Why a Good Communication of Monetary Policy is Important? A Case of Heterogeneous Expectations and Adaptive Learning*

Martin Fukač†,
CERGE–EI‡,
Czech National Bank§

Version: February 2006

Abstract
We reassess the role of monetary policy in a standard New Keynesian business cycle model which we adjust for imperfect, heterogeneous knowledge and adaptive learning. The policy, represented by a forward-looking Taylor rule, is driven by central bank’s own internal forecasts, whereas the core economic dynamics is driven by private agents’ expectations. We study the implications of disagreement between those two. We find that if there is expectations heterogeneity, monetary policy should be less active in its actions in order to be short-run stability improving. This is in contrast to the standard, homogenous imperfect knowledge literature which predicts the opposite.

Abstrakt

Keywords: imperfect and heterogeneous knowledge, adaptive learning, monetary policy
JEL classification: E52

*The author would like to thank Michal Kejak, Sergey Slobodyan, Kristoffer Nimark, and the participants of CFS Summer School 2005 for their helpful comments and suggestions. The usual disclaimer applies.
†Email: martin.fuka@cerge-ei.cz
‡A joint workplace of the Center for Economic Research and Graduate Education, Charles University, Prague, and the Economics Institute of the Academy of Sciences of the Czech Republic. Address: CERGE–EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic
§Research Department, Na Příkopě 28, 115 03 Praha 1, Czech Republic
1 Introduction

Unlike in commonly considered theoretical macroeconomic models, in reality there exists a diversity among economic agents (consumers, businessmen, bankers or stock market brokers, monetary policy authorities etc.). The diversity is often mirrored in the agents’ economic knowledge and how they perceive current and expect future economic development. In ?, we can find some empirical evidence on inflation expectations in the US, which documents heterogeneity in agents’ expectations. Similar observations can also be made in other economies.

In current macroeconomic practice, economic agents are assumed to share the same, complete knowledge. The whole macroeconomic and monetary theory is based on this simplification of reality. Recently, however, there is a growing interest in relaxing that assumption. Agents still share the same knowledge, however, it is not complete any more. Only a part of the complete economic knowledge is available to the economic agents which might also vary among them.

Our objective is to contribute to the discussion on incomplete, heterogenous knowledge and its consequences for monetary policy. Our particular interest is in the relation between policy activity, willingness to learn and the speed of convergence to the first best equilibrium. That is if economic agents share diversified willingness to evaluate information (with the consequence of different expectations), can a central bank effectively improve the short-run economic stability? In other words, we will assess monetary policy recommendations for an imperfect-homogenous-knowledge case under a heterogeneity assumption.

In contrast to the rational and complete-knowledge world, in the incomplete-knowledge world the economy is not in its first best equilibrium. A knowledge improvement is thus welfare improving. The attainment of the first best (rational expectations) equilibrium is linked to the agents’ willingness to learn. In the homogenous-knowledge case, monetary policy can contribute to the knowledge im-
provement (learning). Orphanides & Williams (2003) find that monetary policy ought to be inflation vigilant, favoring policy activity as short-run stability improving. Ferrero (2003) qualifies this conclusion. It holds only in a simple structured model (agents form expectations about only one variable). The answer complexity grows with the model complexity. For instance, if agents already form expectations about two variables, an active policy does not need to be welfare-improving.

What if a private sector employs different model than a central bank and updates it on a different frequency. Using only a simple policy rule, how should a central bank behave in such an environment? Setting up monetary policy, can the central bank affect the learning process and thus improve the short-run economic stability? These and similar question will be addressed in this paper.

The paper is structured as follows. The model and the whole economic environment is introduced in the second section. In the the third section we define knowledge heterogeneity and analyze the model’s properties under the imperfect knowledge assumption. The simulation results are summarized in section four and the fifth section concludes.

2 The Model

The workhorse model follows the standard New Keynesian DSGE schema. On the one hand, there are households who make decisions about consumption, labour, and money holdings in order to maximize and smooth their lifetime welfare. On the other hand, there is a monopolistically competitive production sector that maximizes profits by controlling output, output prices and labour demand. The central bank’s objective is characterized by a forward-looking inflation targeting rule which seeks to anchor the nominal side of the economy.

The linearized model characterizing the aggregate economic dynamics is given
by the IS curve (1) which comes from the households’ Euler equation linearization, and the Phillips curve (2) which is linearized firms’ oligopolistic pricing rule. The central bank’s policy rule is given by (3). In the perfect-knowledge environment, the aggregated sticky-price model takes the form¹

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + v_t, \]

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \]

\[ i_t = \theta_\pi (E_t \pi_{t+1} - \pi^*) + \theta_x E_t x_{t+1} + \theta_u u_t + \theta_v v_t. \]

\( x_t \) is the output gap defined as a deviation of actual output from the output arising in a friction-less environment. \( \pi_t \) is the inflation rate, \( i_t \) is the interest rate set by the central bank. \( v_t \) and \( u_t \) are exogenous shocks assumed to follow AR(1) processes. \( \beta, \sigma, \lambda \) are households’ time preference parameter, risk aversion parameter and inflation-elasticity-with-respect-to-output-gap parameter, respectively. \( \theta_\pi \) and \( \theta_x \) are the weights in the policy rule on inflation and output gap, respectively.

The model (1)-(3) assumes all economic agents have perfect knowledge about the economy structure and all expectations operators \( E_t(\cdot) = E_t(\cdot|\Omega_t) \) stand for the perfect knowledge rational expectations. In our analysis, the assumption that the complete information set \( \Omega_t \) is available to all agents is relaxed. Instead, we will assume agents have imperfect and heterogenous knowledge which will affect the way agents form their expectations. We will consider two groups of agents: (i) private agents - households, firms, and (ii) the central bank. In the following analysis it will be distinguished between the expectation operators which these two groups form. We will assume expectations’ homogeneity within each group but heterogeneity between the groups, that is all households and firms will share the same set of information and beliefs, but, it differs from the information set and beliefs of the central bank. This is a significant relaxation of the original,

¹The derivation of the model is presented in Appendix A.
Consequently, the workhorse model in this paper takes the form

\[ x_t = \hat{E}_t^P x_{t+1} - \sigma \left( E_{t-1}^P i_t - \hat{E}_t^P \pi_{t+1} \right) + v_t, \]

\[ \pi_t = \beta \hat{E}_t^P \pi_{t+1} + \lambda x_t + u_t, \]

\[ i_t = \theta_\pi \left( \hat{E}_t^{CB} \pi_{t+1} - \pi^* \right) + \theta_x \hat{E}_t^{CB} x_{t+1} + \theta_u u_t + \theta_v v_t, \]

where it is specifically distinguished for the form of expectations formed by private agents, \( \hat{E}_t^P(\cdot) = E_t(\cdot|\Omega_t^P) \), and by the central bank \( \hat{E}_t^{CB}(\cdot) = E_t(\cdot|\Omega_t^{CB}) \), where \( \Omega_t^P, \Omega_t^{CB} \subset \Omega_t \). Honkapohja, Mitra & Evans (2003) show that the move from the perfect knowledge model to the imperfect and heterogeneous knowledge model is possible under the so-called Euler-equation learning. If all agents are adaptively learning (using recursive least squares), the originally heterogeneous knowledge \( \Omega_t^P, \Omega_t^{CB} \) enriches over time so that it converges to the perfect knowledge set \( \Omega_t \).

