Money, Interest Rate, and Output in the Open Economy:  
A theoretical and empirical investigation

Comments welcome

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Abstract

We develop open-economy variants of the old Friedman-Schwartz and the new Lucas-Sargent-Wallace monetarist models to investigate the puzzle of monetary neutrality. We further introduce financial aggregation theories into the models – theories that are, in the modern world, consistent with financial liberalization and innovations in the banking payments system. We then study the theoretical and business-cycle relationships between real output and financial aggregates, interest rates, exchange rate, and prices using Canadian quarterly data for the period 1959:1 to 2002:1. We find that the open-economy variants of the monetarist models with aggregation-theoretic financial aggregates perform the best in producing significant sign patterns that are predicted by theory – resolving the ‘twin’ money and interest rate puzzle in previous research. Furthermore, Monte Carlo experiments show that large percentage of real output variance is explained by shocks to aggregation-theoretic financial aggregates relative to other variables -- principally, the rate of interest and the exchange rate. Thus, there is no difference between anticipated and unanticipated monetary shocks. The policy implication is that the correct measurement of money and its opportunity cost as well as a robust specification of the money-output relationship improves the information content of monetary policy.

Keywords: Aggregation theories, user cost of money, long memory, open economy.

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1. Introduction

In the past decades, monetary economics has faced two major challenges: empirical and theoretical. Empirics on the old monetarism – that emphasizes non-neutrality between the level of money stock and the level of aggregate real economic activities (Milton Friedman and Anna Schwartz, 1963; Anderson and Jordan, 1968; Sims, 1972; Christiano and Ljungqvist, 1988) – was challenged due to the weakening of non-neutrality in the 1980s. Among others, Friedman and Kuttner (1992, 1993) have documented that including data from the 1980’s sharply weakens the post-war time-series evidence indicating significant relationship between money and real income.

Theorizing in the new rational expectations monetarism of the 1970s demonstrated that it is surprises in movements in the money stock that generate non-neutrality. Anticipated changes in aggregate demand policy have no output response; thus, deterministic feedback policy rules have no impact on real output (Lucas, 1973; Sargent and Wallace, 1975; also Mishkin, 1982). Ironically, although the old monetarism has more policy relevance than the new monetarism, continued empirical investigation has rejected the proposition of the latter, but the empirical work remained erratic and failed to refute the weakening of non-neutrality of the former.

Empirical work on the old Friedman-Schwartz monetarism (henceforth, MFAS monetarism) ascribed the weakening of evidence on non-neutrality to instability in the demand for money function. This instability was in turn attributed to the liberalization of financial markets and innovations in the banking payments system that distorted the traditional definition of money as a medium of payments (Friedman, 1988; Handa, 2000). Thus, improper financial aggregation\(^1\) and inappropriate choice of the opportunity cost of money\(^2\) were sources of the controversy on non-neutrality of money. Among others, Belongia (1996) documented that inferences about the effects of money on real output depend importantly on the choice of financial index because simple-sum aggregates cannot internalize pure substitution effects. His replications of studies that have challenged an aspect of the “conventional wisdom” about the non-neutrality of money on real activity showed that the qualitative inference in the original study is reversed when a simple-sum financial aggregate is replaced by a divisia index of the same asset collection.

Empirical work testing the neutrality results of the new Lucas-Sargent-Wallace rational expectations monetarism (henceforth, the LSW monetarism) could be grouped into two. Some

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\(^1\) As simple-sums of component assets.
\(^2\) As a return on alternatives to money holdings.
distinguish between the effects of anticipated and unanticipated monetary shocks on real output (Barro, 1977, 1978; Barro and Rush, 1980; Gordon, 1982; Mishkin, 1982), while others distinguish between the asymmetric effects of negative and positive unanticipated changes in money on real economic activities (e.g., De Long and Summers, 1988; Cover, 1992; Ravn and Sola, 2004). Handa (2000) reviews and elaborates the literature in this tradition. Unlike empirical tests of the MFAS monetarism which break down when 1980s data are included in the sample, empirical studies using the LSW framework have documented significant effects of anticipated monetary shocks and the existence of asymmetric effects of unanticipated changes in money on real output – regardless of the sample period.

Previous empirical work on the effects of money on real output in general, however, relies on unadjusted, simple-sum financial aggregates and/or short-term interest rates as measures of policy stance. Yet simple-sum aggregates, which central banks publish, are based on the implicit assumption that the component assets are perfect substitutes – they have the same degree of moneyness. Nevertheless, although financial innovations have blurred the distinction between transactions and savings-type assets, the assets that make up the broader aggregates do not have the same liquidity as currency. Barnett (1980, 1981) has shown that statistical index number approaches – such as superlative indexes, in particular divisia aggregates – provide the correct definition to the aggregates. While simple-sum aggregates presume an equal weight for all component assets of an aggregate, divisia aggregates use the user-cost of money to weight the relative importance of component assets in generating financial services. Despite developments in the aggregation-theoretic literature, the literature on the money-output relationship employs simple-sum aggregates as financial quantities and the return on alternatives to money holdings as financial prices. Thus, empirical results of previous work on the effects of money on real output suffer from biases resulting from measurement error in money and its user cost.

Previous empirical work that tests the money-output relationship in general and the LSW monetarism in particular also suffers from an endogeneity bias. The empirical methodology used to test the effects of anticipated changes in money on real output and the asymmetric effects of unanticipated monetary shocks on real economic activities rests on the contribution of Mishkin (1982). Nevertheless, Mishkin’s methodology, nonlinear least-squares estimation of the joint money and output equations, proceeds with the identifying assumption that the output equation is

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3 In a nutshell, the aggregation-theoretic financial aggregation literature demonstrates that the financial aggregates that are internally consistent and theoretically plausible are not simple-sum aggregates. Nor is the price of money the return on alternatives to money holdings.
a true reduced form. However, as Mishkin himself notes, in the case where only contemporaneous money appears in the output equation, the imposition of this assumption will not invalidate the test statistics of the empirical exercise. Consequently, it is unclear that the reduced-form assumption holds when lagged money enters the output equation. Thus, previous methodology may be subject to endogeneity bias when the output equations include contemporaneous money with endogenous determination, lagged-money, and other explanatory variables.

Previous empirical work on the money-output relationship also suffers from omitted variables bias. Once again, in Mishkin’s setting for example, it is only money that has real effects. However, earlier work on testing the old MFAS monetarism (e.g., Sims, 1980; Friedman and Kuttner, 1992, 1993) has revealed that results change when other explanatory variables for output, such as the price level and the rate of interest, enter the estimating equation. More importantly, among others, Sims (1980) showed that interest rates tend to absorb the effects of money on output. In addition, empirical tests of both the old MFAS and the new LSW monetarism generally rely on closed economy setting. Closed economy setting may be plausible for large economies like the U.S. However, in small open economies like Canada, New Zealand, Australia, and developing countries, external factors could have significant effects on domestic economic activities. Excluding international factors in explaining domestic real economic activities results in omitted variables bias of the estimated coefficients of money.

This paper investigates whether the existing evidence on the money-income relationship in the old MFAS versus the new LSW monetarism has been driven by theoretical, aggregation, and empirical limitations inherent in previous research. The paper contributes to the literature in four principal ways. First, we extend the theory underlying the money-income relationship to incorporate additional domestic factors such as the price level and the rate of interest as well as international factors that have effects on real economic activities in small open economies. Second, we use the aggregation-theoretic approach to measure financial service flows and the user costs of money. Third, we employ empirical methodologies that account for possible endogeneity of all of the variables in the extended money-income relationship – including a long-memory procedure which is very recent in its practical application. Fourth, we use standardized

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5 Appendix table 1 provides a formal treatment of the possible empirical limitations of previous research.
6 Specifically, we study the theoretical relationship between output and its explanatory variables using system short-memory as well as long-memory procedures. We also study the business-cycle dynamics using vector error-correction model. Based on the vector error-correction, we analyze Monte Carlo forecast-error variance decomposition in order to explore the effects of unanticipated changes in money on real output.
and the longest available time-series data on output, money, the rate of interest, prices, and international factors.

We conduct the study on Canadian quarterly data. Canada is one of the most interesting economies for testing the effects of money on real output while controlling for other, mainly international, factors. First, Canadian experience allows the identification of variables other than domestic monetary policy shocks that explain domestic income. For example, since Canada is a small, open economy that relies significantly on international trade, it is affected, among others, by movements in world commodity prices. In addition, Canada has close economic ties with the United States. As a result, Canadian real economic activities are affected by U.S. economic factors such as U.S. monetary policy and U.S. import demand as well as disturbances in U.S. labor and financial markets. By introducing these factors into the Canadian money-income relationships, we can reduce the omitted variables bias in testing the hypothesis on the old MFAS as well as the new LSW monetarism. Second, Canadian monetary policy has been aimed at managing short-term nominal interest rates (Bernanke and Mishkin, 1992; Duguay, 1994; Thiessen, 1995). This allows a flexible movement of the own rates of return of the asset collection in the financial aggregates around the benchmark rate and thus provides variable user-costs of money that facilitate an aggregation-theoretic analysis.

We find that the open-economy variants of the monetarist models with aggregation-theoretic financial aggregates perform the best in producing significant sign patterns that are predicted by theory – resolving the ‘twin’ money and interest rate puzzle in previous research. Furthermore, Monte Carlo experiments show that large percentage of real output variance is explained by shocks to aggregation-theoretic financial aggregates relative to other variables -- principally, the rate of interest and the exchange rate. Thus, there is no difference between anticipated and unanticipated monetary shocks. The policy implication is that the correct measurement of money and its opportunity cost as well as a robust specification of the money-output relationship improves the information content of monetary policy.

The rest of the paper is organized as follows. Section 2 reviews theoretical models of the MFAS and LSW monetarism. In this section, we critically review and extend the models to account for domestic factors other than money and foreign variables that may have effects on domestic real economic activities. Section 3 presents the methodology for constructing aggregation-theoretic financial aggregates and the user-cost of money. Section 4 discusses the empirical methodologies that account for possible endogeneity of all of the variables in the augmented money-output relationship. Section 5 presents the empirical results. And, section 6 concludes.
2. The Monetarist Models and their Extensions

We specify a simple model for the relationship between money and output as advocated by the MFAS and/or the LSW monetarism. We extend the model to accommodate other domestic variables and international factors that affect domestic real economic activities.

Specifically, we allow the level of money and its anticipated and unanticipated components to affect real output. That is,

\[ y_t = y^*_t + a^*(L)[\tau m_t + (1 - \tau)(m_t - m^e_{t-1})] + \epsilon_t \]  

(1)

where \( y_t \) is real output at time \( t \); \( y^*_t \) is the natural level of real output at time \( t \); \( m_t \) is money at time \( t \); \( m^e_{t-1} \) is anticipated money conditional on information available at time \( t-1 \); \( \epsilon_t \) is the error term.

