Monetary union enlargement and international trade∗

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Abstract

This paper studies the effects of monetary union enlargement on international trade in a three-country (two incumbent countries and an acceding country) search monetary model. It is shown that, if the degree of integration among the actual member states and the acceding country is high enough, accessing a currency union is Pareto improving as the increase of international trade is strictly positive for all countries.

Keywords: search equilibrium, trade diversion, trade creation, monetary union

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1 Introduction

Several Eastern European Countries (EECs) are contemplating the option of joining the EMU in the next few years. Should they become part of the Euro area, they will likely be enjoying an increase in the volume of bilateral trade with actual EMU member states. Still, it is unclear whether also actual EMU members will experience a global trade creation effect from new EMU accessions.

The relationship between international trade and currency unions has been extensively studied in the literature. From a theoretical point of view, trade creation effects have been prominently analyzed in customs unions rather than in currency union models (see, [10] and [9].) Empirical analyses as in [7], [1] and [2], for example, have estimated the effects of countries sharing the same currency on bilateral trade.

By developing a two-currency, three-country (two incumbent countries, ICs, and an acceding country, AC) extension of the search equilibrium framework of [6], this paper characterizes the aggregate level of international trade among monetary union participants, and gives conditions for global trade creation to occur as a consequence of new countries accessing a monetary union. The accession of a new member country increases the equilibrium fraction of buyers inside the monetary union, and may result in trade reduction if too much money is issued, unless the degree of integration between the ICs is not too high relative integration with the AC. In this latter case, enlargement of the monetary union increases international trade and is Pareto improving.

The paper is organized as follows. Section 1 describes the model. Section 2 evaluates the trade creation effect by comparing pre-accession and post-accession equilibria. Finally, Section 3 ends the paper with a brief summary of the results.

2 The model

The present model is a straightforward three-country (two ICs and an AC) extension of [6]. Both the ICs, $i = \{1, 2\}$, and the AC country, $i = 3$, are populated by a continuum of $n_i \in (0, 1)$ infinitely lived agents. Total population across countries has unit mass, i.e. $\sum_{i=1}^{3} n_i = 1$. Time is discrete and extends from zero to infinity. Individuals are specialized in production,
consumption and storage of goods that are indivisible. Specialization in production is denoted by the index $j$, in the sense that any agent of type $j$ can only consume the good $j$. The same agent only produces the good $j + 1$ (modulo $k$, where $k$ is the number of different goods), at no cost, so that the act of consuming necessarily involves a transaction between agents. Every agent of type $j$ can costlessly store up to one unit of commodity $j + 1$, without being able of storing any other type of goods. Moreover, the distribution of agents is uniform across $k \geq 3$ types. This specification simply rules out double coincidence of wants so that barter equilibria are not feasible. There are two indivisible fiat monies that are associated with the ICs (type 1 money) and the AC (type 2 money) countries and are introduced as follows. At the initial date, the government of the ICs countries issues one unit of money 1 to a fraction $m_1 \in (0, 1)$ of the population in the ICs countries, and the government of the AC issues one unit of money 2 to a fraction $m_2 \in (0, 1)$ of its citizens. The fraction of the population in country $i$ with a unit of money $j$ is denoted by $m_{ij} \in (0, 1)$. Monies have no intrinsic value and are used for the only purpose of buying goods.

With $u > 0$ denoting the agent’s instantaneous utility from consumption of her own consumption good, and $r$ denoting the discount rate, the value function at time $t$, expected on the information set at time $t$, can be written as

$$V_t = E \left[ \sum_{s=0}^{\infty} \frac{u}{(1 + r)^s} I_{t+s} \mid \Omega_t \right].$$

$I_{t+s}$ is a random indicator function that either equals 1, in the event of consumption occurring at time $t + s$, or zero otherwise.

