Some Perils of Policy Rule Regression: 
The Taylor’s Rule Revisited

Julio A. Carrillo*
University of Toulouse (GREMAQ)

Patrick Fève
University of Toulouse (GREMAQ–CNRS and IDEI)
and Banque de France

Second draft
This version: February, 2006

Abstract

This paper analyzes the potential misidentification involved by the estimations of policy rules trough single equations. We perform this for the Taylor’s rule regression in the context of two model economies, which are only different by the monetary rules they present. We show that one can easily find reduced forms that resemble the Taylor’s rule, even though other policy were in place. Also, we illustrate that in the case of misidentification, limited–information estimation techniques (as GMM) not necessarily deliver consistent estimators for the reduced form parameters. Thus, ruling out policy misspecification requires performing several regressions with important changes in the instrument set. In this sense, the Taylor’s rule given in Clarida, Galí and Gertler (2000) fails this robustness exercise, showing that the misidentification of the rule could be important.

Keywords: exogenous and endogenous policy rules, policy rule regression, nearly observational equivalent models, indeterminacy, policy misspecification

JEL Class.: C22, E40, E52

*Address: GREMAQ–Université de Toulouse I, Manufacture des Tabacs, bât. F, Aile Jean–Jacques Laffont, 21 allée de Brienne, 31000 Toulouse, France. E–mail: julio.carrillo@univ-tlse1.fr. We would like to thank P. Beaudry, F. Collard, M. Dupaigne, T. Kehoe, O. Licandro, F. Portier, J. V. Rios–Rull; B. Djembissi and J. Motis from GREMAQ and two anonymous referees; all the assistants to the X Workshop on Dynamic Macroeconomics, the Bank of France’s Small Monetary Macromodels seminars, and the 9th International Conference on Theory and Methods of Macroeconomics (T2M, 2004) for helpful discussions and comments. All errors are of course our own. J. A. Carrillo gratefully acknowledges the financial support granted by CONACyT (The National Council for Sciences and Technology of Mexico).
Introduction

The aim of this paper is to point out some perils\(^1\) for economic policy misidentification when rules regressions are used, especially when they are conducted from single equations.

In recent years, there has been an increasing interest about the identification of policy rules that the economic authorities are assumed to follow. Notably, applied monetary economics estimates single equations using instrumental variables in order to describe the reaction function of the central bank, which is assumed to respond to changes in expected inflation and the output gap away their targets. For example, Clarida, Galí and Gertler (2000) estimate a monetary policy rule using GMM and document how the change in their estimation accounts for economic (monetary) stability.

The work of Clarida, Galí and Gertler has been criticized by recent contributions as it does not pay sufficient attention to the fact that their single equation would be probably part of a system (see Linde (2002), Lubik and Schorfheide (2004)). Moreover, a single equation approach suffers from a lack of identification (especially when weak instruments are used), contrary to full-information environments (see Mavroeidis (2004a), (2004b), Nason and Smith (2003)). Another problematic issue is presented in Beyer and Farmer (2003, 2004), where it is shown that a single equation approach tell us nothing about the real nature of aggregate fluctuations, \textit{i.e.}, a given policy may be either associated to a model driven by fundamental shocks or a model driven by sunspot shocks.

In this paper, we focus on the peril of misidentify the policy, given the existence of nearly observational equivalent models with similar reduced forms. We also analyze the consistency properties of GMM, as a limited–information–estimation technique, in the presence and absence of misspecification.

In fact, it can be shown rather easily that policy misidentifications may happen more often then expected. In that case, the estimated policy rule is just a reduced form of a model where the empirical parameters are not policy invariant, thus illustrating the \textit{Lucas critique}. The evident pitfall of misidentify the policy concerns spurious economic recommendations.

\(^1\)The reader could easily find some analogy with the paper of Benhabib, Schmitt–Grohe and Uribe (2001b).
Also, when exploring the asymptotic properties of GMM, conditional to a model specification, one can show that in the presence of misidentification the GMM estimators may present important bias. This brings about new insights to the robustness exercises when estimating a policy rule, where it should be preferred changing completely the instrument set, as to get robustness evidence from instruments quantitatively different (in their variance decomposition).

In order to illustrate these findings, we used a simple linear rational expectations model with two different kind of rules to show that they could be observationally equivalent. If only one source of shock is present in the economy, GMM estimation is completely uninformative about the true nature of the policy rule. We then added so more exogenous disturbances into the simple model and looked at the properties of the GMM estimators. It turns out that if the additional exogenous variables explain a large part of the variations of the endogenous variable, the bias in the estimators in the case of misidentification could be quite significant. Therefore, a natural test to elicit misspecification is the change of instruments to test for parameter stability.

Finally, we consider an illustration within the Taylor rule regression that is performed in applied monetary economics. Specifically, we consider an exogenous money growth rule together with technological shocks in a standard cash-in-advance (CIA) economy. A calibration exercise shows that the reduced form of this model is able to match the inflation parameters of the Taylor’s rule founded in the empirical literature for the U.S., so misidentification is not prevented. An empirical exercise to replicate the results of Clarida, Galí and Gertler (2000) about the Taylor’s rule shows that the Taylor’s rule regression may suffer from important misspecification, remarkably concerning the monetary policy inertia terms. These results could be unsatisfactory informative about the structural preferences of the Federal Reserve.

The paper is organized as follows. Section 1 presents a simple rational expectations model with two different policy rules, we illustrate the policy misidentification and the principle of policy rule regression. The asymptotic properties of the GMM estimators are also covered. In section 3, we build two models economies that are exactly the same except for the monetary policy rule the central bank exercises; we then characterize the properties of the estimators used by an hypothetical policy rule regression, where if the rule is correctly identified, any valid instrument imply a consistent estimator. Since ruling out

\footnote{In the sense that the covariances between the regressors and the instrument is not zero.}
misidentification requires performing various regressions with different instruments sets, in section 4 we run the Taylor’s rule as specified in Clarida, Galí and Gertler (2000) to look for misidentification, and report the results. Finally, the last section presents some concluding remarks.

1 Policy Rule Regression

What we call a policy rule regression is an estimation procedure that relies on limited-information estimation techniques, as GMM, in the form of single equations. These methods have gain in popularity because of the agreed upon cliché that “all models are false” and because GMM requires few restrictions on the data. Although specifying the whole model is not necessary for estimation purposes, single equations are not prevented of policy rule misspecification and could undergo the Lucas critique. The obvious consequence is to produce spurious policy recommendations. In this section, we are interested in showing two issues: a) The presence of observational equivalent models make policy misidentification a common and problematic subject, and b) GMM radical robustness exercises may elicit the presence of misspecification.

1.1 One shock world: Two observational equivalent models

In what follows, we present two model economies that have the same reduced forms. Both are deliberately stylized in order to deliver clear results about the policy rule regression. Further references on observational equivalent models can be found in Beyer and Farmer (2003 and 2004).

1.1.1 The exogenous rule economy

We consider an economy that expresses a single endogenous variable $y_t$ as a linear function of the conditional (on the available information at time $t$) expectation of this same variable in period $t + 1$ and a policy instrument $x_t$ governed by an exogenous stationary process. This economy takes the form:

\begin{align*}
y_t &= aE_t y_{t+1} + bx_t \\
x_t &= \rho x_{t-1} + \sigma \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim iid(0, 1)
\end{align*}

where $b \neq 0$, $|a| < 1$, $|\rho| < 1$, $\sigma > 0$ and $E_t$ is the conditional expectation operator with respect to the current and past values of $\{x_t, x_{t-1}, ..., y_t, y_{t-1}, ...\}$. The later assumptions imply that the equilibrium will be determinate. The assumption that $x_t$ is AR(1) is widely used in macroeconomic dynamics. It allows to simply represent the time series behavior
of the forcing variable. Moreover, one must remark that \( x_t \) can be correlated with \( y_t \). In this case, we have just to modify (1), taking into account the feedback effect of \( x_t \) on \( y_t \). The AR(1) specification then represents the pure exogenous component in \( x_t \). A stationary solution would be thus obtained iterating (1) forward.\(^3\)

