Monetary Policy in the Euro Area

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Abstract

The European Monetary Union involves many new interactions. The Fiscal Framework and the Stability and Growth Pact have had great significance since the completion of the European Monetary Union in 1999. The current enforcement and credibility problems, and the discussion about reforming the Stability and Growth Pact, have introduced a new subject for economic research. One of the most surprising observations in recent years is that the larger countries in the EMU have more problems with the budget thresholds of the Stability and Growth Pact than the smaller countries. To explain this ‘stylized fact’ and monetary-fiscal interaction we build a new model. A surprising and new result is the trade-off between the breaching countries and the disciplined countries as well as the trade-off between larger and smaller countries. Moreover we find in that framework that independent monetary policy is not sufficient to discipline fiscal policy behavior in a monetary union. The new insights improve the current modelling situation and helps to find some policy relevant conclusions of the main challenges for the new members in the prospect of joining the Monetary Union, especially for two issues: heterogeneity and timing.

\textit{Key words:} Monetary Policy, Monetary Union, Fiscal Policy, Stability and Growth Pact  
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1 Introduction

The Economic and Monetary Union in Europe has a common central bank that determines monetary policy, but each member country’s government retains simultaneously a large degree of fiscal autonomy. Since 1 January 1999, one of the most problematic issues in the European Monetary Union (EMU) has been the growing interaction between sovereign countries’ fiscal policy and the European Central Bank’s (ECB) monetary policy. Moreover, to ensure European price stability (art. 105 ECT) in the EMU the Maastricht Treaty was supplemented with the European Stability and Growth Pact (SGP) in Amsterdam in 1997. The implementation of the SGP (VO 1466/97 and 1467/97), which aims to be one of the mainstays in the European fiscal framework, introduced additional conflicts. However, since the ECOFIN Council failed in November 2003 both to send early warnings—so called Blue-letters—to Germany and France, and to impose sanctions against sinner states, the SGP has become the subject of lively academic and public debate. Different suggestions and proposals for modifying the current SGP are under discussion (4; 12). One of the most surprising and interesting questions about the SGP that has emerged in the last two years is: Why do the obviously bigger countries have more problems with the budget rules and thus with the SGP (27)? Or: why are countries in breach of the SGP more often larger countries such as Germany, France, and probably also Italy in 2004?

In this paper, we examine fiscal consolidation behavior within the EMU and find some new results and suggestions regarding the design of the European fiscal framework, especially the SGP. Moreover, we try to analyze the trade-offs between the de jure rigidity and de facto flexibility in the current reform discussion about the SGP. We consider a model where fiscal policy reputation, homogeneity, and output variance affect the speed of consolidation, and so explain the problem of huge differences in budget performance in Europe. Countries with good past fiscal reputations, such as Germany (31), consolidate their budget deficit more slowly because of a lower risk-premium on interest rates, higher free-riding incentives in a monetary union, and the well-known signaling effect caused by asymmetric information (10). Delays in consolidation are particularly inefficient, as the longer a country waits the more costly the policy adjustment. The reason is that longer periods of instability imply higher inefficiencies and sanction fees payable under the SGP. This paper studies the economic determinants of delays in the consolidation of fiscal policy adjustment programs.

We present a simple model that describes some determinants of delayed consolidation due to a strategic-interaction game in a monetary union. Concerning the determinants of the speed of budget consolidation, we find that the values of output volatility and homogeneity within fiscal programs are the most
relevant variables for explaining the differences in budget consolidation between the larger and the smaller countries (cf. Alesina and Drazen, 1991 and Alesina and Spolaore (2)). We also explain one unsolved ‘stylized fact’ in empirical macroeconomics (19; 20): why all the empirical evidence points to the presence of a negative relationship between output variability and the size of government.

Moreover in models of monetary policy alone, precommitment leads to better outcomes, and avoids inflation bias (25). Unfortunately that simple relationship is not completely true for fiscal–monetary interaction in a monetary union (18). In the European fiscal framework, particularly in the reform discussion about the SGP, the bias is rather towards more discretion than commitment (15). This development has a strong impact on future consolidation behavior in the EMU. Therefore it is very interesting to understand fiscal policy consolidation behavior in a game-theoretic interaction framework.

The model results suggest that, when there is a difference in budget consolidation speeds in a monetary union, the limits set by the SGP may be useful, on the one hand to reduce the free-rider incentives, and on the other hand to close the gap between the bigger and the smaller countries. Nevertheless the current SGP has not achieved the second objective during the last three years. Moreover von Hagen et al. (32) conclude in an initial empirical assessment about fiscal policy consolidation in the EMU: ‘The fiscal framework of Stage III of EMU will work more effectively in the small European states than in the larger states.’ Thus there are some systematic incentives to play the weak off against the strong. Understanding this phenomenon is an important issue for future reform of the SGP. In sum, the evidence indicates that the SGP needs a more transparent and credible enforcement mechanism.

The remainder of this paper is structured as follows. Section 2 presents a short literature review and discusses several aspects of the current reform literature about the SGP. Section 3 starts with the construction of the model and continues with the discussion of the results. Policy conclusions for the current reform discussion about the SGP are taken up in Section 4. The last, section 5, provides discussion and concluding remarks.

