Inventories and Endogenous Stackelberg Hierarchy in Two-period Cournot Oligopolistic Competition

Sébastien Mitraille*

Toulouse Business School and CMPO (University of Bristol)

Michel Moreaux

Université de Toulouse I (IUF, LERNA, and IDEI)

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Abstract

We show that two-period Cournot competition between \( n \) identical firms producing at constant marginal cost and able to store has only asymmetric pure strategy Nash equilibria. All firms store different quantities to form endogenously a hierarchy of Stackelberg strategic agents, a firm market share decreasing the lower its inventories. When the number of firms goes to infinity, the equilibrium converges to the competitive outcome keeping market shares asymmetric. The use of strategic inventories to exert some leadership over one’s competitors can be connected to dumping practices on the DRAM market, source of a recent international trade dispute.

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*Corresponding author. Toulouse Business School. 20 Bd Lascrosses - BP7010 - 31068 Toulouse Cedex 7 - France. Phone: +33 (0) 5 61 29 48 44. Fax: +33 (0) 5 61 29 49 94. E-mail: s.mitraille@esc-toulouse.fr
1 Introduction

On many mineral or non-mineral commodity markets producers are in imperfect competition. The highly standardized nature of those products regularly triggers aggressive commercial behaviour: producers often modify their strategies to maintain their presence on the market through a high volume of sales relatively to their opponents. The U.S. uranium spot market has for example faced a large recession at the beginning of the 1990’s\(^1\): during that period producers from Kazakhstan have been the object of an anti-dumping duty conducted by the U.S. Department of Commerce\(^2\). A similar inquiry has been conducted on the PC memories market (the DRAM market). On this market, South-Korean producers have been alleged to systematically dump their prices when the market was entering in a downturn, in order to enlarge their market share and sell a higher volume than their American competitors\(^3\) and therefore enlarge their "customer base". The link between the business cycle and the use of dumping strategies is explained explicitly by the report of the U.S. department of Commerce\(^4\),

The DRAM industry is highly cyclical in nature [...]. In the past, the DRAM industry has been characterized by dumping during periods of significant downturn. [...] Because DRAMs are a commodity product, DRAM producers/resellers must price aggressively during a downturn period in order to [...] maintain their customer base. This is especially true during the lowest point in the downturn.

On most if not all of these markets, storage behaviour of producers or consumers are playing a crucial role in the price formation process. On the uranium market for example, "on ground" storage (i.e. extracted from the mine but kept out of the market) has presumably contributed to the decrease of spot prices in the 1990’s. The impact of storage strategies on the DRAM prices has also been pointed out:

\(^1\)Consequence of the correction of the estimated growth of the number of nuclear powerplants, as well as of the large strategic inventories constituted by buyers in the 1980’s to face the predicted and well advertised scarcity of this natural resource


\(^3\)The inquiry started with a denunciation by American producers. See the panel report of the World Trade Organization, WT/DS99/R 29 January 1999.

\(^4\)US/DOC Preliminary Results Third Review, 62 Fed. Reg. at 12796 (Ex. USA-20), par. 16.
market analysts emphasized the strategy of South-Korean producers, consisting in stockpiling when demand started to decrease, to release those quantities in the subsequent periods, inducing an even worse reduction of the market price.

The commitment value of inventories in two-period Cournot duopolistic competition has been recognized in the literature since the path-breaking articles of Arvan [1985] and Ware [1985]. They both identify that once costs of production have been sunk, inventories form a capacity from which firms can sell while producing at a zero marginal cost, making them more aggressive on the market. Moreover inventories create an endogenous discontinuity in the marginal cost the firm is facing when deciding how much to produce and sell on the market. A firm may therefore be committed to sell exactly its inventories for a range of sales of its rival, i.e. its supply becomes locally inelastic to the supply of its opponent, and an adequate choice of inventories may result in a first mover advantage for this firm. This effect, obviously linked to Dixit [1980], implies the existence of asymmetric equilibria even if firms are identical with respect to their production technology, but contrary to Dixit’s game asymmetric equilibria do not rely on an asymmetry in the timing: firms produce and sell simultaneously in every period. In an independent series of papers, Saloner [1987], Pal [1991] and Pal [1996] study duopolistic Cournot competition with advance production and constant returns-to-scale, to show that multiple equilibria may arise, but that this multiplicity may disappear with cost variations. More specifically Saloner shows that any outcome on the outer envelope of the Cournot reaction functions between the two Stackelberg equilibria is a Nash equilibrium in Pure strategy of this game, and Pal shows how cost differentials may help to select an outcome amongst the equilibria of Saloner’s game.

However whether the commitment effect of inventories survives to an increase in the degree of competition, a realistic assumption on many storable commodity markets like the DRAM market, is still an unanswered question. Moreover if it does, what are the properties of the Nash equilibrium, if it exists, and how does this equilibrium evolve when the number of firms increases? The purpose of this paper is to answer these questions.

5See Ware [1984].
We show that the commitment value of inventories does not disappear when the number of firms in competition on a market increases. Under constant returns-to-scale, and in the absence of any capacity constraint or uncertainty, that is without any other incentive than strategic to use inventories, Cournot oligopolistic competition in which $n$ firms build up inventories in a first period before producing again and selling on the second period market has an essentially unique asymmetric pure strategy Nash equilibrium, that is a unique distribution of inventories, a unique aggregate level of inventories and a unique aggregate level of sales. Firms store to form endogenously a hierarchy of Stackelberg strategic agents, each of them acting as a leader over another firm given the inventories of the firm acting as a leader over it. When the number of firms goes to infinity, the equilibrium converges to the competitive outcome, but firms are still storing different amounts: for $n$ sufficiently large, the Nash equilibrium of the game is an asymmetric (quasi-)competitive equilibrium, even if firms are ex-ante identical. This finding is closely related to Boyer and Moreaux [1985] and Boyer and Moreaux [1986] who show that a market with an exogenous hierarchy of Stackelberg firms is competitive when the number of firms increases while leaving asymmetric market shares in any market configuration. We obtain endogenously a hierarchy of Stackelberg leaders in a two-period game, that is without requiring more than one period in which the investment takes place. We then compare our equilibrium to the standard Nash-Cournot outcome obtained in oligopolistic competition: because of an increase in competition, due to the commitment effect of inventories in a strategic substitutes framework, the equilibrium converges quicker to the competitive outcome than the Nash-Cournot one when $n$ increases. However the Hirschman-Herfindhal Concentration index (HHI) is bounded away from zero: leaders persist on the market no matter the number of firms, even if the price-cost margin tends to zero. Due to the strategic use of inventories, it is therefore possible to observe identical firms obtaining different market shares, leaders enjoying up to half of the market and followers enjoying market shares falling down to zero, without any distortion on the Social Welfare.