Without loss of generality, we omit a constant term both in (1) and (2) since the endogenous and exogenous variables can be considered as deviations from their long–term average value. Equation (1) can represent either a linear or a log–linear approximation to the equilibrium conditions of a non–linear model. Despite its simplicity, this simple linear representation embodies several model economies: the Cagan model, the monetary model of exchange rate determination, asset pricing models, dynamic factor demand models, business cycle models, monetary models with a cash–in–advance constraint,\(^4\) and so on. Using forward substitutions, the endogenous variable is expressed by

\[
y_t = bE_t \sum_{i=0}^{T} a^i x_{t+i} + a^{T+1} E_t y_{t+T+1}
\]

Excluding explosive paths, \( i.e., \)

\[
\lim_{T \to \infty} a^{T+1} E_t y_{t+T+1} = 0
\]

the forward solution takes the form:

\[
y_t = bE_t \sum_{i=0}^{\infty} a^i x_{t+i}
\]

Using the exogenous policy rule (2), the solution is

\[
y_t = b \left( \sum_{i=0}^{\infty} a^i \rho^i \right) x_t = \frac{b}{1 - a \rho} x_t \quad \forall \ t \tag{3}
\]

This reduced form expresses the endogenous variable \( y_t \) as a linear function of the exogenous variable \( x_t \). This solution shows that when the parameter \( \rho \) of the exogenous policy rule changes, the reduced form parameter \( b/(1-a\rho) \) will be evidently affected. The lack of policy invariance of the reduced form parameter is a basic illustration of the Lucas critique. Note that we can express the observable reduced form parameter of model (1)-(2) as

\[
x_t = \frac{1 - a \rho}{b} y_t \tag{4}
\]

\[
y_t = \rho y_{t-1} + \sigma_y \varepsilon_t \quad \forall \ t \tag{5}
\]

\(^3\)Following Gourieroux, Laffont and Monfort (1982), among others, this simple model has been widely used in order to study the solutions of linear rational expectations models (see e.g. Blanchard and Fisher (1989), Broze and Szafarz (1991) and Farmer (1999)).

\(^4\)See section 2.
where $\sigma_y = b\sigma/(1 - a\rho)$. We now move to the second economy.

1.1.2 The endogenous (forward–looking) rule economy

We assume that the endogenous variable $y_t$ is still represented by (1). However, in this economy the policy instrument is stated to answer the future values of the endogenous variable. This model is represented by

$$y_t = aE_t y_{t+1} + bx_t$$

(6)

$$x_t = \eta E_t y_{t+1}$$

(7)

where $\eta \neq 0$. The later forward–looking policy rule is said to be endogenous since it reacts to expected changes of the endogenous variable, receiving feedback from the dynamics of the economy and it is not fixed by extraneous process. It is also common in the literature to suggest rules as (7), been one famous example the Taylor’s rule involving the interest rate reacting to forward inflation and output gap in monetary economics. The aggregate dynamics of the above model are obtained by replacing in the endogenous equation the policy rule, which leaves

$$y_t = (a + b\eta)E_t y_{t+1}$$

For the ease of exposition, we assume that the absolute value of $a + b\eta$ is greater than one. Thus, the equilibrium is indeterminate or driven by the sunspot shock $\nu_{y,t} = y_t - E_{t-1} y_t$.\(^5\)

Stating the above equation in a backward perspective, and replacing $E_t y_{t+1}$ in the policy rule equation, delivers the observational reduced form of the above model:

$$x_t = \frac{\eta}{a + b\eta} y_t$$

(8)

$$y_t = \frac{1}{a + b\eta} y_{t-1} + \nu_{y,t} \quad \forall \ t$$

(9)

1.1.3 Policy rule regression

Assume an econometrician is asked to estimate the policy rule behind the behavior of $x_t$. Let figure 1 denote the hypothetical data set the econometrician may find. In this context, a natural question is how can we discriminate between the two policy rules? The answer is that in the single shock economy this is not possible.

It is straightforward to see that if

$$\rho = \frac{1}{a + b\eta} \quad \text{and} \quad Var(\nu_{y,t}) = \sigma_y,$$

\(^5\)For further details about indeterminate models or irregular equilibria, see Farmer (1999) or Beyer and Farmer (2003) and (2004).
both economies are observationally equivalent, since their reduced forms would be the same.

Let us assume that the econometrician is about to estimate a forward–looking policy rule, of the form

\[ x_t = \eta_{prr} E_t y_{t+1} + \varepsilon_{prr,t} \]

where \( \varepsilon_{prr,t} \) is the policy rule estimation error. The estimation properties of the above equation are conditional on the model economy that describes the real world. If the true world is depicted as the exogenous rule economy states, than the reduced form parameter \( \eta_{prr} \equiv (1 - ap)/(bp) \) and \( \varepsilon_{prr,t} \equiv 0 \); the lack of policy invariance of the reduced form parameter provides an illustration of the Lucas critique. In the contrary, if the forward–looking policy rule economy is the true world, than \( \eta_{prr} \equiv \eta \) and \( \varepsilon_{prr,t} \equiv 0 \).

We claim that it is precisely because the policy rule error term, \( \varepsilon_{prr,t} \), is similar in the both models (in this case, equivalent to zero) that we cannot use limited–information–estimation to discriminate between policy rules.

The following is trivial, but it will help to fix ideas on the relevant message to be presented in the next section. As the usual practice, we consider Instrumental Variables
GMM (hereafter, IV–GMM) in order to estimate the policy function (10). To avoid endogeneity issues, empirical studies use a set of predetermined – or weekly exogenous – instrumental variables. We follow here exactly the same empirical strategy. For simplicity and tractability, we assume that the econometrician uses a single instrument, which is enough to fulfilled the condition for identification of the policy rule parameter $\eta_{prr}$. In this case, our set of possible instruments is defined by $Z = \{x_{t-1}, y_{t-1}\}$. In terms of observable variables, (10) is rewritten as:

$$x_t = \eta_{prr}y_{t+1} - \eta_{prr}\nu_{y,t+1} + \varepsilon_{prr,t}$$

(11)

Let $z_t \in Z$ denote a single instrument known in period $t$. The later implies that the probabilistic limit of the IV estimator using instrument $z_t$ is given by

$$\text{plim} \eta_{prr}(z_t) \equiv \frac{E(x_t z_t)}{E(y_{t+1} z_t)} = \eta_{prr}$$

(12)

for any $z_t \in Z$.\(^6\) Using IV is completely uninformative about the true nature of the policy rule, since still we ignore if $\eta_{prr} \equiv (1 - a \rho)/(b \rho)$, according to the first model, or $\eta_{prr} \equiv \eta$ according to the second.

### 1.2 Multiple shocks world: Two nearly observational models

Most of macroeconomic models include several shocks – productivity, government spending, tastes, money supply – in order to improve the specification of the endogenous variable. Moreover, a typical exercise in the business cycle literature is to identify the various sources of aggregate fluctuations and thus to evaluate their relative contribution. We thus extend the previous models to the case of multiple exogenous variables.

#### 1.2.1 Exogenous rule economy

Assume there exists $m$ different exogenous stationary processes, among economic policy decisions and fundamental shocks, affecting the dynamics of the economy. Specifically, the model economy is described by

$$y_t = aE_t y_{t+1} + \sum_{j=1}^{m} b_j x_{j,t}$$

(13)

where as before $|a| < 1$ and each exogenous variable $x_{j,t}$ follows an AR(1) process

$$x_{j,t} = \rho_j x_{j,t-1} + \sigma_j \varepsilon_{j,t} \quad \text{with} \quad \varepsilon_{j,t} \sim \text{iid}(0, 1)$$

(14)

\(^6\)This follows from than fact that, according with the two models presented, $E[(\varepsilon_{prr,t} - \eta_{prr}\nu_{y,t+1}) z_t] = 0$ for any $z_t \in Z$.\]
where $|\rho_j| < 1$ and $\sigma_j > 0$. We assume that the innovations verify

$$E(\varepsilon_{j,t}\varepsilon_{j',t}) = 0 \quad \forall j \neq j',$$

so, the exogenous variables are uncorrelated. For exposition purposes, we denote $x_{1,t}$ as the policy instrument of interest. Iterating (13) forward and using the exogenous policy rules (14), the solution is given by

$$y_t = \sum_{j=1}^{m} \frac{b_j}{1 - a\rho_j} x_{j,t}$$

or, its equivalent representation:

$$x_{1,t} = \frac{1 - a\rho_1}{b_1\rho_1} E_t y_{t+1} - \frac{1 - a\rho_1}{b_1\rho_1} \sum_{j=2}^{m} \frac{b_j\rho_j}{1 - a\rho_j} x_{j,t}$$

Now, suppose that the econometrician wants to estimate the same policy rule as before. Our set of possible instruments changes to $Z = \{x_{1,t-1}, y_{t-1}\}$. In terms of the policy instrument of interest, we have:

7Other instruments, like $x_{j,t}$ are of not concern, since they are uncorrelated with $x_{1,t}$.  

