Incentive Contracts in a Cournot Duopoly

by

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Abstract

We discuss how owners can use incentive contracts to guide a manager in a duopoly. We show how owners can use the terms of the contract as a strategic lever and that asymmetric information increases the advantage to the firm with the lower marginal cost.

The need for a contract depends on the source of asymmetric information and, using this insight, we show why it is important to distinguish the effects of random shocks to the demand curve from the effects of strategic uncertainty. In particular, an incentive contract may expose a manager to strategic risk due to potential deviations by other players where a quota contract would not. We use this distinction to comment on recent proposals concerning corporate governance, which have tended to focus on a single firm in isolation, and to offer some managerial implications.

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Incentive Contracts in a Duopoly

Separating the ownership of a firm from its management can cause the actions of firms to deviate from strict profit maximization. Recent business scandals (e.g. Enron, Parmalat, Nortel) reenergized this discussion sufficiently to cause many governments to pass new laws. A common theme has been to require Chief Executive Officers (CEOs) and managers to disclose more information to their Board of Directors and to their shareholders (e.g. the Sarbanes-Oxley Act in the U.S., http://www.aicpa.org/info/sarbanes-oxley-summary.htm) While most research has rightly focused on how disclosure affects the internal operations of firms and the relationship between shareholders and managers (XX), we think it is important to consider how this information might change the structure of the markets in which these firms operate.

For economists accustomed to thinking of the firm as a “black box”, this consideration may seem insignificant. The separation of ownership and management leads to the use of incentive contracts intended to align the motives of a manager with the profit-maximizing motive of the owner. A manager in a strategic environment needs to be aware that the profits also depend on the actions of competitors and that these actions depend in part on the manager’s strategy. Consequently, managerial incentives can redistribute rents between owners.

The relevance of this analysis depends on a market environment for which an incentive contract is a relevant solution. Without uncertainty, incentives usually have no real importance since there is no asymmetry of information. In an ordinary model of hidden information, a manager may claim that “Don’t punish me if profits are low: the state of demand was low”. An intelligent owner would anticipate this outcome during contract negotiations using an incentive-compatible contract. In a duopoly model, a manager might claim that “Don’t punish me if profits are low: the rival did something unexpected.” In a simple model without information sharing between firms, this claim cannot be refuted and, by definition, cannot be anticipated during negotiations. Sharing information would appear to offer the evidence needed to refute any claims of managerial shirking but sharing information about the actions of different managers can change the nature of the equilibrium.

To investigate the connections among these ideas more precisely, we consider a Cournot game with two owner-manager pairings. Managers hold their position by virtue of having better
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Information about the state of the market, i.e. a situation of asymmetric information: market demand has only two possibilities, high or low where only the managers know the state of the market. During the first stage, the owners solve principal-agent problems. During the second stage, the managers compete with each other by choosing the level of output strategically. We resolve these additional considerations and trade-offs with the use of a Nash Equilibrium concept.

We find that, although the owner wishes to maximize profit, her manager’s maximization includes goals in addition to profit as a means of credibly signaling an intention to increase output. Thus, the market price in a state is lower than in the standard Cournot model. The terms of the contract vary with the distribution of information about the state of the market, especially the likelihood of the high state. We also show that using incentive contracts increases the advantage of a low cost firm since this firm also offers a stronger incentive to increase sales.

This analysis has relatively obvious benefits in terms of market analysis but it also contributes to the growing body of research on corporate governance. Corporate misconduct, scandals around the world and massive revisions in the earnings reported by many companies have reduced investors’ confidence in accuracy of the financial reports disclosed by corporations. Though the laws were motivated by an intention to reduce the abuse of shareholders, the same information can be used by competitors and by financial analysts who study the industry (see Arya and Mittendorf (2005) for a more complete consideration of this issue). Therefore, we ask: will disclosing more information affect the market structure? The answer to this question would be unimportant if activities inside the firm had no impact on the relationship between firms.