When an agent acquires her consumption good, she will immediately consume it, and produce one unit of her production good according to a continuous-time Poisson process with arrival rate $\alpha \to \infty$ (i.e. production is instantaneous). A seller, i.e. an agent with a unit of production good, looks for a buyer, i.e. an agent with a unit of money, to trade with. Individuals meet pairwise and randomly according to a Poisson process with finite arrival rate. When two agents meet, an exchange takes place only if it makes both agents strictly better off. As both monies and goods are indivisible, each agent either holds one unit of money or one unit of production good at every instant of time, and monetary exchange is one-for-one. As in [6], the simplifying assumption that no agent disposes of either money or production good holds.
The term $\beta_{ij}$ indicates the frequency of an agent from country $i$ meeting an agent from country $j$, relative to the frequency of two nationals meeting (it is simply assumed that the chances of nationals meeting are equal across countries.) For example, if an agent from country 1 meets an agent from country 2 with the same frequency that she meets a national, then $\beta_{12} = 1$, and the two economies are said to be perfectly integrated. Conversely, if a meeting between national fellows occurs at a frequency higher than a meeting between foreigners, than the countries are imperfectly integrated, i.e. $\beta_{ij} \in (0, 1)$.

The fraction of population endowed with a unit of production good, money 1 and money 2 in country $i$ is denoted by the vector $M_i = (1 - m_{i,1}, m_{i,1}, m_{i,2})$. Hence, total supply of money 1 and money 2 amounts to $n_1 m_1 = n_1 m_{1,1} + n_2 m_{2,1} + n_3 m_{3,1}$ and $n_3 m_2 = n_1 m_{1,2} + n_2 m_{2,2} + n_3 m_{3,2}$, respectively.

Agents choose strategies that maximize the expected utility given both other individuals’ strategies and inventory distributions. The analysis focuses only on pure strategy equilibria. The game is anonymous since the probability of a double meeting with an agent is zero. The trade strategy of an individual living in country $i$ about the exchange of object $x$ for object $y$ is denoted by $\tau_{i,xy} \in \{0, 1\}$, where $x, y \in \{0, 1, 2\}$ are the production good, money 1 or money 2, respectively. If the individual agrees to exchange the object $x$ for the object $y$ then $\tau_{i,xy} = 1$, otherwise it is zero. In steady state, the strategies and the proportion of agents with a given object, $m_{i,j}$ and $1 - m_{i,1} - m_{i,2}$, are constant in each country.

The transition matrix related to country $i$ is

$$
\Pi_i = \begin{bmatrix}
1 - \pi_{i,01} - \pi_{i,02} & \pi_{i,01} & \pi_{i,02} \\
\pi_{i,10} & 1 - \pi_{i,10} - \pi_{i,12} & \pi_{i,12} \\
\pi_{i,20} & \pi_{i,21} & 1 - \pi_{i,20} - \pi_{i,21}
\end{bmatrix},
$$

where $\pi_{i,xy}$ is the chance of an individual (from country $i$) trading the object $x$ for the object $y$ with some other agent (either from country 1, country 2 or country 3, and holding the object $y$.) As an example, the probability of an agent from country 1 exchanging a unit of money 1 for a unit of the consumption good, $\pi_{1,10}$, is equal to the sum of $a$) the probability of meeting a national agent that holds the consumption good (conditional on the probability of both agreeing on trading) plus $b$) the probability of meeting
an agent from either country 2 or country 3 that holds a unit of consumption
good (conditional on the probability of that agent agreeing on trade.)

The value functions of an agent living in country \( i \) are

\[
\begin{align*}
V_{i,0} &= \frac{\pi_{i,01}V_{i,1}+\pi_{i,02}V_{i,2}+V_{i,0}(1-\pi_{i,02}-\pi_{i,01})}{1+r}, \\
V_{i,1} &= \frac{\pi_{i,10}(V_{i,0}+u)+\pi_{i,12}V_{i,2}+V_{i,1}(1-\pi_{i,12}-\pi_{i,10})}{1+r}, \\
V_{i,2} &= \frac{\pi_{i,20}(V_{i,0}+u)+\pi_{i,21}V_{i,1}+V_{i,2}(1-\pi_{i,21}-\pi_{i,20})}{1+r},
\end{align*}
\]

when she either holds the production good, money 1 or money 2, respectively. For example, the (indirect) utility from holding one unit of production good at time \( t \), \( V_{i,0} \), is equal to the probability of exchanging the production good for money 1 in the next period times the utility from holding a unit of money 1, \( \pi_{i,01}V_{i,1} \), plus the probability of exchanging a unit of production good for a unit of money 2 in the next period times the utility from holding money 2, \( \pi_{i,02}V_{i,2} \), plus the probability of no exchange occurring in the next period, times the utility from holding the production good, \( (1-\pi_{i,02}-\pi_{i,01})V_{i,0} \), each term discounted by the factor \( 1/(1+r) \).

