/\cos^2 \theta$ measures the distortion. That is, the lower $\Upsilon(\varphi, \theta)$, the more the loss due to distortion.

Q.E.D.

**Proof of Proposition 6.**

Next, order every element in $\omega$ that change, such that the first $l$ elements change and the remaining $n - l$ remain unchanged when the performance measure becomes more congruent. The profit given the more congruent performance measure $\hat{\omega}$ is higher than the profit for the initial performance measure sensitivity if

$$\frac{\left( \sum_{i=1}^l \mu_i \hat{\omega}_i + \sum_{i=l+1}^n \mu_i \hat{\omega}_i \right)^2}{2 \left( \sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} + \rho \sigma_P^2 \right)} > \frac{\left( \sum_{i=1}^l \mu_i \hat{\omega}_i + \sum_{i=l+1}^n \mu_i \hat{\omega}_i \right)^2}{2 \left( \sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} + \rho \sigma_P^2 \right)}$$

(41)

Rearranging gives

$$\frac{\left( \sum_{i=1}^l \mu_i \hat{\omega}_i + \sum_{i=l+1}^n \mu_i \hat{\omega}_i \right)^2}{\sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} + \rho \sigma_P^2} > \frac{\sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} + \rho \sigma_P^2}{\sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} + \rho \sigma_P^2}$$

(42)

The expected profit is decreasing the more congruent the performance measure is, if the reversed relation holds.

Q.E.D.

**Proof of Proposition 7.**

Note that $\sum_{i=1}^l \frac{\omega_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\omega_i^2}{\psi_i} = \omega'\Psi$ and $\sum_{i=1}^l \frac{\hat{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^n \frac{\hat{\omega}_i^2}{\psi_i} = \hat{\omega}'\hat{\Psi}$. Thus, the first derivative of the right-hand-side with respect to $\rho \sigma_P^2$ gives

$$\frac{\partial \text{RHS}}{\partial \rho \sigma_P^2} = \frac{\hat{\omega}'\hat{\Psi} (\omega'\Psi + \rho \sigma_P^2) - (\hat{\omega}'\hat{\Psi} + \rho \sigma_P^2) \omega \Psi}{(\omega'\Psi + \rho \sigma_P^2)^2}.$$

(43)
Accordingly, $\frac{\partial \text{RHS}}{\partial \rho \sigma_p^2} > 0$, if
\[
\dot{\omega}' \dot{\Psi} (\omega' \Psi + \rho \sigma_p^2) > (\dot{\omega}' \dot{\Psi} + \rho \sigma_p^2) \omega \Psi.
\] (44)

Rearranging gives $\dot{\omega} \dot{\Psi} > \omega \Psi$ and hence,
\[
\sum_{i=1}^{l} \frac{\dot{\omega}_i^2}{\psi_i} + \sum_{i=l+1}^{n} \frac{\omega_i^2}{\psi_i} > \sum_{i=1}^{l} \frac{\omega_i^2}{\psi_i} + \sum_{i=l+1}^{n} \frac{\omega_i^2}{\psi_i},
\] (45)

and finally,
\[
\sum_{i=1}^{l} \frac{\dot{\omega}_i^2}{\psi_i} > \sum_{i=1}^{l} \frac{\omega_i^2}{\psi_i}.
\] (46)

Q.E.D.
References