The resolution of this game is made by constructing the backward reaction mapping proposed by Novshek [1984] (see also Bamon and Frayssé [1985]). For each distribution of inventories, the second period quantity competition sub-game has a unique Nash equilibrium in pure strategy, and we show how the aggregation of
the non-differentiable backward reaction mappings lead to the determination of this equilibrium.

The remaining of the paper is organized as follows: section 2 presents the model and shows that for any distribution of ordered inventories across firms, it exists a unique sub-game equilibrium in which market shares can be asymmetric. Then section 3 derives the equilibria of the game. Section 4 concludes.

2 The Model and Preliminary Results

We consider an homogenous market with \( n \) Cournot competitors indexed by \( i, i \in I = \{1, \ldots, n\}, n \geq 2 \), competing over two periods indexed by \( t = 1, 2 \). Let \( q^i_t \) be the production level of firm \( i \) in period \( t \), \( q_t = (q^1_t, \ldots, q^n_t) \) the production vector in period \( t \), \( Q_t = \sum_i q^i_t \) the aggregate output and \( Q^{-i}_t = \sum_{j \neq i} q^j_t \) the aggregate output of firms \( j \neq i \).

Production may be undertaken in any period, but the market is open in period 2 only. Let \( s^i \) be the quantity sold by firm \( i \) in period 2, \( s^i \leq q^1_t + q^2_t \), \( s = (s^1, \ldots, s^n) \) be the sales vector, \( S = \sum_i s^i \) the aggregate sales level and \( S^{-i} = \sum_{j \neq i} s^j \) the aggregate sales of firm \( j \neq i \).

Firms have access to the same constant returns production technology and the same factor prices. Each one is a ”small” buyer in the factor market, taking prices as given. Hence all firms have the same constant marginal cost of production denoted by \( c, c > 0 \).

We assume that pure inventory costs are nil, excepted the working capital opportunity cost. Assuming that the capital market is perfectly competitive, and denoting by \( \rho \) the interest rate, the only opportunity cost is the cost of producing in period 1 rather than in period 2, that is \( \rho \ c \ q^i_1 \) in terms of value in period 2. Under a free disposal assumption insuring that inventories unsold at the end of period 2 can be disposed off at zero cost, the total cost of any production and sale plan is given by (C.1), in period 2 value:

\[ (C.1) \] For any firm \( i \in I \) and any plan \( \{(q^i_1, q^i_2, s^i) : s^i \leq q^i_1 + q^i_2\} \), the
total cost incurred in second period, \( C^2_i \), is equal to:

\[
(1 + \rho) \cdot c \cdot q^i_1 + c \cdot q^i_2.
\]

The market demand function is assumed to be linear. Without loss of generality we assume that its slope is equal to \(-1\). Thus:

**Assumption (D.1)** Let \( P(S) \) be the inverse demand function, then

\[
P(S) = \max\{a - S, 0\}, \quad a > 0.
\]

For the ease of the analysis most of the discussion is lead under the two following assumptions:

**Assumption (A.1)** The intercept of the inverse demand \( a \) and the marginal cost of production \( c \) satisfy

\[
c \leq a \leq 3 \cdot c.
\]

**Assumption (A.2)** Firms are indexed by decreasing order of period 1 production levels,

\[
q^1_1 \geq q^2_1 \geq \ldots \geq q^{n-1}_1 \geq q^n_1.
\]

Assuming that any firm can observe all the period 1 production levels but cannot observe period 2 production and sale levels of its competitors, the strategy of firm \( i \), denoted by \( \sigma^i \), is a 3-uple:

\[
\sigma^i = \{q^i_1, \tilde{q}^2_i, \tilde{s}^i\}\quad (1)
\]

where:

\[
\begin{align*}
q^i_1 &\in \mathbb{R}^+, \quad \tilde{q}^2_i : \mathbb{R}^n_+ \to \mathbb{R}^+, \quad \tilde{s}^i : \mathbb{R}^{n+1}_+ \to \mathbb{R}^+. \quad (2)
\end{align*}
\]

\( \tilde{s}^i \) satisfying the following condition:

\[
\forall (q_1, q^i_2) \in \mathbb{R}^{n+1}_+, \tilde{s}^i(q_1, q^i_2) \leq q^i_1 + q^i_2. \quad (3)
\]
To any n-uple of strategy $\sigma$, $\sigma = \{\sigma^1, ..., \sigma^i, ..., \sigma^n\}$, corresponds a payoff function $\pi^i$ for firm $i$ given by, in period 2 value:

$$\pi^i(\sigma^i, \sigma^{-i}) = P \left( \sum_{j \neq i} \tilde{s}^j(q_1, \tilde{q}_2(q_1)) + \tilde{s}^i(q_1, \tilde{q}_2(q_1)) \right) \tilde{s}^i(q_1, \tilde{q}_2(q_1)) - (1 + \rho) c q_1^i - c \tilde{q}_2(q_1).$$

(4)

We now turn to the description of the sub-game equilibrium. Proposition 1 below presents the aggregate oligopolistic sales in equilibrium and corollary 1 the corresponding equilibrium individual sales. Corollary 2 relates the conditions on inventories leading to the different sub-game equilibria to conditions on the marginal revenues. The proof of these results is given in appendix A.1. Let us briefly give its economic intuition.