As in Cochrane (1998), the asterisks on \( a^*(L) \) denote a structural lag polynomial; and \( \tau \) is a pre-specified parameter that varies between 0 and 1.

As \( \tau \) approaches 1, this model in (1) is similar to the specification that there is no difference between anticipated and unanticipated monetary shocks. In this case, the model is in the tradition of the old MFAS monetarism.\(^7\) In this setting, the model has been used by Sims (1972, 1980) and Friedman and Kuttner (1992, 1993) to investigate the money-income relationship. Sims (1980) found that interest rates tend to absorb the role of money. Friedman-Kuttner documented the weakening of non-neutrality when 1980s data are included in the sample.

However, as \( \tau \) approaches 0, the model in (1) specifies that only unanticipated money matters. In this case, the model is in the tradition of the new LSW monetarism. This model has been used by Barro (1977, 1978, 1979), Barro and Hercowitz (1980), Boschen and Grossman (1982), and, in particular, Mishkin (1982). In this vein, Barro (1977, 1978, 1979), Germany and Srivastava (1979), Small (1979), Barro and Rush (1980), Leiderman (1980), and Mishkin (1982) tested the hypothesis on the neutrality implication of the new LSW rational expectations monetarism, that anticipated monetary policy does not matter, and found evidence to the contrary. Similarly, De Long and Summers (1988), Cover (1992), and Ravn and Sola (2004) have tested the asymmetric effects of monetary policy shocks on real economic activities and found strong support for the traditional Keynesian and the menu cost predictions of asymmetry in U.S. data.

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\(^7\) There are two slight differences. First, in the MFAS monetarism proper, there are additional domestic factors that enter (1), albeit at the discretion of the researcher and often without explicit discussion – i.e., just as a tradition. Second, in (1), we are measuring real output as a deviation from its natural level in conformity with modern research.
Nevertheless, in all of the above studies $m$, is a simple-sum aggregate that does not internalize pure substitution effects by measuring only income effects in household asset allocation. Here we improve over previous research by measuring financial service flow using an aggregation-theoretic approach that captures both the substitution and income effects. Furthermore, in its strictest form, as in the methodology of the LSW rational expectations monetarism, model (1) assigns variables other than money that could have direct and indirect effects on output to the error term. Unfortunately, since the omitted variables are at least partially correlated with money, in (1), the latter is correlated with the error term; hence, the estimated parameters on money and its lags are possibly biased. In order to account for this possible omitted variables bias, we assume that the error term follows an AR (1) process and we model it as:

$$\varepsilon_t = b^*(L)x_t + c^*(L)x^*_t + \nu_t, \quad \text{with} \quad \varepsilon_t = \gamma \varepsilon_{t-1} + \eta_t \quad (2)$$

where $x_t$ is a vector of domestic variables other than money, $x^*_t$ is the vector of foreign factors; $\nu_t$ and $\eta_t$ are error terms, and $b^*(L)$ and $c^*(L)$ once again denote structural lag polynomials. What do vectors $x_t$ and $x^*_t$ include?

2.1 Domestic factors: prices and interest rates

What enters the vector of domestic variables $x_t$? It is easier to determine the elements of domestic factors, although the choice among the alternatives to measure each factor requires a thorough discussion. Since Sims (1972), tests for the effects of money on real income usually control for the effects of prices and interest rates. The inclusion of prices in the vector of domestic factors is justified on two grounds. On the one hand, changes in money influence changes in prices. Thus omitting prices overstates the effect of money on real output. On the other hand, lags of the price levels are included because it is apparently real money and real interest rate that affect real variables.8

Evidence on the effects of money on real output shows that the marginal predictive power for income alters when the analysis includes an interest rate. For instance, first Sims (1980) and then Litterman and Weiss (1985) found that the predictive power of money for real income weakens when interest rates are added into the relationship. Friedman and Kuttner (1992, 1993) documented that the predictive power of money for real income breaks down with and without interest rates when the 1980s is included in the sample; yet, they also report that the spread

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8 There is little difference between putting the price level or the inflation rate in the equation, once many lags are used in the exercise.
between the commercial paper rate and the Treasury bill rate consistently contains highly significant information about future movements in real income.

In general, the inclusion of interest rates in empirical work of the money-income relationships is now a tradition. However, in this literature, there has been little discussion about the relevance of the interest rate that is chosen. At the same time, the choice of interest rates has not been consistent. There are different interest rates and their spreads that have been used in the literature: the commercial paper rate, the Treasury bill rate, the Federal funds rate, long-term government bond yield, the paper-bill spread, and the yield spread. It has also been common to use some combinations or group of these rates. For example, among studies that tested the money-income relationship, Sims (1980) and Friedman and Kuttner (1993) used the commercial paper rate, Litterman and Weiss (1985) and Eichenbaum and Singleton (1986) used the Treasury bill rate. Bernanke and Blinder (1992) used the Treasury bill rate, the Federal funds rate, and the long-term government bond yield but found the funds rate to have strong predictive power. Bernanke (1990) used the paper rate, the bill rate, the bond yield, and the paper-bill spread and found the spread to be a strong predictor of real income. Friedman and Kuttner (1992) used the paper rate, the bill rate, and the paper-bill spread and found strong support for the paper-bill spread.

Similarly, among studies that tested the asymmetric effects of unanticipated monetary shocks, Macklem et al. (1996) used the yield spread while Ravn and Sola (2004) used the Federal funds rate. However, with the exception of Freidman and Kuttner (1993) on paper-bill spread, all of these studies did not provide substantive arguments that substantiate their selection of the different rates except that the evidence they gathered supports the interest rate or and the spread of their choice.

The ultimate question is therefore that: “Which interest rate is appropriate and why?” Evidently, different financial aggregates measure different financial market quantities. Equivalently, different interest rates measure different financial market prices. Thus, there is not only a need to measure the right quantity but also the corresponding correct price. Unfortunately, the literature that tests the money-income relationship measures a financial quantity using simple-sum aggregates -- which do not internalize pure substitution effects in household asset selection. It also measures the price of money stock as the return on alternatives to money holdings -- which is an incorrect measure of the foregone return on a financial service flow from a durable asset. Nor does the literature always assign the right prices to the right quantities: a single interest rate is

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9The yield spread equals long-term rate less short-term rate.
used for any financial quantity that is chosen by a researcher. Thus the empirical results in the existing literature might have been driven by measurement problems.10

In this paper, we employ an aggregation-theoretic approach to correct the measurement of a financial aggregate and its opportunity cost. Nevertheless, to conduct a comparable exercise in the tradition of the existing literature, it is necessary to choose among the many interest rates and/or their spreads that have been used in the literature.11 Here we focus on the choice between the short rates, which are used in the money-income relationship and to calculate the yield spread and paper-bill spread. Among others, it is an open question as to whether the corporate paper rate or the Treasury bill rate provides better gauge of the financial prices that matter for the determination of real economic activities (Friedman and Kuttner, 1993).

Clearly, the commercial paper rate is the interest rate on short-term unsecured borrowing by corporations. Analogously, the Treasury bill rate is the unsecured borrowing rate for the government. However, as Friedman and Kuttner (1993) argue, the commercial paper rate more directly reflects the cost of finance corresponding to potentially interest-sensitive expenditure flows than does the Treasury bill rate. As a result, the commercial paper rate is superior to the Treasury bill rate in measuring this effect. That is, the effect of interest rates on real economic activities is transmitted through their influence on the spending behavior of private sector borrowers. Among others, although both interest rates are unsecured, borrowers may default on the paper but do not do so on the bill. Thus, over a business cycle, when changes in the expected default risk of the average business change the paper-bill spread, the commercial paper transmits more information about the borrowing costs that may affect spending flows. However, as far as savers and investors are concerned, who can be assumed to be risk-averse, the Treasury bill rate is more relevant because it nearly represents the return available to most savers (Friedman and Kuttner, 1993).

While this argument makes intuitive sense, its practical application by, among others, Friedman and Kuttner (1992, 1993) themselves is incorrect. They used either the paper rate, or the bill rate, or the paper-bill spread in money-income relationships that included m1, m2, base money, and commercial bank credit to the private sector – all of which are simple-sum aggregates. This is clearly an incorrect measure of the opportunity cost of the different aggregates: a single price for very different quantities. Here, we improve on previous work by employing aggregation-theoretic approach to measure a user cost for each asset in an aggregate.

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10 Appendix table 1 provides a detailed account of the empirical problems of measurement error.
Thus, we contribute to the literature by constructing and using not only aggregation-theoretic quantities but also prices.

2.2 International factors: the exchange rate

What enters the vector of international factors $x^i_t$? As we have argued in the introduction to this paper, although international factors may have little influence on the domestic output of large economies such as the U.S., they have significant influence in small open economies like Canada, New Zealand, Australia, and open-economy developing countries. In Canada in particular, several factors other than domestic monetary policy shocks may have an impact on domestic real output growth. For example, as we discussed earlier, Canada is a small open economy that relies heavily on international (U.S.-Canada) trade. Since Canada is a net exporter of commodities, it is influenced by movements in world commodity prices. In addition, because of Canada’s close economic ties with the U.S., Canadian output growth is likely to be affected by U.S. factors such as U.S. import demand and the Federal Reserve monetary policy. Furthermore, international financial as well as labor market movements have considerable impact on Canadian domestic real economic activities.

How can we capture international factors? Duguay (1994) emphasizes the importance of changes in world commodity prices, shifts in foreign (in particular U.S.) demand, and U.S. monetary policy to Canadian output fluctuations. To this end, as in Macklem, et al. (1996) on the asymmetric effects of monetary policy shocks, different variables such as real non-energy commodity prices, real price of oil, U.S. real output growth, U.S. real import growth, and Boschen and Mills index of U.S. monetary policy stance could be used to proxy external factors.

However, we argue here that the Canada-U.S. exchange rate contains more information than the factors listed above individually or as a group. Exchange rates in small, open economies in general and in Canada in particular could summarize growth in U.S. real output and U.S. import demand, Fed’s monetary policy, and oil price shocks initially through their effects on U.S. real economic activities. In addition, exchange rates capture movements in foreign financial markets over and above commodity market fluctuations. As a result, in Canada for example, a Monetary Conditions Index (MCI), a weighted average of the short-term interest rate and the exchange rate, has been used as a composite indicator for the stance of aggregate demand (Handa, 2000). Therefore, this paper uses the exchange rate to open the closed economy of the old MFAS and the new LSW monetarist models.