Figure 2: Policy rule regression with multiple variables
\[ x_{1,t} = \eta_{prr}E[y_{t+1}] + \varepsilon_{prr} \]

We see immediately that the solution of the model implies that \( \eta_{prr} \equiv \frac{1-a\rho_1}{b_1\rho_1} \) and \( \varepsilon_{prr} \equiv \frac{1}{b_1\rho_1} \sum_{j=2}^{m} \frac{b_j\rho_j}{1-a\rho_j} x_{j,t} \). The later tell us that the plim \( \eta_{prr}(z_t) \) given in (12) will not be free from bias for all \( z_t \in Z \), i.e.

\[
\text{plim}\ \eta_{prr}(z_t) \equiv E(y_{t+1}|z_t) = \frac{1-a\rho_1}{b_1\rho_1} \sum_{j=2}^{m} \frac{b_j\rho_j}{1-a\rho_j} E(y_{t+1}|z_t)
\]

In particular, we have that only the plim \( \eta_{prr}(x_{1,t-1}) \) will be consistent, whereas plim \( \eta_{prr}(y_{t-1}) \) can be stated as

\[
\frac{1-a\rho_1}{b_1\rho_1} \nu
\]

where

\[
\nu = \frac{V(y_{t}/x_{1,t})}{V(y_{t}) + \sum_{j=2}^{m} (\rho_j/\rho_1)^2 V(y_{t}/x_{j,t})}
\]

Since \( V(y_{t}/x_{j,t}) \) denotes the variance of \( y_t \) conditional on \( x_{j,t} \), \( \nu \) can be interpreted as the weighted contribution of \( x_{1,t} \) to the variance of \( y_t \). This is clear when all the weights are the same, i.e. when \( \rho_1 = \ldots = \rho_m \) and \( \nu \) simply reduces to

\[
\nu = \frac{V(y_{t}/x_{1,t})}{V(y_{t})}
\]

If the contribution of \( x_{1,t} \) to the variance of \( y_t \) is large, we have \( \nu \simeq 1 \) and the GMM estimator is the same as the one obtained in the single exogenous variable case. Conversely, if the contribution of \( x_{1,t} \) to the variance of \( y_t \) is very small, we have \( \nu \simeq 0 \) and plim \( \eta_{prr}(y_{t-1}) \simeq 0 \) (see figure 2 for an illustration). We see that changing the instrument set challenge the consistency of the GMM estimators, and thus, the presence of misspecification.

### 1.2.2 The endogenous rule economy

We now study the implied dynamics of the forward–looking rule economy with multiple shocks. Let us assume that the only difference with the model in the preceding subsection is the determination of the \( x_{1,t} \) policy instrument, which follows the rule:

\[ x_{1,t} = \eta E_t(y_{t+1}) \]

After substituting the later into (13), the dynamics of the economy is given by:

\[ y_t = (a + b_1\eta)y_{t+1} + \sum_{j=2}^{m} b_jx_{j,t} - (a + b\eta)\nu_{y,t+1} \]
As before, we assume that $|a + b_1 \eta| > 1$, so the economy present an irregular equilibrium and is given by:

$$y_t = \frac{1}{a + b_1 \eta} y_{t-1} - \sum_{j=2}^{m} \frac{b_j}{a + b_1 \eta} x_{j,t-1} + \nu_{t}$$

If the econometrician does the same job as before, the policy rule parameter $\eta_{prr} \equiv \eta$ and $\varepsilon_{prr,t} \equiv 0$. Unambiguously, plim $\eta_{prr}(z_t)$ is consistent for any instrument, since

$$\text{plim} \eta_{prr}(z_t) = \eta_{prr} - \eta_{prr} \frac{E(\nu_{t+1} z_t)}{E(y_{t+1} z_t)}$$

and $E(\nu_{t+1} z_t) = 0$ for $z_t \in \{x_{1,t-1}, y_{t-1}\}$.

The last two sections insist on two important facts: 1) Policy misspecification may happen more often than expected, due to at least nearly observationally equivalent models, and 2) In the presence of multiple shocks, one should challenge the consistency of the GMM estimations in order to elicit the presence of misidentification. The rest of the paper applies this methodology to the Taylor’s rule of applied monetary economics.

2 The Taylor’s rule revisited

Since Taylor (1993), abundant empirical evidence and a quite number of models have defended the use of the Taylor’s rule as an effective interest rate policy rule to control prices and stabilize the economy. The surprising fit of this equation in terms of inflation and output gap for the Volcker–Greenspan era could perfectly induce to hypothesize that the Federal Reserve Board preferences can be rationalized through a linear equation linking these three variables. Figure 3 shows the stochastic linear relationship between the federal funds rate and one-quarter annualized inflation of the U.S. for 1982:4 through 2004:3.\(^8\)

The estimating equation of Clarida, Gali and Gertler (2000) would become the common wisdom when referring to the Taylor’s rule for the U.S. case, where the nominal interest rate reacts to changes in inflation and output away their targets, plus a term reflecting policy inertia. The stabilization of some macroeconomic variables, specially the price index, during the tenures of Paul Volcker and Alan Greenspan would make economists to think that an aggressive reaction of the funds rate (responding by more than a one-by-one basis) would fight inflation instead of accommodating it. Furthermore, the usual monetary models of sticky-prices\(^9\) accept the so called Taylor principle, in what an aggressive

\(^8\)Source: the Federal Reserve System and the Bureau of Economic Analysis.

\(^9\)See Walsh (2003), ch. 5, for a survey.
reaction of the nominal interest rate to inflation gap brings about determinacy to nominal and real variables.\textsuperscript{10}

However, some sceptics have argued that the estimation of interest rate rules trough single equations might not provide sufficient information about the true behavior of policy-makers (e.g. Beyer and Farmer, 2004; Lubik and Schorfheide, 2004; Mavroeidis 2004a and 2004b). Notably, Hetzel (2000) remarks that the estimated rules could describe rather the final interest rate equilibrium than the central bank’s performance. This is, they could entail, instead, the reaction of agents given their expectations.

A minor interest has been left to discussion about the problems that may arise when there is a monetary rule misidentification, specially for policy recommendations. We claim, as Beyer and Farmer (2004), that this misspecification cannot be ruled out, since there exists - nearly - observational equivalent models with similar reduced forms. In contrast, we also provide a characterization of the IV–GMM estimators (commonly used in single equation analysis) that can help the econometrician to elicit the presence of a misspecified rule.

\textsuperscript{10}Although this is not a general result, since it heavily depends on special arrangements for price-setting and expectations formation.
2.1 The constant-money-growth rule economy

Consider an economy composed by a unit mass of infinite–lived expected utility maximizer agents, indexed by \( i \in [0, 1] \). There is also an infinite number of firms (mass one) producing differentiated final goods, indexed by \( j \in [0, 1] \), with labor as the only input. For the ease of the analysis, let us assume that there is no capital accumulation (as McCallum and Nelson, 1999, recognize that, for short–run dynamics, fluctuations in the stock of capital do not play a major role). Additionally, there exist a government who collects lump–sum taxes and provides money, bonds and lump–sum transfers to the agents in the economy.