Inventories create firm-specific kinks in firms best replies, source of the asymmetry of the Nash equilibrium of the game. Indeed as their cost of production is sunk when selling on the market in second period, inventories form a capacity from which firms can sell without producing. Although this effect of inventories has already been explained\(^6\), it is useful to present it differently from older studies by introducing the effective marginal cost of production of each firm, given by

$$\gamma^i(s^i, q^i_1) = \begin{cases} 0 & \text{if } s^i \leq q^i_1 \\ c & \text{if } s^i > q^i_1. \end{cases}$$

(5)

Even if technologies are ex-ante identical, effective marginal costs differ across firms once inventories have been produced, and present a firm-specific jump at $s^i = q^i_1$. By confronting $\gamma^i(s^i, q^i_1)$ to the marginal revenue $m^i(s^i, S^{-i})$,

$$m^i(s^i, S^{-i}) = a - 2 s^i - S^{-i},$$

(6)

one obtains the best reply of firm $i$ to the aggregate sales of the other firms $S^{-i}$. Given the form of the effective marginal cost of production $\gamma^i$, the individual best reply shows the three different types of behaviour of firm $i$, depending on competitors sales $S^{-i}$ and on initial inventories $q^i_1$: a firm may find profitable either to sell more than its inventories, or to sell exactly its inventories, or to sell less than its inventories. Indeed if given $q^i_1$, competitors sales $S^{-i}$ are such that the marginal revenue to sell $q^i_1$, $m^i(q^i_1, S^{-i})$, exceeds the marginal cost $c$, then firm $i$ produces again

\(^6\)See Arvan [1985] and Ware [1985]
in second period and sells more than \(q_i^1\). If \(m^i(q_i^1, S^{-i})\) is lower than the marginal cost \(c\) but positive, then firm \(i\) sells exactly its inventories. If finally \(m^i(q_i^1, S^{-i})\) is strictly negative, then firm \(i\) is better off selling less than its inventories.

To describe the second period sub-game equilibria, we first restrict the sub-games we analyze by focusing on the sub-games that are not trivially dominated. As we are analyzing an oligopolistic competition, we restrict our attention to inventories lower than the quantity a monopoly minimizing its cost of production would produce, that is \(q^m = (a - c)/2\). Due to the presence of the interest rate \(\rho\) and of \(n - 1\) competitors, no firm will store more than the quantity a monopoly producing in second period would sell. We can now explain how to derive the equilibria. To be in a sub-game equilibrium in which firms 1, 2, ..., \(k\) sell exactly their inventories, and firms \(k + 1, ..., n\) sell strictly more than their inventories, it suffices that given equilibrium sales of competitors,

(1) the marginal revenue of firm \(k + 1\) when selling \(q_{k+1}^1\) is strictly higher than the marginal cost \(c\) (so will it be for firms \(k + 2, ..., n\) who own inventories lower than firm \(k + 1\)),

(2) the marginal revenue of firm \(k\) when selling \(q_k^1\) is strictly lower than \(c\) (so will it be for firms 1, ..., \(k - 1\) who own inventories higher than firm \(k\)) and

(3) the marginal revenue of firm 1 when selling \(q_1^1\) is strictly positive (and so will it be for firms 2, ..., \(k\)).

Figure 1 presents this case,

\[\text{[INSERT FIGURE 1 HERE]}\]

Indeed, remark that when firms \(i = 1, ..., k\) sell each \(q_i^1\), the equilibrium induces firms \(k + 1, ..., n\) to sell the same quantity. Firms \(k + 1, ..., n\) are therefore confronted to the same equilibrium sales of competitors (i.e. the sum of inventories of firms 1, ..., \(k\) plus \((n - 1)\) times the same quantity sold by each of the firms \(k + 1, ..., n\)). Consequently the marginal revenues of firms \(k + 1, ..., n\) are identical functions of individual sales \(s\). Due to the fact that the marginal revenue is decreasing in individual sales \(s\), then if for the highest inventories of the group of firms \(k + 1, ..., n\), i.e. for \(q_{k+1}^1\), the marginal revenue exceeds the marginal cost, then the same is true for all the other firms of the group, explaining condition (1). On the contrary
the marginal revenues of firms 1, ..., k evaluated at the equilibrium sales of their competitors jump upward the higher the firm in the hierarchy. Indeed firm i is confronted aggregate sales in equilibrium equal to the sum of inventories of firms 1, ..., i − 1, i + 1, ...k, plus the identical sales of firms k + 1, ..., n, which is lower the higher inventories of firm i are, that is the higher firm i is in the hierarchy. The value of the marginal revenue at \( s_i = q_i^1 \) is however lower the higher initial inventories, by definition of the marginal revenue\(^7\), which explains why focusing on the comparison between the marginal revenue of firm k when selling \( q_k^1 \) and the marginal cost \( c \) is sufficient (condition (2)). Finally under assumption (A.1), restricting our attention to \( q_i^1 \leq (a - c)/2 \), condition (3) is verified as long as we consider \( k < n \). However when we characterize the sub-game equilibrium in which all firms are selling exactly their inventories, i.e. such that the marginal revenue of firm n is lower than \( c \) when it sells exactly \( q_n^1 \), then condition (3) needs to be verified. If (3) is verified, then the marginal revenues of firms 1, ..., n − 1 are also positive, leading to the sub-game equilibrium we are searching for. The same analysis can be done to characterize the sub-game equilibria in which some firms are selling exactly their inventories and some others less than their inventories.

**Proposition 1** Under assumptions (A.1) and (A.2), restricting the attention to inventories lower than the quantity a monopoly minimizing its costs would produce, \( q_1^1 \leq (a - c)/2 \), equilibrium aggregate sales \( S^* \) are given by

1. If \( q_1 \in B(0) \), then all firms sell more than their inventories, and \( S^*(0) = \frac{n(a-c)}{n+1} \).
2. For \( \ell \in \{1, ..., n-1\} \), if \( q_1 \in B(\ell) \) then firms 1 to \( \ell \) sell exactly their inventories and

   \[
   S^*(\ell) = \frac{(n - \ell)(a - c)}{n - \ell + 1} + \frac{\sum_{i=1}^{\ell} q_i^1}{n - \ell + 1},
   \]

3. If \( q_1 \in B(n) \) then all firms sell exactly their inventories and \( S^*(n) = \sum_{i=1}^{n} q_i^1 \).
4. For \( \ell \in \{n + 1, ..., 2n - 1\} \), if \( q_1 \in B(\ell) \) then firms 1 to \( \ell - n \) sell less than their inventories and

   \[
   S^*(\ell) = \frac{(\ell - n)}{\ell - n + 1} + \frac{\sum_{i=\ell-n+1}^{n} q_i^1}{\ell - n + 1},
   \]

\(^7\)One replaces \(-q_i^1\) by \(-2q_i^1\) in each expression.
5. If \( q_1 \in B(2n) \) then all firms sell less than their inventories and \( S^*(2n) = \frac{n}{n+1}. \)

As for a given level of industry sales there is a unique corresponding level of individual sales on each firm backward reaction mapping, there is a unique vector of individual sales in equilibrium. The detail of this analysis can be found in appendix A.1.