For the later analysis purposes, we assume the policy rule (3) represents optimal discretionary policy. Evans & Honkapohja (2003b) derive an optimal policy rule when a central bank employs internal forecast. The rule takes the form of (3) with parameters \( \{\theta_\pi, \theta_x, \theta_v, \theta_u\} = \{1 + (\lambda^2 + \alpha)^{-1} \sigma^{-1} \lambda \beta, \sigma^{-1}, \sigma^{-1}, (\lambda^2 + \alpha)^{-1} \sigma^{-1}\} \), where \( \alpha \) is the relative preference for output stabilization in the central bank’s quadratic objective function.

### 3 Model Analysis Under Adaptive Learning

Besides the imperfect knowledge and heterogeneity between the private agents’ and central bank’s expectations, we also assume agents are adaptively learning, i.e., they are improving their knowledge about the economy over time, and upon the past mistakes they made in the anticipation of economic movements. Under
certain conditions, if all agents are improving their knowledge over time, the economy converges to the perfect knowledge case eventually. In this light the perfect knowledge case, the rational expectations equilibrium (REE), is a special case of an imperfect knowledge environment.

The minimum-state representation to the structural model (4)-(6) is

\[ Y_t = a + bs_t, \]

where \( Y_t \) is the vector of endogenous variables, \( s_t \) is the vector of exogenous shocks, and \( \{a, b\} \) are the matrices of the structural parameters.

If we say the agents have imperfect and heterogeneous knowledge, we assume the agents’ perception of the economy does not correspond to the perfect knowledge case and further, the knowledge differs between the agents. It is assumed the private agents’ perceived law of motion (PLM) for the economy (4)-(6) takes the form

\[ Y_t = \hat{a}_t^P + \hat{b}_t^P s_t, \]

and the central bank’s MSV is

\[ Y_t = \hat{a}_t^{CB} + \hat{b}_t^{CB} s_t, \]

where \( \{\hat{a}_t^i, \hat{b}_t^i\} \in \Omega_t^i \) for \( i = \{P, CB\} \) are the time-varying matrices of the model primitives. We implicitly assume here, agents have perfect knowledge about the economy structure, i.e., they know what the right-hand side variables are, but they have imperfect knowledge about the true values of model primitives. Though, they are learning about the structural matrices \( \{a, b\} \) over time. The learning mechanism is based on recursive least squares. The mechanism is formalized

\[ \xi_t^i = \xi_{t-1}^i + \kappa_t^i (Y_t - X_t^i \xi_{t-1}^i). \]
where $i = \{P, CB\}$, $\xi_i = [\text{vec}({\hat{a}}^i)'\text{vec}({\hat{b}}^i)']'$ the vector of the perceived-law-of-motion parameters, $X_t$ is the matrix of appropriately stacked exogenous shocks $s_t$, and $\kappa^i_t$ is the information gain. Later in the text we pay a close attention to the gain specification.

**Definition 1. Knowledge heterogeneity** Agents in the model (4)-(6) differ in their knowledge of the structural parameters, and in the speed of updating their knowledge. The individual information sets are defined as

$$\Omega^P_t = \{\hat{a}^P_t, \hat{b}^P_t, \kappa^P_t, s_t\},$$

$$\Omega^{CB}_t = \{\hat{a}^{CB}_t, \hat{b}^{CB}_t, \kappa^{CB}_t, s_t\}.$$

To analyze the conditions under which the imperfect knowledge model (4)-(7) converges to the REE equilibrium, the methodology developed by Evans & Honkapohja (2001) is employed. In principle the methodology consists of two parts. First, the rational expectation equilibrium of a given model is examined. We look for conditions under which the REE is determined. The REE equilibrium is called to be determined if it is found unique and stable. The second part of the methodology is the check for the learnability of REE. The question is, if the economic agents have imperfect knowledge, can they learn, given a learning mechanism, the true RE dynamics? Throughout the paper the recursive, least-squares (econometric) learning mechanism is considered. The conditions that guarantee the REE is attainable under the adaptive learning mechanism are called the $E$-stability conditions. For technical details on the methodology we refer to Evans & Honkapohja (2001) and Evans & Honkapohja (2003a) where the adaptive learning in a homogenous environment is examined and to Honkapohja & Mitra (2003) for the methodology on heterogenous learning.
3.1 REE Determinacy

To examine the rational expectation equilibrium of the model (4)-(6) we begin with rewriting the model into a matrix structural form:

\[ Y_t = A_0 + A_1 \hat{E}_t^P Y_{t+1} + A_2 \hat{E}_t^{CB} Y_{t+1} + A_3 Y_t + s_t, \]  

where \( Y_t = (x_t, \pi_t, \iota^{CB})' \), \( s_t = (v_t, u_t, 0)' \), \( A_0 = \begin{bmatrix} 0 & 0 & -\theta_1 \pi^* \end{bmatrix}' \),

\[ A_1 = \begin{bmatrix} 1 & \sigma & -\sigma \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ \lambda & 0 & \gamma \\ 0 & 0 & 0 \end{bmatrix}. \]

The properties of \( u_t \) and \( v_t \) determine \( s_t \) so that it follows an AR(1) process \( s_t = Fs_t - 1 + w_t \), where

\[ F = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad w_t = \begin{bmatrix} \nu_t^v \\ \nu_t^c \\ 0 \end{bmatrix}, \]

where \( 0 < \rho, \mu < 1 \).

The reduced form to the structural model (8) is

\[ Y_t = M_0 + M_1 \hat{E}_t^P Y_{t+1} + M_2 \hat{E}_t^{CB} Y_{t+1} + P s_t, \]  

where \( M_0 = PM_0, \quad M_1 = PA_1, \quad M_2 = PA_2, \) and \( P = (I - A_3)^{-1} \).

To analyze the REE determinacy, we will assume for now a perfect knowledge environment, \( \hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.) \). Given that, rearranging the reduced form one obtains

\[ Y_t = C + ME_t Y_{t+1} + Qs_t, \]  

where
where \( C = QA_0, \ M = Q(A_1 + A_2), \) and \( Q = (I - A_3)^{-1}. \)

**Proposition 1.** The model (4)-(6) has a unique and stable rational expectations equilibrium if the modulus of the eigenvalues of matrix \( M \) in (10) lies inside the unit circle.