Households

The representative household carries from period \( t - 1 \) into \( t \) the nominal balances \( M_t \), the total revenues from bond holdings \( R_{t-1}B_t \) of a riskless one–period bond issued by the government, where \( R \) is the gross nominal interest rate. In period \( t \), the household has a disposable nominal income \( W_t h_t - T_t \) per \( h_t \) worked hours, where \( W_t \) is the nominal wage and the \( T_t \) denotes lump–sum taxes. Additionally, the household receives from the government the lump–sum transfer \( N_t \) and the profits payed out from firms \( \varpi_t = \int_0^1 \varpi_{j,t}dj \) at the end of the period. All the revenues are used to buy the consumption bundle, money balances and bond holdings for the next period. Consequently, its budget constraint is

\[
B_{t+1} + M_{t+1} + P_t C_t + T_t \leq W_t h_t + R_{t-1}B_t + M_t + N_t + \varpi_t
\]

where \( P_t \) is the general price index. We assume, as Cooley and Hansen (1995), that the household needs cash in order to buy \( C_t \) and \( B_{t+1} \) (financial markets open first), so its CIA constraint takes the form

\[
P_tC_t + T_t + B_{t+1} \leq M_t + R_{t-1}B_t + N_t.
\]

Total profits, \( \varpi_t \), are not included since they are not available till the end of the period.\(^{11}\) Let us assume that the instantaneous utility function over consumption and labor is described by \( U(C_t, h_t) = \log(C_t) - h_t \). Therefore, the representative household maximization problem is simply

\[
\max_{C_t, h_t, M_{t+1}, B_{t+1}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left( \log C_\tau - h_\tau \right)
\]

subject to (18) and (19). The subjective discount factor \( \beta \) lies in the \((0, 1)\) interval, and \( E_t \) denotes the conditional expectations operator given the available information at period \( t \).

\(^{11}\)If the nominal interest rate is positive, (18) and (19) imply that \( M_{t+1} = W_t h_t + \varpi_t \).
The optimal behavior in consumption, labor, money and bonds are resumed by (taking for granted the existence of a positive interest rate)

\[ \frac{1}{W_t} = \beta E_t \frac{1}{P_{t+1}C_{t+1}} \]

and

\[ R_t \beta E_t \frac{1}{P_{t+1}C_{t+1}} = \frac{1}{P_tC_t} \]

Parallel to the lifetime utility maximization, the household must decide how much of each good \( j \) consume. If we assume that the consumption bundle is formed trough the Dixit–Stiglitz aggregator,

\[ C_t = \left[ \int_{0}^{1} C_{j,t}^{\xi-1} dj \right]^{\frac{1}{\xi-1}} \quad \text{(20)} \]

where \( \xi > 1 \) is the elasticity of substitution among the consumption goods, then the minimization of consumption expenditures, i.e. \( \min_{C_{j,t}} \int_{j \in [0,1]} P_{j,t} C_{j,t} \) s.t. (20), leads to the following household demand for good \( j \):

\[ C_{j,t} = \left[ \frac{P_{j,t}}{P_t} \right]^{-\xi} C_t \quad \text{(21)} \]

This implies that the general price index is given by

\[ P_t = \left[ \int_{0}^{1} P_{j,t}^{1-\xi} dj \right]^{\frac{1}{1-\xi}}. \]

**Government and central bank**

The budget constraint of the government is given by

\[ P_t G_t - T_t = M_{t+1} - M_t - N_t + B_{t+1} - R_{t+1}B_t \]

where \( G_t \) is real government spending, which without loss of generality can be normalized to zero, and \( M_0 \) and \( B_0 \) are given.

Money supply is described by

\[ M_{t+1} = \gamma_t M_t. \]

The gross growth rate of money, \( \gamma_t \), follows an exogenous rule that will be given shortly.

**Firms**
We consider a representative firm whose output, \( Y_{j,t} \), is produced by the linear technology
\[
Y_{j,t} = A_t h_{j,t}
\]  
where \( A_t \) is a common technology shock for all firms. Prices are flexible, in the sense that they can be adjusted once per period, but as Galí (1999) we assume that entrepreneurs set their prices at the end of period \( t - 1 \), i.e. before observing any shock of period \( t \). This is the source of stickiness in the model. Firms choose prices according to
\[
P_{j,t} \in \text{argmax} \ E_{t-1} \delta_t \left( P_{j,t} Y_{j,t} - W_t h_{j,t} \right) 
\]  
s.t. (20), (21) and (22).
\[
\delta_t = \beta \frac{1}{C_t A_t} 
\]
where \( \delta_t \) is the appropriate discounting rate for the firm, given that it is owned by households. All firms are essentially equal and at equilibrium they all choose the same price, given by
\[
P_t = \frac{\xi E_{t-1} \frac{Y_t W_t}{C_t A_t}}{(\xi - 1) E_{t-1} \frac{Y_t}{C_t}} 
\]

\[\text{Monetary policy rule and shocks}\]

The gross growth rate of money and the technical progress are stochastically constant or stationary around a long–term value. We consider two fundamental uncorrelated disturbances for each of them. Both are assumed to follow AR(1) processes
\[
\log \gamma_t = \rho_\gamma \log \gamma_{t-1} + \log \bar{\gamma} + \sigma_{\gamma,k} \epsilon_{\gamma,t} 
\]
\[
\log A_t = \rho_a \log A_{t-1} + \log \bar{A} + \sigma_{\alpha,a} \epsilon_{\alpha,t} 
\]
where \( \epsilon_{k,t} \sim \text{iid}(0,1) \), with \( E(\epsilon_{k,t}\epsilon_{k',t}) = 0 \ \forall \ k \neq k' \). Further, \( \sigma_{\alpha,k} > 0 \) and \( |\rho_k| \in (0,1) \) for all \( k \in \{\alpha, \gamma\} \).

**Definition 1 (Equilibrium)** An equilibrium is a sequence of prices \( \{P_{j,t} : j \in [0,1]\}, W_t, R_t \}_{t=0}^\infty \) and a sequence of allocations \( \{C_{j,t}, Y_{j,t}, h_{j,t} : j \in [0,1]\}, M_{t+1}, B_{t+1} \}_{t=0}^\infty \), such that given prices, these sequences maximize profits, households’ utility, and clears all markets every \( t \). 

\[\text{This can be saw also as a variation of the Mankiw and Reis (2002)'s sticky–information model, in which price–sluggishness comes from the lags in the available information of firms; here this lag is reduced to a single period. It is worth noting that the sticky–information assumption allows for some inflation inertia (see Mankiw and Reis, 2002; and Carrillo, 2006). We avoid the usual Calvo pricing to allow for a discussion later on about the choice of a “suitable” monetary policy rule.}\]
A log-linear approximation around the deterministic steady state characterizes the solution of the model in terms of the forcing variables denoted by the vector $\left[ \rho_a \rho_\gamma \sigma_{\varepsilon,a} \sigma_{\varepsilon,\gamma} \right]'$. The following relations characterize the aggregate dynamics at equilibrium (hat denotes the percentage deviations from the steady state):

\[
\hat{y}_t = \hat{c}_t \quad (27)
\]
\[
\hat{w}_t = E_t [\hat{p}_{t+1} + \hat{c}_{t+1}] \quad (28)
\]
\[
\hat{R}_t = E_t (\hat{\pi}_{t+1} + \hat{c}_{t+1}) - \hat{c}_t \quad (29)
\]
\[
\hat{p}_t = E_{t-1} (\hat{w}_t - \hat{a}_t) \quad (30)
\]
\[
\hat{\pi}_{t+1} = \gamma_{t+1} - \hat{y}_{t+1} + \hat{y}_t \quad (31)
\]

with (25) and (26) to complete the system. Equations (28) and (29) come from the decision rules of the household, (30) from the price choice of firms, and (31), where $\pi_t = p_t - p_{t-1}$ is the inflation rate, is an implication from the CIA constraint and the government budget constraint. The aggregate dynamics at equilibrium of the model economy are presented in the next proposition.

**Proposition 2.1 (Equilibrium for the constant-money-growth rule economy)**

The aggregate dynamics of the model economy denoted by equations (25)-(31) present a regular or determinate equilibrium, giving the following solutions for output, inflation, and the interest rate:

\[
\hat{y}_t = \gamma_t - \rho_\gamma (1 + \rho_\gamma) \gamma_{t-1} + \rho_a \hat{a}_{t-1} \quad (32)
\]
\[
\hat{\pi}_t = \gamma_{t-1} + (1 - L) \left( \rho_\gamma (1 + \rho_\gamma) \gamma_{t-1} - \rho_a \hat{a}_{t-1} \right) \quad (33)
\]
\[
\hat{R}_t = \rho_\gamma \gamma_t \quad (34)
\]

where $L$ is the lag operator, such that $Lx_t = x_{t-1}$.