**Corollary 1 (to Proposition 1)** For each equilibrium level of aggregate sales \( S^*(\ell) \), \( \ell = 0, ..., 2n \), there is a unique equilibrium vector of individual sales \( s^*(\ell) \) given by

1. If \( q_1 \in B(0) \), \( s^*(0) = \frac{a-c}{n+1} \) for any \( i \in I \),

2. For \( \ell \in \{1, ..., n-1\} \), if \( q_1 \in B(\ell) \) then \( s^*(\ell) = q_1^\ell \) for \( i = 1, ..., \ell \) and

\[
s^*(\ell) = \frac{a-c}{n-\ell+1} - \frac{\sum_{i=1}^{\ell} q_1^i}{n-\ell+1} \quad \text{for} \quad i = \ell + 1, ..., n,
\]

3. If \( q_1 \in B(n) \) then \( s^*(n) = q_1^n \) for all \( i \in I \),

4. For \( \ell \in \{n+1, ..., 2n-1\} \), if \( q_1 \in B(\ell) \) then \( s^*(\ell) = q_1^\ell \) for \( i = \ell - n + 1, ..., n \) and

\[
s^*(\ell) = \frac{a}{\ell - n + 1} - \frac{\sum_{i=\ell-n+1}^{n} q_1^i}{\ell - n + 1} \quad \text{for} \quad i = 1, ..., \ell - n,
\]

5. If \( q_1 \in B(2n) \) then \( s^*(2n) = \frac{a}{n+1} \) for any \( i \in I \).

Finally it is possible to re-write the sequence of sets \( \{B(\ell)\}_{\ell=0, ..., 2n} \) in terms of conditions on the marginal revenues.

**Corollary 2 (to Proposition 1)** The sequence of conditions on inventories \( \{B(\ell)\}_{\ell=0, ..., 2n} \) can be expressed as a sequence of conditions on the marginal revenue

1. \( B(0) = \{q_1/m^1 (q_1^1, (n-1)(a-c)/(n+1)) \geq c\} \),

2. For \( \ell \in \{1, ..., n-1\} \),

\[
B(\ell) = \{q_1/m^\ell \left( q_1^\ell, (n-\ell)(a-c-\sum_{i=1}^{\ell} q_1^i)/(n-\ell+1) + \sum_{i=1}^{\ell-1} q_1^i \right) < c, \quad m^{\ell+1} \left( q_1^{\ell+1}, (n-\ell-1)(a-c-\sum_{i=1}^{\ell} q_1^i)/(n-\ell+1) + \sum_{i=1}^{\ell} q_1^i \right) \geq c\},
\]

3. \( B(n) = \{q_1/m^n (q_1^n, \sum_{i=1}^{n-1} q_1^i) < c, m^1 (q_1^1, \sum_{i=2}^{n} q_1^i) \geq 0\} \),

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4. For $\ell \in \{n + 1, \ldots, 2n - 1\}$,

$$B(\ell) = \{q_1/m^{\ell-n}(q_1^{\ell-n}, (\ell - n - 1)(a - \sum_{i=\ell-n+1}^{n} q_i^n)/(\ell - n + 1) + \sum_{i=\ell-n+1}^{n} q_i^n) < 0,$$

$$m^{\ell-n+1}(q_1^{\ell-n+1}, (\ell - n)(a - \sum_{i=\ell-n+1}^{n} q_i^n)/(\ell - n + 1) + \sum_{i=\ell-n+2}^{n} q_i^n) \geq 0\}.$$  

5. $B(2n) = \{q_1/m^n(q_1^n, (n-1)a/(n+1)) < 0\}$.

The proof of these two last results can be found in appendix A.1.

3 Oligopolistic inventories in equilibrium

Let us start to establish what cannot be an equilibrium of the game we study. The lemma below shows that firms never store quantities that lead them to produce again in the second period. It implies directly that they will not produce in the second period to sell the same quantity on the market. Remark it does not rule out the fact they may produce and store the same quantity in first period.

Lemma 1. Any n-uple of inventories $q_1 = (q_1^1, \ldots, q_1^n)$ such that

(i) Some firms are selling strictly less than their inventories,

(ii) Some firms are selling the same quantity strictly higher than their inventories,

(iii) Firm $n$ with the lowest level of inventories holds a quantity $q_1^n$ strictly positive,

cannot be an equilibrium.

Proof. Since inventories are costly to produce, if they were selling strictly less than their inventories, firms would be strictly better off reducing unilaterally their inventories. Situations in which some firms are selling less than their inventories cannot be an equilibrium: (i) holds. To prove (ii), consider a situation in which firms $1$ to $k$ are selling exactly their inventories and firms $k+1$ to $n$ are selling the same quantity $(a - c - \sum_{i=1}^{k} q_i^1)/(n-k+1)$ (as shown in corollary 1). To be an equilibrium, it must be the case that $q_1 \in B(k)$. Examine the case of firm $k+1$. It could increase its inventories to a level such that it sells exactly its inventories, as firms $1$ to $k$ do, but in which the remaining firms ($k+2$ to $n$) are still selling $(a - c - \sum_{i=1}^{k} q_i^1)/(n-k+1)$.

For an interest rate sufficiently close to 0, the profit differential from this deviation
is strictly positive. This argument can be repeated for any firm \( k \), proving (ii). Finally (iii) is obviously granted: as \( \rho > 0 \), firm \( n \) with the lowest level of inventories prefers to minimize its cost of production by setting \( q^n_1 = 0 \) and producing its sales in second period. ||

This lemma has several important implications. Even if it does not rule out a priori equilibria in which all firms apart one select the same level of inventories, it directly leads to consider inventories and sales in the sub-game of the form

\[
S^*(n-1) = (a - c + \sum_{i=1}^{n-1} q_i^1)/2, \quad s^i*(n-1) = q_i^1 \quad \text{for} \quad i = 1, ..., n-1,
\]

and

\[
s^n*(n-1) = (a - c - \sum_{i=1}^{n-1} q_i^1)/2
\]

(7)

where inventories \( q_1 \) belong to \( B(n-1) \). It therefore rules out the possibility for group of more than one firm to act as a leader (even if choosing their inventories non cooperatively) over another group of more than one firm reacting identically in the second period: there is at most one ultimate follower. Consequently the equilibrium of the game, if existing, will be asymmetric with (at least) one firm acting as a follower with respect to \( n-1 \) leaders. Second, remark that in the range of inventories above, firms sales are not modified by an increase in their opponent inventories: that is inventories commit their actions on the second period market.