2 Literature review

Our approach is related to the literature on dynamic games between a monetary and a fiscal authority, initiated by Kydland and Prescott (1977) and Barro and Gordon (3). The paper relates to two analyses of delayed stabilization: (A) Tabellini (29) considers a war of attrition that is played out between the fiscal and the monetary authorities—an unsustainable combination of mone-
tary and fiscal policies is in place until one side concedes; and (B) Alesina and Drazen (1) also build a war of attrition model; however, they shift the focus to a game between interest groups. They show why stabilizations are delayed.

Our paper differs from Tabellini (1986, 1987) and Alesina and Drazen (1991) in several ways. First, we concentrate on the consolidation of deficits and debts, and therefore abstract from pure political–economic determinants. Second, we try to analyze a strategic situation in a monetary union that fits the situation in the EMU with the current SGP since 1999. Finally, and most importantly, the model attempts to explain not only the fact that consolidation speeds are delayed and variable in the EMU, but also to show why consolidation is different between larger and smaller countries in the EMU.

The results illustrate that larger countries consolidate more slowly than smaller countries because of greater differences in the public sector and output variations. Indeed, the model focuses on a few details to explain the current empirical case in the EMU. Together with the paper by Alesina and Drazen (1991), it provides a reasonable explanation for the current phenomenon of breaching countries and refers to the discussion on the SGP (von Hagen et al., 2001).

There is also a substantial literature about the economic impact of and reasons for the European SGP and the new fiscal–monetary interaction relationships. An early attempt to model the SGP is provided by Beetsma and Uhlig (7). They present in a two-period model of a monetary union where governments have incentives to issue more debt than a social planner would choose. They conclude therefore that the incentives to restrain debt accumulation are diminished in a monetary union, and hence the excessiveness of debt will be exacerbated. Thus the spillover effect arises through increasing public debt in a country, which leads to a looser common monetary policy and therefore affects all the union participants. Similar to Beetsma and Uhlig (1999) is the work by Chari and Kehoe (13, 14), who explore the need for debt restrictions in a two-country model of a monetary union. They conclude that restrictions on public debt are needed, because union members do not fully internalize the welfare effects of an increase in nominal debt on the common union-wide inflation rate. ¹ Also Dixit and Lambertini (17, 18) and Beetsma and Jensen (9) model a monetary union with fiscal–monetary interaction. ² The main results of these models in relation to the SGP are: (A) fiscal discretion eliminates the gains of monetary commitment, but monetary discretion does not completely eliminate the gains of fiscal commitment within prescribed rules; and (B) shock-contingent budgetary targets (or sanctions) lead to increasing free-riding behavior and thus eliminate discipline.

¹ cf. Giovannetti, Marimon and Teles, who extend the paper of Chari and Kehoe into various directions.
² But without implementing fiscal restrictions like the SGP.
The common point of all the papers mentioned so far is that the Union’s central bank is not only concerned with low inflation, but also with other objectives. Debrun (16), in contrast, provides a rationale for short-run (deficit-based) fiscal constraints, despite the assumption that the ECB is totally committed to its objective (6). The important point here is that fiscal policies affect aggregated demand and supply, hence the price level in the monetary union. Through a lack of commitment in monetary and fiscal policy the public deficit is biased upwards: first, governments try to stimulate aggregate demand by expansive fiscal policy; and second, they use deficits to move the common inflation rate into the direction they individually prefer. This model’s prediction is perhaps an empirical rationale for the fact that France and Germany, which have very low ‘national’ inflation rates and growth rates, breach the SGP. Also Herzog (21) found that the current SGP does not really work for securing price stability. He shows that if more than one country breaches the Pact, a deficit spiral (debt spiral) to more excessive government spending will be induced. Moreover Herzog (2004) shows that monetary policy in the EMU in combination with the current SGP is insufficient for punishing undisciplined fiscal policy. That implies under specific circumstances a higher optimal inflation rate than intended by the ECB. The theoretical analysis of that topic explains on the one hand the need for fiscal restrictions, and on the other hand the implementation problems of the current SGP. Despite this, Beetsma (8) concludes that the theoretical literature has no clear verdict on the SGP: ‘Therefore, the pros and cons of the SGP need to be assessed using qualitative arguments.’ However, we show here a further argument for the necessity of an efficient and strict SGP in the EMU.

The model framework consists of three interacting institutions (7). The first is the centralized monetary policy (European Central Bank, ECB). The primary objective of monetary policy is to maintain price–stability (art. 105 ECT). The monetary policy mainly interact with fiscal policies through the determination of price–level (cf FTPL) and interest rates. The second important institutional framework is the decentralized fiscal policy. The main difference between monetary and fiscal policy is that the fiscal policy retain a large degree of responsibility in the own national sovereignty. That implies three different interactions:

(i) Fiscal policy interact with monetary policy. The budget decisions about deficit and debt have an impact on price–stability and thus on the monetary policy (cf FTPL).

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3 If the actual inflation rate target is to be too tight, they boost aggregate demand further, which increases inflation.

4 Strengthening the SGP corresponds with the recent proposal by the EU Commission (2004).
(ii) Fiscal policy interact with the other fiscal policies in the monetary union because of the competition around the public good ‘price–stability’ provided by monetary policy. Thus one fiscal policy can undertake free-rider behavior against the other participating member states within the monetary union. That free-riding incentives even increase in the framework of EMU (5). To eliminate the discipline the free-rider behavior in the European Monetary Union the so-called ‘Stability and Growth Pact’ was implemented.