**Proof:** See appendix.

The constant-money-growth rule economy does not present any indeterminacy as long as the growth rate of money follow an exogenous stationary process. Despite its simple resolution, the preset-price assumption or lagged information arrival, is able to make inflation persistent at least for one period, since the inflation rate does not answer immediately to a monetary policy shock.\(^\text{13}\) We now present the second economy, governed by a Taylor’s rule form.

\(^{13}\) The same qualitative results can be found in Mankiw and Reis (2002), although in a larger magnitude.
2.2 The Taylor’s rule model economy

The behavior of households, firms and the fiscal authority is completely analogous as the constant-growth-rule model. The central bank, however, is assumed to follow the next Taylor type rule

$$\hat{R}_t = \eta E_t \hat{\pi}_{t+1} + \sigma_u u_t$$

(35)

where $u_t$ is an iid(0,1) policy shock and $\sigma_u > 0$. We consider this rule, that relates the nominal interest rate only with inflation, as the implied dynamic properties will depend on a single parameter. Moreover, previous empirical results suggest that the estimates of $\eta$ are unambiguously significant and positive for the Volcker–Greenspan era, while the output gap has shown a minor contribution in predicting the target interest rate (see Clarida et al. 2000).\footnote{We have investigated also rules that include policy inertia terms, or lagged interest rate terms, and all the results are qualitative unaltered, though unnecessary cumbersome.}

Since the growth rate of money is now endogenous, the Taylor’s rule model economy is summarized by the system of equations (26)–(31) plus (35). As the recurrent assumption, we set $\eta > 1$ to represent an aggressive interest rate rule. The dynamic properties of the economy are described by the proposition below.

**Proposition 2.2 (Equilibrium for the Taylor’s rule economy)** If $\eta > 1$, the aggregate dynamics of the model economy described by (26)–(31) and (35) present an irregular or indeterminate equilibrium, with two independent sunspot shocks, giving the following solutions for output and inflation:

$$\hat{y}_t = \frac{1}{\eta} \left[ \hat{y}_{t-1} + (\eta - 1) \rho_a \hat{a}_{t-2} + \sigma_u u_{t-1} + (\eta - 1) \nu_{w,t-1} + \nu_{y,t} \right]$$

(36)

$$\hat{\pi}_t = \frac{1}{\eta} \left[ \hat{\pi}_{t-1} + \rho_a (1 - L) a_{t-2} - \sigma_u u_{t-1} + (1 - L) \nu_{w,t-1} - \nu_{y,t-1} \right]$$

(37)

**Proof:** See appendix. 

where $\nu_{w,t} = (\hat{w}_t - E_{t-1} \hat{w}_t)$ and $\nu_{y,t} = (\hat{y}_t - E_{t-1} \hat{y}_t)$ are two independent sunspot shocks. The first shock in beliefs, $\nu_{w,t}$, is a nominal sunspot that is carried into the real variables; this is an example of a nominal indeterminacy that is translated into real indeterminacy.

The second sunspot, $\nu_{y,t}$, results of having an aggressive coefficient in the interest rate rule; indeed, in a preset–price economy, an active Taylor rule leads to real indeterminacy.
This contrasts the Taylor principle statement of achieving determinacy through an aggressive rule found in some Calvo pricing models.

Indeed, the so-called Taylor principle breaks when we consider environments where prices are assumed to be more flexible. Recall that the sluggishness in the price index in the preset-price model comes from the lagged available information of firms, that prevent them to change their prices instantaneously to current innovations in the shocks of the economy. A result of the sticky-information models, from which the preset-price is the simplest representation, is that they can explain some inflation inertia without imposing exogenous restrictions in price-setting, such as Calvo pricing (Walsh 2003, ch. 5) or rule-of-thumb indexation (Galí and Gertler 1999). Whereas one model specification is better than the other is a discussion that lies beyond this paper. We just want to point out that the Taylor’s principle is not as general as it is usually perceived, and that its efficiency to stabilize inflation depends on specific assumptions about price-setting.

3 Policy rule regression: The Taylor’s rule

3.1 The econometrics of the constant-money-growth economy

Assume that an econometrician is asked to retrieve the preferences of the central bank into a policy rule equation. In this subsection, we assume that the “true” world is the one described by the constant-money-growth rule economy. Our interest is on the risk and consequences of misspecification, so we let the econometrician to perform a Taylor’s rule regression, i.e., to estimate the following equation:

\[ \hat{R}_t = \eta_{prr} E_t \hat{\pi}_{t+1} + \epsilon_{prr,t}, \]

(38)

where the subindex \( prr \) denotes the “policy rule regression” objects. Note, however, that if the true world is given by system (25)–(31), which imply the solution of proposition 2.1, one can find a linear relationship between the nominal interest rate and inflation gap as:

\[ \hat{R}_t = \frac{1}{\rho_\gamma} E_t \tilde{\pi}_{t+1} - \frac{1 + \rho_\gamma}{\rho_\gamma} \sigma_{\epsilon,\gamma} \epsilon_{\gamma,t} + \frac{\rho_a}{\rho_\gamma} (1 - L) \hat{\alpha}_t \]

(39)

\[ \hat{\pi}_t = \gamma_t + (1 + \rho_\gamma)(\rho_\gamma \gamma_t - \gamma_t + \epsilon_{\gamma,t}) - \rho_a (\hat{\alpha}_t - \hat{\alpha}_{t-1}), \] since \( \hat{R}_t = \rho_\gamma \gamma_t \), solving the later for the interest rate gives the expression below.

\[ \hat{R}_t = \eta_{prr} E_t \hat{\pi}_{t+1} + \epsilon_{prr,t}, \]

Note that solution for inflation in proposition 2.1 can be rewritten as \( \hat{\pi}_{t+1} = \gamma_t + (1 + \rho_\gamma)(\rho_\gamma \gamma_t - \gamma_t + \epsilon_{\gamma,t}) - \rho_a (\hat{\alpha}_t - \hat{\alpha}_{t-1}), \) since \( \hat{R}_t = \rho_\gamma \gamma_t \), solving the later for the interest rate gives the expression below.
Some worthy observations are as follows. First, the relation above is an equilibrium condition, not a policy rule. Second, in the preset-price model, the nominal interest rate will react by more than a one-by-one basis to inflation gap as long as the growth rate of money be a stationary process, taken as granted that no shock is present. Third, if the above were to be estimated by IV–GMM, as commonly done in the literature, not all of the predetermined instruments would fulfill the orthogonality conditions.

This latter remark is formally presented as following. For the ease of the exposition, assume that the econometrician uses a single instrument, denoted by $z_t$, to estimate the policy rule parameter through IV–GMM. Since in this world $\eta_{prr} \equiv \frac{1}{\rho}$ and $\varepsilon_{prr,t} \equiv \frac{1+\rho_\gamma}{\rho_\gamma} \sigma_{\gamma,\tau} \varepsilon_{\tau,t} - \frac{\rho_\gamma}{\rho_\gamma} (1 - L) \hat{\alpha}_t$, the GMM estimator of $\eta_{prr}$ would be the ratio between two covariances, as\(^{16}\)

\[
\text{plim } \hat{\eta}_{prr}(z_t) \equiv \frac{E(\hat{R}_t z_t)}{E(\hat{\pi}_{t+1} z_t)} = \eta_{prr} + \frac{E(\varepsilon_{prr,t} z_t)}{E(\hat{\pi}_{t+1} z_t)}
\]

(40)

The last term constitutes the potential bias of the policy rule regression. The classic assumption is that this term is equal to zero for the consistency of the GMM estimator. In the presence of misidentification, this is not necessarily the case.