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12 They find external factors to have no contribution to the asymmetric effects of monetary policy shocks.
2.3 Summary: the augmented empirical model

Thus, given (1) and (2), our augmented model for the empirical exercise in the paper can be written as:

\[
y_t = y_t^* + a^*(L)[m_t^c + (1-\tau)(m_t^c - m_t^{c1})] + b^*(L)x_t^* + c^*(L)x_t^* + \nu_t,
\]

where \( y_t \) is domestic real output at time \( t \), \( y_t^* \) is the natural level of real output at time \( t \), \( m_t^c \) is aggregation-theoretic correct (c) money, \( m_t^{c1} \) is expected aggregation-theoretic correct money based on information available at time \( t-1 \), \( x_t^c \) is a vector of domestic variables other than money and includes prices and an aggregation-theoretic correct opportunity-cost of money, \( x_t^* \) is a vector of international factors summarized by the Canada-U.S. exchange rate, and \( \nu_t \) is the error term. Asterisks on \( a^*(L) \), \( b^*(L) \) and \( c^*(L) \) denote structural lag polynomials; and \( \tau \) is a pre-specified parameter that varies between 0 and 1.

As \( \tau \) approaches 1, (3) specifies that there is no difference between anticipated and unanticipated money. This specification is similar to the old MFAS monetarism, but augmented to account for explicit domestic factors, namely, prices and interest rates, and foreign factors summarized by the exchange rate. Furthermore, we are measuring output as a deviation from the full employment level in line with modern research. We also use aggregation theory to measure financial quantities and their prices.

As \( \tau \) approaches 0, however, (3) specifies that only unanticipated monetary policy matters. This is similar to the new LSW rational expectations monetarist model, augmented to take into account prices, interest rates, and the exchange rate. Here too, we measure output as a deviation from full employment, and we use aggregation theory to measure financial quantities and their prices.

3. Financial Aggregation Theory

The introduction of microeconomic and aggregation-theoretic foundations into monetary economics was marked by Barnett (1980, 1981). In his work, he introduced the theoretical linkages between aggregation theory and monetary theory. Based on microeconomic aggregation theory, he developed and applied the theory needed to construct monetary aggregates -- on the recognition that financial assets are durable goods. He demonstrated that a financial aggregate that reflects the flow of financial services during a holding period is not equal to the financial
stock -- because a durable good does not depreciate fully during one time period. He proposed weighting individual assets in the collection of an aggregate to reflect the flow of services.

Consequently, the price of the flow of financial service is the user cost or the opportunity cost of holding financial assets per unit of time. The real user-cost price is therefore the present value of the interest foregone by holding the asset in the presence of a higher rate of return on a benchmark asset — which is a pure investment asset providing no financial services (Barnett, 1980). Thus, the user-cost of financial asset \( i \) during period \( t \) is defined as:

\[
\pi_i = \frac{(r_t - r^*_t)}{(1 + r^*_t)},
\]

where \( r^*_i \) is the own rate of return on asset \( i \), and \( r^*_t \) is the yield available in the economy on a benchmark asset.

Given a well-defined flow of financial service and its user-cost, the aggregation-theoretic financial aggregate can be correctly measured using statistical index number theory. The class of “superlative index numbers” defined by Diewert (1976) is a class of statistical index numbers to follow aggregator functions. The divisia index is a well-recognized superlative index with extensive application to financial aggregation. As a result, in this paper, we use the divisia index.

With the user-cost and quantity data, the expenditure share on financial asset \( i \) in period \( t \) \( m^*_i \) is \( s^*_i = \pi_i m^*_i / \sum_j \pi_j m^*_j \). A divisia quantity index computes the growth rate of the financial aggregate as the share-weighted average of its components. It is thus defined as:

\[
\Delta \log m_t = \sum_{i=1}^n s^*_i \Delta \log m^*_i,
\]

where \( s^*_i = (1/2) (s^*_i + s^*_{i-1}) \). Thus, (3) yields a chain-weighted measure. Further, the same index number results regardless of what the aggregation-theoretic aggregate approximates – e.g., the output of a utility function or of a consumption function (Barnett, et al [1984]). Therefore, the growth rate of the divisia index is a weighted average of the growth rates of the component financial assets. The weight of each component asset is the share of that component in expenditure on financial services. Assuming the relative share of each component in the total expenditure on financial assets to be constant at \( s_j \) and dropping the time subscript \( t \), the divisia aggregate in levels becomes (see Handa, 2000):

\[
m_j = \prod_{i=1}^n m^*_i.
\]

The corresponding real divisia user-cost price index \( \pi_j \) in growth rates and levels is defined, respectively, as in (6a) and (6b):
\[ \Delta \log \pi_t = \sum_{i=1}^{n} s_i \Delta \log \pi_{it}. \]  \hspace{1cm} (6a)

\[ \pi_t = \prod_{i=1}^{n} \pi_{it}^{s_i}. \]  \hspace{1cm} (6b)

The implication of (6a) or (6b) is that the asset holder responds by substituting towards the asset with relatively lower user costs should there be a change in the interest rate on a component financial asset. By treating component assets as perfect substitutes, a simple sum aggregate measures income effects but fails to capture the inter-asset substitution effects. Since component assets in a divisia index are imperfect substitutes, divisia aggregates internalize the substitution effects as well as the income effects. Consequently, the exact change in the financial service flows is captured by the divisia aggregate. Thus, neither the financial aggregates that are internally consistent and theoretically plausible nor the price of money are the simple-sum aggregate and the return on alternatives to money holdings, respectively, but have been used by the existing empirical literature on the old MFAS and the new LSW monetarism. In this paper, we use (5) and (6) to measure, respectively, the correct financial quantities and their prices.

Currency equivalent (ce) index, proposed by Rotemberg, et al. (1995), is a more recent alternative index number which has been applied to financial aggregation. It is defined as

\[ ce_t = \sum_{i=1}^{n} \left( \frac{[r_i - r_g]}{r_i} \right) m_{it}. \]  \hspace{1cm} (7)

As can be seen, this index is the simple-sum index with a weight that varies between 0 and 1. If a component asset is more liquid and pays no interest (e.g., currency), the asset will be added to the stock with a unitary weight. As the return of the individual asset rises towards \( r_i \), however, its weight declines towards zero. In this case, the asset behaves less like money and more like the benchmark asset, which is a means to transfer wealth. Notice that the ce measures the stock of financial wealth, while the divisia index in (5a) measures the flows of financial services.

In this paper we use Canadian quarterly simple-sum, ce, and divisia financial aggregates and their aggregation-theoretic user-costs to investigate the money-income relationship in the open-economy variants of the old MFAS and the new LSW monetarist traditions. Many publications have reported that using aggregation-theoretic financial aggregates resolves many of the “puzzles” in macroeconomics literature.\(^\text{13}\) We contribute to the literature by introducing aggregation theory into the old MFAS and the new LSW frameworks that are relevant for policy analysis.

4. Empirical Methodology

Unlike previous work, we allow possible endogeneity of all of the variables in the money-output relationship. We study the long-run as well as the short-run money-output relationships with a methodology that resolves problems of possible endogeneity of all of the variables in the relationships. The relationship between income and the levels of money, or other domestic variables and foreign factors, is especially relevant in the case of what is termed in the literature as the “base drift” problem (Walsh, 1986; Friedman and Kuttner, 1992). In particular, there are two possible decisions that a central bank may take whenever actual money growth differs from the growth rate targeted \textit{ex ante}. The central bank may decide to continue conducting monetary policy so as to best achieve the targeted growth rate \textit{ex post}. Alternatively, the central bank may chose to return the aggregate itself to the previously targeted path. This alternative requires offsetting the initial unplanned deviation of actual from targeted money growth by a systematic reversal of the deviation in the opposite direction.

Yet offsetting in full all observed deviations of actual from targeted money growth, which is endogenous over short time horizons, is optimal only if all disturbances to the money-income relationship are transitory (Friedman and Kuttner, 1992). Thus, we first study the long-run relationship between output and its determinants using the maximum likelihood procedure proposed by Johansen and Juselius (1990). We then study the existence of long memory in the long-run relationships using Geweke and Porter-Hudak’s (1983) (GPH) fractional cointegration procedure -- a procedure which is very recent in its practical application. We then represent the growth in money-income relationship using vector error-correction technique proposed by Johansen-Juselius. Based on the vector error-correction model, we conduct Monte Carlo real-output variance decomposition in order to investigate the effects of unanticipated monetary shocks on real output. While the Johansen-Juselius system cointegration and vector error-correction procedures as well as variance decomposition analysis are common in the literature, the practical application of GPH’s fractional co-integration technique is new. However, the GPH test is conducted based on the equilibrium error from the Johansen-Juselius procedure. Therefore, for expositional purposes, we will discuss the two methodologies respectively in 4.1 and 4.2.

4.1 System cointegration and vector error-correction

We employ the maximum likelihood procedure proposed by Johansen and Juselius (1990) for estimating the co-integrating vectors. In a nutshell, identifying the co-integrating vectors amounts to estimating a vector error-correction of the form:
\[ \Delta x_t = \delta + \sum_{i=1}^{k} \phi_i \Delta x_{t-i} + \Pi x_{t-k} + \theta d_t + \eta_t, \] (8)

where \( x \) is a \((k \times 1)\) vector of \( I(1) \) variables, \( \delta \) a vector of constants, \( d \) a vector of seasonal dummies, and \( \eta \) a stochastic error term. In our extended empirical model (3), \( x' = [y, m, p, c, e] \), where \( y \) is real income, \( m \) is a financial aggregate, \( p \) is the price level, \( c \) is the opportunity cost variable, and \( e \) is the exchange rate. \( \Pi x_{t-k} \) contains information about the long-run relationships among the variables represented in the vector \( x \). This term will be consistent with a stationary VAR only if it is itself stationary; that is, if there is at least one linear combination of the variables of interest that is stationary—equivalently, if the variables are cointegrated. Such a linear combination would represent a stable long-run relationship between the variables. In this case, if the elements of \( \Pi \) can be efficiently estimated, equation (8) provides an efficient way of estimating the short-run parameters, \( \phi_i \), since all the differenced variables in this equation are stationary. Note that equation (8) includes \( k \) separate regressions, of which we are interested in the regression for \( y \) and/or that for \( m \).

