Market Distortions and Public Enterprise Strategies in an
International Mixed Oligopoly

Richard C. Cornes∗ Mehrdad Sepahvand†

Abstract

This study investigates the possible sources of distortions in an international mixed oligopoly. We extend the existing linear/quadratic model to a general framework and show that a public enterprise may either serve as a regulatory device or may itself create an additional level of distortion. Which of these is the case depends critically on the timing of firms output decisions. We then extend the basic quantity setting game to incorporate a preplay stage at which firms can choose the timing of action, rather than moving in an exogenously imposed sequence, in order to determine endogenously the equilibrium sequence of moves. We argue that the distortions associated with a public enterprise and the welfare gain of privatization found in earlier studies can be attributed to an arbitrary and unjustified modeling assumption concerning the order of play, rather than to public ownership.

JEL: D43, F12, L33

Keywords: Privatization, International Mixed Oligopoly, Strategic Trade Policy

∗Correspondence address: School of Economics, The University of Nottingham, University Park, NG7 2RD. Tel: +44 (0)115 9515480, Fax: +44 (0)115 951 4159. Email: richard.cornes@nottingham.ac.uk
†Leeds University Business School, Leeds, LS2 9JT, UK. Tel:+44(0)113436263 Fax:+44(0)113434465, Email: M.Sepahvand@lubs.leeds.ac.uk.
1 Introduction

Privatization that involves the transfer of ownership from government to the private sector entails changes in the firm’s monitoring and incentive systems as well as changes in the structure of market.

From the theoretical point of view, if markets (including the political market) were competitive and complete contracts could be written and enforced, ownership structure would not matter (Sappington and Stiglitz, 1987). Models that deal with the effects of ownership on efficiency must depart from one or both of these assumptions. If markets are not competitive but complete contracts are possible, then the public enterprise can serve perfectly as a policy instrument to pursue social goals and overcome market failure. If competition prevails but contracts are incomplete, then firms face principal-agent problems at different levels of their organizations which are arguably more severe in public sector firms. Therefore, in a relatively competitive market due to organizational failure of government firms, as theoretical analysis and empirical evidence assert, private ownership is most efficient (Vickers and Yarrow, 1988, see Megginson and Netter, 2001, for a recent survey on empirical research).

The mixed oligopoly literature that deals with public enterprise behavior in an imperfect market has concluded that, even in the absence of organizational failure, privatization of a public enterprise still can be justified (De Fraja and Delbono, 1989). That is because a public enterprise that strives to correct, or reduce, the detrimental effects of the market imperfection may itself raise an additional level of distortion. If the market is not highly concentrated, this may outweigh the beneficial effect of eroding underprovision by the private firms. Pal and White (1998) incorporate international competition into the model and find that, regardless of the degree of competition intensity, it is always preferable to privatize the domestic public enterprise and use other policy instruments such as production subsidies to regulate an imperfect market. This result holds even when the government uses a complete set of strategic trade policy instruments that matches all possible sources of distortions apart from public enterprise itself (Sepahvand, 2004).

This observation has far-reaching implications for policy making. The World Bank Policy Research (1995) reports that, despite the declining share of public enterprise investment in recent decades due to vast privatization programs, public enterprises still play a significant role in the world economy. This is particularly evident in developing countries. For example, the shares of public enterprises in China and Vietnam in 2000 were 26% and 48% of their GDPs respectively, while the latter, which has been trying to open up its market to international competition, is still reluctant to reduce the share of public enterprises in the industrial sectors of its economy (IMF Report, 2003). The above findings relating to the distortionary effects of the public enterprises behavior in the presence of foreign competition may encourage these countries to revise their position on government ownership.

In some sectors, such as banking businesses, direct state intervention remains very strong. La
Porta et al. (2002) report that in mid-1990s, even after a large wave of privatization, about one quarter of the assets of the largest banks in developed countries and half of the assets of the largest banks in the developing countries were still under state control. Also due to the financial liberalization over the last decade in many countries foreign-owned banks have become the main players in the domestic financial market. For example, over a period of just five years from 1994 to 1999, among countries of Latin America the share of foreign banks increased from 13 percent to 45 percent (Demirguc-Kunt et. al. 2004). If state banks are themselves a source of distortion and that is expected to be more detrimental in the presence of foreign participants, then further privatization should be invoked in the banking industry.