**Proof** follows from the properties of the stable FODE system.

### 3.2 E-Stability

The second issue is to analyze the conditions under which the REE is learnable. We will follow the methodology by Evans & Honkapohja (2003a) for the heterogenous adaptive learning based on recursive least squares. If the REE is determined, the model has the minimum state variable (MSV) representation

\[
Y_t = a + bs_t. \tag{11}
\]

\( a, \) and \( b \) are the (3x1) and (3x3) matrices of the model primitives. Their exact form is derived in Appendix B.

Next suppose, the agents have the following perceived law of motion (PLM). The private agents’ PLM is

\[
Y_t^P = \hat{a}_t^P + \hat{b}_t^P s_t, \tag{12}
\]

and of the central bank’s PLM

\[
Y_t^{CB} = \hat{a}_t^{CB} + \hat{b}_t^{CB} s_t. \tag{13}
\]

The subscript \( t \) at matrices indicates the time dependence of the matrices as the agents are learning using (7). The private agents and central bank use their PLMs
to form expectations

\[
\begin{align*}
\hat{E}_t^P Y_{t+1} &= \hat{a}_t^P + (\hat{b}_t^P F + \hat{c}_t^P) s_t, \\
\hat{E}_t^{CB} Y_{t+1} &= \hat{a}_t^{CB} + (\hat{b}_t^{CB} F + \hat{c}_t^{CB}) s_t.
\end{align*}
\]

(14) (15)

Substituting (14)-(15) back to the reduced form (9), one obtains the economy’s actual law of motion

\[
Y_t = \left[ M_0 + M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB} \right] + \\
\left[ P + M_1 (\hat{b}_t^PF + \hat{c}_t^P) + M_2 (\hat{b}_t^{CB} F + \hat{c}_t^{CB}) \right] s_t.
\]

(16)

The mapping from PLM to ALM is formalized to

\[
T[a, b, c] = \left[ M_0 + M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB}, \\
P + M_1 (\hat{b}_t^P F + \hat{c}_t^P) + M_2 (\hat{b}_t^{CB} F + \hat{c}_t^{CB}) \right].
\]

(17)

The E-stability is determined by the differential equation

\[
\frac{d}{dt} (a, b, c) = T[a, b, c] - (a, b, c).
\]

(18)

Evans & Honkapohja (2001) prove the E-stability exists if (22) is locally stable. Honkapohja & Mitra (2003) and Evans & Honkapohja (2003a) show the map (17) can be simplified. They show the E-stability conditions in the case of heterogenous expectations are equivalent (under least squares learning) to the homogenous expectations case. Thus (17), assuming \( \hat{j}_t^P = \hat{j}_t^{CB} = \hat{j}_t \) for \( j = \{a, b, c\} \), simplifies to

\[
T[a, b, c] = \left[ M_0 + (M_1 + M_2) \hat{a}_t, P + (M_1 + M_2) (\hat{b}_t F + \hat{c}_t) \right].
\]

(19)
Proposition 2. The REE of the model (4)-(7) is E-stable under heterogenous expectations if and only if the corresponding model with homogenous expectations is E-stable. Hence the modulus of eigenvalues of

\[
DT_a(a) = I \otimes (M_1 + M_2)
\]
\[
DT_b(b) = F' \otimes (M_1 + M_2)
\]

must lie inside the unit circle.

Proof see Evans & Honkapohja (2003a) for the proof of the first statement and Appendix C for the derivation of \(DT_a(a), DT_b(b),\) and \(DT_c(c).\)

3.3 Numerical evaluation of the Conditions

Bullard & Mitra (2002) provide an exhaustive analysis of determinacy and E-stability conditions for the model (4)-(5) and different policy rules. In the adaptive learning literature it is well established that the model equilibrium is determined and E-stable under imperfect, homogeneous knowledge if the Taylor principle holds. Honkapohja & Mitra (2003) show this holds also under heterogenous knowledge. The Taylor principle, implied in Propositions 1 and 2, is formally defined as

\[
\theta_\pi + (1 - \beta)\lambda^{-1} > 1.
\]

A numerical evaluation of the determinacy and E-stability conditions from Propositions 1 and 2 are summarized in Figure 1. For the simulation purposes, we calibrate the model using the results from Clarida, Gali & Gertler (2000). We set \(\beta = 0.99, \lambda = 0.075, \sigma = 4, \theta = 0.75.\) The red area [1] in Figure 1 represents the policy-parameter space where determinacy and E-stability conditions are both satisfied. We can see that the space corresponds to the Figure 3 in Bullard & Mitra (2002).
Figure 1: Numerical evaluation of the REE determinacy and E-stability conditions when the credit channel is closed ($\gamma = 0$). Legend: [1] determinate and E-stable, [2] indeterminate and E-stable, [3] indeterminate and E-unstable region.

4 Model simulation

4.1 Methodology

In this section, we focus on addressing three research questions motivated in the introductory part, i.e., (i) *should the central bank be more inflation or output-gap averse in order to improve a short-run economic stability?*, (ii) *what implications does a different willingness to learn for a short-run economic stability?*, and (iii) *can monetary policy influence the speed of learning/convergence to the REE?* In the fashion of adaptive learning literature, the questions will be addressed by a means of numerical simulation.

In our analysis we focus both on a short-run and long-run horizon. The results presented below are obtained for 1,000 Monte Carlo experiments for which the system (4)-(7) is simulated on the horizon of 2,000 observations. As initial conditions for the learning we set $\xi_0 = 0$ and disregard first 50 periods.

To address the first research question, three policy specifications are considered: the policy is inflation-fluctuation averse ($\alpha = 0.2$), policy with equal preferences
(α = 0.5), and policy is output-gap-fluctuation averse (α = 0.8). To compare our results with the findings of Orphanides & Williams (2003), we simulate the system for both a heterogeneous and homogeneous knowledge case. The measure we use as a criteria to asses the answer is the variability of inflation and output gap.

To address the second question, we consider a modification to the gain in (7). In all simulations we assume an econometric learning algorithm which is consistent with $\kappa_i^t = c_i t^{-1}$, where $t$ denotes time, $i = \{CB, PA\}$, and $c_i$ is a positive constant and represents a bias in the gain. If $c_i = 1$, $\kappa_i^t$ represents the standard econometric learning. If $c_i > 1$, it implies a greater willingness to update the model parameters than under standard econometric learning. If for instance $c_{CB} = 1$ and $c_{PA} = 1.5$, we say that private agents are more willing to update their forecasting models than a central bank does. As $\kappa_i^t \to 0$ as $t \to \infty$, the effect of $c_i \neq 0$ is relevant only at the beginning of the sample and does not affect the REE.

The third question is analyzed by approximating the speed of convergence to the REE. In particular, we focus on the speed of inflation and output gap variability to theirs long-run values. We employ the same approach as Marcet & Sargent (1995) and approximate the speed as

$$\delta_y = \frac{1}{\log k} \log \left[ \frac{E(y_t - y^*)^2}{E(y_{tk} - y^*)^2} \right]^{\frac{1}{2}},$$

where $\delta_y$ is the speed of convergence parameter, $y_t = \{\pi_t, x_t\}$, $y_t = \{\pi^*, x^*\}$ are their corresponding values in REE, and $k > 1$. 