We follow Johansen and Juselius (1990) in estimating the elements of \( \Pi \). Consider a process of the form given in equation (8), where the order of integration, both the elements of \( x \) and of the VAR itself, are known. The rank of the matrix \( \Pi \) will be determined by the number of stationary linear combinations of the variables represented by \( x \). If there are \( k \) such combinations, then \( \Pi \) is of full rank. In this case, the variables represented by \( x \) are not \( I(1) \). If there are no such combinations, then the variables are all \( I(1) \), but not cointegrated. In this case, the \( \Pi x_{t-k} \) term is redundant and the error-correction isomorphism cannot be employed. However, if the rank of \( \Pi \) is positive but less than \( k \), then \( \Pi \) can be factorized as \( \Pi = \alpha \beta' \), where \( \alpha \) and \( \beta \) are \((r \times k)\) matrices, with \( \beta \) containing the \( r \) stationary linear combinations and \( \alpha \) their corresponding (feedback) parameters. Johansen and Juselius (1990) derive maximum likelihood estimators for \( \alpha \) and \( \beta \), along with a test statistic for the hypothesis that there are, at most, \( m \) stationary linear combinations in \( \beta \); \( r \) is chosen as \((m+1)\), where \( m \) is the lowest number at which the hypothesis can be rejected.

Note that the VAR above contains \( k \) variables, all of which are potentially endogenous. This makes the procedure appropriate for analyzing the old MFAS and the new LSW monetarist views with possible problems of “base drift” and endogeneity of policy variables in the short-run that is discussed above.
4.2 Fractional integration and fractional cointegration

Granger (1986) initially suggested fractional cointegration as a concept, although fractional differencing was already introduced into the literature by, among others, Granger and Joyeux (1980) and Hosking (1981). Nevertheless, studies illustrating the potential of fractional cointegration as a practical tool are very recent. For example, Cheung and Lai (1993) and Masih and Masih (2004) studied purchasing power parity using fractional cointegration.

The system cointegration approach by Johansen and Juselius (1990) assumes the equilibrium error to follow an I(0) process with an autocorrelation function that damps exponentially, i.e., very rapidly. In this case, the ARMA has short memory and it may be predicted at short horizons. However, the equilibrium error, or any other series for that matter, may exhibit long memory in which case it is neither stationary I(0) nor is it a unit root I(1) process – it is an I(d) process, with d a real number. A series exhibiting long memory, or persistence, has an autocorrelation function that damps hyperbolically, more slowly than the geometric damping exhibited by ‘short memory’ (ARMA) processes. Thus, it may be predictable at long horizons. A process with a hyperbolically-damping autocorrelation function is referred to as a long-memory or fractionally-cointegrated process, while one that damps exponentially is referred to as a short-memory process.

In short, a process $x_t$ is integrated of order d if it has an invertible ARMA representation after being differenced d times. A ‘stationary’ series is indicated by I(0), whereas a ‘non-stationary’ series in levels, but stationary in first difference, is indicated by I(1). If d is non-integer, then the series is referred to as fractionally-integrated, and is indicated by I(d). To further illustrate, consider a linear combination $z_t = \beta[x_{1t}, x_{2t}]$ with $\beta$ representing a vector of coefficients such that $z_t$ is I(d-b) with b > 0. Then ‘if the equilibrium error $z_t$ can be found to be I(d-b) with b > 0, though not necessarily I(0), the $x_t$s are said to be fractionally cointegrated’ (Granger, 1986). The integration parameter d can be estimated by a procedure proposed by Geweke and Porter-Hudak (1983) [GPH]. The GPH approach uses nonparametric methods – a spectral regression – to evaluate d without explicit specification of the ‘short memory’ (ARMA) parameters of the series.

Consider the problem of estimating the parameter d in the general integrated series model. Suppose $(1 - B)^d x_t = \nu_t$, where $\nu_t$ is a stationary linear process with spectral density function $f_\nu(\omega)$. The spectral density function of $\{x_t\}$ is $f(\omega) =$
\[(\sigma^2 / 2\pi)\{4 \sin^2(\omega)\}^{-d} f_\gamma(\omega)\]. Therefore, \(d\) could be obtained as an OLS parameter in a regression:

\[
\ln\{I(\omega_{j,T})\} = c - d \ln\{4 \sin^2(\omega_{j,T} / 2)\} + \eta_j, \forall j = 1, \ldots, n
\]

(9)

where \(n = g(T) < T; \omega_{j,T}\) is the harmonic frequency with \(\omega_{j,T} = 2\pi j / T, j = 0, \ldots, T - 1\), \(T\) is the length of the series; \(I(\omega_{j,T})\) is the periodogram at ordinate \(j\); \(\eta_j\) is independent across harmonic frequencies and equal to \(\ln\{I(\omega_{j,T})/ f(\omega_{j,T})\}\) where \(f(\omega_{j,T})\) is the spectral density.

Note that \(n = g(T)\) is an increasing function of \(T\). Usually, \(g(T) = T^\lambda\) for \(0 < \lambda < 1\).

Nevertheless, the choice of \(\lambda\) in the GPH spectral regression is subjective. A choice of \(\text{root}(T)\), or power = 0.5, is often employed. To evaluate the robustness of the GPH estimate, a range of power values (from 0.4 to 0.75) is commonly calculated as well. OLS estimate of the spectral regression yields a consistent estimate of \(d\). The GPH procedure tests whether the error process follows I(0) or I(\(d\)) with \(d = 0 < d < 1\). Once \(d\) is tested for, an error-correction model could represent the short-run relationship – in the same way as in the system cointegration approach.

As a result, we depart from much of the previous studies (on money-output relationship) by relaxing the strict I(0) or I(1) distinction of the equilibrium error. This is facilitated by the fact that cointegration requires the equilibrium error \(z_t\) to be only mean-reverting. Since the equilibrium error need not be exactly I(0), there exist significant possibilities for analytically extracting a rich class of low frequency dynamics (Cheung and Lai, 1993). That is, unless \(z_t\) reverts back to its mean, a shock to one of the variables in the system, \(x_{1t}\) or \(x_{2t}\), will result in the error deviating from equilibrium permanently.

Note, however, that although \(d < 1\) implies mean reversion, \(0.5 < d < 1\) implies covariance nonstationary: i.e., the variance of the process will not be finite. Nevertheless, even in the case where \(0.5 < d < 1\), the process will exhibit mean reversion as mean reversion is a subset of covariance nonstationary processes. In effect, even in this latter case, a shock to \(x_{1t}\) or \(x_{2t}\) will have no permanent effect on \(z_t\). However, when \(d = 1\), the process is classified as both nonmean-reverting and covariance nonstationary. In this case, the effect of a shock is permanent.
5. Empirical Results

This section presents the empirical results of the paper. In 5.1, we describe the behavior and sources of the data used in the analysis. We discuss detailed results in 5.2.

5.1 The Data

The Bank of Canada uses five financial aggregates, namely, m1, m1+, m1++, m2, and m3. As shown in appendix table 2, the Bank’s aggregates are constructed by means of additive accounting that begins with m1 and adds a group of items to m1 until the broadest aggregate m3 is reached. However, should we strictly follow the Bank’s methodology, some of the interest rate series used to construct adjusted financial aggregates and their user costs are unavailable. In effect, we follow Haug and Lucas (1996) in constructing our (simple-sum) aggregates. Haug-Lucas construct two aggregates for Canadian money demand analysis: m1 (the sum of currency and demand deposits) and m2 (m1 plus saving and time deposits). We further make additional modifications in defining the aggregates in order to facilitate comparison with the literature on the old MFAS and the new LSW monetarist empirical work.

First, the Bank’s aggregates include personal fixed-term deposits in the broadest measure (see appendix table 2). However, it is evident that personal fixed-term savings deposits do not influence spending over a business cycle. We, therefore, drop fixed-term savings deposits from our m2 aggregate. Second, as mentioned above, we follow Haug-Lucas and standardize the accounting by aggregating currency, demand deposits, and savings deposits each as a group. This is because the proxies for the own rate of return of the components of each group are either unavailable or vary over time, and hence tracking those rates introduces more noise than signal. Under this procedure, and based on appendix table 2, for our simple-sum aggregates, demand deposits include personal checking accounts and current accounts; and saving deposits include personal checkable savings deposits, non-personal checkable notice deposits, personal non-checkable savings deposits, and non-personal non-checkable notice deposits. Then, following Haug-Lucas, m1 includes currency and demand deposits and m2 includes m1 and saving deposits. Based on these definitions, we construct currency-equivalent and divisia aggregates following the methodologies outlined in section 3.

Thus, our notation and data sources are as follows. We use m1 for the sum of currency and demand deposits (dd); and m2 for the sum of m1 and saving deposits (sd). The corresponding currency-equivalent aggregates are cem1 and cem2; and the divisia aggregates are dvm1 and dvm2. We use constant 1996 price GDP to proxy real output and the Consumer Price Index to
proxy the price level. To calculate user costs, we assume currency to have a zero return. We use long-term government bond yield as the return on the illiquid (benchmark) asset. We proxy the own rate of return on dd by the commercial paper rate. We use the Treasury bill rate as a return on sd.

We obtained data from CANSIM I and II and cross-checked with the International Financial Statistics data base. The data are quarterly from 1959:01 to 2002:01.

5.2 The Financial Aggregates and Monetarism

This section presents detailed results of the empirical work of the paper in six categories. Overview of the financial aggregates is in 5.2.1, short memory results are in 5.2.2, long memory results are in 5.2.3, discussion on the theoretical relationships is in 5.2.4, and the short-run dynamics is in 5.2.5. And results on Monte Carlo variance decomposition are discussed in 5.2.6.

5.2.1 Overview of the financial aggregates

We have constructed three financial aggregates: the simple-sum aggregate, the currency-equivalent aggregate, and the divisia aggregate. Table 1 contains a preliminary comparison of the different financial aggregates in terms of their growth rates. In the table, measures of central tendency and dispersion show a similar picture. The mean growth rate of m2 and its currency-equivalent and divisia aggregates are higher than their m1 counterparts, over the 42 year period considered. Specifically, the mean growth rates of simple-sum m1 and currency-equivalent m1 aggregates are pretty the same at about 1 percent per quarter whereas that of dvm1 is around 2 percent. However, the standard deviation of the growth rates of m2, cem2, and dvm2 are higher than that of the growth rates of their m1 counterparts. For example, the standard deviation of dvm2 is 4.2% compared with 2.9% for dvm1. The high standard deviation for dvm2 could be attributed to the variation in the moneyness of the component assets over the period of financial liberalization and innovations in the banking payments system – the period covering the 1980s. Hence, the mean and standard deviation of growth in the different financial aggregates reveal difference in the behavior of the aggregates and a need to assess their performance in explaining the money-output relationship.