The paper proceeds as follows. The next section offers a literature review. Section 3 presents the game which describes the relationships amongst the owner, manager and the competitor’s owner and its manager. We solve for equilibria to three variations on this model. The first variation is closest to a traditional principal-agent model. In the second and third variations, information is shared and, as a result, changes the equilibrium contracts. In one variation, both owners make a minor change to the type of contract being negotiated; as a result,
they expose their managers to strategic risk. In the other variation, both owners use a more radical contract, akin to complete oversight in a traditional principal agent model; as a result, they lose the ability to credibly commit to expand output. Our results, their policy implications and the managerial implications are summarized in Section 5.

**Literature Review**

Many authors have studied many aspects of the relationship between owners and managers. The simplest story for why a manager would want to increase output at the expense of profits is because he wants to. Baumol (1958) used the idea that the corporations seek to maximize sales, subject to a minimum profit constraint, to obtain testable predictions. Though still presented in some textbooks used by undergraduate business students, this literature asserts a motive rather than deriving it as a special case of the more familiar motive. It has been replaced by a literature which offers more detailed insights into the operations.

Economists have studied many versions of the principal-agent relationship. Harris and Raviv (1979), Grossman and Hart (1983), Hart and Holmstrom (1987) focused on the structure of the optimal incentive scheme based on a one-to-one relationship between the principal and the agent, when the principal acts as a monopolist. Jensen and Meckling (1976) is generally credited with introducing the concept of “agency cost”, which recognizes that delegation through imperfect an incentive contract adds to the cost of operating a firm. Other papers, such as Holmstrom (1982), Demski and Sappington (1984), and Mookherjee (1984), considered single principal-many agents problem while Bernheim and Whinston (1986) considered the common agency (many principals- single agent) problem. Each of these papers showed how incentive contracts could be designed to resolve selected issues.

A few papers have considered market structures other than a monopoly. Fershtman (1985) explained why a profit-oriented owner would want to deviate from strict profit maximization when delegating decisions in an oligopolistic market. In a duopolistic Cournot game, where the best response functions of both firms slope downward, a firm can credibly commit to increasing output by giving the manager an appropriate incentive. An “appropriate” incentive implies that the manager should not be encouraged to concentrate solely on
maximizing profits. As is well-known, the managers’ actions in a Cournot model are strategic complements of each other; Sklivas (1987) considered a parallel model of Bertrand competition and found comparable results.

Aggarwal and Samwick (1999) updated these ideas and tested some predictions. Empirically, they showed that the compensation given to a Chief Executive officer (CEO) is sensitive to the performance of rivals and that the coefficient is smaller in more concentrated industries (p. 2022). They also confirmed a common finding that, as a percentage of the firm’s profits, CEO incentives are not very large. Their theoretical development is noteworthy for its detail, and for what it omits. The main text presented a Cournot model and a Bertrand model of competition in a duopoly with differentiated products. Their appendices considered several variations on the problem.

Our paper differs from these papers in a couple of ways. First, Aggarwal and Samwick considered a model of hidden action with a random and common shock to the marginal cost of production for both firms. Second, they focused on the case of profit-based incentives and excluded the possibility of compensation based on sales. We consider a model of hidden information where managers can make difference decisions in different states of nature and they are compensated using a contract that can, in principle, use incentives more precisely. These differences are important because, in the familiar model of hidden information, owners can use reports on revenue or profits to accurately infer what the state of the market must have been and can use this inference to determine compensation. A common argument in the agency literature is that, without uncertainty about the state of demand, any incentive contract is weakly dominated by a contract with a suitably chosen production quota. We offer several reasons why this argument should be considered more carefully in a strategic environment.

Jones (1989) and Katz (1991) discussed how principals can use agency contracts to establish credibility when it would not otherwise exist. Jones used a general Nash bargaining model to study what the kind of agent a principal would select as a representative. He showed

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Fershtman and Jud (1987) extended Fershtman (1985) and, in its conclusion, noted that a “major weakness … [was] the absence of a detailed asymmetric information structure which motivates the use of contracts in the first place.” (p. 939). Fershtman and Jud thought that a moral hazard model would be more interesting.
that whether the agent is “tougher” or “weaker” than the principal varies with the details of the
game. Katz studied a game which the principal may or may not use an agent and focused on the
effects of the contract being unobservable. He noted that the benefit to a principal of delegating
decisions to an agent tends to be highest when only one principal is permitted to delegate. To
simplify his findings, he found that the use of an agent, even if compensated using an unobserved
contact, benefits a principal if the gain in efficiency between principal and agent is evident to
other players.