We now prove that a firm facing a leader does not find profitable to increase its inventories in order to force him to reduce its sales, that is does not put him with redundant inventories in Arvan’s terminology.

**Lemma 2** It is not profitable for any firm \( k \) to increase its inventories to force firms \( 1 \) to \( k-1 \) to sell strictly less than their inventories.

**Proof.** To see that consider a fixed hierarchy such that (7) holds, and first look at whether any firm can force the leader 1 to sell less than his inventories. Consider a deviation of firm 2 consisting in increasing its inventories in such a way that this firm sells exactly her inventories \( \tilde{q}_2^1 \) while the leader sells \( s^{1*} = \frac{a}{2} - \frac{1}{2} \tilde{q}_2^1 - \frac{1}{2} \sum_{i=3}^{n-1} q_i^1 \) that is strictly less than her inventories \( q_1^1 \). Note that the ultimate follower with no inventories is not able to produce anymore since its residual marginal revenue falls
below its constant marginal cost. The profit obtained by firm 2 is equal to

$$\tilde{\Pi}^2 = \left( \frac{a - 2c}{2} - \frac{1}{2} \sum_{i=3}^{n-1} q_i - \frac{1}{2} \tilde{q}_1^2 \right) \tilde{q}_1^2 - \rho c \tilde{q}_1^2$$

for a deviation satisfying

$$\tilde{q}_1^2 \geq a - 2q_1^1 - \sum_{i=3}^{n-1} q_i^i$$

If it sticks to $q_1^1$ such that all $n - 1$ firms are selling exactly their inventories, firm 2 obtains

$$\Pi^2 = \left( \frac{a - c}{2} - \frac{1}{2} \sum_{i=3}^{n-1} q_i - \frac{1}{2} q_1^1 - \frac{1}{2} \tilde{q}_1^2 \right) q_1^2 - \rho c q_1^2$$

Remark that the condition for the deviation to put the leader firm 1 with redundant inventories is equivalent to

$$q_1^1 \geq \frac{a}{2} - \frac{1}{2} \sum_{i=3}^{n-1} q_i^i - \frac{1}{2} \tilde{q}_1^2$$

that is firm 2 increases its inventories in such a way that firm 1 inventories are above the set of choices, given $\sum_{i=3}^{n-1} q_i^i$, that maximizes firm 1’s payoff if it was able to sell a 0 cost. Under the assumption (A.1) that $a \leq 3c$, this set is strictly above the set

$$q_1^2 = \frac{a - c}{2} - \frac{1}{2} \sum_{i=3}^{n-1} q_i - \frac{1}{2} q_1^1$$

in the graph $(q_1^2, q_1^1)$, that is strictly above the set that maximizes the profit $\Pi^2$ that include the cost of production of inventories, i.e. the total profit. Meaning that in order to put a rival with larger inventories with redundant inventories, a firm must do some losses: this threat is void. The same argument applies to firms 3,..., $n - 1$, since they all have the same expression of the profit. A similar argument also prevents firm $k$ to put firms 1 to $k - 1$ with redundant inventories.||

As stressed in the proof above, this result follows directly from the assumption that consumers willingness to pay is not too large compared to the marginal cost of production. If it is not the case, asymmetric equilibria may still prevail, but instead of being extremely close to Stackelberg outcomes, the leader now chooses a lower level of inventories such that the follower cannot put him with redundant inventories.
inventories. The remaining of the paper consists in showing that the equilibrium exists in pure strategies, and is such that firms store different amounts and create a hierarchy of Stackelberg leaders. Before stating this result, let us explain why the Cournot equilibrium cannot be an equilibrium of the game.

Consider that all firms but one are storing the same Cournot quantity, \((a - c)/(n + 1)\). Then in the absence of an interest rate, \(\rho = 0\) the best reply of the remaining firm is to store \((a - c)/(n + 1)\). All firms would sell these quantities in the market. In the presence of a positive interest rate, \(\rho > 0\), this holds no longer: it would be less costly for this firm to produce the Cournot quantity in second period rather in the first one to minimize its cost of production. Consequently the Cournot outcome cannot be an equilibrium of our game. It is worth noting the role of \(\rho\) as an equilibrium selection device: by making first period production slightly more costly than second period production, one is able to rule out of the set of equilibria the weakly dominated ones (like the Cournot outcome if firms were indifferent between the two periods). This natural selection device allows to select the equilibria in the duopoly game (Saloner [1987]), where any pair of inventories on the outer envelope of the Cournot reaction functions between the two Stackelberg outcomes is an equilibrium in pure strategies. It is a special case of Pal [1991] and Pal [1996] who consider the duopoly game with cost differentials between periods.

**Theorem 1** The two-period Cournot oligopolistic competition with constant marginal cost of production \(c\) possesses a unique asymmetric Nash equilibrium in pure strategies, in which inventories of firm of rank \(i = 1, \ldots, n\) satisfy

\[
q_i^{*} = \frac{a - c}{2^i} \quad \text{for all } i = 1, \ldots, n - 1 \quad \text{and} \quad q_1^{n*} = 0
\]

individual and aggregate sales \(s^{*}\) and \(S^*\) are given by

\[
s_i^{*} = \frac{a - c}{2^i} \quad \text{for all } i \in I, \quad S^* = (a - c) \left(1 - \left(\frac{1}{2}\right)^n\right)
\]

\(8\)See Mitraille [2005]. In the duopoly game with \(\rho\) close to 0, this level is equal to the intersection between the curves \(q_1^1 = \frac{a}{2} - \frac{1}{2}q_1^2\) and \(q_2^1 = \frac{a - c}{2} - \frac{1}{2}q_1^1\) that is \((q_1^1, q_1^2) = (\frac{a + c}{2}, \frac{a - 2c}{2})\). The assumption (2) is too restrictive on purpose: it would presumably be sufficient to assume that at the Stackelberg leader production, the second firm in the hierarchy cannot put the leader with redundant inventories, that is assume that the usual Stackelberg outcome is below the locus \(q_1^1 = \frac{a}{2} - \frac{1}{2}q_1^2\), where the intercept of the demand contains all the inventories of the \(n - 2\) followers.