Figures 1 to 4 depict the growth rates of simple-sum money and its currency-equivalent and divisia aggregates. Figure 1 comares simple-sum m1 and currency-equivalent m1. The figure shows that fluctuations in cem1 were greater than fluctuations in m1 during the sample period. While the turning points of m1 and cem1 were the same in most of the sample sub-periods, the turning points of cem1 precede that of m1 at all points of divergence (e.g., from 1970 to 1985). Figure 2 compares growth rates of m1 and dvm1. Between 1970 and 1990, there are
several occasions when growth rates of m1 and dvm1 exhibited opposite turning points. In other sample sub-periods, the two seem to move together. This substantiates the difference between unadjusted simple-sum and adjusted currency-equivalent and divisia aggregates.

Figures 3 and 4 compare the growth rates of m2 and its currency-equivalent and divisia aggregates. Figure 3 compares growth rates of m2 and cem2. In this figure, the fluctuation in cem2 is greater than that of m2; this difference is more amplified between 1970 and 1990. Figure 4 compares growth rates of m2 and dvm2 during the 42 years period. In this figure, the fluctuation of dvm2 is greater by far than that of m2; there are also episodes of turning points in opposite directions. In addition, very interestingly, the difference in fluctuation between the two aggregates was the highest during the period covering the 1980s. Thus, the period of high fluctuation in the aggregates captures the period of financial liberalization and innovations in the payments system – the period when, based on evidence using simple-sum aggregates, the non-neutrality of money in the old MFAS tradition has been weakened (Friedman and Kuttner, 1992, 1993). It is therefore interesting to assess the difference in aggregation in capturing the neutrality/non-neutrality controversy in the period covering the 1980s.

Figures 5 and 6 depict the growth rates of the user costs of m1 and m2, and figures 7 and 8 compare these growth rates with that of the commercial paper rate, the latter being the most preferred rate in predicting expenditures over a business cycle. As can be seen from figures 5 and 6, the user costs of m1 and m2 exhibited the highest fluctuation during the period covering the 1980s. Comparison of the growth rates of the user costs of dvm1 and dvm2 with the growth rate of the paper rate in figures 7 and 8 reveals greater fluctuation of the former than the latter. There are also differing turning points in the growth rates of the user costs and the commercial paper rate. Therefore, the two measures of financial opportunity cost reveal different behavior. Finally, in figure 9, we compare the commercial paper rate, the Treasury-bill rate, and the government bond yield. This figure reflects the usual yield curve, with stable government bond yield and fluctuating bill and paper rates.

In a nutshell, the most interesting feature of the figures is that the 1980s exhibit different fluctuations in the adjusted financial aggregates and their user costs. This may explain the weakening of non-neutrality during this period, because of the failure to adjust the financial aggregates for variations in the user costs of the components, as we have argued in preceding sections of the paper. We will evaluate the implications of the variation in the financial aggregates and their opportunity costs in explaining the money-output relationships.

Before estimating the money-income relationships, we ascertained the order of integration of the series of interest. We used two methods: the augmented Dickey-Fuller (ADF)
parametric test and the Phillips-Perron (PP) non-parametric test. The results are reported in table 2. The ADF test shows that the CPI and the GDP deflator are I(2), and the other series are stationary in first differences (i.e., they are I(1)) with trend and drift. The PP test also substantiates the ADF test. The first stage in the estimation of the money-output relationship will, therefore, be a search for stationary linear combinations of all of the variables. We employ the Johansen and Juselius (1990) system cointegration technique in order to account for the possible endogeneity of all of the variables. Here, as in Watson (1994), we are interested in a stationary linear combination of I(1) and I(0) variables. We then test the existence of long memory in the money-output relationship using the Gewek-Porter-Hudak methodology. We finally model the short-run dynamics using the vector error-correction technique. We then proceed to Monte Carlo real-output-variance decomposition, to study the effects of unanticipated shocks on real output.

5.2.2 Maximum likelihood test results of the long-run relationships

The maximum likelihood procedure proposed by Johansen-Juselius possesses several advantages in testing for cointegration. First, rather than, a priori, assuming the existence of at most a single cointegrating vector, it explicitly tests for the number of cointegrating relationships. Second, it assumes all variables in the system to be endogenous. Third, it is insensitive to the variables being normalized on. Fourth, it is established on a unified framework for estimating and testing co-integrating relations within the vector error-correction formulation. And fifth, it provides the appropriate statistics and the point distributions to test hypothesis on the number of co-integrating vectors and tests of restrictions upon the coefficients of the vectors.

Here the Johansen-Juselius procedure involves the identification of the rank of the 5x5 matrix $\Pi$ in the specification given by: $\Delta x_t = \delta + \sum_{i=1}^r \phi_i \Delta x_{t-i} + \Pi x_{t-k} + \alpha t_i + \epsilon_t$, where $x_t$ is a column vector of the five variables: y (output), m (money), p (inflation), c (the opportunity cost), and e (the exchange rate). If $\Pi$ has zero rank, no stationary linear combination can be identified. That is, the variables in $x_t$ are noncointegrated. However, if the rank is r, there exist r possible stationary linear combinations. Johansen and Juselius offer two likelihood ratio tests. First, in the case of the maximum eigenvalue test, the null of exactly r cointegrating relationships is tested against an explicit alternative hypothesis such as $r = 0$ against $r = 1$, $r = 1$ against $r = 2$, and so on. Second, in the case of the trace test, the null is associated with at most r cointegrating relationships, against an alternative that is more general. Furthermore, in the Johansen-Juselius procedure it is expected that both tests provide consistent witness concerning the null.

The Johansen-Juselius test results are reported in table 3; in the table, the 5% critical values for the maximum eigenvalue and the trace test statistics are from Osterwald-Lenum
(1992). Our results indicate that, without the imposition of any a priori restrictions upon the coefficients of the variables entering the money-output relationships, only in the case of the models with divisia financial aggregates do both the maximum eigenvalue and trace consistently reject noncointegration. For m2, the null of zero cointegrating vectors cannot be rejected by either of the tests, whereas for the remaining models only the trace test can provide any indication of the presence of cointegration. Hence, these mixed results do not provide convincing evidence of support for the money-output relationships across all models. In general, however, the test results favor the aggregation-theoretic financial aggregates, specifically the divisia ones.

The multivariate, maximum likelihood cointegration technique tests the error for I(0) versus I(1). It does not allow for non-integer, fractional cointegration. Therefore, we will use the Geweke and Porter-Hudak (1983) methodology to investigate the existence of possible fractional cointegration in the money-income relationship.

5.2.3 Geweke-Porter-Hudak-based test results for fractional cointegration

After the assessment of the evidence from the multivariate system cointegration, we conducted Geweke-Porter-Hudak tests using residuals of the cointegrating equation from the Johansen-Juselius test results. In standard tests of cointegration, Stock (1987) showed that the OLS estimator of the parameters in the cointegrating equation is consistent. In the case of fractional cointegration, Cheung and Lai (1993) show that the OLS estimate is also consistent. Since the periodogram is used as an estimator of the spectral density, $d$ may be estimated by OLS regression, as in equation (9). However, a choice of a truncation parameter needs to be made for the number of low frequencies, $n$, to be used in the spectral regression. In this regard, Geweke and Porter-Hudak (1983) recommend using $n = T^{0.5}$ where $T$ is the number of observations. Nevertheless, to avoid subjective choices that are too high or too low, we follow Cheung and Lai (1993) and choose a range of values with powers ranging from 0.5 to 0.65.

Estimation results for fractional cointegration are reported in table 4. Next to the estimates, figures in parenthesis are t-statistics testing the hypothesis $H_0: d = 0$ against a two-sided alternative of $H_1: d \neq 0$. The results indicate uniformity across all values of the power parameter with all estimates of $d$ lying between 0 and 1. Moreover, in each case of the varying power parameter, the hypothesis tests of $d = 0$ are convincingly rejected at the 10% or better. These rejections are clearer in models with dvm1 and dvm2. Therefore, these results provide clear evidence of possible fractional cointegration and show a strong support for the existence of a long memory money-output relationship compared with the Johansen-Juselius tests.

Notice however that the values of $d$ for most of the values of $\lambda$ are in the $0.5 < d < 1$ range. For $\lambda = 0.5$, only three models cem1 ($d = 0.2$), cem2 ($d = 0.39$), and dvm2 ($d = 0.35$) have
stationary error with $0 < d < 0.5$. For $\lambda = 0.55$, only two models dvm2 ($d = 0.39$) and cem1 ($d = 0.35$) have stationary errors. For $\lambda = 0.6$ and $\lambda = 0.65$, only one model dvm2 ($d = 0.31$ and $d = 0.37$, respectively) has stationary error. This shows that only a few models exhibit stationary error process for the different values of $\lambda$. In addition, with the exception of dvm2, none of the other models exhibited a stationary error process for all values of $\lambda$. Therefore, we conclude that, except in dvm2, the presence of fractional cointegration in the money-output relationship is not supported. That is, in most of the models, shocks result in nonstationary deviation from the long-run relationship established by the cointegrating vector. However, all of the estimated values of $d$ are consistently smaller than one for all values of $\lambda$. Values of $d$ smaller than unity for all values of $\lambda$ indicate that the error process is a mean-reverting one. Hence, most of our estimated $d$s are in the nonstationary but mean-reverting range ($0.5 < d < 1$). These results support, albeit weakly, a long-run money-output relationship. Notice further that divisia aggregates have the most significant $d$ for all values of $\lambda$ and that only dvm2 exhibits fractional cointegration for all values of $\lambda$.

Once we have obtained significant long-run money-output relationships, we will proceed to assess the sign patterns of the cointegrating vectors. This is important because we are interested in understanding how the financial aggregates and the exchange rate fare in the long-run relationships predicted by theory. We investigate whether error-in-variance and omitted variables are serious problems in previous research. For instance, if the sign pattern of the cointegrating coefficients are incorrect in a significant long-run relationship, our results are not of any importance in exploring theoretical, measurement, and identification issues.

### 5.2.4 Maximum likelihood estimates of the long-run money-output relationships

We follow Chrystal and MacDonald (1995) and Sinkie-Gebregiorgis and Handa (2005) in assessing whether the long-run relationships produced the sign pattern predicted by theory. In table 5, we report the long-run money-output relationships estimated using the Johansen-Juselius maximum-likelihood, system cointegration procedure. Here, we are interested in investigating the sign pattern of individual coefficients of the variables in the money-output relationships in general and that of the different financial aggregates, the rate of interest, and the exchange rate in particular. That is, we are interested in examining how financial aggregation and open economy modeling change the evidence on the weakening of the relation between money and output, by resolving the bias emanating from error-in-variable and/or omitted variables, respectively.

We modeled six financial aggregates: sum m1 and m2, currency-equivalent m1 and m2, and divisia m1 and m2. For each aggregate, we estimated two models: a four-variable closed-economy model with $y$ (real output), $m$ (financial aggregate), $p$ (inflation), and $c$ (the opportunity
cost variable) and a five-variable open-economy model with the exchange rate added to the former. The results are in table 5 -- in rows 1 to 4 for sum aggregates, rows 5 to 8 for currency equivalent aggregates, and rows 7 to 12 for divisia aggregates.