A larger literature may explore these same ideas in different disguises. For example,
many of the issues involved in the separation of ownership and management are similar to the
issues involved in the geographical separation of goods manufactured in one country and
consumed in another country: a person located in one country tends to be well informed about
either the production process or the selling process but not both. Thus, some part of the large
literature on foreign investment and international trade may be applicable or become an
application of our work. A brief investigation of this literature suggests that our model has not
been presented in this context. Markusen (1995) discussed different ways to organize a firm and
to mark its “boundaries”. More recently, Helpman and Grossman (2004) studied how (a
continuum of) heterogeneous firms in industry might jointly choose location and organization.
As we note below, our analysis depends the market being oligopolistic. For obvious reasons,
these authors were less concerned about policies or experiments which change the information
structure of a market.

A Model

Consider a Cournot duopoly model with each firm having one owner and a separate
manager. We assume that the inverse market demand function is:

$$P = A - b \times X$$

$$X = X_1 + X_2$$

where $P$ is the market price and $X_i$ denotes firm $i$’s output, $i=1, 2$. Each firm has constant
marginal cost $c_i \geq 0$, $i=1, 2$. We also suppose there are only two possible market demand levels
$\{a_h, a_l\}$, where market demand is high, i.e. $A = a_h$, with probability $k$. $0 \leq k \leq 1$ and the
probability that the market demand level is low equals $1-k$. $b, c_1, c_2$ and $k$ are commonly known
by all owners and managers, but the realization of $A$ is known only by the managers. $\max(c_1, c_2) < a_L$ ensures that both firms want to produce something in equilibrium for both each states.

Each owner hires a manager to operate the firm and, hopefully, to make some profit. Based on his incentive contract and his observations of the market demand level, manager $i$ chooses $X_i$ to maximize his payment. During the first stage of the game, the owners simultaneously set incentive contracts for their managers. These contracts must ensure each manager gets at least his reservation utility. During the second stage, the competing managers learn the state of the market and, knowing each other’s incentive contracts, simultaneously choose their outputs.

An important issue is the functional form of the incentive contract and the performance measures to be used in that function. Many measures can be considered and, in the common principal-agent model, the one-to-one relationship between sales, profits and output makes the choice unimportant. In a strategic environment, where $\partial (\partial \pi_i/ \partial X_i)/ \partial X_j \neq 0$, the actions of the other firm may create differences between these measures that are significant to an owner. For this reason, we propose that each contract is based on a linear combination of profits $\pi_i$ and sales revenue $R_i = P X_i$ for $i = 1, 2$ and $j = 3 - i$: manager $i$ is paid

$$G_i = M_i + N_i S_i \quad (2)$$

where

$$S_i = \alpha_i \pi_i + (1-\alpha_i) R_i \quad (3)$$

$$= \alpha_i (A - bX_i - bX_j - c_i) X_i + (1-\alpha_i) (A - bX_i - bX_j) X_i$$

$$= R_i - \alpha_i c_i X_i$$

Owner $i$ chooses $M_i$, $N_i$ and $\alpha_i$. In practice, changes in $N_i$ have no effect on any trade off relevant to the manager and the choices for $M_i$ and $N_i$ are unimportant so long as $N_i>0$ since, for any $N_i$, there exists $M_i$ that satisfies the manager’s participation constraint. Aggarwal and Samwick (1999) found a similar characteristic. Thus, an owner’s influence over her manager’s action depends on her choice of $\alpha_i$. If $\alpha_i = 1$, an owner wants her manager to maximize profits. As $\alpha_i$ falls, the weight given to sales performance increases.
We assume that the each manager knows the terms of these contracts given to the other manager but that this knowledge cannot be verified in a way that would make them contractible. This assumption can be justified by noting that the contract negotiated between a CEO and a Board of Directors is rarely published or told to shareholders. Alternatively, a manager in one company might know the terms of the contract given to another manager because they competed for the same position.