Notwithstanding the valuable contribution of mixed oligopoly models and the policy relevance of the results, so far they have not attracted the attention in the theoretical debates over privatization that the topic merits. This is partly because few have gone beyond simple linear-quadratic models. More significantly, little attention has been paid to checking the robustness of results with respect to their basic modelling assumptions.

We extend the basic model applied in the literature from a linear-quadratic case to a fairly general framework that accommodates more general forms of demand and cost functions. Furthermore, we show that only when the public enterprise acts as a Cournot player will it generate a distortion. But with a first-mover advantage the public enterprise can serve as an effective regulatory device comparable with a production subsidy. This allows us to suggest an alternative explanation for the welfare gain of privatization in an international Cournot mixed oligopoly found by earlier studies. We then consider which of the above models (Cournot or Stackelberg) is consistent with the firms preferences and objectives and may lead to a self-enforcing equilibrium. So the quantity-setting game is extended to a preplay stage where firms can choose their timing of action rather than moving in an ordered time. We adopt the extended game with observable delays suggested by Hamilton and Slutsky (1990) and developed by Amir and Grilo (1999) in order to determine endogenously the sequence of moves. The result indicates that the natural order of play in an international mixed oligopoly model with subsidization is not the simultaneous play that characterizes Cournot competition. However, under some circumstances, including the linear demand model that has widely been used in the literature, public Stackelberg leadership is the subgame perfect Nash equilibrium of the extended game with endogenous order of moves. This has important implications for debates over privatization and market distortions as we will discuss shortly.

2 The Framework

Consider a domestic market for a homogeneous product. The rest of the world is labelled as foreign. While the foreign firms supply their products to the country in question, we assume the domestic
firms only operate domestically. The number of firms is limited hence both foreign and domestic private firms enjoy some degree of market power. The trade policy of the home country includes an import tariff, $t$, and a production subsidy for domestic firms, $s$. Both are defined as specific rates i.e. as amounts of money per unit of output. In an international oligopoly without public enterprise, these policy instruments are enough to correct distortions (Dixit, 1984). Adopting this setting as a benchmark, we examine if the introduction of a domestic state-owned enterprise can generate another level of distortion.

The home market is served by $m+1$ domestic private firms and $n$ foreign private firms. Output of the $i$ th domestic private firm is $q_i^d$, $i = (0, 1, ..., m)$, and that of the $j$ th foreign firm is $q_j^f$, $j = (1, ..., n)$. Total output is $Q = \sum_{i=0}^{m} q_i^d + \sum_{j=1}^{n} q_j^f$. The inverse demand function, $p(Q)$, is decreasing, continuous and twice-differentiable, and is derived from a log-concave demand. Thus it satisfies $p'' + p' \leq 0$.

The log-concavity of demand implies the Hahn stability condition and also that the marginal revenue of each profit-maximizing firm decreases as the output of any other firm increases. We assume firms share an identical technology that can be represented by a convex cost function $c = c(q)$ where $c(0) = 0$. Let $\pi^d$ be the profit of each domestic firm $\pi^d = p(Q)q^d - c^d(q^d) + sq^d$ and the profit of each foreign firm be $\pi^f = p(Q)q^f - c^f(q^f) - tq^f$. The industry revenue cannot increase indefinitely as there is a level of $\bar{Q}$ for which $p(\bar{Q}) = 0$. Let $g(Q)$ denote the gross benefits to home consumers i.e. the area under the inverse demand curve. Thus

$$g(Q) = \int_{Q}^{\infty} p(u)du, \quad g'(Q) = p(Q)$$

and the home country’s welfare function is given by net consumer surplus plus domestic firm surpluses minus the net costs of trade policy:

$$W = g(Q) - p(Q)Q + (m + 1)[p(Q)q^d - c^d(q^d) + sq^d] - [(m + 1)q^d - \bar{Q}q^f].$$

Now consider a two stage game. At the first stage, the government announces the strategic trade policy instruments. At the second stage, taking the announced subsidy and tariff as given, all domestic and foreign firms select their outputs to maximize their own profits under Cournot-Nash assumptions. Cournot equilibrium requires each firm equates its marginal cost and marginal revenue from the sales. Therefore the equilibrium conditions are

$$p + p'q^d + s = c^{\text{nd}}$$

$$p + p'q^f = c^{\text{nf}} + t.$$  

Using (3) and (4) to substitute for $s$ and $t$, the welfare function can be written as

$$W = g((m + 1)q^d + nq^f) - np'(m + 1)q^d + nq^f[Q^2 - nc^{\text{nf}}(q^f)q^f - (1 + m)c^{\text{nd}}(q^d)].$$
The trade policy instruments directly influence firms output decisions. The change in home welfare resulting from a small change in firms decisions in equilibrium is

\[ dW = \left[ p + np''q^f - c^d \right] (m + 1) dq^d + \left[ p + q^f (np''q^f + 2p' - c^d') - c^f \right] ndq^f. \tag{6} \]