An agent exchanges with another agent only if she is strictly better off, i.e.
\( (i) \; \tau_{i,0x} = 1 \Leftrightarrow V_{i,0} < V_{i,x} \) for any \( x = 1, 2 \); \( (ii) \; \tau_{i,x0} = 1 \Leftrightarrow V_{i,x} < V_{i,0} + u \) for any \( x = 1, 2 \); \( (iii) \; \tau_{i,xy} = 1 \Leftrightarrow V_{i,x} < V_{i,y} \) such that \( x \neq y \) and \( x, y \neq 0 \). The production process always yields positive utility if the production good can be exchanged at least with one type of money, i.e. \( \max\{V_{i,1}, V_{i,2}\} > V_{i,0} > 0 \). From the zero production and storage costs assumption it follows that \( V_{i,0} \geq 0 \). Finally, the inequality \( V_{i,1}, V_{i,2} < V_{i,0} + u \) means that it is always profitable for an individual with a unit of money to acquire the consumption good.

The level of imports of country \( i \) from country \( j \), denoted as \( T_{i,j} \), is obtained by multiplying the proportion of moneyholders living in country \( i \), at the beginning of the period, times the probability of a moneyholder in country \( i \) acquiring her consumption good from a resident in country \( j \) during the same period. If \( i = j \), one obtains the level of transactions among agents of the same country \( i \) (domestic trade).

The following assumptions hold throughout the paper:

(A1) \( r = 0; \)

(A2) \( n_i = 1/3 \) for any \( i = \{1, 2, 3\}; \)

(A3) \( m_i = m \) for any \( i = \{1, 2\}; \)

(A4) \( \beta_{1,3} = \beta_{2,3} = \beta_3. \)
\( (A1) \) is made for ease of analysis. \((A2)\) and \((A3)\) restrict the analysis to the simplest case in which countries have symmetric population and money supply. By \((A4)\), both countries 1 and 2 share the same degree of integration with country 3. These assumptions enable to focus the analysis on the trade diversion and creation effects when a new country enters a monetary union.

By \((A2)\), it follows that \(m_1 = m_2\). Then, using \((A3)\), it also holds that

\[
m = \sum_{i=1}^{3} m_{i,j},
\]

where \(i\) denotes the country, and \(j\) the money type.

### 3 Trade creation effects

This section characterizes steady state equilibria, before and after an additional country acceding the monetary union. Pre-accession equilibria are characterized by the fact that both money 1 is the only money circulating inside the monetary union and money 1 is not circulating inside the AC. In the accession equilibrium both currencies are perfect substitutes throughout the enlarged monetary union.

#### 3.1 Pre-accession equilibrium

In the steady state pre-accession equilibrium it holds that \(m_1 = m_{1,1} + m_{2,1}\), \(m_2 = m_{3,2}\), \(m_{1,2} = m_{3,1} = m_{2,2} = 0\), and \(\tau_{1,02} = \tau_{2,02} = \tau_{3,01} = 0\). Hence, the related transition probabilities are

\[
\begin{align*}
\pi_{1,02} &= \pi_{2,02} = \pi_{3,01} = 0, \\
\pi_{1,01} &= \pi_{2,01} = \frac{m(1 + \beta_{1,2})}{6k}, \\
\pi_{3,10} &= \frac{\beta_{3}(2-m)}{3k}, \\
\pi_{3,20} &= \frac{(1-m)}{3k}, \\
\pi_{3,02} &= \frac{m}{3k}.
\end{align*}
\]  