**Solution**

With this description of the game, we solve for the Subgame Perfect Nash Equilibrium. An equilibrium is described by \((\alpha_1^*, \alpha_2^*, X_1^*(\alpha_1, \alpha_2), X_2^*(\alpha_1, \alpha_2))\) where \((\alpha_1^*, \alpha_2^*)\) represent each owner’s best responses to the contract offered by the other owner and \((X_1^*(\alpha_1^*, \alpha_2^*), X_2^*(\alpha_1^*, \alpha_2^*))\) is each manager’s best response to the strategy of the other manager given the contracts.

First, we solve for the equilibrium to the last stage. Since no information is hidden between the managers, the stage is comparable to a standard Cournot game. Given \(\alpha_i, b, c_1, c_2\) and knowing the realization of \(A\), manager \(i\) chooses \(X_i\) to maximize \(S_i\). The first order condition to (3) shows that manager \(i\)’s best response is

\[
X_i^{BR} = \frac{(A - b X_j - \alpha_i c_i)}{2b}
\]

where \(i = 1, 2\) and \(j = 3 - i\). Combining the managers’ reaction functions, the quantity for each manager is:

\[
X_i^* = \frac{(A - 2 \alpha_i c_i + \alpha_j c_j)}{3b}.
\]

Deriving other parts of the second-stage equilibrium shows that

\[
P = \frac{A + \alpha_1 c_1 + \alpha_2 c_2}{3}
\]

\[
\pi_i = (P - c_i) X_i
\]

\[
= \frac{(A + \alpha_i c_1 + \alpha_2 c_2 - c_i)(A - 2 \alpha_i c_i + \alpha_j c_j)}{3b}
\]

\[
= \frac{(A + \alpha_i c_1 + \alpha_2 c_2 - 3c_i)(A - 2 \alpha_i c_i + \alpha_j c_j)}{9b}
\]

where \(i = 1, 2\) and \(j = 3 - i\).
The equilibrium solutions for $X_i^*$ ($i = 1, 2$) are functions of $(\alpha_1, \alpha_2)$ and the first stage of the game is needed to derive solutions for $(\alpha_1, \alpha_2)$. $A$ may be either $a_H$ or $a_L$ and the owner never knows the level of demand with certainty. Thus, the owner must choose $\alpha_i$ to maximize expected profits $E\pi_i$,

$$E\pi_i = k \pi_i(A = a_H) + (1 - k) \pi_i(A = a_L) \quad (8)$$

subject to participation incentive compatibility constraints. The first order condition, $\partial E\pi_i/\partial \alpha_i = 0$, implies that the best response function for owner $i$ is:

$$\alpha_{i, BR} = \frac{[ka_H + (1 - k)a_L] + 6c_i - \alpha_jc_j}{4c_i} \quad (9)$$

where $i = 1, 2$ and $j = 3 - i$. We assume that both owners choose their contracts simultaneously and, in equilibrium, $\alpha_i$ and $\alpha_j$ must be compatible. That solution is:

$$\alpha_i^* = \frac{[ka_H + (1 - k)a_L] + 8c_i - 2c_j}{5c_i} \quad (10)$$

for $i = 1, 2$ and $j = 3 - i$. Solutions for $M_i$ and $N_i$ can be found by combining equation (2) with a participation constraint.

**Proposition 1**

i) An equilibrium exists and is unique.

ii) If $c_1 < c_2$ then $\alpha_1^* < \alpha_2^*$.

iii) Either $\alpha_1$ or $\alpha_2$ or both are less than 1 in equilibrium.

**Proof**

i) We constructed the equilibrium solution.

ii) If $c_1 < c_2$ then $k a_H + (1 - k) a_L)/(5c_1) > (k a_H + (1 - k) a_L)/(5c_2)$ and $c_2/c_1 > c_1/c_2$. Thus, Equation (10) implies that $\alpha_1^* < \alpha_2^*$.

iii) The parameters are restricted to ensure that both firms produce positive outputs in
both states: $a_h > a_L > \max(c_1, c_2)$. Accordingly $(k a_h + (1-k) a_L)/(5c_i) > 1/5$. If $c_1 = c_2$, then equation (10) shows that $\alpha_1 = \alpha_2 < 1$.