There are four general results that suggest the presence of biases in previous research originating from omitted variables, measurement error, and endogeneity. First, appropriately measured money is not neutral in the 42 years quarterly data of the Canadian economy. Second, including the exchange rate improves the sign pattern of the long-run money-output relationship that is predicted by theory. Third, the open-economy models with divisia aggregates have better explanatory power of the money-output relationship in terms of elasticities. Fourth, the narrower aggregates produce better sign pattern than the broader ones. These are better results than previous research which could be attributed to our accounting for measurement error, omitted variables, and endogeneity.

In particular, in rows 1 and 2 of table 5, sum m1 enters the relationship with the correct sign. The exchange rate also has the correct sign. However, sum m2 in rows 3 and 4 enters the money-output relationship with an incorrect sign, both without and with the exchange rate. But the exchange rate itself has the correct sign. This could be attributed to the failure of sum aggregates to internalize pure substitution effects and measuring only income effects, thereby resulting in systematic error in the measurement of money that attenuates the estimated coefficient on money. In rows 5 and 6, cem1 enters the relationship with the correct sign, but the opportunity cost variable and the exchange rate have the wrong sign. In rows 7 and 8, cem2 enters the cointegrating relationship with the wrong sign in the closed economy model, but the sign changes when the exchange rate is added. The opportunity cost variable has the right sign in both rows. However, the magnitudes of the long-run elasticities associated with cem2 and the exchange rate in the open-economy model (row 8) are implausibly large. Therefore, cem2 performs poorly in producing the sign pattern predicted by theory. Notice however that the estimated coefficient on money rises as money is correctly measured and as the exchange rate is added to the model.

Finally, divisia aggregates with the user cost of money performed the best, with a clear distinction between the closed and open economies – open economy models yielded better results. The results are presented in rows 9 and 10 for dvm1 and in rows 11 and 12 for dvm2. For example, in row 9, the closed economy money-output relationship for dvm1 has the wrong sign; but in the open economy money-output relationship in row 10, not only the money measure but also the opportunity cost and the exchange rate have the correct sign. The same is true for dvm2 in rows 11 and 12: money, opportunity costs, and the exchange rate have the correct sign.
In conclusion, appropriately measured money is not neutral in this data – and there is no difference between anticipated and unanticipated money, consistent with the old MFAS monetarism. Divisia aggregates with the user cost of money produced a better sign pattern of the long-run money-output relationship. The exchange rate diametrically improved the long-run money-output relationship. Except for the divisia index, where both the narrow and broad financial aggregates performed well, the narrow aggregates produced a better sign pattern than the broad aggregates. This could be plausibly explained by the declining moneyness of the components of broader aggregates. Therefore, improvements in measuring financial aggregates and their user costs as well as in specifying the money-output relationships increase the predictive power of monetary policy. In other words, systematic measurement error and omitted variables bias have been serious problems in previous research.

5.2.5 The short-run dynamics and exogeneity

We further estimated a vector error-correction model, where the lagged error-correction term is possibly fractional from the preceding analysis. Initially, eight lagged-difference terms of each of the I(1) regressors were used, before choosing an optimal lag of three based on Akaike’s Information Criteria.

Estimates of the parameters are presented in table 6. The table shows that the error-correction term is negative and significant at least at the 5% level for the output equation. (The exception here is the model with m2 that has an error-correction coefficient which is significant only at the 10% level.) This shows that output moves to restore equilibrium between the five variable relationships. The error-correction coefficient indicates the single period response of the output shock. However, the magnitude of the coefficient varies across models. It ranges from -0.005 in the case of the model with m2 to -0.180 and -0.240 in the case of the models with, respectively, dvm1 and dvm2. The practical implication of this is that between 0.5% and 24% of the imbalance in real output is corrected within three months (a single quarter). For example, in the case of the model with dvm1, after a shock to the implied money-output relationship, it takes real output approximately 15 months (5 quarters) to adjust in order to re-establish its equilibrium value.

However, as can be seen from table 6, none of the business-cycle channels of causality, for all models, are strong as indicated by the significance levels associated with F-tests of joint restrictions. This is further illustrated by a detailed presentation of our preferred models in box 1 below. In the box, we present vector error-correction results from dvm1 and dvm2 models. In both models the error-correction term has the right sign and is significant at the 5% level for dvm1 and at the 1% level for dvm2. However, none of the business-cycle channels of causality,
Box 1: Vector error-correction models for divisia aggregates.

\[ \Delta y_t = -0.180 \Delta c_{t-1} + 0.243 \Delta y_{t-1} + 0.207 \Delta y_{t-2} + 0.176 \Delta y_{t-3} - 0.015 \Delta m_{t-2} - 0.009 \Delta m_{t-3} - 0.017 \Delta p_{t-1} \]
\[ (3.22) \quad (2.16) \quad (1.77) \quad (0.13) \quad (1.26) \quad (0.78) \quad (3.263) \]
\[- 0.014 \Delta p_{t-3} - 0.020 \Delta c_{t-1} + 0.022 \Delta c_{t-2} - 0.009 \Delta e_{t-1} - 0.004 \Delta e_{t-3} + 0.005 \]
\[ (1.04) \quad (1.09) \quad (1.19) \quad (0.29) \quad (0.14) \quad (2.08) \]

Adj. R-sq = 0.28  s.e. = 0.008  Log likelihood = 413.982

\[ \Delta y_t = -0.241 \Delta c_{t-1} + 0.027 \Delta y_{t-1} + 0.087 \Delta y_{t-2} + 0.02 \Delta y_{t-3} - 0.136 \Delta m_{t-2} - 0.15 \Delta m_{t-3} - 0.24 \Delta p_{t-1} \]
\[ (3.32) \quad (0.18) \quad (0.53) \quad (0.13) \quad (0.836) \quad (3.74) \quad (5.32) \]
\[- 0.034 \Delta p_{t-3} + 0.033 \Delta c_{t-1} + 0.033 \Delta c_{t-2} - 0.014 \Delta e_{t-2} - 0.012 \Delta e_{t-3} + 0.004 \]
\[ (1.037) \quad (1.52) \quad (0.606) \quad (0.412) \quad (0.34) \quad (1.31) \]

Adj. R-sq = 0.47  s.e. = 0.007  Log likelihood = 395.632

5.2.5 Unanticipated monetary shocks and real output

Table 7 reports forecast-error variance decomposition which shows the percentages of the 4-quarter and the 8-quarter forecast-error variance of real output that is explained by innovations to the different variables, together with their approximate standard errors. This table shows the respective contributions of innovations in money, price, the opportunity cost variable, and the exchange rate in accounting for the forecast variance of output. Variance decompositions are computed via Monte Carlo experiments, with 500 draws. The orthogonalization order is the opportunity cost of money, the different financial aggregates, prices, the exchange rate, and real output. In effect, as in Serletis and Molik (1996), we assumed that the opportunity cost is determined before the money supply – an interest targeting operating procedure consistent with the Bank of Canada’s experience (see Bernanke and Mishkin, 1992; Duguay, 1994; Thiessen, 1995).

Perhaps the most important result in the table is that innovations in money, and even the other variables, as expected, explain a very small percentage of the variance of real output. Specifically, innovations in money and the opportunity cost explain a very small percentage of variance of output when sum aggregates are used. Their contributions however rise when currency equivalent and divisia aggregates are employed. The most striking result is that innovations in cem2 account for 36% of output variance at the 8-quarter forecast horizon. The
contributions of divisia aggregates range from 7.2% at the 4-quarter horizon for dvm2 to 18% at the 8-quarter forecast horizon for dvm1. The contributions of innovations in prices and exchange rate in explaining output variance are mixed. In the case of prices, it is as high as 17.4% for a model with cem1 at the 8-quarter horizon. In the case of exchange rates, it is as high as 13% for a model with cem2 at the 8-quarter horizon. However, their contributions dramatically fall in models with divisia aggregates. Equally important is the high precision with which the variance decompositions are measured: the standard error ranges from 0.01 to 0.03.

In general, innovations in money and other explanatory variables in our open-economy models account for a very small variance of output. The output variance accounted for by the financial aggregates rises from sum to currency-equivalent and to divisia aggregates. Conversely, the contributions of interest rates and exchange rates decline from sum to divisia aggregates. This contrasts with previous work on money-output relationship (such as Sims, 1980) that reported that interest rates tend to absorb the effects of money on output. Here, the output variance captured by properly measured financial aggregates is greater than that of the rate of interest and the exchange rate.

6. Conclusions

Two of the contributions of the paper are noteworthy. First, we extended the old MFAS and the new LSW monetarist models to account for domestic factors other than money and international factors that may affect domestic real output. This reduces omitted variables bias in the estimated coefficients of the variables in the money-output relationships. Second, this paper improves the measurement of money and the opportunity cost variable based on an aggregation-theoretic approach. In the augmented versions of both the old MFAS and the new LSW monetarism, money and the variables that determine money demand -- among others, the rate of interest -- affect real output. As a result, we introduced the aggregation-theoretic financial aggregates and their user costs. This is equivalent to solving the problem of error-in-variable which results in attenuation bias of the estimated coefficients of the money-output relationship.

In short, this paper developed the open-economy variants of the old MFAS and the new LSW monetarist models and evaluated for the Canadian economy the determination of real output using aggregation-theoretic and long-memory techniques. First, we measured financial quantities and prices using an aggregation-theoretic technique in order to internalize pure substitution effects in household asset allocation over and above income effects. Plots of these aggregates
against the simple sum ones show that the former diverges from the latter during periods of financial deregulation and innovations in the banking payments system, mainly in the 1980s.

Second, we analyzed the long-run relationship between real output and its determinants using maximum likelihood and fractional cointegration techniques in the closed- as well as open-economy versions of the money-output relationships. Our results show that there are very significant and robust, albeit weak, long-run relationships between output and its determinants in the Canadian data. Nevertheless, open-economy versions of the analysis dramatically improve the magnitude and sign pattern of the money-income relationship that is predicted by theory.

Third, we also modeled the short-run dynamics of the determination of real output in the open economy using vector error-correction technique. We found a highly significant adjustment mechanism of the relationship around the long-run fractional equilibrium. However, the quarterly causality of all of the explanatory variables is very weak.

Fourth, we conducted Monte Carlo real-output variance decomposition of the open-economy money-output relationship. We find that innovations to aggregation-theoretic financial aggregates account for a large fraction of output variance compared with sum aggregates as well as other explanatory variables for real output.