The coefficients of \((m + 1)dq^d\) and \(ndq^f\) respectively tell us the welfare effects of altering the volume of \(s\) and \(t\). The government of the domestic country increases (decreases) the rates of tariff and subsidy if the values of these coefficients are greater (smaller) than zero. Therefore, the equilibrium solution satisfies the following conditions across all domestic and foreign firms

\[
\left\{
\begin{array}{l}
\text{(i)} \quad p + np''q^f - c^d = 0 \\
\text{(ii)} \quad p + q^f (np''q^f + 2p' - c^d') - c^f = 0
\end{array}\right.
\tag{7} \]

We assume that the welfare function is strictly concave in \((q^d, q^f)\) or any linear and positive transformation of it, so the conditions \((7i - ii)\) are necessary and sufficient for an allocation to be an optimum solution for the government s problem. The policies \(t\) and \(s\) are implicitly defined by the above conditions. For a linear example, the optimal subsidy is

\[ s^* = \frac{\theta^d p}{(m + 1)\epsilon} \tag{8} \]

where \(\theta^d = Q^d / Q\) is the share of domestic production, \(Q^d\), in total supply and \(\epsilon = -(\partial Q / \partial p)(p/Q)\) is the price elasticity of demand. This implies that the level of optimal subsidy is decreasing in the number of domestic firms and the elasticity of demand in absolute value. Also the greater is the share of home production in total output, the higher is the level of the subsidy. It can be checked that the level of optimal tariff is always positive with a log-concave demand and convex cost functions.

3 International Mixed Oligopoly and Market Distortions

To investigate the effect of public enterprise behavior on market efficiency in this section, we will replace one of the domestic firms by a welfare maximizing public firm in the model and compare the results.

To begin with, following Fjell and Heywood (2002), Matsumura (2003) and Cornes and Sepahvand (2003), we assume that the public firm has a first-mover advantage. Consider a two stage game. At an initial stage, the government commits itself to selected rates of \(s\) and \(t\) while at the same time it sets the output of the public firm \(q_o\) to maximize domestic welfare given by (2). At the second stage, taking the government's policy instruments as given, \(m + n\) domestic and foreign private firms choose their outputs to maximize their own profits. We are led to the following result:
Proposition 1: If the public &rm acts as a Stackelberg leader in an international mixed oligopoly with strategic trade policy, it adds no distortions to the market.

(See Appendix A for the proof.)

After change in ownership, the zero-th &rm follows the same pattern of behavior presented by \((7-i)\). As the results of the game still support the unique optimum solution where all distortions are corrected, we may conclude that the presence of a dominant public enterprise adds no distortions to the market.

Notice that the public enterprise in a linear example always follows marginal cost pricing. However in general it produces where its marginal cost is greater or equal to the market price\(^4\). This is also the case for the foreign &rns, as the term in bracket in \((7-ii)\) is always negative.

The next step is to check the impact of the timing of the public &rm’s behavior on market outcomes. That is because earlier studies in the mixed oligopoly literature that insist on the distortionary effects of the public enterprise behavior typically adopt a different modelling assumption, namely Cournot competition.

Lemma 1: If the public enterprise moves simultaneously with the private &rns at the second stage of the game, then the economy cannot achieve at its optimum.

(See Appendix B for the proof).

Since we have already had the optimum solution under public Stackelberg leadership, any difference in results implies that the economy is diverging from its optimum. With Cournot competition the public enterprise adds to market distortions because &rns output decisions do not satisfy the optimum conditions. Thus, it is not surprising to observe privatization that returns the model to a standard oligopoly with strategic trade policy being able to improve welfare of the domestic country when the public enterprise acts as a Cournot player.