If $c_1 < c_2$, then using equation (9), these inequalities imply that $\alpha_1 < 3/2 - 1/4 - \alpha_2/4$. Thus, if $\alpha_2$ were greater than 1 then $\alpha_1 < 1$. If $\alpha_2$ were less than 1, then part ii) shows that $\alpha_1 < \alpha_2 < 1$. Q.E.D.

Corollary 1

If $c_1 = c_2 = c$ then $\alpha_1 = \alpha_2 = 6/5 - (k a_h + (1-k) a_L)/(5c)$. Proof Insert $c_1 = c_2 = c$ into equation (10). Q.E.D.

Corollary 1 uses the case of equal marginal cost to create a benchmark for the incentives. Proposition 2 explores how changes in demand uncertainty affect the equilibrium contract.

Proposition 2

i) An increase in $k$ decreases $\alpha_i^*$, if $0 < k < 1$.

ii) An increase in $a_h$ or $a_L$ increases the weight given to sales in the incentive contract if $0 < k < 1$.

Proof

i) From equation (10), $\frac{\partial \alpha_i^*}{\partial k} = -(a_h - a_L)/c_i < 0$.

ii) Equation (10) shows that $\frac{\partial \alpha_i^*}{\partial a_h} = -k/(5c_i) < 0$, and that $\frac{\partial \alpha_i^*}{\partial a_L} = -(1-k)/(5c_i) < 0$: $\alpha_i^*$ decreases with $a_h$ and with $a_L$.

In both parts of this proposition, $k$ should not equal 0 or 1 since, if it were true, then a situation of asymmetric information would not exist. Contracts that rely on monitoring would become feasible and, perhaps, preferred. Q.E.D.

Manager i acts as though $\alpha_i c_i$ is his marginal cost, instead of $c_i$. A decrease in $\alpha_i$ causes the manager’s best response function to shift out beyond the ordinary profit-maximizing level: $\frac{\partial X_i^*}{\partial \alpha_i} = -2c_i/3b < 0$ and $\frac{\partial X_i^*}{\partial \alpha_j} = c_j/b > 0$. Encouraging a manager to increase output shifts rents between the firms but the rents available for shifting vary with demand, with the high demand state producing more total rent. The same encouragement in the low demand state may
force the equilibrium price below marginal cost. In equilibrium, an owner must weigh these
effects according to their probabilities. Thus, it is not surprising that an increase in the
probability of the high state increases the encouragement that owners give to managers to
increase production. Similarly, an increase in the demand in the either state would lead an owner
to encourage her manager to increase production.

This result differs from what would be expected if an owner delegated in the absence of
asymmetric information. If the demand curve shifted from low to high and both owners and
managers knew this then, effectively, $k$ changes from 0 to 1 in equation (10). Thus managers
would perceive sales to become a higher priority goal when demand is higher.

Corollary 1 implies that an increase in $c_1 = c_2$ increases $\alpha_1 = \alpha_2$ because an increase in
cost reduces the profit that can be redistributed using a carefully chosen, but distorting, strategy.
If the firms have different costs, each owner faces a trade off between the advantages of shifting
rents and the disadvantages of excess production. Proposition 3 shows that the owner of the low
cost firm would use the terms of the contract to exploit this advantage.

**Proposition 3**

If $c_2 > c_1$ then the market share of the low cost firm in this model exceeds the market share
of a low cost firm in a standard Cournot model.

**Proof**

Computation shows that the expected market share of the low cost firm in a market using
the demand curve shown in equation (1) is

$$\{1 + 3(c_2 - c_1)/([k a_H + (1-k) a_L] - [c_1 + c_2])/2. \quad (11)$$

In the equilibrium to our model, manipulating equation (5) shows that that market share of the
low cost firm increases to

$$\{1 + 5(c_2 - c_1)/([k a_H + (1-k) a_L] - [c_1 + c_2])/2. \quad (12)$$

Comparing these formulas establishes the proposition. Q.E.D.
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Sharing Information

This section compares the results of duopoly Cournot competition with and without information sharing. Sharing information enables an owner to offer incentives based on sources other than the manager’s report. Thus, it is possible to reduce the rents being earned by a manager in certain states by using a contract which matches the manager’s input to the state more carefully. In addition, we note that the activities being reported are endogenous to the market. Sharing information can be expected to change those activities and, depending on the type of contract being used, may expose a manager to the strategic risk associated with a deviation, and, thus, change the equilibrium.