\(9\)Under some extra assumptions on demand and cost missing in Saloner’s work relating to assumption (A.1).
and aggregate inventories are

\[ I^* = (a - c) \left( 1 - \left( \frac{1}{2} \right)^{n-1} \right) \]

\[ \| \]

Proof. From lemmas 1 and 2, we know that we can restrict our attention to inventories leading to (7). As firms are committed it suffices to optimize ”sequentially” the payoffs. Each firm acts as a monopoly over the residual demand left by firms acting as leaders over it, integrating the influence on its followers. The result is straightforward to derive.\[ \| \]

Let us establish the welfare impact of inventories in Cournot competition.

**Corollary 3** When the number \( n \) of firms in competition over the two periods increases, the aggregate sales \( S^* \) converges to the competitive sales \( S^C = a - c \) quicker than in the situation where Cournot competitors are unable to store. Consequently the Social Welfare converges quicker to the optimum when firms are able to store than when they are not.

Proof. Consider the difference between the competitive outcome and our aggregate sales,

\[ (a - c) - S^* = (a - c) \left( \frac{1}{2} \right)^n \]

and the same difference for the traditional static Cournot oligopoly;

\[ (a - c) - \frac{n}{n+1}(a - c) = (a - c) \frac{1}{n+1} \]

It is immediate to check that \( \left( \frac{1}{2} \right)^n \) converges to 0 quicker than \( \frac{1}{n+1} \).\[ \| \]

For example if the number of firms is equal to 4, Cournot competitors are selling an aggregate quantity equal to 80% of the social optimum, while Cournot competitors able to store are selling an aggregate quantity equal to 93.75% of the social optimum!

**Corollary 4** Inventories are distributed over \([0, \frac{a - c}{2}]\) and market shares are asymmetric independently of the number of firms in competition. The average level of inventories hold by a \( n \)-firm oligopoly is equal to

\[ \bar{q}_i = \frac{a - c}{n} \left( 1 - \left( \frac{1}{2} \right)^{n-1} \right) \]
and converges to 0 when the number of firm increases.

Proof. From theorem 1, we know that the leader on the market always stores \( \frac{a-c}{2} \), no matter the number of firms in competition. Since the ultimate follower prefers not to store, inventories are distributed over \([0, \frac{a-c}{2}]\). Every firm sells exactly the amount of inventories it is holding, and the ultimate follower who is not storing acts as a monopolist on the residual demand left by the cascade of leaders. Let us briefly compute the average inventories.

\[
\bar{q}_1 = \frac{1}{n} \sum_{i=1}^{n-1} q_i = \frac{1}{n} \sum_{i=1}^{n-1} \frac{a-c}{2} = \frac{a-c}{n} \left( \sum_{i=0}^{n-1} \frac{1}{2^i} - 1 \right) = \frac{a-c}{n} \left( 1 - \left( \frac{1}{2} \right)^{n-1} \right)
\]

When \( n \) goes to infinity, the first element of the product goes to 0, the second to 1. Therefore in the limit average inventories are equal to 0.\(\|^1\)

An interesting feature of the distribution of inventories can be remarked. First note that the usual leader quantity \( \frac{a-c}{2} \) is stored with a strictly positive probability, although its likelihood in the population, \( \frac{1}{n} \), decreases the higher the number of firms. An histogram representation of the distribution of inventories would clearly depict a distribution asymmetric to the left, strictly decreasing on \([0, \frac{a-c}{2}]\), with median inventories lower than average inventories. The HHI index will not converge to 0, as the next corollary establishes.

**Corollary 5** When the number of firms competing in period 1 tends to infinity, the equilibrium becomes competitive but firms still enjoy asymmetric market shares and leaders are present on the market. The HHI concentration index is

\[
HHI = \lim_{n \to +\infty} \sum_{i=1}^{n} \left( \frac{s_i}{S} \right)^2 = \frac{1}{3} \times 10,000 > 0
\]

Proof: It suffices to compute the limit of the summation in the corollary.\(\|^1\)

## 4 Conclusion

Without any other incentive than strategic to build up inventories, and in a perfectly symmetric setting, this paper shows that one may expect to observe different levels of inventories and different market shares on an oligopolistic market, no matter the
number of firms in competition. As pointed out in Boyer and Moreaux [1985] and [1986], observing a leader on a market is not incompatible with perfect competition. However they assume that the hierarchy is exogenous, which does not address the important question of how firms may act as leaders. In this paper we endogenize the leadership through inventories to show that firms may use large but different inventories to constrain their opponents, act as leaders, while being on a competitive market even if actions are chosen simultaneously every time players have to take a decision. The commitment value of inventories does not disappear the higher the degree of competition.

The current results have been derived using a positive but close to 0 level of interest rate. If the interest rate grows, then the likelihood of strategic inventories diminishes to leave the market in a symmetric equilibrium. Introducing a cost of storage would have a similar effect. It is important to remark that the result we are pointing out is extreme in our setting, but would be smoother in a more sophisticated setting. Indeed we obtain that all firms apart one store their entire future sales, the last one not storing at all. If we were to allow for uncertain demand and risk neutral firm, there would not be any other incentive than strategic to store, and we would still obtain the fact that "big" (in terms of market share) firms store a larger part of their future sales than "small" firms.

Remark that a longer dynamic with several consecutive markets under constant returns-to-scale would not jeopardize the use of strategic inventories: with a constant marginal cost of production, firms objectives are separable between periods. However the firm would try to get a higher rank in the hierarchy as soon as possible, case in which the interest rate and the cost of storage would play a larger role.

References


Appendices

A. Proofs

A.1 Proof of proposition 1

The proof is done in 3 steps. (Step 1) shows how to simplify each firm second period problem to enlighten the role of period 1 production (i.e. inventories) and derives the individual sales each firm chooses as a best reply to the aggregate sales of its competitors, \( \tilde{s}^i(S^{-i}, q^i_1) \), which depends on inventories. First, as we study an oligopolistic competition, it is possible to restrict one’s attention to period 1 inventories lower or equal to the individual production of a monopoly minimizing its cost of production, denoted \( q^m \). Given the opportunity cost of producing in period 1 instead of period 2, and given the demand and costs parameters assumed before, this quantity is equal to \( q^m = (a - c)/2 \). Consequently, the vector of period 1 production satisfies assumption (A.2) and is such that \( q_1 \in [0, (a - c)/2]^n \).