4 Endogenous Order of Moves

In the last section we observed that the existence of a distortionary affect associated with the presence of a public enterprise depends on the basic modelling assumptions regarding the order of moves. Most of the economic literature assumes Cournot competition with simultaneous play as the natural order of play in a quantity-setting game\(^5\). Existing analyses typically argue that the assumed order of play should also be consistent with the players preferences over the timing of actions. As public enterprises are typically non-profit maximizing &rns, their preferences are not the same as private &rns. So one cannot simply assume that in a mixed oligopoly &rns compete in Cournot fashion without first justifying this assumption.

We focus now on an international mixed oligopoly in its simplest form where there is just one
&rmand each type; one domestic public &rmand one domestic private &rmand one foreign private &rmand.

We assume that the government only uses a production subsidy or we set $t = 0$. We are interested
in this setting because in earlier studies relating to mixed oligopoly, privatization is justiﬁed only if
the public enterprise is operating in a sufﬁciently competitive market (De Fraja and Delbono 1989,
Cremer et. al. (1989). By contrast, in an international mixed Cournot oligopoly with subsidization,
privatization always improves welfare regardless of the intensity of competition. However, this result
is sensitive to the timing assumption (Sepahvand, 2004).

**Corollary 1:** The results of Proposition 1 and Lemma 1 hold if the tariff is equal to zero. However, in this event the equilibrium solution differs and satisﬁes the following conditions:\(^7\):

\[
\begin{align*}
\text{(i)} & \quad p + p'(1 + \frac{\partial q^f}{\partial q^f})q^f - c^d = 0 \\
\text{(ii)} & \quad p + p'q^f - c^f = 0
\end{align*}
\]

where the optimal subsidy is

\[
s^* = -p'[Q^d/2 + (1 + \frac{\partial q^f}{\partial q^f})q^f].
\]

The SNP equilibrium level of subsidy is always positive since with a log-concave demand the slope
of all proﬁt-maximizing &rmand s best response function, $BR$, always belongs to $[-1, 0]$ (Vives, 1999).

Following Hamilton and Slutsky (1990), let us extend the basic quantity-setting game of an
international mixed oligopoly with subsidization to a preplay stage where &rmands choose their timing
of action rather than moving in an exogenously imposed order. This hypergame is denoted by
$\Gamma(.)$ and it consists of two stages: a preplay stage and a basic stage. In the preplay stage, &rmands
simultaneously decide whether to choose actions in the basic game at the ﬁrst opportunity and move
early, denoted by $E$, or to wait until observing their rival’s action and move late, denoted by $L$.
So the set of action times is $T = (E, L)$ and a combination of timing decisions is $\psi \equiv (\tau_0, \tau_p, \tau_f)$
where $\tau_i \in T$ and the subscripts $i \in \{0, p, f\}$ where $0, p$ and $f$ stand for domestic public enterprise,
domestic private &rmand foreign &rmand respectively. The set of all possible timings is $\Psi^8$.

In the basic stage, there are two phases: the regulation phase and the action phase (Figure 1).
In the regulation phase, after the announcement, the government of the host country observes the
announced timing of the game and chooses the subsidy from a non-negative &nite subset of real
numbers $B$ in advance of the &rmands moves. Then in the action phase of the basic stage, &rmands select
their outputs knowing when the others will make their choices and the announced level of subsidy
associated with each order of moves. For example, the output choice of the domestic public &rmand
when it moves simultaneously with other &rmands at the early stage of action phase is $q_i^{(EE)}(s)^9$. The
strategy of each &rmand is $S_i = (\tau_i, q_i^0(s))$ and the strategy proﬁle is $S = \Pi S_i$. 

\[7\]
The government observes the announced timing of actions, $\psi$, and sets $s$. Firms announced moving early, observe $\psi$ and $s$ and choose $q_i$. Firms simultaneously choose their time of action $t_i$ and announce them as commitments. Firms announced moving late observe $\psi$ and $s$ and $q_i$ and choose $q_j$.

Figure 1: The extended game

We do not need to consider all possible strategies of firms at the basic game in order to solve the game, because Nash equilibrium in subgames eliminates non-equilibrium output choices. Therefore, we can confine ourselves to Nash equilibria in subgames denoted by $e^\psi$. All payoffs in subgames Nash equilibria are shown in Table 1.