(iii) The Stability Pact is the third institution in the EMU. On the one hand it try to discipline fiscal policy and the free-rider behavior. On the other hand it helps the monetary policy to maintain the primary objective ‘price–stability’. Hence, the objective of the SGP is twofold and thus represent a intermediary institution.

The main task in the following paper is to analyze the interactions or interrelations in the European Monetary Union between these three institutional agents. We choose a dynamic concept that uses differential equations. The existing economic literature analyze fiscal–monetary interaction (Dixit and Lambertini 2003, Beetsma and Bovenberg 1999) in a game theoretic framework. The first approach to analyze the Stability and Growth Pact (Beetsam and Uhlig 1999) again uses a game theoretic framework but without the real fiscal–monetary interaction structure. Moreover the economic approaches focuses more on monetary and real variables and their developments in the monetary union (30). But nobody try to analyze the institutional interaction in the European Monetary Union and simultaneously using a dynamic framework.\(^5\)

To illustrate the model framework graphically look to Figure 4.4. Fiscal policy can influence the SGP and the monetary policy through to lax deficit and debt policy. The incentive to do this are: national interest, increase of re-election probability, national output stabilization, reaction to asymmetric and idiosyncratic shocks and the new free–riding behavior \(\zeta\) which summarize all the incentives.

The next section try to model the interaction relationships between all three institutions with differential equations. The stringent modelling of that complex framework helps us to learn something new about the interactions, impacts and causalities of the 'European Monetary Union'.

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\(^5\) An non-technical or analytical institutional analysis is done by Ohr (26).
The following section describes the basic interaction model between the European fiscal policy and the Stability and Growth Pact. The primary target is to understand the evolution of breaching countries \(x(t)'\). Modelling the dynamics result in \((x(t) \geq 0)\):

\[
x' = (g - p * s)x, \quad t > 0, \quad x(0) = x_0
\]

with \('g'\) as the benefit through free-rider behavior of fiscal policy in the European monetary union (5) \(^7\) and \('s'\) represents disciplining sanctions from the 'Stability and Growth Pact'. The parameter \('p'\) is the probability of imposing sanctions. The intuition behind the first-order differential equation is:

- the increasing free-rider behavior \(g > 0\) in a monetary union increase with the number of countries that violate (against) the Stability and Growth Pact (SGP) because of the expected benefits.
- the sanction procedure \(s > 0\) of the Stability and Growth Pact try to reduce or discipline the free-rider behavior of national fiscal policies and thus reduce the number of breaching countries. But this mechanism works only efficient if the probability of imposing sanctions \(p > 0\), is sufficiently large.

The solution of that model is \(x(t) = x_0 e^{(g - p * s)t}\). This implies an increasing number of breaching countries in the SGP, if free-riding incentives \(g\) are larger than the disciplining sanctions \(s\). In the current fiscal–SGP interaction system is the probability to impose sanctions very small. \(^8\) That implies \(g > p * s\) and thus the number of breaching countries might be increasing. \(^9\) But this model approach is simplified in the sanction mechanism and their impact on the national fiscal policy. A more realistic sanction mechanism looks like:

\[
s = s(x) = s_0 + a * x, \quad s_0, a \geq 0,
\]

\(^6\) cf (author?) (Schmeiser) and \(^7\).

\(^7\) cf Beetsma and Bovenberg (1999) show in the paper that free-riding behavior even increase in a monetary union.

\(^8\) cf the failures of imposing early warnings against Germany, France (2002) and for example Italy (2004) and no sanctions against sinner states as Germany and France (2003) confirm that.

\(^9\) That describes empirically the current situation in the EMU. The new breaching countries are Netherlands, United Kingdom, Greece and some of the new EAC.
where $s_0$ represent the basic sanction amount and $a$ the marginal sanction rate or the idiosyncratic influence of the national fiscal policy. Substitute equation (4.4.2) in equation (4.4.1) yields:

$$\frac{dx}{dt} = x' = (\zeta^F - p*a*x)x, \quad with \quad t > 0, x(0) = x_0, \quad (3)$$

and $\zeta^F := g - p*s_0$. The differential equation above is a so-called logistic-differential equation (or ‘Verhulst–Model’). The logistic modelling framework show also the sustainability of the number of breaching countries ‘$x(t)$’. The solution of that differential equation through integration is:

$$t = \int_0^t d\tau = \int_{x(0)}^{x(t)} \frac{dx}{(\zeta - pa*x)x} = \int_{x_0}^{x(t)} \frac{1}{\zeta} \left( \frac{1}{x} + \frac{pa}{\zeta - pa*x} \right) = \frac{1}{\zeta} \left( \ln \left[ \frac{x(t)}{x_0} \right] + pa \ln \left[ \frac{\zeta - pa*x_0}{\zeta - pa*x(t)} \right] \right). \quad (4)$$

To solve the last term to $x(t)$ result in:

$$x(t) = \frac{\zeta * x_0}{pa * x_0 + (\zeta - pa*x_0)e^{-\zeta t}}. \quad (5)$$

For $t \to \infty$:

$$x(t) \to \begin{cases} \zeta/(p*a) : \zeta > 0, \\ 0 : \zeta < 0. \end{cases} \quad (6)$$