The markovian steady state property \(M_i \Pi_i = M_i\) implies that \(m_{1,1} = m_{2,1} = \frac{m}{7}\). The sufficient conditions for the existence of the pre-accession equilibrium are

\[
\begin{align*}
V_{1,2} &< V_{1,0}, \\
V_{2,2} &< V_{2,0}, \\
V_{3,1} &< V_{3,0},
\end{align*}
\]
which turn out to be satisfied if and only if both of the following inequalities hold. Using (3), the trade-flow matrix can be written as

\[
T_P = \begin{bmatrix}
\frac{m(2-m)}{12k} & \frac{\beta_3 m(2-m)}{12k} & 0 \\
\frac{\beta_3 m(2-m)}{12k} & \frac{m(2-m)}{12k} & 0 \\
0 & 0 & \frac{4m(1-m)}{12k}
\end{bmatrix}.
\] (6)

This matrix is symmetric, because of equal population size and money supply across countries. The \(i\)-th element on the principal diagonal is the level of domestic trade in country \(i\). As shown in (6), the level of international trade between country 3 and countries 1 and 2, denoted as \(T^P_{31} + T^P_{13}\) and \(T^P_{32} + T^P_{23}\) respectively, is zero in the pre-accession equilibrium. The reason for this being that any transaction involving agents from countries 1 and 2, on one hand, and country 3, on the other hand, cannot occur since a country 3 agent is not willing to accept money 1 and a resident either in country 1 or in country 2 does not accept money 2 in transactions.

### 3.2 Accession equilibrium

In the accession equilibrium, the buyers’ value function is the same regardless the type of money held. This implies that monies 1 and 2 are perfect substitutes.

In steady state it holds that, \(m_{1,1} = m_{1,2} = m_{2,1} = m_{2,2} = m_{3,1} = m_{3,2}\), and the transition probabilities are

\[
\begin{align*}
\pi_{1,01} &= \pi_{2,01} = \pi_{1,02} = \pi_{2,02} = \frac{(1+\beta_{1,2}+\beta_3)m}{9k}, \\
\pi_{1,10} &= \pi_{1,20} = \pi_{2,10} = \pi_{2,20} = \frac{(3-2m)(1+\beta_{1,2}+\beta_3)}{9k}, \\
\pi_{3,01} &= \pi_{3,02} = \frac{(1+2\beta_3)m}{9k}, \\
\pi_{3,10} &= \pi_{3,20} = \frac{(3-2m)(1+2\beta_3)}{9k}.
\end{align*}
\] (7)

Therefore, in steady state, the following holds

\[m_{i,j} = \frac{m}{3}, \quad i = 1, 2, 3, \quad j = 1, 2\]
with $i$ and $j$ denoting country and money types, respectively. The associated trade-flow matrix is

$$T^A = \begin{bmatrix}
\frac{2m(3-2m)}{2k} & \frac{2\beta_m(3-2m)}{2k} & \frac{2\beta_m(3-2m)}{2k} \\
\frac{2\beta_m(3-2m)}{2k} & \frac{2m(3-2m)}{2k} & \frac{2\beta_m(3-2m)}{2k} \\
\frac{2\beta_m(3-2m)}{2k} & \frac{2\beta_m(3-2m)}{2k} & \frac{2m(3-2m)}{2k}
\end{bmatrix}$$

(8)

in the accession equilibrium. As $\max\{V_{i,1}, V_{i,2}\} > V_{i,0}$ holds when monies 1 and 2 are equally valued, the accession equilibrium exists.

The following Lemma shows that the fraction of buyers is greater in the enlarged monetary union.

**Lemma 1** The pre-accession fraction of buyers is $m_{i,1} + m_{i,2} = m$, $i = 1, 2$, and $m_{3,1} + m_{3,2} = m_{3,2} = m$. The accession fraction of buyers is $m_{i,1} + m_{i,2} = \frac{2}{3}m$, $i = 1, 2, 3$.

**Proof.** (Pre-accession equilibrium.) In the pre-accession equilibrium the following expressions are satisfied, $m_{3,2} = m_2 = m$, $m_{1,1} = m_{2,1} = \frac{m}{2}$ and $m_{1,2} = m_{2,2} = m_{3,1} = 0$.