13
4.2 Results

Tables 1 and 2 summarize the results for the first two research questions and the homogenous and heterogenous knowledge case, respectively. Table 3 summarizes the results for the third research question analysis. The complete set of simulated results are graphically summarized in the Appendix D.

Table 1: Implied inflation and output gap variability under homogenous knowledge

<table>
<thead>
<tr>
<th>α</th>
<th>(c_{PA} = 1), (c_{CB} = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>std((\pi))</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0037</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0059</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0234</td>
</tr>
</tbody>
</table>

Table 2: Implied inflation and output gap variability under heterogenous knowledge

<table>
<thead>
<tr>
<th>α</th>
<th>(c_{PA} = 1.5), (c_{CB} = 1)</th>
<th>(c_{PA} = 1), (c_{CB} = 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>std((\pi))</td>
<td>std((x))</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0252</td>
<td>0.6433</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0189</td>
<td>0.6592</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0184</td>
<td>0.6633</td>
</tr>
</tbody>
</table>

Table 3: Speed of convergence under homogenous knowledge

<table>
<thead>
<tr>
<th>α</th>
<th>(c_{PA} = 1), (c_{CB} = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\delta_{\pi})</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4424</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4399</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4392</td>
</tr>
</tbody>
</table>

Table 4: Speed of convergence under heterogenous knowledge

<table>
<thead>
<tr>
<th>α</th>
<th>(c_{PA} = 1.5), (c_{CB} = 1)</th>
<th>(c_{PA} = 1), (c_{CB} = 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\delta_{\pi})</td>
<td>(\delta_{x})</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9964</td>
<td>0.8447</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3082</td>
<td>0.8199</td>
</tr>
<tr>
<td>0.8</td>
<td>1.4235</td>
<td>0.8137</td>
</tr>
</tbody>
</table>

The simulation results suggest 'the world is simpler if knowledge and beliefs are homogenous'. The functioning of homogenous-knowledge models is well understood. For instance, the recommendation by Orphanides & Williams (2003) of
being inflation hawk can be applied. As shown in Tables 1 and 3, if the knowledge is homogenous, inflation hawkiness (lower $\alpha = 0.2$) helps to decrease the inflation variability and to speed up the learning of the REE dynamics.\(^2\) If knowledge and beliefs are heterogenous, the results in Table 2 and 4 suggest the policy ought not to be an inflation hawk in the short run as the variability increases and the speed of convergence prolongs. For the central bank to play its role effectively in the heterogenous information world and help the economy converge to the first best equilibrium, the policy ought to be conservative and focus on the information and knowledge homogenization in the economy.

We can draw three conclusions from the analysis above:

1. the lower the willingness to learn (low $c_i$), the higher economic persistence (inflation persistence in this case);

2. if the monetary authority is less willing to learn, active monetary policy does not affect the speed of convergence in a considerable way. A high economy persistence remains;

3. if, however, monetary policy is willing to learn (less economic persistence), an active policy actually improves the speed of economic convergence.

These findings are crucial for monetary policy based on calibrated models. If monetary policy relies on a calibrated large-scale structural model which, due to its scale, is updated on a lower frequency, it may be in theory harmful to the economic stability. It is the case, especially, if other economic agents use for instance a simple statistical models. Such models are often updated whenever a new observation arrives.\(^3\) We focus on the problem where agents perceive their model as the true economy model and ignore the fact that there might be also other models.

\(^2\)Ferrero (2003) provides an excellent analysis in this respect.

\(^3\)Certainly, in reality, there is a discrepancy in models updating. The discrepancy is the bigger, the less monetary policy is credible or understood by private agents.
In fact, the model uncertainty is usually high. Economic agents can never be certain about their forecasting model validity. Given the model uncertainty, if a central bank insists on its model and is less willing to learn, it leads to an increase in the economic volatility by pushing the economy away from the REE towards the ’equilibrium’ given by the bank’s model. Moreover, if the policy is not interest rate smoothing, i.e., interest rates are changed in an aggressive way, it harms the economy.

Assume a situation where an economy is initially in a long-run equilibrium. The inflation rate is zero. Both the central bank and private agents use models that correspond to the REE model. The central bank is aware of that and is unwilling to change its model. Private agents are, however, uncertain about their model, and they favor the doubt. Now, an inflationary shock arrives. Both the central bank and private agents had expected the equilibrium (zero) inflation before. The central bank does not put any weight on the unanticipated inflation and sets the interest rate so that it brings the inflation back to the REE equilibrium (to zero). Because the central bank believes (in this set-up) that the private agents use the same model to form their forecasts, the central bank believes that this interest rate delivers zero inflation in the next period. The private agents are, however, uncertain about their model and given the unanticipated inflation, they update their model and believe that so does the central bank. It leads them to believe that since the inflation was high today, it is going to be high tomorrow too. No further shock arrives. The actual inflation rate is a convex combination of central bank’s and private agents’ expectations. Thus, the actual inflation rate will be higher than what the central bank expects but smaller than what private agents expect. This is given by ... The adjustment/learning process continues in the same fashion until the REE is achieved eventually. Certainly, the tougher monetary policy is on the inflation, the faster the convergence back to the REE will be.
This was an example when the central bank knows the REE. What happens, however, if the REE is not known? If a central bank insists on its model (its view of the world), using the above logic, it can harm the economy since the central bank may push away or slow down the convergence to the REE. The risk increases with the monetary policy activity. This cannot be economic-stability improving behavior.
5 Final Remarks

_Old truth_ What we discuss in this paper is an old truth. Already classics have recommended: the less policy makers know about the economy functioning, the less they should interfere. In our case, it is demonstrated within a simple model framework.

_A good policy communication to gain credibility_ We find that homogenous-knowledge economy is dominant to its heterogenous knowledge counterpart in terms of short-run economic-stability improvement. How the knowledge homogeneity can be achieved, can be seen in two ways. First, the central bank adopts private agents expectations or, second, private agents acquire the central bank’s expectations. Abstracting from the theory world, none of those is a simple task. The former will require reliable observations of such expectations. Central banks run surveys of private sector’s expectations about the future economic development. There is a question, however, whether the information that such surveys yield is economically reliable, i.e., whether the data collected truly represent market expectations (which are those being important and employed in macro models), and they are not subject to biases instead (due to the inaccuracy of responses, a collusion-game behavior of some respondents, etc.). In fact, the central bank can never be sure, the data being collected are useful for immediate policy decisions. In this respect, the latter seems to be more appealing.