Table 1: The reduced strategic form of the game in an international mixed triopoly model

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<th>Early</th>
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<tr>
<td><strong>Domestic Firm</strong></td>
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<td><strong>Private Firm</strong></td>
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<tr>
<td>$E$</td>
<td>$(W^{EEE}, \pi_p^{(EEE)}, \pi_f^{(EEE)})$</td>
<td>$(W^{EEL}, \pi_p^{(EEL)}, \pi_f^{(EEL)})$</td>
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<tr>
<td>$L$</td>
<td>$(W^{ELE}, \pi_p^{(ELE)}, \pi_f^{(ELE)})$</td>
<td>$(W^{ELL}, \pi_p^{(ELL)}, \pi_f^{(ELL)})$</td>
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<tr>
<td>$E$</td>
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<td>$(W^{LEL}, \pi_p^{(LEL)}, \pi_f^{(LEL)})$</td>
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<tr>
<td>$L$</td>
<td>$(W^{LLE}, \pi_p^{(LLE)}, \pi_f^{(LLE)})$</td>
<td>$(W^{LLL}, \pi_p^{(LLL)}, \pi_f^{(LLL)})$</td>
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The timing choice of firms can be obtained by comparing the payoffs of each timing choice with the payoffs of any feasible timing of play at that subgame. Three possibilities arise: 1) only one of the sequential outcomes Pareto dominates the simultaneous move outcome and it will be the unique equilibrium of the extended game, 2) both sequential outcomes Pareto dominate the simultaneous move outcome, 3) neither of the sequential outcomes Pareto dominates the simultaneous moves. The possibility of the simultaneous play, which is the basic assumption of Cournot competition, arises only in the third case.
Corollary 1 ensures that a simultaneous play with all &rms moving late, $e^{(LLL)}$, cannot be an SPN equilibrium of the extended game. However we may still have a simultaneous play as an equilibrium if all &rms choose to move early, $e^{(EEE)}$. But we will show in the next lemma and proposition that the foreign &rm has an incentive to deviate from this setting.

**Lemma 2** In the extended game $\Gamma(,)$, if the foreign &rm chooses moving late and the public &rm chooses moving early, the equilibrium level of the home industry output is unchanged regardless of the domestic private &rm's choice of action time.

**Proof.** If $q^d_p$ is the equilibrium output level of the domestic private &rm at $e^{(EEL)}$, it should solve

$$\frac{d\pi_p}{dq^d_p} = p'(1 + \partial q^f / \partial q^d)q^d_p + p - c^d + s = 0.$$  

In the regulatory phase of the basic game the public enterprise maximizes welfare with respect to $q^d_o$, while taking the $BR_f$ and the level of the subsidy and $q^d_p$ as given. So its behavior still supports (9i–ii). As there exists a positive subsidy that supports the optimum,

$$s^*(EEL) = -p'(1 + \partial q^f / \partial q^d)(q^d_p + q^f),$$  

we may conclude that the SPN equilibrium level of the home industry output in both settings is the same because of the uniqueness of the optimum along the $BR_f$. Therefore, $Q^d(EEL) = Q^d(ELL)$. $\blacksquare$

Lemma 2 simply says that if the foreign &rm plans to move late and the public &rm chooses to move early, any change in the timing of action by the domestic private &rm just changes the level of the subsidy. But the SPN equilibrium level of the home industry output and the share of each domestic &rm in the total home industry output remains unchanged. The following proposition summarizes our findings about the simultaneous order of moves.

**Proposition 2** In an extended game $\Gamma(,)$, the simultaneous order of moves is not an SPN equilibrium outcome.

**Proof.** The simultaneous order of moves where all &rms move late is not an SPN equilibrium of the extended game because of Corollary 1. As Lemma 2 shows $Q^d(EEL) = Q^d(ELL)$. But from Corollary 1 we also know that $Q^d(EEE) > Q^d(ELL)$. Recall $\pi_f$ is decreasing in $Q^d$. This ensures us that the foreign &rm always prefers moving late rather than moving simultaneously with domestic &rms. Hence the simultaneous order of moves in the 1st period, $e^{(EEE)}$, cannot be the SPN equilibrium outcome of the extended game too. $\blacksquare$

So far, we have shown that simultaneous play is not the SPN equilibrium of the extended game. Now we want to evaluate an alternative assumption about the order of moves, namely public &rm
leadership. The feasible alternatives for the firms in this setting are \( e^{(LLL)} \), \( e^{(EEL)} \) and \( e^{(ELE)} \) (see Table 1). The domestic public enterprise who seeks to maximize domestic welfare prefers \( e^{(ELL)} \) to \( e^{(LLL)} \) setting due to Proposition 2. The following lemma shows that from the domestic private firm’s point of views public firm leadership is always preferable to \( e^{(EEL)} \), so the domestic private firm has also no incentive to deviate from that setting.