(Accession equilibrium.) In the accession equilibrium it holds that $m_{i,j} = \frac{m}{3}$, $i = 1, 2, 3$, $j = 1, 2$ where $i$ is the country and $j$ is the money type.

The entailed change in the equilibrium fraction of buyers affects the volume of trade between ICs as in the following

**Proposition 2** $m < \frac{6}{7}$ is a necessary and sufficient condition for inter-ICs trade creation (i.e. between IC countries).

**Proof.** (Sufficiency.) Assume

$$m < \frac{6}{7}$$

(9)

is true. Expression (9) can be rewritten as

$$6 - 7m > 0$$

(10)

which, can be rewritten as

$$24 - 16m - 18 + 9m > 0$$

(11)
or
\[
8(3 - 2m) - 9(2 - m) > 0. \tag{12}
\]
Since \(\beta_{1,2}, m \in (0, 1)\) and \(k > 0\), it holds that
\[
\frac{11}{9}\frac{\beta_{1,2}m}{k} [8(3 - 2m) - 9(2 - m)] > 0 \tag{13}
\]
which is equivalent to
\[
\frac{\beta_{1,2}m}{k} \left[ \frac{2(3 - 2m)}{27} - \frac{(2 - m)}{12} \right] > 0 \tag{14}
\]
or
\[
\frac{2\beta_{1,2}m (3 - 2m)}{27k} - \frac{m\beta_{1,2}(2 - m)}{12k} > 0, \tag{15}
\]
so that \(T_A^{1,2} - T_P^{1,2}\) is strictly positive.

(Necessity.) Assume inter-ICs trade creation (i.e. between the IC countries) occurs, that is (from (6) and (8))
\[
\frac{2\beta_{1,2}m (3 - 2m)}{27k} - \frac{m\beta_{1,2}(2 - m)}{12k} > 0, \tag{16}
\]
which can be rewritten as
\[
\frac{\beta_{1,2}m}{k} \left[ \frac{2(3 - 2m)}{27} - \frac{(2 - m)}{12} \right] > 0, \tag{17}
\]
or
\[
\frac{11}{9}\frac{\beta_{1,2}m}{k} [8(3 - 2m) - 9(2 - m)] > 0. \tag{18}
\]
Since parameters \(m, \beta_{1,2}\) and \(k\) are all strictly greater than zero, (18) implies
\[
8(3 - 2m) - 9(2 - m) > 0 \tag{19}
\]
or, upon rearranging and simplifying,
\[
m < \frac{6}{7}. \tag{20}
\]

Proposition 2 states that a new accession reduces bilateral trade between pre-accession monetary union members if the fraction of money holders is
too large. The reason is clear from Lemma 1: enlargement of the monetary union in the form of dual currency circulation increases the fraction of buyers across countries. Hence, if \( m \) is large enough, a new accession reduces trade opportunities between ICs as the event of an ICs buyer matching an ICs seller becomes less likely.

**Proposition 3** \( m < \frac{6(\beta_{1,2} + 4\beta_3)}{7\beta_{1,2} + 16\beta_3} \) is a necessary and sufficient condition for global (i.e. among the ICs and AC) trade creation.

**Proof.** *(Sufficiency.)* Assume that

\[
m < \frac{6(\beta_{1,2} + 4\beta_3)}{7\beta_{1,2} + 16\beta_3}\]  

(21)

holds. Expression (21) can be rewritten as

\[
6\beta_{1,2} + 24\beta_3 - 7\beta_{1,2}m - 16\beta_3m > 0
\]

(22)

which, adding and subtracting \( 18\beta_{1,2} \) and \( 9\beta_{1,2}m \) to both members of (22) yields

\[
24(\beta_{1,2} + \beta_3) - 18\beta_{1,2} - 16m(\beta_{1,2} + \beta_3) + 9\beta_{1,2}m > 0,
\]

(23)

or

\[
8(\beta_{1,2} + \beta_3)(3 - 2m) - 9\beta_{1,2}(2 - m) > 0.
\]

(24)

As \( m \) and \( k \) are strictly positive, (24) implies

\[
\frac{m}{k} \left( \frac{1}{12} \frac{1}{9} \left[ 8(\beta_{1,2} + \beta_3)(3 - 2m) - 9\beta_{1,2}(2 - m) \right] \right) > 0
\]

(25)

or, equivalently,

\[
(\beta_{1,2} + \beta_3) \frac{2m(3 - 2m)}{27k} - \beta_{1,2} \frac{m(2 - m)}{12k} > 0.
\]

(26)

so that \( T_{1,2}^A + T_{1,3}^A - (T_{1,2}^P + T_{1,3}^P) \) is strictly positive.