Forming its own expectations/forecasts, a central bank avoids the need of collecting private sector’s expectations and verifying their reliability. Instead, a central bank can concentrate its capacities on producing the best expectations/forecast on the market and to gain a credibility of its actions. A central bank producing the best forecasts on the market, i.e. private sector cannot systematically outperform them, appears to be the first step to the expectations homogeneity. This is not sufficient, of course.
Another important element for making expectations homogenous across econ-
omy is policy credibility. As it is argued in the standard monetary theory, a nec-
essary requirement for gaining credibility is a good discussion, clarification and
justification of past policy errors, and of further policy steps to be taken. This im-
plicitly concerns central bank’s expectations which stand behind policy decisions.
In short, to make private sector adopt central bank’s views would require a good
communication of central bankers’ views. If a central bank communicates well, it
gains credibility, and can afford to be more active in its policy and to contribute
to an economic stability.
References


Appendix A

In this appendix we derive the model (1)-(5) from first principles.

Agents

**Households** The households’ objective is to maximize lifetime utility. The consumption bundle, \( c_t \), and leisure, \( (1 - N_t) \), deliver the utility. To meet the objective, a household does not only decide about how much to consume and how much to work, but it also decides about how much money to hold, since money is the means of transaction in this economy and serves the consumption-smoothing purposes. Households face two constraints in their decisions. First, following Fuerst (1992), they need to hold cash in advance in order to purchase consumption goods. The decision about \( M^c_t \) is made at the end of the period \( t - 1 \). Disposable income in period \( t \) is \( W_t N_t \), where \( W_t \) is the nominal wage and \( N_t \) is the hours worked. A budget constraint is second constraint the households face. It equates the current period income from labour \( (W_t N_t) \), financial assets \( (M^c_t + (1 + i^d_t)M^d_t) \) and the ownership of firms \( (\Pi^f_t) \) and banks \( (\Pi^b_t) \), to the value of current period consumption \( (P_t c_t) \) and financial portfolio carried to the next period \( (M_{t+1}) \). The representative household’s problem can be formally written as

\[
\max_{\{c_t, N_t, M^c_{t+1}, M^d_{t+1}\}} \sum_{t=0}^{\infty} \beta^t E_0 \left( \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp(\varepsilon_t^c) - \psi \frac{N_t^{1+\phi}}{1+\phi} \right)
\]

subject to

\[
M^c_t + W_t N_t \geq P_t c_t, \quad (21)
\]

\[
M_{t+1} + P_t c_t = M^c_t + (1 + i^d_t)M^d_t + W_t N_t + \Pi^f_t + \Pi^b_t, \quad (22)
\]

\[
M_t = M^c_t + M^d_t. \quad (23)
\]
Here $c_t$ represents the CES composite index (Dixit-Stiglitz aggregator) of real consumption, $c_t = \left( \int_0^1 c_t(i)^{\frac{1}{\epsilon}} di \right)^{\frac{1}{\frac{1}{\epsilon}}} \text{ with } c_t(i)$ being the consumption of differentiated good $i$ and $\epsilon > 1$; $P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{\epsilon}}$ is the corresponding nominal price index, with $P_t(i)$ being the price of the differentiated good $i$. $N_t$ is the hours worked, $M^c_t$ is cash money, $M^d_t$ is deposit money, $\Pi^f_t$ is the profit coming from the firm ownership, $\Pi^b_t$ is the profit from the bank ownership, $W_t$ is the nominal wage, and $i^d_t$ is the nominal return on the deposit money. $\varepsilon^c_t$ is the preference shock which is assumed to follow an AR(1) process $\varepsilon^c_t = \rho \varepsilon^c_{t-1} + \nu^c_t$, with $\nu^c_t$ being iid with zero mean and finite variance, and $0 < \rho_c < 1$. $\beta$, $\phi$ and $\psi$ are scalars between 0 and 1, and $\sigma > 1$.

Setting up the Lagrangian function

$$L(c_t, N_t, M^c_{t+1}, M^d_{t+1}) = \sum_{t=0}^{\infty} \beta^t E_t \left( \frac{c_t^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\sigma}} \exp(\varepsilon^c_t) - \psi N_t^{1+\phi} \right) + \lambda_{1,t} (M^c_t + W_t N_t - P_t c_t) + \lambda_{2,t} \left[ M^d_t + (1 + i^d_t) M^d_{t+1} + W_t N_t + \Pi^f_t + \Pi^b_t - P_t c_t - M^c_{t+1} - M^d_{t+1} \right].$$

and maximizing it gives a set of first order conditions

$$\frac{\partial L(.)}{\partial c_t} = \beta^t c_t^{-1/\sigma} \exp(\varepsilon^c_t) - \lambda_{1,t} P_t - \lambda_{2,t} P_t = 0 \quad (24)$$
$$\frac{\partial L(.)}{\partial N_t} = -\beta^t \psi N_t^{\phi} + \lambda_{1,t} W_t + \lambda_{2,t} W_t = 0 \quad (25)$$
$$\frac{\partial L(.)}{\partial M^d_{t+1}} = -\lambda_{2,t} + (1 + i^d_t) \lambda_{2,t+1} = 0 \quad (26)$$
$$\frac{\partial L(.)}{\partial M^c_{t+1}} = -\lambda_{2,t} + \lambda_{1,t+1} + \lambda_{2,t+1} = 0 \quad (27)$$

Combining (24), and (25) gives the Euler equation for the household’s labour supply

$$\frac{c_t^{-1/\sigma} \exp(\varepsilon^c_t)}{\psi N_t^{\phi}} = \frac{P_t}{W_t}. \quad (28)$$
Combining (24), (26) and (27) gives the Euler equation for consumption

\[ c_t^{-1/\sigma} \frac{P_t}{\sigma} \exp(\epsilon c_t) = \beta(1 + \epsilon d_t) E_t \left( c_{t+1}^{-1/\sigma} \frac{P_{t+1}}{\sigma} \exp(\epsilon c_{t+1}) \right). \]  

(29)

Having the relation for the aggregate consumption, we also have to solve for the individual demand for differentiated goods \( c_t(i) \). Here the household solves

\[ \max_{c_t(i)} c_t = \left( \int_0^1 c_t(i)^{\epsilon - 1} di \right)^{\frac{1}{\epsilon - 1}} \]  

subject to the budget constraint

\[ P_t c_t = \int_0^1 c_t(i) P_t(i) di, \]  

(31)

where \( P_t c_t \) are the expenditures on the consumption bundle \( c_t \), and \( P_t(i) \) is the price of an individual good. Solution to this problem is the individual good demand

\[ c_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} c_t. \]  

(32)

In summary, constraints (21)-(23) and equations (28), (29), and (32) describe the household’s optimal decisions.

**Firms** Firms operate in a monopolistically competitive environment. As such, to maximize their profits, they choose how much to produce, what price to charge, and how much labour to demand. Following the timing in Fuerst (1992), management’s decisions are taken after the shocks to the economy are realized. We assume labour is the only production factor. To start production, a firm goes to the labour market to hire workers. Once the output is produced, the labour is paid out. The firm goes to a bank and applies for a credit to cover the wage bill. When the revenues from selling the output are collected, the firm re-pays the credit and
transfers its net financial position to households.