The policy implication is that policy-makers could foresee economic performance using appropriately measured financial quantities and prices and other explanatory variables at their disposal. However, there is no straight forward formula to do so, particularly over a business cycle. Thus, improvements in measuring financial aggregates and their user costs as well as in specifying the money-output relationships increase the predictive power of monetary policy. Investigating further extensions of the old MFAS and the new LSW monetarist models to capture variables measuring government fiscal policies and asset prices remains a topic of future research by the author.
Table 1: Descriptive statistics for alternative measures of money growth rates, Canada: 1959:1 to 2002:1.

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>Financial aggregates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m1</td>
</tr>
<tr>
<td>Mean</td>
<td>0.009</td>
</tr>
<tr>
<td>Median</td>
<td>0.011</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.057</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.050</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.018</td>
</tr>
<tr>
<td>Obs.</td>
<td>171</td>
</tr>
</tbody>
</table>

*Note:* m1 and m2 are sum aggregates, cem1 and cem2 are currency-equivalent aggregates, and dvm1 and dvm2 are divisia aggregates.
Table 2: Unit root test results.

| Variable | ADF test | | | | PP test | | |
|----------|----------|---|---|---|---|---|---|---|---|---|
|          | level    | Δ  | Δ² | 1% critical | level | Δ  | Δ² | 1% critical |
| y        | -0.05    | -4.07 | -4.02 | 2.05 | -8.08 | -3.47 | 1% critical |
| m1       | -0.49    | -5.18 | -3.47 | 3.27 | -18.50 | -4.01 | 1% critical |
| m2       | -1.72    | -4.20 | -3.47 | 1.46 | -15.83 | -4.01 | 1% critical |
| cem1     | -3.38    | -6.08 | -4.02 | 1.19 | -10.33 | -4.01 | 1% critical |
| cem2     | -3.95    | -6.04 | -4.02 | 0.63 | -9.87 | -4.01 | 1% critical |
| dvm1     | -3.07    | -6.10 | -4.02 | 3.59 | -11.36 | -4.01 | 1% critical |
| dvm2     | -2.14    | -5.94 | -4.02 | 1.21 | -12.60 | -4.01 | 1% critical |
| cpi      | -1.48    | -2.80 | -6.98 | -3.47 | -3.83 | -6.30 | -4.01 | 1% critical |
| gdpdef   | -0.74    | -2.43 | -6.78 | -4.02 | -3.44 | -5.72 | -4.01 | 1% critical |
| gby      | -1.08    | -6.08 | -4.02 | 1.47 | -6.49 | -4.01 | 1% critical |
| er       | -3.59    | -5.58 | -4.02 | 3.07 | -13.03 | -4.01 | 1% critical |
| tbr      | -1.21    | -5.34 | -4.02 | 2.14 | -9.37 | -4.01 | 1% critical |
| cpr      | -1.50    | -6.01 | -4.02 | 2.00 | -1033 | -4.01 | 1% critical |

Note: y is real output, m1 and m2 are sum aggregates, cem1 and cem2 are currency-equivalent aggregates, dvm1 and dvm2 are divisia aggregates, cpi is the Consumer Price Index, gdpdef is GDP deflator, gby is government bond yield, er is exchange rate, tbr is Treasury bill rate, cpr is commercial paper rate.
Table 3: Johansen and Juselius MLE test results.

<table>
<thead>
<tr>
<th>Model</th>
<th>Test</th>
<th>Null hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r = 0$</td>
</tr>
<tr>
<td>m1</td>
<td>LR1</td>
<td>33.489</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>130.541**</td>
</tr>
<tr>
<td>m2</td>
<td>LR1</td>
<td>25.443</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>85.666</td>
</tr>
<tr>
<td>cem1</td>
<td>LR1</td>
<td>36.412</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>125.339**</td>
</tr>
<tr>
<td>cem2</td>
<td>LR1</td>
<td>34.617</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>126.128**</td>
</tr>
<tr>
<td>dvm1</td>
<td>LR1</td>
<td>45.627**</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>142.223**</td>
</tr>
<tr>
<td>dvm2</td>
<td>LR1</td>
<td>48.459**</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>135.668**</td>
</tr>
<tr>
<td>5% critical</td>
<td>LR1</td>
<td>44.91</td>
</tr>
<tr>
<td></td>
<td>LR2</td>
<td>124.25</td>
</tr>
</tbody>
</table>

Notes: Model refers to a cointegrating relationship which included the respective financial aggregates. m1 and m2 are sum aggregates, cem1 and cem2 are currency-equivalent aggregates, and dvm1 and dvm2 are divisia aggregates. All of the cointegrating relationships include exchange rate. LR1 refers to the maximum eigenvalue test and LR2 refers to the trace test. $r$ indicates the number of co-integrating relationships. The optimal lag structure for the VAR was selected by minimizing Akaiki’s Information Criteria. Based on AIC, the optimal lag length is four for each model. ** indicates significance at the 5% critical values. The reported 5% critical values are from Osterwald-Lenum (1992).
Table 4: Geweke-Porter-Hudak tests for fractional co-integration.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda = 0.50$</th>
<th>$\lambda = 0.55$</th>
<th>$\lambda = 0.60$</th>
<th>$\lambda = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>0.64(2.46)**</td>
<td>0.68(3.06)**</td>
<td>0.50(2.66)**</td>
<td>0.56(3.41)**</td>
</tr>
<tr>
<td>m2</td>
<td>0.50(1.85)*</td>
<td>0.65(2.38)*</td>
<td>0.65(2.50)**</td>
<td>0.61(2.80)*</td>
</tr>
<tr>
<td>cem1</td>
<td>0.20(2.10)**</td>
<td>0.38(3.25)**</td>
<td>0.50(4.37)*****</td>
<td>0.69(6.31)*****</td>
</tr>
<tr>
<td>cem2</td>
<td>0.39(3.80)*</td>
<td>0.66(3.84)*</td>
<td>0.66(5.15)*</td>
<td>0.61(6.34)*</td>
</tr>
<tr>
<td>dvm1</td>
<td>0.71(3.87)*****</td>
<td>0.71(4.78)****</td>
<td>0.58(4.26)*****</td>
<td>0.57(5.00)*****</td>
</tr>
<tr>
<td>dvm2</td>
<td>0.39(3.94)*****</td>
<td>0.39 (4.11)*****</td>
<td>0.31 (5.54)*****</td>
<td>0.37 (6.82)*****</td>
</tr>
</tbody>
</table>

Notes: Estimates contained in parenthesis represent approximate t-statistics testing the hypothesis: $H_0: d = 0$ against $H_1: d \neq 0$. m1 and m2 are sum aggregates, cem1 and cem2 are currency-equivalent aggregates, and dvm1 and dvm2 are divisia aggregates. Critical values are constructed by way of simulation experiment based on 10,000 replications. ***, **, and * indicates rejection of the null at 1%, 5%, and 10% levels of significance.
Table 5: Maximum likelihood estimates of cointegrating vectors.

\[ y = \alpha_1 m + \alpha_2 p + \alpha_3 c + \alpha_4 e \]

<table>
<thead>
<tr>
<th>Models</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \alpha_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1-1</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.042</td>
<td>-</td>
</tr>
<tr>
<td>m1-2</td>
<td>0.14</td>
<td>0.11</td>
<td>-0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>m2-1</td>
<td>-0.20</td>
<td>0.13</td>
<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>m2-2</td>
<td>-0.61</td>
<td>0.27</td>
<td>0.01</td>
<td>0.67</td>
</tr>
<tr>
<td>cem1-1</td>
<td>0.17</td>
<td>0.09</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>cem1-2</td>
<td>0.17</td>
<td>0.09</td>
<td>0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>cem2-1</td>
<td>-0.56</td>
<td>0.16</td>
<td>-0.10</td>
<td>-</td>
</tr>
<tr>
<td>cem2-2</td>
<td>2.08</td>
<td>0.13</td>
<td>-0.34</td>
<td>-1.03</td>
</tr>
<tr>
<td>dvm1-1</td>
<td>-1.50</td>
<td>0.12</td>
<td>-0.005</td>
<td>-</td>
</tr>
<tr>
<td>dvm1-2</td>
<td>0.93</td>
<td>0.07</td>
<td>-0.07</td>
<td>0.86</td>
</tr>
<tr>
<td>dvm2-1</td>
<td>-0.75</td>
<td>0.15</td>
<td>-0.13</td>
<td>-</td>
</tr>
<tr>
<td>dvm2-2</td>
<td>0.33</td>
<td>0.13</td>
<td>-0.21</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Notes: m = money as defined in the first column, p is the inflation rate (the first difference of the consumer price index), c = the opportunity cost variable is the corporate paper rate, e = the exchange rate defined as the home currency price of a unit of foreign currency. The numbers in this table are estimates of the cointegrating vectors derived from the number of significant cointegrating vectors noted in table 3, where the relationships have been normalized on the real output.
Table 6: Summary results from vector error-correction models of the real output on money, prices, opportunity cost, and the exchange rate.

<table>
<thead>
<tr>
<th>Models</th>
<th>Significance levels of F-statistic</th>
<th>ect(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta y_t$</td>
<td>$\Delta m_t$</td>
</tr>
<tr>
<td>m1</td>
<td>0.044</td>
<td>0.034</td>
</tr>
<tr>
<td>m2</td>
<td>0.065</td>
<td>0.0002</td>
</tr>
<tr>
<td>cem1</td>
<td>0.033</td>
<td>0.045</td>
</tr>
<tr>
<td>cem2</td>
<td>0.017</td>
<td>0.303</td>
</tr>
<tr>
<td>dvm1</td>
<td>0.987</td>
<td>0.810</td>
</tr>
<tr>
<td>dvm2</td>
<td>0.856</td>
<td>0.840</td>
</tr>
</tbody>
</table>

Notes: The lagged error correction terms ect(-1) were derived by normalizing the cointegrating equation on the real output. Estimates of significance levels are those associated with F-tests of joint restrictions that all lagged difference terms of the respective variables are statistically insignificant. The lag structure for VECMs was determined based on Akaiki’s Information Criteria. ***, **, and * indicates significance at the 1%, 5%, and 10% levels.
Table 7: Real output forecast-error variance decomposition.