If the sanction mechanism is fully credible i.e. the implementation probability ‘$p$’ and sanction ‘$s$’ are high, then the number of breaching countries converge to zero. But if free-riding behavior ‘$g$’ dominance the disciplining mechanism ‘$(p*a)$’ then $\zeta > 0$ and thus the number of breaching countries convergence to ‘$\zeta/(p*a)$’, a positive figure. The final number of violating countries increase with higher free-riding incentives and decrease if the sanctions are more credible and the economic impact of fiscal policy ‘$a$’ in the MU is relatively high.\textsuperscript{10} The intuition behind the last term is that higher influence of fiscal policy ‘$a$’ in MU implies normally a stronger sanction procedure (higher sanction amount or a punishment through the monetary policy) because of the increasing inflation danger. That might be a strong disciplining effect for member countries to reduce the fiscal policy below the 3% deficit and 60% debt thresholds. That

\textsuperscript{10} vice versa for a high policy impact of fiscal policy member states.
finding suggest a sanction–threshold that should depend on the national GDP rate. The term \( \zeta / (\rho a) \) could be interpreted as the natural intake capacity of breaching countries in a Monetary Union.

The next section extend the simple model with the monetary interaction level. Monetary policy interact both with fiscal policy and the Stability and Growth Pact. Now we take into account monetary policy and analyze the full interaction framework.\(^{11}\)

### 2.2 Full–Interaction–Model

Similar to the model description in section 4.4.2, we extend now the model with the monetary authority. Analyzing the compleat–complex system explains the current European fiscal–monetary interaction framework and the connection with the Stability and Growth Pact in a more realistic way as all the other existing economic models.

To model the evolution of monetary policy \( y(t) \), we follow a similar approach with differential equations:

\[
y'(t) = (\zeta^M - d^{-1} \ast y)y, \quad t > 0, \quad y(0) = y_0 \tag{7}
\]

where \( y \) is monetary policy (for instance interest rates) and \( d \geq 0 \) reflects the independence of monetary policy (or a weight i.e. the possibility to follow also other objectives as output stabilization). In the following section, we define \( c := d^{-1} \). The intuition behind equation (4.4.7) is:

- if the free–riding behavior is dominant in the MU \( \zeta^M > 0 \) then monetary policy might punish fiscal policy additionally with higher interest rates.
- on the other hand if the monetary policy is fully independent \( (d \to \infty) \) then the primary objective 'price stability' \( (\zeta^M) \) has the whole weight. A more dependent Common Central Bank (CCB) \( (d \to 0) \) implies that output targets are more important. That have an explicit negative impact to interest rates (i.e. decline).

In a more realistic interaction framework, free–rider incentives \( \zeta \) depend also on the current number of breaching fiscal policy countries (22)\(^{12}\):

\(^{11}\) cf because the independent European monetary policy can also discipline the fiscal policy for instance with higher interest rates.
\(^{12}\) cf Fiscal Theory of Price Level, Woodford (33).
\[ \zeta = \zeta^M(x) = -\zeta_1^M + \zeta_2^M x, \quad \text{with} \quad \zeta_1, \zeta_2 \geq 0 \quad (8) \]

where \( \zeta_1 \) represents disciplining incentives (for the number of non-breaching countries) and \( \zeta_2 \) describes the 'Cascade to the top' effect which was new explained by Herzog (2004a). Moreover the fiscal policy free-rider incentive \( \zeta^F \) depend also on monetary policy:

\[ \zeta^F(y) = \zeta_3^F - \zeta_4^F y, \quad \text{with} \quad \zeta_3, \zeta_4 \geq 0 \quad (9) \]

where \( \zeta_3 \) represents the increasing free-rider behavior in the Monetary Union (Beetsma and Bovenberg 1999) and \( \zeta_4 \) describes the 'Disciplining–Monetary–Policy' effect (interest rate effect).

Substituting equation (4.4.8) in equation (4.4.7) and also equation (4.4.9) in equation (4.4.1) yields the following system of differential equations. That system is very similar to the so-called 'Lotka–Volterra equations':

\[
\begin{align*}
x' &= (\zeta_3^F - \zeta_4^F y - pa \times x) x \quad t > 0 \quad x(0) = x_0 \\
y' &= (-\zeta_1^M + \zeta_2^M \times x - c \times y) y \quad t > 0 \quad y(0) = y_0
\end{align*}
\]

(10)

To understand how the solution of the system evolves, we first simplify the system und assume \( a = c = 0 \).

\[
\begin{align*}
x' &= (\zeta_3^F - \zeta_4^F y) x \quad t > 0 \quad x(0) = x_0 \\
y' &= (-\zeta_1^M + \zeta_2^M \times x) y \quad t > 0 \quad y(0) = y_0
\end{align*}
\]

(11)

That system of differential equations have two possible solutions \((x_1, y_1)'\) and \((x_2, y_2)'\):

\[
\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \zeta_1^M / \zeta_2^M \\ \zeta_3^F / \zeta_4^F \end{pmatrix}
\]

To show the (asymptotic) stability or instability of the two solutions, we define the function \( F(x, y) \) and calculate the eigenvalues of the system:

\(^{13}\) Goes back to Alfred James Lotka (1880–1949) and Vito Volterra (1860–1949).
\[ F(x, y) = \begin{pmatrix} (\zeta_3^F - \zeta_4^F * y)x \\ (-\zeta_1^M + \zeta_2^M * x)y \end{pmatrix}, \quad x, y \geq 0, \quad (12) \]

and the derivatives in the associated points are:

\[ F'(0, 0) = \begin{pmatrix} \zeta_3^F & 0 \\ 0 & -\zeta_1^M \end{pmatrix}, \quad \text{and} \quad F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right) = \begin{pmatrix} 0 & -\frac{\zeta_3^F}{\zeta_4^F} \\ \frac{\zeta_2^M}{\zeta_2^M} & 0 \end{pmatrix}. \]

Now we calculate the eigenvalues of \( F'(0, 0) \):

\[ \det \left| F'(0, 0) - \lambda I \right| = -(\zeta_3^F - \lambda)(\zeta_1^M - \lambda) = 0, \quad (13) \]

that imply \( \lambda_1 = \zeta_3^F \) and \( \lambda_2 = \zeta_1^M \). Because of the assumption that all \( \zeta_i > 0 \) \( \forall i \), the two eigenvalues are positive. Hence, there is an unstable equilibrium point \( P_1(0, 0) \).\(^\text{14}\)

To determine the eigenvalue for \( F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right) \), we have to solve the following equation:

\[ \det \left| F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right) - \lambda I \right| = \lambda^2 + \zeta_2^M * \frac{\zeta_3^F}{\zeta_4^F} * \frac{\zeta_1^M}{\zeta_2^M} = \lambda^2 + \zeta_3^F * \zeta_4^F = 0, \quad (14) \]

the system is also unstable if \( \text{Re} \lambda_1 < 0 \) and \( \text{Re} \lambda_2 > 0 \) (28). Therefore the system is unstable around the second point \( P_2(\zeta_1^M/\zeta_2^M, \zeta_3^F/\zeta_4^F) \). However, from (4.4.14) follows directly that the eigenvalues are: \( \text{Im} \lambda_{1,2} \). The possibility of complex eigenvalues imply no real solution. To describe the solution behavior of the system near the point \( (\zeta_1^M/\zeta_2^M, \zeta_3^F/\zeta_4^F) \), we rewrite the differential equation system (4.4.11) in the following shape:

\[ \frac{dx}{dy} = \frac{dx}{dt} \frac{x'}{y'} = \frac{(\zeta_3^F - \zeta_4^F * y)x}{(-\zeta_1^M + \zeta_2^M * x)y}, \quad (15) \]

\(^\text{14}\) The instability can also be seen from: \( \det \left| F'(0, 0) \right| < 0. \)
and after integration we can rewrite the equation above as,

\[-ln[x^{\zeta_1}] + \zeta_2 \ast x = \int \frac{-\zeta_1 + \zeta_2 \ast x}{x} \, dx = \int \frac{\zeta_3 - \zeta_4 \ast y}{y} \, dy = \ln[y^{\zeta_3}] - \zeta_4 \ast y - \alpha,\]

(16)

where \(\alpha\) is an integration constant. Thus all the solutions \((x(t), y(t))'\) satisfy the implicit solution:

\[ln[x(t)^{\zeta_1}] + \ln[y(t)^{\zeta_3}] - \zeta_2 \ast x - \zeta_4 \ast y = \alpha \quad \forall t \geq 0.\]

(17)

The integration constant \(\alpha\) can be calculated from the initial condition \((x_0, y_0)\):

\[\alpha = \ln[x_0^{\zeta_1}] + \ln[y_0^{\zeta_3}] - \zeta_2 \ast x_0 - \zeta_4 \ast y_0.\]

(18)

We suggest that the solution set \((x(t), y(t))\) satisfy a closed-form solution in the environment \((\epsilon, \delta)\) around the point \((x_2, y_2)\):

\[x(t) = \frac{\zeta_1}{\zeta_2} + \epsilon \ast \sin[\omega t], \quad \land \quad y(t) = \frac{\zeta_3}{\zeta_4} + \delta \ast \cos[\omega t],\]

(19)

with \(\epsilon > 0, \delta \ll 1\) and \(\omega > 0\). For \(t = 0\) and after trivial aggregation it result:

\[\alpha = \zeta_1 \ln \left[\frac{\zeta_1}{\zeta_2}\right] + \zeta_3 \ln \left[\frac{\zeta_3}{\zeta_4}\right] - \zeta_1 - \zeta_3 + |O(\delta)| \quad (\delta \rightarrow 0).\]

(20)

The next step is now the approximation of the general solution \((x(t), y(t))\) (with second-order Taylor series) in the environment of \(x_2 = \zeta_1/\zeta_2\) and \(y_2 = \zeta_3/\zeta_4\) (cf Appendix):

\[\zeta_1 \ast \ln[x(t)] + \zeta_3 \ast \ln[y(t)] - \zeta_2 \ast x(t) - \zeta_4 \ast y(t)\]

(21)

is equivalent to:

\[= \zeta_1 \ln \left[\frac{\zeta_1}{\zeta_2}\right] + \zeta_2 \epsilon \sin \omega t + \zeta_3 \ln \left[\frac{\zeta_3}{\zeta_4}\right] + \zeta_4 \delta \cos \omega t + \frac{\zeta_2}{2\zeta_1} \epsilon^2 \sin^2 \omega t + \frac{\zeta_1}{2\zeta_3} \delta^2 \cos^2 \omega t - \zeta_1 - \zeta_2 \epsilon \sin \omega t - \zeta_3 - \zeta_4 \delta \omega t + O(\epsilon^3 + \delta^3),\]