Each firm, distinguished as an \(i\) firm, produces one type of good and solves the following problem

\[
\max_{\{N_t(i), P_t(i), B_t(i)\}} E_0 \sum_{t=0}^{\infty} \Phi_{t+1} \Pi_t^f(i) 
\]

where \(\Pi_t^f(i) = P_t(i)y_t(i) - W_tN_t(i) - i_t^kB_t\) is the firm’s \(i\) nominal profit and \(\Phi_{t+1}\) is the stochastic discount factor defined as \(\beta^{t+1}/(c_{t+1}P_{t+1})\).\(^4\) \(N_t(i)\) is the labour demanded by the firm \(i\), \(P_t(i)\) is the firm-specific price charged on the output \(y_t(i)\), \(B_t(i)\) is the demand for credit, and \(i_t^k\) is the interest rate paid for the credit. Note that the firm’s problem is in fact static and thus the firm maximizes only \(\Pi_t^f(i)\) subject to

\[
y_t(i) = A_tN_t(i), \quad y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} c_t, \quad W_tN_t(i) \leq B_t(i).
\]

(34) is the firm’s production function where labour is the only production factor. The technology associated to the labour is captured by \(A_t = \exp(\varepsilon_t^A)\), where \(\varepsilon_t^A = \rho_A \varepsilon_{t-1}^A + \nu_t^A\) is the aggregate technology shock, \(\nu_t^A\) is iid, zero mean and finite variance disturbance, \(0 < \rho_A < 1\). (35) is the demand function for the consumption good \(c_t(i)\) the firm produces. The firm also faces a cash-in-advance constraint (36) which requires to pay wages in advance, i.e., after the output was produced but before it is sold.

Since in equilibrium (36) holds with equality, we substitute all the constraints

\(^4\)It follows that if the firm acts in the best interest of the shareholder, the discount factor corresponds to the representative household’s relative valuation of consumption across time.
into the profit function and suitably rearrange to obtain

$$\max_{P_t(i)} \Pi'_t = P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} c_t - (1 + \epsilon b_t \frac{W_t}{A_t} \left( \frac{P_t(i)}{P_t} \right)^{\epsilon} c_t$$

The first order condition follows

$$\frac{d\Pi'_t}{dP_t(i)} = (1 - \epsilon) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} c_t + \epsilon (1 + \epsilon b_t \frac{W_t}{A_t} P_t(i)^{-1} \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} c_t = 0.$$ 

Rearranging it and using constraints (34)-(36) gives a set of conditions characterizing the optimal behavior of the $i$'th firm:

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} MC_t,$$  \hspace{1cm} (37)

$$\frac{W_t}{P_t(i)} = \frac{\epsilon - 1}{\epsilon - 1 + \epsilon b_t A_t},$$ \hspace{1cm} (38)

$$B_t(i) = W_t N_t(i).$$ \hspace{1cm} (39)

$MC_t$ are the firm’s nominal marginal costs, $MC_t = \frac{(1 + \epsilon b_t W_t N_t(i))}{\mu_t(i)}$. (37) is the standard pricing rule in the monopolistic competition. The price is a fixed markup over marginal costs, (42) is the labour demand, and (39) constitutes the credit demand function. Note that these conditions characterize the firm’s optimal behavior in a frictionless environment.

To introduce a persistence into the prices in the model, Calvo’s pricing scheme is assumed. The production sector is monopolistically competitive and as such has control over prices. Calvo’s pricing mechanism assumes that in every period only a fraction of firms, $\theta \in (0, 1)$, can adjust its price. The rest of the firms, $(1 - \theta)$, charge the same price as in the previous period. $\theta$ is often viewed as a price-stickiness measure. The higher its value, the higher the degree of price persistence. Since the pricing mechanism is well known and described in the literature, we will limit ourselves to its optimal solution.
Introducing Calvo’s pricing mechanism, the firm’s problem is no longer a static one. If a firm $i$ is allowed to change price in period $t$, it chooses to charge the optimal price

$$P_t^*(i) = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t(MC_{t+k}),$$

which is the discounted sum of the future expected marginal cost. Since we are in a monopolistically competitive environment, note that the marginal cost here meets the first order condition (37). This specification fully corresponds to the one employed in Gali & Gertler (1999). $\beta$ is the subjective discount factor from the households problem. In this specification, the firm takes into account the possibility it might not be allowed to change the price for some time from now on.

Introducing the price persistence in the economy, the set of conditions (37)-(39) characterizing the firm’s optimal behavior in a monopolistically competitive environment is extended by the time dependent Calvo pricing rule (40). The firm applies it only if it wins the lottery and is allowed to change the price. Otherwise the firm charges the same price as in the previous period.

At this point, it is useful to determine the aggregate price level since later we will be particularly interested in the aggregate dynamics. As stated above, the aggregate price level is computed as

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

The aggregate level in the sticky-price environment is a weighted average of past prices and new prices. The weights are given by the portion of firms allowed to change the prices. The aggregate price level becomes

$$P_t = \left[ (1 - \theta)P_t^{(1-\epsilon)} + \theta P_{t-1}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

In summary, in the frictionless environment, the optimal behavior of firm is
given by equations (37)-(39). If the Calvo pricing rule is introduced, (40) also applies. It is employed if the firm is allowed to change its price. Otherwise, it charges the price from the last period.

**Banking Sector** The role of the competitive banking sector is two-fold. First, to collect deposits from households and, second, to provide credit to firms. The operation schedule of the private bank is as follows. The bank enters the period $t$ with the deposit money from the households. The only money source in the economy is the central bank. Once the shocks to the economy are realized, and the central bank makes its decision about the policy rate, and firms decide on their production and credit demand, the private bank goes to the central bank and (i) puts the collected deposit money into its accounts and (ii) asks for a credit to cover the firms’ demand. The central bank charges the same interest rate on both the deposit money and credit.

Households visit a bank before exogenous shocks are realized, i.e., at the end of period $t - 1$. A private bank collects deposits and puts them on interest-bearing accounts at the central bank. As the central bank sets its rates after shocks are realized and the private bank has to sign contracts before that, the private bank has to form expectations about the central bank’s future rate. From the perfect competition and zero profit condition it follows that the private bank sets its price as

$$i_t^b = i_t^{CB}, \quad (42)$$

$$i_t^d = E_t^{-1}i_t^{CB}, \quad (43)$$

where $i_t^b$ is the interest rate charged on the credit provided to the firms, and $i_t^d$ is the interest rate offered on deposits.
Monetary Authority  The monetary policy, in order to anchor the nominal side of the economy, is assumed to follow the targeting rule

\[ i_t^{CB} = \theta_\pi (E_t \pi_{t+1} - \pi^*) + \theta_x E_t x_{t+1}, \]  

(44)

where \( i_t^{CB} \) is the policy instrument, \( \pi_{t+1} \) is the inflation rate between periods \( t \) and \( t+1 \), \( x_{t+1} \) is the output gap in the \( t+1 \) period (see the definition below), \( \pi^* \) is the inflation target. The target is set exogenously by the central bank and constitutes a nominal anchor to the economy. According to the rule, the central bank sets its policy instrument \( i_t^{CB} \) on the basis of expected deviation of inflation from the target in the next period, and the expected output gap. \( \theta_\pi \) and \( \theta_x \) characterize the bank’s preferences with respect to inflation stabilization and/or to the output gap stabilization. The higher value of \( \theta \)’s, the more vigilant the bank is. The reason for the choice of policy rule (44) is twofold. First, the choice is motivated by the empirical evidence by Clarida et al. (2000) who argue for this type of rule. Second, Bullard & Mitra (2002) find that this type of rule is robust to deliver the rational expectations equilibrium determinacy and E-stability, which is required for the analysis below.