Lemma 3) If the public firm moves first and the foreign firm moves later, the payoff of the domestic private firm is always higher when it chooses moving late.

Proof. A comparison of (12) and (10) shows that

\[
s^{(ELL)} - s^{(EEL)} = p' q_f (\partial q_f / \partial q_d) \geq 0
\]  

(13)
since \( \partial q_f / \partial q_d \in [-1, 0] \) when demand function is log-concave and \( p' < 0 \) everywhere. Therefore, while the domestic private firm produces the same level of output, it will receive a greater subsidy when it chooses to move late.

The payoff of the foreign firm after deviation from \( e^{(ELL)} \) depends on the effects of such a change on the home industry output. So we need to look at the timing of the foreign firm’s action and its effects on the home industry.

First let us identify the set of feasible output plans in two settings, \( e^{(ELE)} \) and \( e^{(ELL)} \). In \( e^{(ELL)} \), it seems that the set of feasible output plans does not depend on the subsidy. Let \( V \) denote the set of all points along the \( BR_f \) for any given level of the public enterprise output. The set of feasible output plans, the set of all points in \((Q_d(s), q_f)\) that the home government can choose from by setting various level of subsidies, equals \( V \). That is because all that a subsidy does is to shift the \( BR_p \) along the \( BR_f \).

Let \( U \) denote the set of all feasible output plans in \( e^{(ELE)} \) for any given level of \( q_d^0 \). Any element of this set depicts a point along the \( BR_p \) that maximizes the foreign firm’s profit for a given level of subsidy. Note that both \( BR_p \) and \( BR_f \) are downward sloping and a foreign firm’s iso-profit contours touch the \( BR_p \) curves at the downward sloping part of the contours. A change in the level of subsidy shifts the \( BR_p \). As the downward sloping part of the foreign firm’s iso-profit contours are above the \( V \), thus \( U \) is located inevitably above the \( V \) for any given level of the home industry output in the \((Q_d, q_f)\) space.

Figure 2 illustrates three possible points of \( U \) in the two-dimension space for a given level of \( q_d^0 \). \( BR_p \) is tangent to the foreign firm’s iso-profit contour at point \( A \) if the level of subsidy is \( s_1 \). The government can choose points \( B \) or \( C \) if it sets the level of subsidy at \( s_2 \) or \( s_3 \). Therefore \( U \) is the locus of tangencies traced out as the subsidy varies. One can expand the above analysis into a three-dimension space to include the public enterprise output choices. However, that would not affect our findings\(^{10} \).
The following lemma shows sufficient conditions for the home country preferences that guarantee an increase in the home industry output when the foreign &rm deviates from $e^{(ELL)}$.

**Lemma 4** If the demand function is concave and $p' - p''q_f - C^{(d)} < 0$, then $Q^d(e^{(ELL)}) \geq Q^d(e^{(ELE)})$.

(See Appendix C for the proof)

Lemma 4 guarantees that the solution of $e^{(ELE)}$ is located to the northeast of the $e^{(ELL)}$. This implies that the foreign &rm faces a higher level of home industry output and hence a reduction in its profit due to deviation from $e^{(ELL)}$.

We can now establish the following proposition:

**Proposition 3** In the extended game $\Gamma(\cdot)$, if the conditions of lemma 4 hold, then public Stackelberg leadership is the SPN equilibrium solution of the game.

**Proof.** First, the public enterprise cannot be better off if it chooses to move late instead of moving early at $e^{(ELL)}$ since $W^{(ELL)} > W^{(ELL)}$ (Corollary 1). Second, the domestic private &rm will not change its timing choice in $e^{(ELL)}$ as this results in a reduction in the level of optimal subsidy but leaving its SPN equilibrium output unchanged (Lemma 3). Third, if the foreign &rm were to deviate from $e^{(ELL)}$ and the lemma 4 conditions were hold, the home industry would produce more. But $\pi_f$ is decreasing in $Q^d$. So the foreign &rm will also be worse off by deviating from $e^{(ELL)}$. As no &rm has an incentive to deviate from the order of moves under public Stackelberg leadership, $e^{(ELL)}$ is the only SPN equilibrium of the extended game.