Second and most importantly, inventories create firm-specific kinks in firms best replies, source of the asymmetry of the Nash equilibrium of the game. Indeed as their cost of production is sunk when selling on the market in second period, inventories form a capacity from which firms can sell without producing. That is their effective marginal cost of production is given by

\[
\gamma^i(s^i, q^i_1) = \begin{cases} 
0 & \text{if } s^i \leq q^i_1 \\
\gamma & \text{if } s^i > q^i_1.
\end{cases} \tag{8}
\]

The best reply of firm \( i \) to aggregate sales of its competitors \( S^{-i} \) can be obtained by confronting the effective marginal cost \( \gamma^i(s^i, q^i_1) \) to the marginal revenue

\[
m^i(s^i, S^{-i}) = a - 2s^i - S^{-i} \tag{9}
\]

If \( S^{-i} \) is such that \( m^i(q^i_1, S^{-i}) \), the marginal revenue firm \( i \) obtains from selling exactly its inventories, exceeds the marginal cost of production \( c \), then firm \( i \) is
better off producing again in period 2 to sell more than its period 1 inventories \( q_i^1 \).

If \( S^{-i} \) is such that \( m^i(q_i^1, S^{-i}) \) is lower than the marginal cost \( c \) but strictly positive, then firm \( i \) does not produce in period 2 and sells exactly \( q_i^1 \) on the market. Finally, if \( S^{-i} \) is such that \( m^i(q_i^1, S^{-i}) \) is negative, firm \( i \) is better off selling less than its inventories \( q_i^1 \). For a given level of period 1 production, the best reply of firm \( i \) to aggregate sales \( S^{-i} \) of its competitors is therefore given by

\[
\hat{s}^i(S^{-i}, q_i^1) = \begin{cases} 
\frac{1}{2}(a - S^{-i}) & \text{if } S^{-i} \leq a - c - 2q_i^1 \\
q_i^1 & \text{if } S^{-i} \in [a - c - 2q_i^1, a - 2q_i^1] \\
\frac{1}{2}(a - S^{-i}) & \text{if } a - 2q_i^1 \leq S^{-i}
\end{cases}
\]

(10)

As period 1 production of any firm \( i \) is lower than \((a - c)/2\), the bounds \( a - c - 2q_i^1 \) and \( a - 2q_i^1 \) are non-negative and consequently the best reply is continuous and has exactly two kinks: depending on competitors market behavior, each firm can either sell more than (first line of equation (10)), or sell exactly (second line of (10)), or sell less than (third line of (10)) its inventories.

In Step 2, we aggregate all the best replies to find the equilibrium aggregate sales of the industry. To do so, we construct the best reply of each firm to the aggregate quantity sold by the industry, \( \hat{s}^i(S, q_i^1) \), and we sum these functions over all firms to obtain the industry best reply to an aggregate sales level, \( \sum_{i \in I} \hat{s}^i(S, q_i^1) = \hat{S}(S, q_i^1) \).

To construct the best reply \( \hat{s}^i(S, q_i^1) \), also known as the backward reaction mapping (from Novshek 1984 terminology) we first determine the cumulative reaction to \( S^{-i} \) for firm \( i \), \( S_i^*(S^{-i}, q_i^1) = \{ s^i + S^{-i} / s^i = \hat{s}^i(S^{-i}, q_i^1) \} \).

\[
S_i^*(S^{-i}, q_i^1) = \begin{cases} 
\frac{1}{2}(a - c + S^{-i}) & \text{if } S^{-i} \leq a - c - 2q_i^1 \\
q_i^1 + S^{-i} & \text{if } S^{-i} \in [a - c - 2q_i^1, a - 2q_i^1] \\
\frac{1}{2}(a + S^{-i}) & \text{if } S^{-i} \geq a - 2q_i^1 
\end{cases}
\]

(11)

Then we invert it to obtain the inverse cumulative best response function \( S_i^{-i}(S, q_i^1) \) for firm \( i \). As \( S_i^*(S^{-i}, q_i^1) \) is strictly increasing it has a unique inverse,

\[
S_i^{-i}(S, q_i^1) = \begin{cases} 
2S - (a - c) & \text{if } S \in \left[ \frac{a - c}{2}, a - c - q_i^1 \right] \\
S - q_i^1 & \text{if } S \in \left[ a - c - q_i^1, a - q_i^1 \right] \\
2S - a & \text{if } S \geq a - q_i^1 
\end{cases}
\]

(12)

Finally we solve for the individual sale \( s^i \) in \( \{ s^i / S - s^i = S_i^{-i}(S, q_i^1) \} \), to obtain
the backward reaction mapping,

\[
\hat{S}(S, q_i^1) = \begin{cases} 
(a - c) - S & \text{if } S \in [(a - c)/2, a - c - q_i^1] \\
q_i^1 & \text{if } S \in [a - c - q_i^1, a - q_i^1] \\
a - S & \text{if } S \geq a - q_i^1
\end{cases}
\]  

(13)

where as in (10), given some industry sales \( S \), firm \( i \) can either sell more than (first line of (13)), or sell exactly (second line of (13)), or sell less than (third line of (13)) its inventories \( q_i^1 \). Under assumptions (A.1) and (A.2), summing all the backward reaction mappings to obtain \( \hat{S}(S, q_1) \) can be done easily. Indeed for any level of industry sales \( S \), all firms are either selling at least their inventories or selling at most their inventories, but it is not possible that some of them sell strictly more than their inventories, while some others are selling strictly less. To put it differently the cut-off values for \( S \) determining the reaction of an individual firm in (13) are ”nicely” ranked across firms. To see this, first remark that from (A.2),

\[
(a - c)/2 \leq a - c - q_1^1 \leq ... \leq a - c - q_n^1 \leq a - c
\]  

(14)

and

\[
a - q_1^1 \leq ... \leq a - q_n^1.
\]  

(15)

As justified in step 1, there is no loss of generality to restrict our attention to period 1 productions lower than \( q^m = (a - c)/2 \). Consequently the lower bound in the sequence of inequalities (15) can be minored, \( a - q_1^1 \geq (a + c)/2 \). It suffices to remark that as a consequence of (A.1), the upper bound of the sequence of inequalities (14) is lower than the lower bound of the sequence (15), i.e. \( a - c \leq (a + c)/2 \), to be able to rank across all firms the cut-off values at which there are kinks in the backward reaction mappings (13)

\[
a - c - q_1^1 \leq ... \leq a - c - q_n^1 \leq a - q_1^1 \leq ... \leq a - q_n^1
\]  