As in previous empirical works, let us consider the following universal set of possible weakly exogenous or predetermined instruments $Z = \{\hat{\pi}_{t-1}, \hat{y}_{t-1}, \hat{R}_{t-1}\}$. We are particularly interested in the special effect of changing the estimating instrument. Proposition 3.1 shows the probabilistic limits for the IV–GMM estimators of $\eta$, conditional on each of the above instruments and given that the world is the constant-money-growth economy.\(^{17}\)

**Proposition 3.1 (IV–GMM estimators)** The probabilistic limit of the GMM estimators $\text{plim } \hat{\eta}(z_t)$ for $z_t \in Z = \{\hat{\pi}_{t-1}, \hat{y}_{t-1}, \hat{R}_{t-1}\}$ of $\eta_{prr}$ are given by:

\(^{16}\) Since all variables are zero-mean, the covariance between two variables is given by the crossed expectation. In this sense, multiplying instrument $z_t$ into (38) and taking total expectations yields: $E(\hat{R}_t z_t) = \eta_{prr} E(\hat{\pi}_{t+1} z_t) + E(\varepsilon_{prr,t} z_t)$. The rest is just solving the later for $\text{plim } \hat{\eta}_{prr}$.

\(^{17}\) Clarida, Galí and Gertler (2000) include up to four lags in the inflation rate. To keep tractable results, we do not introduce over-identifying conditions.
\[
\plim \hat{\eta}(\hat{\pi}_{t-1}) = \frac{1}{\rho_\gamma} + \frac{\rho^4_a \text{Var}(\hat{\alpha}_t)(1 - \rho_a)^2}{\rho_\gamma \left( \rho^4_\gamma \text{Var}(\hat{\gamma}_t)(1 + \rho_\gamma (1 - \rho_\gamma^2)) - \rho^4_{a} \text{Var}(\hat{\alpha}_t)(1 - \rho_a)^2 \right)}
\]

\[
\plim \hat{\eta}(\hat{y}_{t-1}) = \frac{1}{\rho_\gamma} - \frac{\rho^4_a \text{Var}(\hat{\alpha}_t)(1 - \rho_a)}{\rho_\gamma \left( \rho^4_\gamma \text{Var}(\hat{\gamma}_t)(1 - \rho_\gamma^2) + \rho^4_{a} \text{Var}(\hat{\alpha}_t)(1 - \rho_a)^2 \right)}
\]

\[
\plim \hat{\eta}(\hat{R}_{t-1}) = \frac{1}{\rho_\gamma}
\]

where \( \text{Var}(k_t) = \frac{\sigma^2_{k_t}}{1 - \rho_k^2} \) for \( k \in \{\gamma, a\} \).

**Proof:** The moments are deduced from the solution equations (32)–(34). Since the fundamental shocks are independent from each other, \( E(x_t x_{t+s}) = \text{Cov}(x_t, x'_{t+s}) = 0 \) for any integer \( s \) and \( x \neq x' \), we have only to worry about the autocovariances of money growth and technology. Thus, according to equation (41), the relevant moments can be rewritten as

\[
E[(\hat{\alpha}_t - \hat{a}_{t-1})\hat{\pi}_{t-1}] = \rho^2_a \text{Var}(\hat{\alpha}_t)(1 - \rho_a)^2
\]

\[
E[(\hat{\alpha}_t - \hat{a}_{t-1})\hat{y}_{t-1}] = -\rho^2_a \text{Var}(\hat{\alpha}_t)(1 - \rho_a)
\]

\[
E[(\hat{\alpha}_t - \hat{a}_{t-1})\hat{R}_{t-1}] = 0
\]

\[
E(\hat{\pi}_{t+1}\hat{\pi}_{t-1}) = \rho_\gamma^2 \text{Var}(\hat{\gamma}_t)(1 + \rho_\gamma (1 - \rho_\gamma^2)) - \rho^4_{a} \text{Var}(\hat{\alpha}_t)(1 - \rho_a)^2
\]

\[
E(\hat{\pi}_{t+1}\hat{y}_{t-1}) = \rho_\gamma^2 \text{Var}(\hat{\gamma}_t)(1 - \rho_\gamma^2 (1 + \rho_\gamma)) + \rho^4_{a} \text{Var}(\hat{\alpha}_t)(1 - \rho_a)
\]

The plim of the IV–GMM estimators is just the sum between the true reduced form parameter \( \eta_{prr} \) and the specific instrument bias.

In this example, two out of three instruments not deliver consistent estimators. This comes from the fact that the policy rule error \( \varepsilon_{prr} \) is not an uncorrelated innovation, given its intrinsic relation with the persistence of technology. Just the interest rate not
imply any bias, since in the preset–price model as stated in the constant–money–growth economy this variable is purely determined by the money growth.

It is clear that the instruments considered are qualitatively different, and that these differences surely influence the absolute size of the bias. For example, if the output gap is mostly driven by technology shocks than money growth disturbances, we could expect that the size of the bias will be greater for this instrument than for anything else.

3.1.1 A calibrated exercise

The aim of this subsection is to illustrate numerically, within some standard values of the literature for the U.S., the probabilistic limits provided in proposition 3.1. Quite surprisingly, two out of three calibrated estimators are not so different from their empirical counterpart of the estimated Taylor’s rule for the U.S. Table 1 presents the parametrization that we consider for this quantitative exercise.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\rho_a, \sigma_{\varepsilon,a})$</td>
<td>$(0.98, 0.0072)$</td>
<td>King and Rebelo (1999)</td>
</tr>
<tr>
<td>$(\rho_{\gamma}, \sigma_{\varepsilon,\gamma})$</td>
<td>$(0.49, 0.0089)$</td>
<td>Cooley and Hansen (1995)</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Table 2: The calibrated and estimated policy rule parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant-money-growth economy</td>
</tr>
<tr>
<td>$\text{plim } \hat{\eta}<em>{prr}(\hat{\pi}</em>{t-1})$</td>
</tr>
<tr>
<td>$0.50$</td>
</tr>
<tr>
<td>$\text{plim } \hat{\eta}<em>{prr}(\hat{y}</em>{t-1})$</td>
</tr>
<tr>
<td>$2.17$</td>
</tr>
<tr>
<td>$\text{plim } \hat{\eta}<em>{prr}(\hat{R}</em>{t-1})$</td>
</tr>
<tr>
<td>$2.04$</td>
</tr>
</tbody>
</table>


The first three lines of table 2 contains the calibrated values for the different plim $\eta_{prr}$ conditional to the instrument used; the bottom of the table shows the estimation of Clarida et al. (2000) of the inflation coefficient for the Taylor’s rule in the Volcker–Greenspan era. As noted before, only the nominal interest rate yields a consistent IV–GMM estimator, whereas the others are not free from biases. In fact, the greatest of them is obtained with the output gap, which reaches $-1.54$ estimate points. This should not be very surprising
since this variable is mostly driven by technology shocks, as it is illustrated by its variance decomposition in table 3, where after 100 periods the technology innovations accounts for 93.2 per cent of the total variance of output. Notice as well that output reacts immediately to a money shock, but the effects of the latter are not persistent and decay at increasing rates over time, been extensively dominated by the technology disturbance.

Table 3: Variance decomposition (in %)

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$\hat{\pi}_t$</th>
<th>$\hat{R}_t$</th>
<th>$\hat{y}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>17.3</td>
<td>82.7</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>17.3</td>
<td>82.7</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>17.3</td>
<td>82.7</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>17.3</td>
<td>82.7</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>17.4</td>
<td>82.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: The variance decomposition was performed given a one-standard-deviation realization of each shock.

Table 3 also shows the variance decomposition for inflation and the nominal interest rate, which are mostly affected by money shocks; more than 80 per cent of their variation is explained by money disturbances at every period. This is the reason for which their biases are rather small and null, respectively. It is also remarkable the similarity between the magnitudes of these GMM plim estimators and the baseline point estimate of Clarida et al. (2000), shown in table 2. This observation naturally rises the question of a possible misidentification on the current estimations of Clarida et al., since even a simple preset–price model with a constant-money-growth rule is able to match the empirical value of the inflation coefficient in the Taylor’s rule.

3.2 The econometrics of the Taylor’s rule economy

If the true world is the Taylor’s rule economy, consistency for any predetermined instrument is trivially achieved following the assumptions detailed in section 2.2. Lemma 3.2 resumes these results.