Proposition 3 asserts that public Stackelberg leadership Pareto dominates the simultaneous play and the other feasible alternatives. Accordingly, it is the only SPN equilibrium of the extended game with observable delays. Nevertheless, in general we may have other SPN equilibrium solutions with
different orders of play.

The majority of papers that deal with the welfare effects of public enterprise behavior in mixed oligopoly use a linear demand model. As the linear demand model satisfies the Lemma 4’s conditions, we may legitimately apply the above results.

**Example 1** Suppose the inverse demand function is given by \( p = 10 - Q \) and the firms share an identical cost function represented by \( c(q) = 1/2q^2 \). Table 2 shows the Nash equilibria in the subsequent subgames in this simple example.

Table 2: International mixed triopoly: a numerical example

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<th>Foreign Firm</th>
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<td>Domestic Firm</td>
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<tr>
<td>Early</td>
<td>33.96</td>
<td>15.4</td>
<td>1.92</td>
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<tr>
<td>Late</td>
<td>34.16</td>
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<td>Public &amp;rm moves early (E)</td>
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<td>14.39</td>
<td>2.03</td>
</tr>
</tbody>
</table>

The above payoff matrix reveals that the public Stackelberg leadership is the SPN equilibrium of the extended game in the linear example. A comparison of the payoffs shows that no player in the \( e^{(ELL)} \) setting can unilaterally change its timing of action in a way that improves its payoff.

5 Conclusion and Remarks

There are two distinct sources of market distortion in a mixed oligopoly with foreign competition. Market distortions in the domestic economy arise partly because of oligopolistic nature of the competition within the domestic industry, and partly because of the market power of the foreign &rms; they can set price above marginal costs and transfer the profits of the industry abroad.

It is well known from the theory of trade that in an imperfect market of an open domestic economy, the optimum trade policy calls for a combination of subsidies and tariffs to address each type of distortion directly. Some studies argue that the presence of a public enterprise in the domestic
market raises a new source of distortion that cannot be corrected by a full set of strategic trade policy instruments. It follows then that privatization can enhance welfare in the domestic country even if it does not improve internal (organizational) efficiency.

This paper shows that this argument is valid only if the public enterprise acts as a Cournot player and moves simultaneously with private firms in a quantity-setting game. But if it sets its outputs in advance of the private firms and acts as a Stackelberg leader the operation of a public enterprise is comparable to a production subsidy deployed to correct distortions within the home industry. Moreover the public enterprise produces where the market price is greater or equal to its marginal cost.

The application of the extended game with observable delay to endogenize the order of moves shows that the assumption of simultaneous quantity choice by public and private firms does not lead to a self-enforcing equilibrium. This result raises a serious question about those works in the existing literature that condition their models upon the Cournot assumption without justifying this assumption. Second, under some circumstances, including a linear demand model, the public Stackelberg leadership is the unique SPN equilibrium of the extended game of mixed oligopoly with subsidization\(^1\). The intuition behind our results is simple. If, following the founders of mixed oligopoly models, we consider the public enterprise as an instrument to regulate an industry from inside, then we should recall that the necessary assumption is that the government can credibly commit to its policy choice before the firms make their choices... [this is] reflected by the assumption that the government moves first in the game tree (Brander, 1995).

Thus in conclusion this paper shows that the distortions raised by the presence of a public enterprise in international mixed Cournot oligopoly with subsidization cannot be attributed to the firms ownership but rather it can be fully explained by an unjustified assumption about the timing of the game.

Notes

\(^1\) Those banks that are at least 50 percent foreign.

\(^2\) It is very difficult to trace the sources of distortions with the heterogeneous cost structure across the firms. This unrealistic assumption helps us to abstract from organizational efficiency and also makes our results comparable with the existing literature.

\(^3\) One can assume \(c(0) \neq 0\) to endogenize the number of firms. However, as the number of firms in our model is fixed and we are not dealing with entry/exit problem for expository simplicity, we assume there is no sunk cost in this model.
Fjell and Pal (1996) state that the public enterprise follows marginal cost only if the number of foreign firms is zero. This sounds like \( n = 0 \) is a necessary condition for marginal cost pricing while they use a linear demand model. The advantage of our general framework is that it reveals the limits of their analysis.

That is because without establishing a mechanism that provides a precommitment for the leader (and only for the leader) to a certain strategy, the only credible precommitment for the profit-maximizing firms is a simultaneous play strategy (Wolfstetter, 1999, p. 79).