(22)
\[ = \zeta_1 \ln \left( \frac{\zeta_1}{\zeta_2} \right) + \zeta_3 \ln \left( \frac{\zeta_3}{\zeta_4} \right) - \zeta_1 - \zeta_3 + \frac{\zeta_2^2}{2\zeta_1} \epsilon^2 \sin^2 \omega t + \frac{\zeta_2^2}{2\zeta_3} \delta^2 \cos^2 \omega t + O(\epsilon^3 + \delta^3), \]

\[ = \alpha + O(\epsilon^2), \]

if we choice

\[ \frac{\zeta_2^2}{2\zeta_1} \epsilon^2 = \frac{\zeta_2^2}{2\zeta_3} \delta^2. \]

Thus you can conclude that our specified solution (4.4.19) solve the general system \((x(t), y(t))\) until a error term of order \(O(\epsilon^2)\). Moreover we can see that the Trajectories \(\{(x(t), y(t)) : t \geq 0\}\) are approximative ellipse around the point \((x_2, y_2)\) (cf figure 1 in the graphical appendix).

The intuition in short term: The simplified system–dynamics imply that the number of breaching countries increase so long as the monetary policy see no danger for price–stability in future. After the reaction of the monetary policy (increase in interest rates) the number of breaching countries decrease.

But the most interesting case is the general model (4.4) with \(a \neq c \neq 0\).

Now we calculate the general solution and proof the stability of the associated differential equation system. Start from the bottom, we are no ready to solve and analyze the interaction relationship between all interaction institutions: Fiscal, Monetary and the Stability and Growth Pact.

The general model is described through the function \(F(x, y)\):

\[ F(x, y) = \begin{pmatrix} (\zeta_3^F - \zeta_4^F \ast y - pa \ast x)x \\ (-\zeta_1^M + \zeta_2^M \ast x - c \ast y)y \end{pmatrix} x, y \geq 0, \tag{23} \]

There are the following four solutions for the general model:

\[ \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \& \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\zeta_1/c \end{pmatrix} \quad \& \quad \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} \zeta_3/(pa) \\ 0 \end{pmatrix} \]

and \((x_4, y_4)'\) is the solution of the linear equation system:
\[
\begin{pmatrix}
pa \cdot 4 \\
\zeta_2 - c
\end{pmatrix}
\begin{pmatrix}
x_4 \\
y_4
\end{pmatrix}
= 
\begin{pmatrix}
\zeta_3 \\
\zeta_4
\end{pmatrix}
\]

using Cramer’s-rule result in:

\[
\begin{pmatrix}
x_4 \\
y_4
\end{pmatrix}
= 
\frac{1}{pa c + \zeta_2 \cdot 4}
\begin{pmatrix}
\zeta_3 \cdot c + \zeta_4 \cdot \zeta_1 \\
\zeta_3 \cdot \zeta_2 - pa \cdot \zeta_1
\end{pmatrix}
\]

with \(A := \zeta_3 \cdot c + \zeta_4 \cdot \zeta_1\) and \(B := \zeta_3 \cdot \zeta_2 - pa \cdot \zeta_1\).

The second solution \((x_2, y_2)\) is a non possible stationary point because we have assumed \(x, y \geq 0\). To find out the stability of the other three solutions, we deviate the function \(F(x, y)\):

\[
F'(x, y) = \begin{pmatrix}
\zeta_3 - \zeta_4 - 2ax & -\zeta_4 x \\
\zeta_2 y & -\zeta_1 + \zeta_2 x - 2cy
\end{pmatrix}.
\]

Similar to the model in section 4.3 the point \((x_1, y_1)' = (0, 0)'\) is a non stationary solution because of \(\text{Re} \lambda > 0\). The point \((x_3, y_3)\) is unstable, if \(\zeta_1/\zeta_2 < \zeta_3/a\), and asymptotic stabil, if \(\zeta_1/\zeta_2 > \zeta_3/a\). The point \((x_4, y_4)\) is positiv i.e. \(x, y \geq 0\) for \(\zeta_1/\zeta_2 < \zeta_3/a\). The eigenvalues from \(F''(x_4, y_4)\) are:

\[
\lambda_{1,2} = -\frac{1}{2} (aA + cB) \pm \sqrt{\frac{1}{4} (aA + cB)^2 - (\zeta_2 \zeta_1 + ac)AB},
\]

with \(A, B > 0\). Because of \(\text{Re} \lambda_{1,2} < 0\) the point \((x_4, y_4)\) is asymptotic stable, i.e. \((x(t), y(t)) \rightarrow (x_4, y_4)\) for \(t \rightarrow \infty\). That implies that the number of breaching countries convergence after a necessary time to:

\[
x_4 = \frac{\zeta_3 c + \zeta_4 \zeta_1}{pa c + \zeta_4 \zeta_3}
\]

The following equation system summarize the general model results:

\[\text{cf} \text{ Heuser (23, 24) because of Lipschitz-stetig (steady) or Bronstein et al. (11).}\]
\[
\frac{\zeta_1}{\zeta_2} > \frac{\zeta_3}{pa} : \quad x(t) \to \frac{\zeta_3}{pa} \quad \text{and} \quad y(t) \to 0 \quad \text{for} \quad t \to \infty;
\]
\[
\frac{\zeta_1}{\zeta_2} < \frac{\zeta_3}{pa} : \quad x(t) \to \frac{\zeta_3c + \zeta_4\zeta_1}{pac + \zeta_4\zeta_3} \quad \text{and} \quad y(t) \to \frac{\zeta_3\zeta_2 - pa\zeta_1}{pac + \zeta_3\zeta_4} \quad \text{for} \quad t \to \infty.
\]

The interpretation of the results are delegated to the next subsection.