If an owner can observe the state of market demand then it is a common argument that an owner in a monopolized market would force her manager to produce a pre-selected level of output using a “quota contract”: produce \( Q(A) \) in state \( A \). The owner benefits because \( q(.) \) would be chosen so that the manager would receive no more than his reservation payoff in all states. If the owner must compete with another managed firm then this argument suggests that both owners would use a quota contract. The actual level of the quota demanded by each owner would be determined as a best response to the actions of the other owner-manager pair. Proposition 4 shows that the owners lose the advantage of using a management contract to credibly increase output.

**Proposition 4**

Suppose that both owners know the state of demand. If both owners use quota contracts, then firm \( i \) produces

\[
X_i^*(A) = \frac{(A - 2c_i + c_j)}{(3b)} \quad \text{where } A = a_H \text{ or } a_L; \ i = 1, 2; \ j = 3 - i
\]

in equilibrium.

**Proof**

When both owners use quota contracts, both owners solve a simple Cournot game knowing the level of \( A \). An equilibrium is defined by \( (Q_1(A), Q_2(A)) \) where, in state \( A \), \( Q_i(A) \) is owner \( i \)’s best response to \( Q_j(A) \) and \( Q_i(A) \) is owner \( i \)’s best response to \( Q_j(A) \). The managers have become order takers and, assuming that the penalties for not acting according to the quota are sufficiently severe, they are not independent economic actors.
Given this definition of an equilibrium, this game is a standard Cournot game where the equilibrium output for each firm is found in the usual way. If \( Q_i(A) = X_i^*(A) \) then the definition of an equilibrium is satisfied. Q.E.D.

The equilibrium price and the profit, conditional on \( A \), are

\[
P(A) = \frac{A + c_1 + c_2}{3} \quad (14)
\]

\[
\pi_i(A) = bX_i^2 = \frac{(A - 2c_i + c_j)^2}{9b} \quad (15)
\]

where \( A = a_H \) or \( a_L; \ i = 1, 2; \ j = 3 - i \). Thus, the expected output and profit are

\[
EX_i = \frac{[ka_H + (1-k)a_L] - (2c_i - c_j)}{3b} \quad (16)
\]

\[
E\pi_i = \frac{\text{var}(A)}{9b} + \frac{\{[ka_H + (1-k)a_L] - (2c_i - c_j)\}^2}{9b} \quad (17)
\]

The chart below compares this equilibrium with the equilibrium using incentive contracts derived above when there was asymmetric information. Notice that expected profits are higher in the equilibrium with quota contracts because, though privately profitable, the ability to shift rents by means of incentive contracts led to a Prisoner’s Dilemma situation.

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<th>Cournot Competition with Incentive Contracts</th>
<th>Cournot Competition with Quota Contracts</th>
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<tbody>
<tr>
<td>( EX_i )</td>
<td>( \frac{2{[ka_H + (1-k)a_L] - (3c_i - 2c_j)}}{5b} ) &gt; ( \frac{[ka_H + (1-k)a_L] - (2c_i - c_j)}{3b} )</td>
<td></td>
</tr>
<tr>
<td>( P(A) )</td>
<td>( \frac{5A - 2[ka_H + (1-k)a_L] + 6(c_1 + c_2)}{15} ) &lt; ( \frac{A + c_1 + c_2}{3} )</td>
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<tr>
<td>( E\pi_i )</td>
<td>( \frac{\text{var}(A)}{9b} + \frac{2{[ka_H + (1-k)a_L] - (3c_i - 2c_j)}^2}{25b} ) &lt; ( \frac{\text{var}(A)}{9b} + \frac{{[ka_H + (1-k)a_L] - (2c_i - c_j)}^2}{9b} )</td>
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The preceding discussion assumed that owners knew the state of the market but was a bit
vague about how that knowledge could be obtained. The introduction noted that many countries have recently passed laws to increase the financial disclosure by managers to shareholders. Making this information public implies that owners have can get a more complete picture of their market ex post. Industry associations also play this role if by reporting statistics compiled from their members. In either case, compensation or quotas can vary with this information.²