<table>
<thead>
<tr>
<th>Percentage of output variance</th>
<th>s.e.</th>
<th>Δc</th>
<th>Δm</th>
<th>Δp</th>
<th>Δe</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1 4 quarters ahead</td>
<td>0.02</td>
<td>1.17</td>
<td>0.69</td>
<td>0.05</td>
<td>1.55</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.03</td>
<td>3.63</td>
<td>0.63</td>
<td>2.30</td>
<td>3.02</td>
</tr>
<tr>
<td>m2 4 quarters ahead</td>
<td>0.02</td>
<td>0.50</td>
<td>0.70</td>
<td>0.60</td>
<td>1.92</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.03</td>
<td>2.05</td>
<td>1.67</td>
<td>3.24</td>
<td>4.22</td>
</tr>
<tr>
<td>cem1 4 quarters ahead</td>
<td>0.02</td>
<td>0.48</td>
<td>2.25</td>
<td>8.73</td>
<td>0.61</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.03</td>
<td>1.36</td>
<td>7.84</td>
<td>17.36</td>
<td>1.88</td>
</tr>
<tr>
<td>cem2 4 quarters ahead</td>
<td>0.01</td>
<td>2.25</td>
<td>6.10</td>
<td>0.24</td>
<td>9.02</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.02</td>
<td>3.25</td>
<td>35.91</td>
<td>2.44</td>
<td>12.92</td>
</tr>
<tr>
<td>dvm1 4 quarters ahead</td>
<td>0.02</td>
<td>2.51</td>
<td>11.55</td>
<td>0.23</td>
<td>0.03</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.03</td>
<td>4.99</td>
<td>17.60</td>
<td>0.19</td>
<td>0.11</td>
</tr>
<tr>
<td>dvm2 4 quarters ahead</td>
<td>0.02</td>
<td>2.50</td>
<td>7.16</td>
<td>0.21</td>
<td>0.23</td>
</tr>
<tr>
<td>8 quarters ahead</td>
<td>0.03</td>
<td>4.89</td>
<td>7.73</td>
<td>0.21</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: The models have been estimated with three lags. Variance decompositions are computed via Monte Carlo experiments, with 500 draws. s.e is the standard error with which the accounted variances are computed. The orthogonalization order is the opportunity cost, money, price, the exchange rate, and real output. That is, we assumed that the interest rate is determined before the money supply – an interest rate targeting operating procedure.
Fig 1: Growth rates of m1 and cem1.

Fig 2: Growth rates of m1 and dvm1.
Fig 3: Growth rates of m2 and cem2.

Fig 4: Growth rates of m2 and dvm2.
Fig. 5: Growth rate of the user cost of m1 (ucm1).

Fig. 6: Growth rate of the user cost of m2 (ucm2).
Fig. 7: Growth rates of the user cost of m1 (ucm1) and the commercial paper rate (cpr).

Fig. 8: Growth rates of the user cost of m2 (ucm2) and the commercial paper rate (cpr).
Fig. 7: Commercial paper rate (cpr), Treasury-bill rate (tbr), and government bond yield (gby).
References:


There are various potential explanations for the empirical finding of a weak money-output relationship as, for example, in Friedman and Kuttner (1992, 1993). First, the estimated elasticity of real output with respect to money may be downward biased because of simultaneity, arising from possible endogeneity of the money supply. This criticism implies that the validity of any attempt to estimate structural money-output relationships can be questioned and, in effects, the analysis of monetary transmission should rather focus on the effect of the exogenous or unsystematic component of monetary policy. As a result, at least partly, the bulk of the literature on money-output relationship is now based on VAR impulse response analysis. This approach aims at identifying and simulating the effect of monetary shocks to disentangle the demand effect of monetary policy from its endogenous response to economic activity. However, the limitation of the VAR approach is that it provides evidence only for the effect of monetary policy shocks, which account for a negligible share of overall money supply movements, while nothing is learnt about the effect of systematic monetary policy measures.

Another explanation for the weak money-output relationship is that empirical models are misspecified. For example, other variables besides the money supply may affect aggregate demand. Among others, in small open economies, exchange rates have significant influence in communicating shocks between foreign and domestic economies. A depreciation of the real exchange rate makes domestic goods more competitive and therefore has a positive effect on net exports. The exchange rate is often considered to be the most important determinant of aggregate demand besides the money supply and the interest rate, in small, open economies like Canada, Australia, New Zealand, and open-economy developing countries where net exports account for a large share of aggregate demand.

An additional explanation of the weak money-income relationship is error in measuring the monetary variable – the money supply and the rate of interest. As we discussed in the body of the paper, empirical work on money-income relationship relies on unadjusted, simple-sum financial aggregates and/or short-term interest rates. However, simple-sum aggregates that are published by central banks are based on the assumption that the component assets are perfect substitutes. Although financial innovations have blurred the distinction between transactions and savings-type assets, the assets that make up the broader aggregates do not have the same liquidity as currency. As a result, aggregating, with equal weight, interest-bearing assets with currency introduces systematic error in measuring money and its user cost and hence results in attenuation bias in the estimated coefficient of money. Divisia aggregates, by weighting component assets by their user costs, correct the error-in-variance problem inherent in simple-sum aggregates.

Problems of omitted variables and measurement error can be illustrated by the help highly stylized models. First, consider the case of omitted variables. If other variables besides real money also affect aggregate demand, the estimated elasticity on money in the money-output relationship may be biased. The direction of the bias depends on the correlation between money and the omitted variable. This point can be illustrated by the help of highly stylized model. Suppose that aggregate demand is determined by a relationship of the form:

$$y_t = \alpha_1 m_{t-1} + \alpha_2 x_{t-1} + \eta_t$$

where $y$ is real output, $m$ is money, and $x$ is some other variable that also affects aggregate demand, most importantly the real exchange rate in small open economies. If we estimate the equation by omitting $x$, the OLS estimator of $\alpha_1$ would be given by

$$\hat{\alpha}_1 = \alpha_1 + \alpha_2 \frac{\text{cov}(m, x)}{\text{var}(m)}$$

The omission of a variable $x$ would therefore give rise to a biased estimate of the effect of money on output if $\text{cov}(m, x) \neq 0$. The coefficient $\alpha_1$ gives the direct effect of money on output. However, money supply changes may also be transmitted via variable $x$. For example, an increase in money

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14 This model is similar to Woodford (1994) which is also used by Goddhart and Hofmann (2005)
15 When $\text{cov}(m, x) \neq 0$, this is not a reason to worry.
supply is expected to depreciate the exchange rate. If there is a relationship between \( m \) and \( x \) of the form:

\[
x_i = \delta m_i
\]

then the estimator of \( \alpha \) will be given by \( \hat{\alpha} = \alpha + \alpha \delta \). Since \( \delta \) is positive in this case, the estimate of \( \alpha \) is biased downwards to zero. On the other hand, if \( \delta \) is negative, the estimate of \( \alpha \) is biased upwards.

Second, consider the case of error in measuring money and its opportunity cost. Although, random errors tend to average out in aggregate data, systematic errors are less likely to disappear. If money is measured with error, the elasticity of output with respect to money will be attenuated. This could be illustrated using a classical model of measurement error. Suppose we have data on \( m_i \) and \( y_i \), where \( m_i \) is real money and \( y_i \) is real output. The variables \( m_i \) and \( y_i \) may or may not equal the correctly-measured variables the researcher would like to have data on, which we denote \( *m_i \) and \( *y_i \). The error in measuring the variables is simply the deviation between the observed variable and the correctly-measured variable. For example, \( \nu = m_i - m_i^* \), where \( \nu \) is the measurement error in \( m_i \). Under classical measurement error, \( \nu \) is assumed to have the properties \( \text{cov}(\nu, m_i^*) = E(\nu) = 0 \). That is, the measurement error is just mean-zero “white noise.”

As we will show, the strongest implication of classical measurement error is that when the explanatory variable is measured with error, its estimated coefficient will be “attenuated.” Consider the case of measurement error in an explanatory variable. For simplicity, we focus on a bivariate regression, with mean zero variables so we can suppress the intercept. Suppose \( y_i \) is regressed on the observed variable \( m_i \), instead of on the correctly-measured variable \( m_i^* \). The population regression of \( y_i^* \) on \( m_i^* \) is:

\[
y_i^* = m_i^* \beta + \epsilon_i,
\]

while if we make the additional assumption that the measurement error (\( \nu_i \)) and the equation error are uncorrelated, the population regression of \( y_i^* \) on \( m_i \) is:

\[
y_i^* = m_i \kappa \beta + \tilde{\epsilon}_i,
\]

where \( \kappa = \text{cov}(m_i^*, m_i) / \text{var}(m_i) \). If \( m_i \) is measured with classical measurement error, then \( \text{cov}(m_i^*, m_i) = \text{var}(m_i^*) \) and \( \text{var}(m_i) = \text{var}(m_i^*) + \text{var}(\nu_i) \), so the regression coefficient is necessarily attenuated, with the proportional “attenuation bias” equal to \( (1 - \kappa) < 1 \). The quantity \( \kappa \) is often called the “reliability ratio.” A higher reliability ratio implies that the observed variability in \( m_i \) contains less noise.

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16 First, consider a situation in which the dependent variable is measured with error. Specifically, suppose that \( y_i = y_i^* + \mu_i \), where \( y_i \) is the observed dependent variable, \( y_i^* \) is the correctly-measured, desired, or “true” value of the dependent variable, and \( \mu_i \) is classical measurement error. If \( y_i \) is regressed on one or more correctly-measured explanatory variables, the expected value of the coefficient estimates is not affected by the presence of the measurement error. Classical measurement error in the dependent variable leads to less precise estimates: because the errors will inflate the standard error of the regression, but does not bias the coefficient estimates.
Appendix table 2: Bank of Canada’s Components of Monetary Aggregates.

<table>
<thead>
<tr>
<th>Monetary aggregate</th>
<th>Component</th>
<th>CANSIM series number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CANSIM I</td>
</tr>
<tr>
<td>m1</td>
<td>Currency outside banks</td>
<td>B2001</td>
</tr>
<tr>
<td></td>
<td>Personal checking accounts</td>
<td>B486</td>
</tr>
<tr>
<td></td>
<td>Current accounts</td>
<td>B487</td>
</tr>
<tr>
<td>m1+</td>
<td>Personal checkable savings deposits</td>
<td>B452</td>
</tr>
<tr>
<td></td>
<td>Non-personal checkable notice deposits</td>
<td>B472</td>
</tr>
<tr>
<td>m1++</td>
<td>Personal non-checkable savings deposits</td>
<td>B453</td>
</tr>
<tr>
<td></td>
<td>Non-personal non-checkable notice deposits</td>
<td>B473</td>
</tr>
<tr>
<td>m2</td>
<td>Personal fixed-term savings deposits</td>
<td>B454</td>
</tr>
<tr>
<td>m3</td>
<td>Non-personal term deposits</td>
<td>B475</td>
</tr>
<tr>
<td></td>
<td>Foreign currency deposits</td>
<td>B482</td>
</tr>
</tbody>
</table>

*Source:* Based on Serletis and Molik (1996).