(16)

For a level of industry sales \( S \) lower than \( a - c - q_1^1 \), all firms are selling more than their period 1 production and consequently the sum of all the backward reaction mappings is simply equal to \( n \) times the expression in the first line of (13), \( (a - c) - S \). For \( S \) higher than \( a - c - q_1^1 \) and lower than \( a - c - q_2^1 \), firm 1 is selling exactly its inventories and firms 2 to \( n \) are selling strictly more: the sum of the backward reaction mappings is equal to \( q_1^1 \) plus \( n - 1 \) times \( (a - c) - S \), ... and so on.
industry sales higher than \( a - c - q_i^1 \) and lower than \( a - q_i^1 \), all firms are selling exactly their inventories, and \( \sum_{i \in I} \hat{S}(S, q_i^1) = \sum_{i \in I} q_i^1 = Q_1 \). For \( S \) higher than \( a - q_i^1 \) and lower than \( a - q_i^2 \) firm 1 sells less than its inventories and firms 2 to \( n \) sell exactly their inventories: \( \sum_{i \in I} \hat{S}(S, q_i^1) \) is equal to \( a - S + \sum_{i \geq 2} q_i^1 \), ... and so on to complete the summation. To summarize, \( \hat{S}(S, q_i^1) \) is given by

\[
\hat{S}(S, q_i^1) = \begin{cases} 
(a - c) - S & \text{if } S \in [(a - c)/2, a - c - q_i^1] \\
(n - k)(a - c - S) + \sum_{i=1}^{k} q_i^1 & \text{if } S \in [a - c - q_i^k, a - c - q_i^{k+1}] \\
\sum_{i=1}^{n} q_i^1 & \text{if } S \in [a - c - q_i^n, a - q_i^1] \\
k(a - S) + \sum_{i=k+1}^{n} q_i^1 & \text{if } S \in [a - q_i^k, a - q_i^{k+1}] \\
n(a - S) & \text{if } S \geq a - q_i^n 
\end{cases}
\]

(17)

Step 3 determines the fixed points of \( \hat{S}(S, q_i^1) \). As there are \( 2n \) cut-off values determining the different expressions of \( \hat{S} \), there are \( 2n + 1 \) different expressions and potentially \( 2n + 1 \) different sub-game equilibria to find. We index the consecutive lines from (17) by \( \ell = 0, \ldots, 2n \) at line 0 all firms sell more than their inventories, at line 1 firm 1 sells exactly its inventories and the others more,... and so on. At line \( n \) all firms sell exactly their inventories and at line \( n + 1 \) firm 1 sells less than its inventories while the others sell exactly their inventories, until line 2\( n \). We derive the fixed points of \( \hat{S}(S, q_i^1) \) line by line: for every line \( \ell = 0, \ldots, 2n \), there is a unique equilibrium aggregate sales level \( S^*(\ell) \). To this equilibrium industry sales \( S^*(\ell) \) corresponds a unique set of period 1 inventories \( B(\ell) \) such that if the vector of firms inventories \( q_i^1 \) belongs to \( B(\ell) \), then the equilibrium is \( S^*(\ell) \). Let us describe these fixed points and the sets that are associated to them. The equilibrium in which all firms sell more than their inventories (line \( \ell = 0 \)) is characterized by \( S^*(0) = n(a - c)/(n + 1) \). Inventories must be such that \( S^*(0) \leq a - c - q_i^1 \) that is must belong to \( B(0) \) given by

\[
B(0) = \left\{ q_i^1/q_i^1 \leq (a - c)/(n + 1) \right\}.
\]

(18)

For \( \ell = 1, \ldots, n - 1 \), equilibrium aggregate sales are \( S^*(\ell) = ((n - \ell)(a - c) +
\[ \sum_{i=1}^{\ell} q_1^i / (n - \ell + 1), \text{ and } q_1 \text{ must belong to } B(\ell) \text{ given by} \]
\[ B(\ell) = \left\{ q_1/q_1^\ell \geq (a - c - \sum_{i=1}^{\ell-1} q_1^i) / (n - \ell + 2), \quad q_1^{\ell+1} \leq (a - c - \sum_{i=1}^{\ell} q_1^i) / (n - \ell + 1) \right\}. \] (19)

For \( \ell = n \), \( S^*(n) = \sum_{i=1}^n q_1^i \) and \( q_1 \) must belong to \( B(n) \)
\[ B(n) = \left\{ q_1/q_1^n \geq (a - c - \sum_{i=1}^{n-1} q_1^i) / 2, \quad q_1^1 \leq (a - \sum_{i=2}^{n} q_1^i) / 2 \right\}. \] (20)

For \( \ell = n + 1, \ldots, 2n - 1 \), \( S^*(\ell) = (\ell - n) a + \sum_{i=\ell-n+1}^{n} q_1^i ) / (\ell - n + 1) \) and \( q_1 \in B(\ell) \) such that
\[ B(\ell) = \left\{ q_1/q_1^{\ell-n} \geq (a - \sum_{i=\ell-n+1}^{n} q_1^i) / (\ell - n + 1), \quad q_1^{\ell-n+1} \leq (a - \sum_{i=\ell-n+2}^{n} q_1^i) / (\ell - n + 2) \right\}. \] (21)

Finally for \( \ell = 2n \), \( S^*(2n) = na / (n + 1) \) and \( q_1 \in B(2n) \) such that
\[ B(2n) = \{ q_1/q_1^n \geq a/(n + 1) \}. \] (22)

The intersection between (the interior of) two sets is empty, \( B(\ell) \cap B(\ell') = \emptyset \) for \( \ell \neq \ell' \), and the reunion of all sets \( \bigcup_{\ell=0, \ldots, 2n} B(\ell) \) encompasses exactly all the cases for \( q_1 \) we are interested in. We complete this proof by expressing the conditions on inventories in terms of conditions on the marginal revenues to obtain corollary 2.

Obviously,
\[ q_1^1 \leq \frac{a - c}{n + 1} \Leftrightarrow m^1 \left( q_1^1, \frac{n - 1}{n + 1}(a - c) \right) \geq c \]
and so on...

B. Figures
Figure 1: Graphical analysis of the sales sub-game