Now the model is exactly the same as the model used in Pal and White (1998, Section 3) but here in a generalized form.

We omit the proof here as it has been given elsewhere.

Note that the set \( \Psi \) has 8 elements as there are 8 possible ways to put 3 elements into 2 cells.

Formally, here the set of one firm, say domestic public firm, strategies is \( S_o = T \times \Phi_o \) where \( \Phi_o \) is the set of functions that maps the information set of the public firm into the action set of public firm, \( A_o \). Here the information set of the public firm is \( \{(EEE), (EEL), (ELL), (LLL), (LEE) \times A_p \times A_f, (LLE) \times A_f, (LEL) \times A_p \times B\} \) where \( A_p \) and \( A_f \) are the action spaces of domestic private firm and the foreign firm respectively. The set of strategies profile is \( S = S_o \times S_p \times S_f \).

Then set \( U \) is a plane that still for any given value of \( q^d_o \) is located above \( V \) that is also a plane with decreasing slope in both \( q^d_p \) and \( q^d_o \) in \( R^3 \) space.

Capital \( C \) represents the cost function of home industry. It can be defined as \( C(Q^d) = \min \{c(q^d_p) + c(q^d_o), Q^d = q^d_p + q^d_o\} \).

References

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A- The equilibrium conditions at the second stage of the game for all domestic firms except the zero-th firm is presented in (3). A small change of welfare in equilibrium with a dominant welfare maximizing public enterprise is

\[ dW = \left[ p + np''qf^2 - c_{d0} \right] dq_0 + \left[ p + np''qf^2 - c_{p0} \right] m \frac{\partial q_d^d}{\partial Q} dq_0 \]

\[ + \left[ p + qf (np''qf + 2p - c''f) - c_{d1} \right] m dq_d^d + \left[ p + np''qf^2 - c_{p1} \right] m dq_p^d \]

\[ + \left[ p + qf (np''qf + 2p - c''f) - c_{d1} \right] ndq_f. \]  

At equilibrium where the government makes a proper use of its policy instruments in order to induce firms to produce the desired level of outputs, the firms output levels satisfy the following conditions,

\[ \begin{cases} 
  i) & p + np''qf^2 - c_{d0} = 0 \\
  ii) & p + np''qf^2 - c_{p0} = 0 \\
  iii) & p + qf (np''qf + 2p - c''f) - c''f = 0 
\end{cases} \]  

which are exactly the same as equilibrium conditions at optimum presented in (7 i - ii).

B- In Cournot competition the public enterprise sets \( q_0^d \) at the second stage of the game to maximize welfare. So the equilibrium at the second stage is characterized by (3) and (4) and also

\[ p - np'qf^1 = c_{d0}^d. \]  

Define \(-p'qf^1\) from (B-1) and insert it in (4). Then evaluate it for \( t \) and plug the result in (2) to get
Following the same logic as Appendix A, the equilibrium solution for the &rms outputs is required to satisfy these conditions,

\[
\begin{align*}
\text{i)} & \quad p - np'q^f - c_{0d}^d = 0 \\
\text{ii)} & \quad p + p'q^f - mc_{0p}^d = 0 \\
\text{iii)} & \quad 2p + q^f(p' - mc_{0p}^d) - nc^f - c_{0d}^d = 0
\end{align*}
\]

(B-2)

that differ from (7 i – ii) the conditions of optimum solution.

C- Let \( Z \equiv \{ (Q^d, q^f) \in R^2 | Q^d \in \arg \max W(Q^d, q^f) \} \) denote all the points where for any given level of imports the domestic welfare is maximized. If the graph of \( Z \) is upward sloping then the iso-welfare contours at the far distance from origin in northeast direction are associated with a higher level of welfare. The slope of \( Z \) is equal to

\[
-(\frac{\partial^2 W/\partial q^d\partial q^f}{\partial^2 W/\partial^2 q^f}) = -\frac{-p''q^f}{p' - p''q^f - C^{ad}}.
\]

which is positive if the demand function is concave and \( p' - p''q^f - C^{ad} < 0 \). As the graph of \( U \) is decreasing in home industry output and it is located above \( V \), the upper iso-welfare curves touch it at the northeast of the public Stackelberg leadership solution. This implies that \( Q^d(ELE) \geq Q^d(ELL) \).