**Lemma 3.2 (Consistency)** If the true monetary policy is given by the Taylor’s rule, then, for any predetermined instrument $z_t \in Z = \{\hat{\pi}_{t-1}, \hat{y}_{t-1}, \hat{R}_{t-1}\}$ and given that the policy shock $u_t$ is an iid process, the plim $\eta_{\text{prf}}(z_t) = \eta$. 
Proof: The policy rule regression equation (38) in terms of observable variables is

\[ \hat{R}_t = \eta_{prr} \hat{\pi}_{t+1} + \eta_{prr} \nu_{\pi,t+1} + \varepsilon_{prr,t}. \]

Equation (35) means \( \varepsilon_{prr,t} = \sigma_u u_t \); solution (37) implies that \( \nu_{\pi,t+1} = (\hat{\pi}_{t+1} - E(\hat{\pi}_{t+1})) = 0. \) Thus, the plim of the IV–GMM estimator of \( \eta_{prr} \) given \( z_t \) is written as

\[
\text{plim} \hat{\eta}_{prr}(z_t) \equiv \frac{E(\hat{R}_t z_t)}{E(\hat{\pi}_{t+1} z_t)} = \eta + \sigma_u \frac{E(u_t z_t)}{E(\hat{\pi}_{t+1} z_t)} \quad (41)
\]

Given the solutions (36) and (37), we see immediately that \( E(u_t z_t) = 0, \) for any choice of \( z_t \in Z \) and \( E(\eta_{prr}) = \eta. \) There is no bias caused by the instruments and all the three estimators are consistent, provided that \( E(\hat{\pi}_{t+1} z_t) \neq 0. \)

The statement of lemma 3.2 is general for any specification of the Taylor’s rule, given that the monetary policy shock; \( u_t, \) be an uncorrelated process. And is also general to the character of the equilibrium of the model economy, i.e. determinate or indeterminate. Thus, if the policy rule regression is to be performed, the lack of robustness when varying from one set of instruments to another could embody three different causes: a) The policy rule is misspecified; b) The policy shock presents some degree of persistency; or c) Both, a) and b) happen.

We emphasize the fact that any predetermined instrument will yield a consistent estimator if the assumptions of lemma 3.2 are verified, no matter the quantitatively information contained in each of them, i.e., no matter by which fundamental or sunspot shock the fluctuations of the instrument are mostly driven. It is straightforward to recommend, then, that when performing a robustness test in a policy rule regression, a dramatically change in the set of instruments must be preferred in order to clearly ruling out misspecification.

3.2.1 An empirical exercise

This subsection is intended to explore the threat of policy rule misidentification with real data. We analyze the case for the U.S. using different set of instruments: the Clarida et al. (2000)’s original instrument set,\(^{18}\) and the rest composed separately by the lags of the federal funds rate, inflation rate of the GDP deflator, the Congressional Budget Office’s output gap, the unemployment rate, and the labor income share, the last three as different

\(^{18}\)Conformed by four lags in the federal funds rate, annualized inflation, CBO’s output gap, commodities price inflation, the growth rate of money stock (M2), and the short-run–long-run interest rate spread.
measures of real activity; \(^{19}\) the period considered corresponds to the Volcker–Greenspan era, i.e. 1979:3–2004:3, quarterly data.

At first stage, we estimate equation (35), hereafter called the reduced Taylor’s rule, and only afterwards we look to reproduce the estimations of Clarida et al. (2000), CGG hereafter, to study the effects of changing the instrument set on their extended version of the Taylor’s rule. The reduced Taylor’s rule is estimated using IV techniques with a single instrument within five different predetermined variables: interest rate, inflation, output gap, unemployment and labor income share lagged one period.

Table 4: Estimations of the reduced Taylor’s rule, variation in the instrument

<table>
<thead>
<tr>
<th>Instrument</th>
<th>(\hat{\eta}_{prr})</th>
<th>(SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation lag</td>
<td>1.63</td>
<td>(0.28)</td>
</tr>
<tr>
<td>Interest rate lag</td>
<td>2.41</td>
<td>(0.50)</td>
</tr>
<tr>
<td>Output gap lag</td>
<td>1.44</td>
<td>(1.19)</td>
</tr>
<tr>
<td>Unemployment lag</td>
<td>2.18</td>
<td>(0.70)</td>
</tr>
<tr>
<td>Labor income share lag</td>
<td>1.86</td>
<td>(0.46)</td>
</tr>
</tbody>
</table>


Table 4 summarizes the results (standard deviations are stated in parenthesis). All estimates look significant, except when using the output gap as instrument. Indeed, they could all, precluding \(\hat{\eta}_{prr}(\hat{y}_{t-1})\), be considered as statistically similar. \(^{20}\)

To explain why the estimation using the output gap is not statistically different from zero, we could argue that this may be due to a weak covariance between \(\hat{n}_{t+1}\) and \(\hat{y}_{t-1}\), which would entail a correlation between these two variables near to zero. In fact, this is could be the case, since the correlation coefficient reaches -0.13 and is not statistically different from zero at 5 per cent of significance.

\(^{19}\)Unemployment is considered as an alternative measure of the output gap in Clarida et al. (2000) estimations, whereas the labor income share is a proxy to labor costs, which is strongly correlated with the output gap (see Galí and Gertler (1999) for a discussion).

\(^{20}\)Denoting unemployment and the labor income share as \(u_n\) and \(ls\), respectively, the 95 per cent confidence interval of all these estimates have un interception at some regions: \(\hat{\eta}_{prr}(\hat{n}_{t-1})\) ∈ [1.08, 2.18], \(\hat{\eta}_{prr}(\hat{R}_{t-1})\) ∈ [1.42, 3.40], \(\hat{\eta}_{prr}(u_{n,t-1})\) ∈ [0.79, 3.56] and \(\hat{\eta}_{prr}(ls_{t-1})\) ∈ [0.95, 2.77].
The stability of the coefficient of inflation holds even when one changes to instruments that are not so correlated with the inflation rate, as unemployment and the labor share (the contemporaneous correlation coefficients are 0.33 and 0.64).

The relative stability of the reduced Taylor’s rule imply that the true policy may be a reaction function of the interest rate to expected inflation. This could be challenged by using a larger set of instruments.

We now turn to the CGG’s extended Taylor’s rule to apply the same methodology as before. The extended Taylor’s rule includes the output gap and monetary policy inertia into its policy rule regression equation, and it writes as following (constant terms are omitted):

\[
\hat{R}_t = (1 - \rho_R) [\eta_\pi E_t \hat{\pi}_{t+1} + \eta_y E_t \hat{y}_{t+1}] + \rho_R(L) \hat{R}_{t-1} + \varepsilon_{prr,t},
\]

(42)

where \(\rho_R(L) = \rho_{R,1} + \rho_{R,2}L\) is the lag–polynomial to describe the inertia of monetary policy and \(\rho_R \equiv \rho_R(1)\).\(^{21}\) These terms are assumed to represent the “Federal Reserve’s tendency to smooth changes in the interest rate [CGG, pp. 152]”. The period considered is the same as before, 1979:3–2004:3. The instruments used in the first estimation are those used by CGG described earlier. We then change the instrument sets, allowing for 8 lags in the inflation rate, interest rate, the CBO’s output gap, unemployment rate and the labor income share, each by separate sets, to test for parameter stability.\(^{22}\) The estimation method used is IV–GMM using the 12 lags to estimate the optimal weighting matrix trough the Newey–West procedure. The next table resumes the results, with the standard deviation in parenthesis and the p-value of the J-statistic under it.

Despite the estimations using the output gap instrument set, which is completely uninformative since it may be considered as weak instruments, all the other estimations provide interesting results. The first is that the coefficient of inflation is quite stable and significant for all instrument set, except for the output gap’s. The second observation is that in fact, the later is the only stable coefficient in the whole experience. Notably, the rejection of the policy inertia parameters when using the interest rate instruments rises serious doubts about the good specification of the interest rate persistence and the existence of any monetary policy inertia. Also, the irrelevance of the output gap coefficient for all instruments set may provide evidence that the weight of this variable in the Taylor’s rule is rather small or null.

\(^{21}\)See Clarida et al. (2000) for further details.