### III. MODEL IMPLICATIONS

Now we discuss the mathematical analysis above a show some numerical simulations. Moreover that proofs the robustness and stability of our theoretical results, also in a more complex model framework.\(^\text{16}\)

The first part of our general results is very similar to the findings in the basic model in section 4.3. However, the implications from the assumed constellation \(\frac{\zeta_1}{\zeta_2} > \frac{\zeta_3}{(pa)}\) is not so realistic because the monetary policy variable convergence to zero \(y(t) \to 0\). Despite that problem we can show that even in that case the number of breaching countries convergence against a fixed ratio. That is a really surprising finding because it illustrates that monetary policy is not sufficient to discipline breaching countries in a monetary union.

Moreover assume that the free–rider incentives in a MU are small \((\zeta_3 \to 0)\) and the number of disciplined member countries within the SGP are big \((\zeta_1 \to \infty)\). Hence, the ratio above exceeds then the ratio of fiscal policy \(\frac{\zeta_3}{(pa)}\). That illustrates our first proposition:

**Proposition 1** The number of breaching member states depends on real benefit from free–riding \((\zeta_3^p)\) and the probability (credibility) of sanctions \(p\) as well as the influence of the fiscal policy to the aggregate variables \(a\).

The proof follows directly from the first part of the model.

**Remark 2** Hence, you can see that a high sanction probability or a high influence of monetary variables (to the big countries) reduce the number of breaching countries. On the other hand the free–riding incentives induce the problem of lax fiscal policy behavior in that framework. It is clear that the implementation of sanctions within the Stability and Growth Pact depends on the probability and credibility of the enforcement mechanism. On the one hand is the current sanction procedure within the SGP worse because of the to low sanction probability and to weak credibility in the enforcement procedure. On

\(^{16}\) cf figure 2 and 3 in the graphical appendix.
the other hand is the partisan influence within the ECOFIN–council and the pretty vague fiscal–institutional framework in the EMU a reason for the past failures in the SGP. That situation implies an increasing number of breaching countries despite of the fact that the good member countries dominate the EMU per definition.

The second part of our results are more interesting, because of the following more realistic assumptions:

(a) The impact of an individual country to monetary policy is relative small \((a \rightarrow 0)\) and the sanction probability ’p’ within the SGP is rather low. Moreover the public good ’price–stability’ induce a strong incentive to free–riding \((\zeta_3^F \rightarrow \infty)\) as shown by Beetsma and Bovenberg (6). Thus the following ratio converges to infinity \((\zeta_3^F/(pa) \rightarrow \infty)\). Otherwise the intended disciplining ratio \(\zeta_1^M/\zeta_2^M\) is lower because of the weak fiscal consolidation effect in the MU, ’\zeta_1’ and the so–called new ’Cascade–to–Top’ effect within the interaction framework, ’\zeta_2’.

(b) Moreover in the observed case exists for both solution variables \((x(t), y(t))\) a stable and strict positive outcome.

First, we discuss the determinants of ’\(x(t)\)’: 

Is monetary policy sufficient to constrain the number of fiscal policy breaching states in the EMU? To answer that question, we find a really trade–off in the monetary–fiscal interaction framework. The determinants of the breaching countries depend on the monetary independence variable ’c’ and on the fiscal policy impact variable ‘p*a’ to monetary policy. That illustrates an trade–off between central bank independence and the credibility of the SGP. Unfortunately is that trade-off not discussed in the whole reform discussion of the Stability and Growth Pact.

**Proposition 3** The monetary policy independence ’c’ and the fiscal policy impact to monetary policy ‘p*a’ can influence the number of breaching countries. Moreover monetary policy ’\(\zeta_4\)’ reduce the number of breaching countries but the consolidation incentives (good guys) from the SGP increase the number of breaching countries ’\(\zeta_1\)’.

Derivation of \(x(t)\) is:

\[
\frac{\partial x(t)}{\partial c} = \frac{\zeta_4(\zeta_3^F–pa\zeta_1)}{(pac+\zeta_4^2)^2} > 0, \text{ because of the assumption } \zeta_3, \zeta_4 \rightarrow \infty \text{ and } a \rightarrow 0.
\]

Increasing monetary independence \(c \downarrow\) imply a reduction of breaching countries. The impact of \(\zeta_2\) is independent from the number of
breaching countries. This is immediately clear from equation (4.19). But a higher impact of fiscal policy 'a' reduce the number of breaching countries in the interaction framework through a more restrictive monetary policy ($\zeta_4$). The disciplining effect through monetary policy is:

$$\frac{\partial x(t)}{\partial \zeta_4} = \frac{c(p*a\zeta_1 - \zeta_4^2)}{(pac + \zeta_3\zeta_4)^2} < 0$$

On the other hand generates the fiscal policy framework especially the Stability and Growth Pact ' $\zeta_1$' an increasing number of undisciplined countries in the EMU. See also equation (4.20)

**Remark 4** A very interesting and new insight is the impact of $\zeta_1$. This variable describes the impact of the good guys (non-breaching countries) or the incentives of 'sound' and 'sustainable' fiscal policy. If the number of good guys increase in MU ' $\zeta_1 \uparrow$' that imply immediately a simultaneous increase of the breaching countries ' $x(t) \uparrow$' because of the increasing free-riding incentives and the influences of the declining sanctions. The main problem for that paradox finding is again the redistribution of sanction revenues to the other member countries (graphical appendix: figure 1).