*(Necessity.)* Suppose global trade creation holds, i.e. by (6) and (8)

\[
(\beta_{1,2} + \beta_3) \frac{2m(3 - 2m)}{27k} - \beta_{1,2} \frac{m(2 - m)}{12k} > 0
\]

(27)

or

\[
\frac{m}{k} \left( \frac{1}{12} \frac{1}{9} \left[ 8(\beta_{1,2} + \beta_3)(3 - 2m) - 9\beta_{1,2}(2 - m) \right] \right) > 0
\]

(28)
Figure 1: the new accession is Pareto improving for parameter values to the right of the plotted surface.

Since both parameters \( m \) and \( k \) are strictly positive, (28) implies

\[
8 (\beta_{1,2} + \beta_3) (3 - 2m) - 9\beta_{1,2} (2 - m) > 0
\]

which, rearranging terms, can be rewritten as

\[
24 (\beta_{1,2} + \beta_3) - 18\beta_{1,2} - 16m (\beta_{1,2} + \beta_3) + 9\beta_{1,2}m > 0
\]

or, simplifying,

\[
6\beta_{1,2} + 24\beta_3 > 7\beta_{1,2}m + 16\beta_3m,
\]

which implies

\[
m < \frac{6 (\beta_{1,2} + 4\beta_3)}{7\beta_{1,2} + 16\beta_3}.
\]

Proposition 3 states that an IC’s country global trade is increased by a new member accessing the monetary union if and only if \( m \) satisfies (32).

The last result of this paper is a sufficient condition for global trade creation to occur that is independent of the amount of money issued.

**Proposition 4** If \( \frac{\beta_{1,2}}{\beta_3} < 8 \), then the newly created volume of trade between ICs and the AC more than offsets the trade diversion effect between ICs.
Figure 2: the surface in Figure 1 collapses to the depicted line when $m$ equals 1.

**Proof.** Assume $\frac{\beta_{1,2}}{\beta_3} < 8$. This is equivalent to

$$\beta_{1,2} + \beta_3 > \beta_{1,2} + \frac{\beta_{1,2}}{8},$$

which is equivalent to

$$(\beta_{1,2} + \beta_3) \frac{2m(3 - 2m)}{27k} > \left( \beta_{1,2} + \frac{\beta_{1,2}}{8} \right) \frac{2m(3 - 2m)}{27k}$$

or, upon rearranging terms,

$$(\beta_{1,2} + \beta_3) \frac{2m(3 - 2m)}{27k} > \beta_{1,2} \frac{m(3 - 2m)}{12k}$$

which implies that

$$(\beta_{1,2} + \beta_3) \frac{2m(3 - 2m)}{27k} > \beta_{1,2} \frac{m(2 - m)}{12k},$$

since $(3 - 2m)$ can be rewritten as $(2 - m) + (1 - m)$, with $0 < (1 - m) < 1$ by definition.

Proposition 4 states that the enlargement of a monetary union always results in trade creation if the degree of integration between each IC and the AC is not too small relative to the inter-ICs degree of integration.
4 Conclusions

Within a three-country (two ICs and an AC) extension of the search equilibrium framework of [6], this paper characterizes conditions for global trade creation to occur as a consequence of new countries accessing a monetary union. The accession of a new member country increases the equilibrium fraction of buyers inside the monetary union, and may result in trade reduction if too much money is issued, unless the degree of integration between the ICs is not too high relative integration with the AC. In this latter case, enlargement of the monetary union increases international trade and is Pareto improving.

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