Formally, we assume that each owner knows $P$, $X_1$ and $X_2$ after the fact and use equation (1) to estimate $A$:

$$A_e(P, X_2) = P + b(X_1 + X_2).$$

(18)

Since we assume a particularly simple demand curve, this estimate is accurate. Thus, each owner could solve for what her (temporarily better informed) manager’s action should have been in the subgame and punish any deviation. Mimicking equation (4), that quota for manager 1 should be

$$Q_1(P, X_2) = (A_e(P, X_2) - bX_2 - c_1)/2b.$$  

(19)

This equation specifies a quota that depends on measures not controlled by manager 1 and revealed after the manager makes his decision. At the same time, the owner and the manager of firm 2 would be thinking through the same set of issues to solve for $Q_2(P, X_1)$. In a pure strategy equilibrium, this punishment can be as severe as imaginable since it never needs to be implemented on the equilibrium path.³

The only caveat to this argument is that manager i chooses his best response action by anticipating manager j’s action but the actual action of manager j may differ from what manager i anticipated. Thus, a player may face a strategic risk, rather a risk due to a random state of nature, in the sense that players at the other firm may deviate from a plan of action. A quota

² More informally, during year-end performance reviews, the manager may be able to use such evidence to renegotiate the initial contract. Katz (1991) argued that renegotiation undermines the commitment effect of any particular contract but, since renegotiation can be expected to increase ex post efficiency, other parties to the game should anticipate this fact. Beaudry and Poitevin (1995) studied the effect of renegotiation in a single principal-single agent model and found that its effect on the set of contracts was less than expected.

³ It might be possible to create a contract which specifies payment in only two possible outcomes corresponding to the two possible states of nature. We ignore this solution because it seems to exploits the simplicity of our model unfairly. It also reinforces our argument about the role of potential deviations and “strategic risk”. A manager who signs a contract that specifies a (potentially severe) punishment in all but two outcomes demonstrates a strong belief about what the other owner and other manager will do. It might be interesting to investigate the properties of this model in the context of an evolutionary model which starts with less restrictive assumptions.
contract is special because the manager is compensated or punished based on a particular choice of output. In a principal-agent model with only one owner-and manager, the profit or sales revenue implied by a particular choice of output is clear. Thus, it is equivalent to punish a manager for failing to produce an output quota or a target profit.

In a duopoly, the link between a manager’s choice of output, its profit, revenue and the actions of the other firm is less clear because

\[ \frac{\partial (\frac{\partial \pi_i}{\partial X_i})}{\partial X_j} < 0 \quad \text{and} \quad \frac{\partial (\frac{\partial R_i}{\partial X_i})}{\partial X_j} = -b < 0, \]

for \( j \neq i \), in a Cournot model: the first inequality is true because an increase in \( X_j \) decreases \( i \)’s preferred level of output and the second inequality is true because the marginal revenue curve associated with the residual demand curve is downward sloping. The following proposition extends this idea to show that, in an environment where information is shared and managers are compensated using a contract that depends on shared information, the equilibrium outputs produced when using quota contracts in Proposition 4 is no longer an equilibrium.

**Proposition 5**

Suppose that managers are paid according to equations (2) and (3) where \( \alpha_i \) is allowed to vary with \( A^e \). Then \( \alpha_i \neq 1 \) in equilibrium.

Proof (by contradiction)

If information is shared and the incentive contract were equivalent to the quota contract, then comparing equations (4) and (13) shows that a necessary condition is that \( \alpha_i(A^e) = 1 \). If so, then the other owner would recognize that deviating from this equilibrium by decreasing \( \alpha_i(A^e) \) would change manager \( i \)’s subsequent behavior and since \( \frac{\partial (\partial \pi_i/\partial X_i)}{\partial X_j} \neq 0 \). This change in behavior creates a first order change in the profits for the other owner because it shifts the residual demand curve. Q.E.D.