Second, we discuss the determinants of ' $y(t)$':

Assume an initial constellation of parameters, where monetary policy can increase the interest rates. The following proposition shows that monetary policy is very constraint in the European Monetary Union to punish the breaching countries ' $x(t)$':

**Proposition 5** An increasing fiscal policy impact $\zeta_2$ increase monetary policy $y(t)$ but restricted to: (i) Monetary impact on reducing the free-riding incentives $\zeta_4$ and (ii) the number of good guys i.e. the fiscal rules like the SGP $\zeta_1$.

The derivation is: $\frac{\partial y(t)}{\partial \zeta_2} = \frac{\zeta_3}{(pac + \zeta_3\zeta_4)} > 0$
rider behavior in the European Monetary Union. Hence, if the number of disciplined member states (good guys) decreases or the fiscal framework reveals several weaknesses to discipline the free-riding behavior, then a strict monetary policy CB —committed to 'price-stability'— will fail. The reason for that case are the limitations and constraints in the fiscal–monetary institutional interaction framework within the European monetary union. Again such finding shows: How important a sound and efficient fiscal framework as the Stability and Growth Pact is. A strong and independent 'Common Central Bank' is not enough to solve the 'new' incentives to more free-riding behavior in the European Monetary Union (graphical appendix figure 2 and 3).

The paradoxical situation with a full independent monetary policy but unable to discipline lax fiscal policy confirms the necessity of a strong and efficient fiscal–coordination framework in the European Monetary Union. Some modification proposals to the current SGP are in the next section.

Last but no least we discuss short the results for a theoretically compleat independent monetary policy. The result (4.20) changes to:

\[
\frac{\zeta_1}{\xi_2} < \frac{\zeta_3}{\mathcal{P}A} : \quad x(t) \rightarrow \frac{\zeta_1}{\xi_3} \quad \text{and} \quad y(t) \rightarrow \frac{\zeta_3 \xi_2 - \mathcal{P}A \zeta_1}{\xi_3 \xi_4} \quad \text{for} \quad t \rightarrow \infty. \tag{27}
\]

**Remark 7** The last case illustrates that the number of breaching countries depend only on the impact of fiscal policy free-riding incentives \(\zeta_3\) and the good guys (i.e. fiscal policy rule; SGP) \(\zeta_1\). Hence, if the number of disciplined countries (good guys) is larger and/or if the fiscal rule is sufficiently strong then the number of breaching countries might be increasing. Moreover in the full independent case is the monetary policy more contractive but also more restricted. So a weak fiscal framework '\(\zeta_1\)' and a low impact to discipline fiscal policy '\(\zeta_4\)' are both big limits for monetary policy in reducing the free-riding incentives in the EMU. This is a really surprising result and show the clear disadvantage of a full independent monetary policy within a monetary union framework and a weak Stability and Growth Pact. Basically this finding suggests a clear benefit from more fiscal policy coordination because of the strong limits of monetary policy independence in the EMU.

The puzzling question of why some of the EMU member countries do not consolidate immediately, once it becomes apparent that current policies are unsustainable, could partially be explained using the above model. Large deficits imply an explosive path of government debt, and it is apparent that such deficits would have to be eliminated at some stage because of the SGP excessive deficit procedure. The spirit of our analysis is similar to recent attempts to
explain other stylized facts of fiscal policy. Starting from these results we dis-
cuss in the following section some policy conclusions for the reform discussion
about the SGP.

3 Conclusions

This paper has explained new constraints of monetary policy in a currency
area. The new insights improve the current modelling situation and helps
to find some policy relevant conclusions of the main challenges for the new
members in the prospect of joining the Monetary Union. We conclude the
paper by discussing some generalizations and by touching on some issues that
the model did not address but that are important in explaining the main
challenges new and old member states facing in monetary union.

First, our argument is much more general than initially considered. The result
shows monetary policy is insufficient to discipline lax fiscal policy behavior.
Thus a strict fiscal framework is essential in a monetary union. However dif-
ferent countries have different problems with the thresholds. Therefore each
new member country joining monetary union has to focus on the challenges of
the fiscal framework. A second innovativ finding explains the new constraints
of monetary policy in a currency union despite its independence. The major
omission is a closer endogenous political-economic description of the model,
considering for instance important political events such as veto power.

Our model suggests that successful new member states in the prospect of
joining the Monetary Union have to incorporate the new fiscal-monetary in-
teraction channels and the constraints of monetary policy. We can explain
that heterogeneity is one reason to delay the timing for the the new member
states to join the Monetary Union. A major message is that monetary policy
in a monetary union and the fiscal framework are complementary constraints
and not as often assumed substitutive for all participating member states.

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