Model Equilibrium

Definition 2. The flexible-price equilibrium is given by an allocation

\[ \{c_t, N_t, M_{t+1}^d, M_{t+1}^c, B_t\}_{t=0}^\infty \] and set of \( \{P_t, P_t(i), \bar{i}_t^b, \bar{i}_t^d, i_t^{CB}\}_{t=0}^\infty \) such that

1. households maximize their lifetime welfare (24) subject to constraints (25)-(26);

2. monopolistically competitive firms maximize their present-value profit (37) constrained by (34)-(36);

3. perfect competitive private banks maximize their profit;
4. central bank meets its inflation target and zero-output-gap objectives; and

5. labour market, money market, and goods market clear.

**Definition 3.** The **sticky-price equilibrium** is given by an allocation
\[ \{c_t, N_t, M_{t+1}^d, M_{t+1}^c, B_t\}_{t=0}^{\infty} \] and set of \( \{P_t, P_t(i), i^b_t, i^d_t, i^{CB}_t\}_{t=0}^{\infty} \) such that

1. households maximize their lifetime welfare (24) subject to constraints (25)-(26);

2. monopolistically competitive firms maximize their present-value profit (37) constrained by (35)-(36), and Calvo’s pricing principle allows the firm to set an optimal price according to (40) if it is allowed to change its price, otherwise \( P_t(i) = P_{t-1}(i) \);

3. perfect competitive private banks maximize their profit;

4. central bank meets its inflation target and zero-output-gap objectives; and

5. labour market, money market, and goods market clear.

**Log-Linearized Model and Aggregate Equilibrium**

From now on we focus our attention particularly on the aggregate dynamics. We log-linearize the sticky-price model and describe its aggregate-level dynamics. Because we concentrate specifically on the dynamics of output and inflation, we concentrate on the IS and Phillips curves.

First we derive the IS curve which characterizes the dynamics of output around its steady state. The derivation is straightforward and follows the same strategy as Ravenna & Walsh (2003) and Malik (2004). We log-linearize the Euler equation from the household’s problem (33) to get

\[ c_t = E_t c_{t+1} - \sigma (i^d_t - E_t \pi_{t+1}) + \sigma \varepsilon^c_t. \]
From the market clearing condition it follows that $c_t = y_t$. If we define the output gap as $x_t = y_t - y^f_t$, then (49) becomes

\[ x_t = E_t x_{t+1} - \sigma(i^d_t - E_t \pi_{t+1} - r^f_t) + \sigma \varepsilon^c_t, \]

where $r^f_t$ is the real interest rate that arises in the frictionless equilibrium and $y^f_t$ is the output in the frictionless equilibrium. Both are defined as

\[ r^f_t = \left( \frac{1}{\sigma} \right) E_t \left( y^f_{t+1} - y^f_t \right) + \varepsilon^c_t, \]
\[ y^f_t = \frac{\sigma}{1 + \sigma \phi} \left[ \ln \left( \frac{\epsilon - 1}{\epsilon} \right) - \ln \psi + (1 + \phi) \varepsilon^A_t + \varepsilon^c_t - i^d,f_t \right], \]

where $i^d,f_t$ is the nominal interest rate in the frictionless equilibrium. For computational convenience and without loss of generality, we will assume that this rate is equal to zero.

Eliminating $r^f_t$ from the above equation for the output gap we get

\[ x_t = E_t x_{t+1} - \sigma(i^d_t - E_t \pi_{t+1}) + v_t, \quad (46) \]

where $v_t = \frac{\sigma(1+\phi)(1-\rho_\Delta)}{1+\sigma \phi} \varepsilon^A_t - \frac{\sigma(1+\rho_c-2\sigma \phi)}{1+\sigma \phi} \varepsilon^c_t$. Recalling the properties of $\varepsilon^A_t$ and $\varepsilon^c_t$ and further assuming $\rho_\Delta = \rho_c = \rho$, $v_t$ follows an AR(1) process\(^5\). Equation (46) constitutes the IS curve as a function of expected future output gap and the ex ante real interest rate.

Second, we derive for the New Keynesian Phillips curve. Log-linearizing and combining (44) and (45) we obtain

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta \beta)}{\rho} m e_t, \quad (47) \]

\(^5\)The process is $v_t = \rho v_{t-1} + \nu^v_t$, where $\nu^v_t = \frac{\sigma(1+\phi)(1-\rho_\Delta)}{1+\sigma \phi} \varepsilon^A_t - \frac{\sigma(1+\rho_c-2\sigma \phi)}{1+\sigma \phi} \varepsilon^c_t$. 

30
where $mc_t$ is the log of real marginal costs. To eliminate the marginal costs, we plug in (37) to (42) and divide both sides by $P_t$; we obtain the real marginal costs. Log-linearizing that under the perfect knowledge assumption gives

$$mc_t = w_t - p_t + i_t^b - \varepsilon_t^A.$$  \quad (48)

Substituting in (52) for the log-linearized labor supply function (32), gives

$$mc_t = \frac{1 + \sigma \phi}{\sigma} y_t - (1 + \phi) \varepsilon_t^A + i_t^b.$$  

We deduct $y_t^f$ from $mc_t$ to obtain $mc_t$ in terms of the output gap

$$mc_t = \frac{1 + \sigma \phi}{\sigma} x_t + i_t^b + \ln \left( \frac{\epsilon - 1}{\epsilon} \right) - \ln \psi + \varepsilon_t^c.$$  

Substituting this expression back to (51) gives the New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + \gamma i_t^b + u_t,$$  \quad (49)

where $\gamma = \frac{(1-\theta)(1-\theta \beta)}{\theta}$, $\lambda = \gamma \frac{1 + \sigma \phi}{\sigma}$, and $u_t = \varepsilon_t^c$, assuming $\epsilon = \frac{1}{1-\psi}$. 