The fundamental problem in a situation with strategic interaction and agency contract is that an owner wants to use the contract to get her agent to react appropriately to changes in the state of nature while the other owner exploits the incentives embedded in the first contract during the second stage. Corollary 2 shows where these effects find balance each other.
Corollary 2

Suppose that managers are paid according to equations (2) and (3) where \( \alpha_i \) is allowed to vary with \( A^e \). Then, in equilibrium,

\[
\alpha_i(A^e) = \frac{8}{5} \cdot \frac{A^e}{5c_j} - \frac{2c_j}{5c_i}.
\]

Proof

This result is derived from equation (10) under the assumption that both owners are fully informed about the state of demand. Q.E.D.

Concluding Remarks

This paper explores the role of incentive contracts in a duopoly Cournot game. As previous authors have found, we find that each owner exploits the separation of ownership and management to create a credible commitment to increase output and to shift rents. We also find that the incentives within the competing firms interact with each other in meaningful ways. Most importantly, we show that the effect of incentives depends on why incentives are needed: e.g. the presence of asymmetric information. We show how the terms of the contract vary with parameters associated with asymmetric information: e.g. that an increase in the probability of a good outcome magnifies the incentive given to the managers to increase output.

We also show that if firms are forced to disclose more information and share it with a wider audience, then the asymmetry of information between an owner and a manager disappears. But, the effect on the equilibrium depends on how that information is used. In a simple principal-agent model, every quota contract can be replicated by a suitable incentive contract. In a duopoly setting, this argument is incomplete since the surplus to be divided between the owner and her manager varies with the actions of the other firm. Depending on the form of the contract, it must also deter deviations from the equilibrium path. Consequently, if owners use a quota contract then a manager’s compensation is immune to actions of other firm but they lose the ability to shift rents relative to the Cournot equilibrium. Because this variation on the game has a flavor of a Prisoner’s Dilemma, losing this ability actually increases expected profits. If owners use an incentive contract then the terms of the contract need to change because the surplus available in each state depends on the other firm and because the contract needs to
recognize the strategic risk associated with the other firm deviating from what is anticipated. This kind of risk might be higher in markets where the participants change often.

We note that these results are special to the situation of an oligopoly with asymmetric information. If the market were a monopoly then the results of classic theory apply. If the market were perfectly competitive then the market price is a sufficient indicator of the state of demand and no individual can influence it. Even in an n-player Cournot game, as n approaches infinity, the significance of strategic risk falls to zero because the difference between the equilibrium price if another owner deviates and the price without deviation falls to zero.

Our analysis focused on asymmetric information about the state of demand. We think that asymmetric information about the cost of production would have different effects. Solving this problem may be easier since many, but not all (see Aggarwal and Samwick’s (1999) theory), of the sources for cost shocks seem to be firm-specific. It is not unreasonable to suppose that a manager should be made responsible for their solution. Our analysis suggests that if the contract contains incentives related to profits or sales then the rivalry between the two managers would change because each would be exposed to strategic risk.

Finally, this paper was motivated by a desire to understand the owner-manager relationship when there is asymmetric information in a strategic environment. This kind of model should help to understand the effect of laws passed after recent corporate scandals in many parts of the world. A common reaction to the scandals has been to increase the oversight of managers by increasing the amount of information that is disclosed (e.g. the Sarbanes-Oxley Act in the US). People seem to be concerned about this issue mostly because of the effects on financial markets. The fact that we focus on sales revenue as a significant aspect of the incentive contract shows that any effects of sharing information on how financial markets allocate scarce financial capital or on the investment plans of a firm are independent effects.

The effects of sharing information on market structure are more subtle. Since asymmetric information increases the advantage to a low cost firm, requiring firms to disclose more information should reduce the strategic advantage of a low cost firm. In our Cournot
model, this change also raises the equilibrium price though this finding would change if we replaced the Cournot model with a Bertrand model. Disclosing information may reduce the rents that can be extracted by managers but we show that it also has dramatic effects on the tactics used to shift rents between competitor firms, which may or may not also benefit consumers.
References


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