\(^{22}\)We choose 8 lags in the alternative instrument rates as to control for excessive moments conditions. Nevertheless, the results are qualitatively invariable to the choose of larger horizons.
Table 5: Estimations of the extended Taylor’s rule, different set of instruments

<table>
<thead>
<tr>
<th>Instruments</th>
<th>$\hat{\rho}_R$</th>
<th>$\hat{\eta}_e$</th>
<th>$\hat{\eta}_y$</th>
<th>$J$–stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGG instruments</td>
<td>0.86</td>
<td>2.21</td>
<td>0.46</td>
<td>7.81</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.19)</td>
<td>(0.10)</td>
<td>0.99</td>
</tr>
<tr>
<td>Inflation lags</td>
<td>0.65</td>
<td>1.84</td>
<td>0.0018</td>
<td>3.59</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.35)</td>
<td>(0.35)</td>
<td>0.46</td>
</tr>
<tr>
<td>Interest rate lags</td>
<td>-0.22</td>
<td>2.21</td>
<td>-0.67</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(0.43)</td>
<td>(0.36)</td>
<td>0.85</td>
</tr>
<tr>
<td>Output gap lags</td>
<td>1.02</td>
<td>32.25</td>
<td>-0.18</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(199.13)</td>
<td>(3.19)</td>
<td>0.67</td>
</tr>
<tr>
<td>Unemployment lags</td>
<td>0.57</td>
<td>1.71</td>
<td>0.79</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.07)</td>
<td>(0.56)</td>
<td>0.91</td>
</tr>
<tr>
<td>Labor income share lags</td>
<td>0.64</td>
<td>2.01</td>
<td>0.29</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.46)</td>
<td>(0.62)</td>
<td>0.85</td>
</tr>
</tbody>
</table>


More experiences like the one presented might be enlighten to elucidate any misspecification in policy rule regression analysis. Here, we have some evidence that the extended Taylor’s may be misidentified, since only the inflation coefficient seems to be stable across different instruments set.

4 Concluding Remarks

This paper provides a general overview of the policy rule regression analysis. Our main finding is that, within the framework of a rational expectations model, policy rule regression that uses a limited information estimation method tell us very little about the real structural behavior of policy–making. In the presence of nearly observational equivalent models, policy misidentification may happen more often then expected. Therefore, a series of robustness experiences may be useful to elicit any misspecification.

In fact, when the policy rule is misidentified, the consistency of the GMM estimators may be challenged. Using very different instruments, such as their variance decomposition be qualitatively different, could be a powerful tool to rule out misspecification.

In the case of the Taylor’s rule regression, we show that even a preset–price model with exogenous money growth rule can match the value of the inflation coefficient in the Taylor’s rule. Therefore, misidentification cannot be ruled out in advantage. Some empirical robustness test to the CGG extended Taylor’s rule show that the stability of parameters fails for all variables and all instrument sets, except for the inflation coefficient. Notably,
there could be evidence against the monetary policy inertia hypothesis, since is not hold by the interest rate instrument set. More extensive exercises as the one performed in this paper may be useful to ruling out misidentification.

REFERENCES


**Appendix**

A Solution for the constant–money–growth economy

Since the preset-price model presented in section 2 does not allow for any predetermined endogenous variable, i.e. there is no capital accumulation, the aggregate dynamics of the constant-money-growth economy collapse into a single equation. As a solution strategy, we make use of the following auxiliary variables: \( p_t^* = \hat{p}_t - \hat{m}_t \) and \( w_t^* = \hat{w}_t - \hat{m}_t \), where \( \hat{m}_t = \log(M_t/\bar{M}_t) \) and \( \bar{M}_t = \bar{M}_{t-1} \gamma \) is the stock of money if there is no shocks in the money growth rate. The later implies that \( \hat{m}_{t+1} - \hat{m}_t = \gamma_t \) and \( E_t \hat{m}_{t+1} = \hat{m}_{t+1} \). Accordingly, equations (28), (30) and (31) can be rewritten as:

\[
\begin{align*}
  w_t^* &= E_t (p_{t+1}^* + \hat{y}_{t+1}) + \gamma_t \\
  p_t^* &= E_{t-1} (w_t^* - \hat{a}_t) \\
  p_t^* &= \gamma_t - \hat{y}_t
\end{align*}
\]

The last two expressions imply

\[
\hat{y}_t = \gamma_t + \rho_a \hat{a}_{t-1} - E_{t-1} w_t^* 
\]

Placing the later and (45), forwarded one period, into (43) yields

\[
w_t^* = (1 + \rho_\gamma) \gamma_t
\]

which in (46) give us the solution for output, expressed in proposition 2.1. The inflation and nominal interest rate solutions are straightforward retrieved from equations (29) and (31).
B Solution for the Taylor’s rule economy

Since (44) implies

\[ w_t^* = p_t^* + \rho_a \hat{a}_{t-1} + \nu_{w,t}, \quad (48) \]

where \( \nu_{w,t} = w_t - E_{t-1} w_t \), equation (43) can be stated as

\[ E_t (p_{t+1}^* - p_t^*) = \rho_a \hat{a}_{t-1} - E_t y_{t+1} - \gamma_t + \nu_{w,t}. \quad (49) \]

Substituting \( p_t^* \) on the later, according to (45), yields:

\[ E_t \hat{\gamma}_{t+1} = \rho_a \hat{a}_{t-1} - \hat{y}_t + \nu_{w,t}. \quad (50) \]

(27), (29) (31) imply that \( \hat{R}_t = E_t \hat{\gamma}_{t+1} \). Since the nominal interest rate is set according to the rule (35), this means that output is governed by the following dynamic law:

\[ \hat{y}_t = \eta \hat{y}_{t+1} - (\eta - 1) (\rho_a \hat{a}_{t-1} + \nu_{w,t}) - \sigma_u u_t - \eta \nu_{y,t+1}, \quad (51) \]

where \( \nu_{y,t+1} = y_{t+1} - E_t y_{t+1} \). To characterize the determinacy conditions, setting (30) one period ahead and taking it into (28) yields:

\[ \hat{w}_t = \hat{w}_{t+1} + \hat{y}_{t+1} - \rho_a \hat{a}_t - \nu_{w,t+1} - \nu_{y,t+1}. \quad (52) \]

The two later expressions conform the system

\[ X_t = AX_{t+1} + BF_t + CS_t \quad (53) \]

where \( X_t = \begin{pmatrix} \hat{w}_t \\ \hat{y}_t \end{pmatrix} \) is the vector of independent endogenous variables, the first nominal and the second real; \( F_t = \begin{pmatrix} \hat{a}_t \\ u_t \end{pmatrix} \) contains the fundamental disturbances and \( S_t = \begin{pmatrix} \nu_{w,t+1} \\ \nu_{y,t+1} \end{pmatrix} \) the sunspot shocks.

\( A, B \) and \( C \) are matrices of parameters, equal to \( \begin{pmatrix} 1 & 1 \\ 0 & \eta \end{pmatrix} \), \( - \begin{pmatrix} \rho_a & 0 \\ (\eta - 1)\rho_a L & \sigma_u \end{pmatrix} \) and \( - \begin{pmatrix} 1 & 1 \\ (\eta - 1)L & \eta \end{pmatrix} \) respectively, where \( L \) is the lag operator. The dynamic properties of the above system depend on the eigenvalues of \( A \), which are equal to 1 and \( \eta \). Since one eigenvalue lies within the unit circle, we have at least one degree of indeterminacy, denoted by \( \nu_{w,t+1} \). In other words, regardless of the dynamic environment of the real variables, a nominal sunspot shock will be carried into the real economy; this comes from the indetermination of the growth rate of money.\(^{23}\) If the second eigenvalue \( \eta \) is outside the unit circle, then a second degree of indeterminacy will appear in the solution, denoted by \( \nu_{y,t+1} \). Eventually, this is the assumption we made in section 2.2 when we chose an aggressive interest rate rule.

---

\(^{23}\)Notably, we can determine the solution for \( E_t \hat{\gamma}_{t+1} \), but not for \( \hat{\gamma}_{t+1} \) itself, which leaves free the innovation \( \nu_{\gamma,t+1} \), that is translated into prices and nominal wages.
Since the output gap will be driven not only by the fundamental disturbances, as technology or the policy shock, but also by the sunspot (or beliefs) innovations, solution (36) in proposition 2.2 comes simply from stating (51) in a backward perspective. For the inflation rate, first note that (52) can be rewritten as (one period backward)

\[ E_{t-1} \hat{w}_t - \hat{w}_{t-1} = \rho_o \hat{y}_{t-1} - E_{t-1} \hat{y}_t. \]  

(54)

Substituting the later and the solution for output into the first–order difference of equation (30) yields the solution for the inflation rate as given in proposition 2.2.