31
Appendix B

MSV representation

Using the method of undetermined coefficients, we derive the exact form of the minimum state variable (MSV) representation for the model considered in the text. Starting with the reduced form (9) and assuming rational expectations, i.e., $\hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.)$, we get

$$y_t = M_0 + (M_1 + M_2)E_t y_{t+1} + M_3E_{t-1}y_t + P\epsilon_t, \quad (50)$$

where

$$\epsilon_t = F\epsilon_{t-1} + \epsilon_t.$$

Now assume the MSV form takes the form

$$y_t = a + b\epsilon_t + c\epsilon_{t-1}. \quad (51)$$

Taking the appropriate expectations needed in (50) one obtains

$$E_{t}y_{t+1} = a + (bF + c)\epsilon_t,$$

$$E_{t-1}y_t = a + (bF + c)\epsilon_{t-1}.$$

Plugging these expectations back into (50) yields

$$y_t = M_0 + (M_1 + M_2 + M_3)a + [(M_1 + M_2)(bF + c) + P]\epsilon_t + M_3(bF + c)\epsilon_{t-1}. \quad (52)$$
Using the method of undetermined coefficients, it follows that the MSV solution must satisfy

\[ M_0 + (M_1 + M_2 + M_3)a = a, \]
\[ (M_1 + M_2)(bF + c) + P = b, \]
\[ M_3(bF + c) = c. \]

Solving for the matrices \( a, b, \) and \( c \) we get

\[ a = (I - M_1 - M_2 - M_3)^{-1}M_0, \]  
(53)

\[ vec(b) = [I - F' \otimes (M_1 + M_2) - F' \otimes (M_1 + M_2)(I - M_3)^{-1}M_3]^{-1}vec(P), \]  
(54)

\[ c = (I - M_3)^{-1}M_3bF. \]  
(55)
Appendix C

Here we derive the matrices used in Proposition 2 on page 14.

Having the map from the PLMs to ALM

\[
T[a, b, c] = \left[ M_0 + (M_1 + M_2 + M_3)\hat{a}_t, P + (M_1 + M_2)(\hat{b}_t F + \hat{c}_t), M_3(\hat{b}_t F + \hat{c}_t) \right].
\]

we take derivatives with respect to \(\hat{a}_t\), \(\hat{b}_t\), and \(\hat{c}_t\). Using the rules for the derivatives of matrices we get

\[
DT_a(a) = \frac{d}{d\hat{a}_t} [M_0 + (M_1 + M_2 + M_3)\hat{a}_t] = I \otimes (M_1 + M_2 + M_3),
\]

\[
DT_b(b) = \frac{d}{d\hat{b}_t} \left[ P + (M_1 + M_2)(\hat{b}_t + \hat{c}_t) \right] = F' \otimes (M_1 + M_2),
\]

\[
DT_c(c) = \frac{d}{d\hat{b}_t} \left[ M_3(\hat{b}_t F + \hat{c}_t) \right] = I \otimes M_3.
\]
Appendix D

Here the details on simulation results are provided. The results presented below are obtained for the following numerical calibration: $\beta = 0.99$, $\lambda = 0.075$, $\sigma = 4$, $\gamma = 0.1$, $\theta_x = 1.5$, $\theta = 0.4$, $\pi^* = 0$, $\rho = 0.8$, $\sigma_{\nu} = 0.1$, $\sigma_{\nu}^c = 0.2$.

The REE Model Dynamics

Given the numerical calibration, we simulate the impulse response function for the standard New Keynesian model and for the credit channel extension. The impulse responses to the productivity and demand shocks are summarized in Figure 3. In Table 3 we report on the moments, correlation and autocorrelation of the simulated variables. The numbers are obtained for 5,000 replications.

![Impulse response functions](https://via.placeholder.com/150)

Figure 2: Impulse response functions when the credit channel is opened ($\gamma = 0.1$) and closed ($\gamma = 0$).

The economy with the credit channel opened is less responsive to the exogenous shocks than the economy where the channel is closed. The only exception is the response of output gap to the technology shock, which is about as twice as big in
contrast to the no-credit market economy. Surprisingly, the credit channel does not affect the persistence of any model variable. Indeed, this holds only *ceteris paribus*. A detailed investigation on the model properties we leave for future research.

### Table 5: Moments, correlation, and autocorrelation of simulated variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Correlation</th>
<th>Autocorrelation (lag)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( x )</td>
<td>( \pi )</td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{std}(x) )</td>
<td>0</td>
<td>1.77</td>
<td>1</td>
<td>-0.99</td>
</tr>
<tr>
<td>( \text{std}(\pi) )</td>
<td>0</td>
<td>1.56</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td>( \text{std}(i) )</td>
<td>0</td>
<td>1.39</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0.1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{std}(x) )</td>
<td>0</td>
<td>2.79</td>
<td>1</td>
<td>-0.99</td>
</tr>
<tr>
<td>( \text{std}(\pi) )</td>
<td>0</td>
<td>2.47</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td>( \text{std}(i) )</td>
<td>0</td>
<td>2.21</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Imperfect Knowledge Experiment Results (details)**

In Figures 4 - 11 below, we present detailed simulation results on inflation, output gap, interest rate variability and mean-square forecast errors (*MSFE*) of private agents and central bank. The results are obtained for time series of 100 observations and 500 experiments. The summary results presented in the paper are based on these simulations. One observation follows immediately, there is no monotone relation between \( \kappa_{CB}^t, \kappa_P^t \) and \( \text{std}(.) \) or \( MSFE^P(.) \). Other observations can be summarized as:

1. The policy parameter combination \{1.5, 0.4\} appears to be the most effective in delivering the lowest variation in inflation and output gap, although for some combinations of \{\( \kappa_{CB}, \kappa_P \)\} a higher value of \( \theta_\pi \) may deliver better results. These cases are marginal though.

2. The policy parameter combination \{1.5, 0.4\} appears to help the forecast efficiency. Both private agents and central bank form better forecast under
that policy configuration.

Figure 3:

Figure 4:
Figure 5:

- Case 1: $\theta_\pi = 1.5$, $\theta_x = 0.4$
- Case 2: $\theta_\pi = 2$, $\theta_x = 0.4$
- Case 3: $\theta_\pi = 2$, $\theta_x = 0.2$
- Case 4: $\theta_\pi = 1.5$, $\theta_x = 0.2$

Figure 6:

- Case 1: $\theta_\pi = 1.5$, $\theta_x = 0.4$
- Case 2: $\theta_\pi = 2$, $\theta_x = 0.4$
- Case 3: $\theta_\pi = 2$, $\theta_x = 0.2$
- Case 4: $\theta_\pi = 1.5$, $\theta_x = 0.2$
Case 1: $\theta_\pi = 1.5$  $\theta_x = 0.4$
Case 2: $\theta_\pi = 2$  $\theta_x = 0.4$
Case 3: $\theta_\pi = 2$  $\theta_x = 0.2$
Case 4: $\theta_\pi = 1.5$  $\theta_x = 0.2$

Figure 7:

Case 1: $\theta_\pi = 1.5$  $\theta_x = 0.4$
Case 2: $\theta_\pi = 2$  $\theta_x = 0.4$
Case 3: $\theta_\pi = 2$  $\theta_x = 0.2$
Case 4: $\theta_\pi = 1.5$  $\theta_x = 0.2$

Figure 8:
Figure 9:

Figure 10: