Abstract

We develop a micro-founded dynamic general equilibrium model in which firms are endogenously concerned with their market share position. Introducing customer flow dynamics into the firm’s problem yields a tradeoff between increasing the profit margin in the short run and investing in a stock of customers that will allow the firm to sustain higher profits in the long run. We show that in an environment where firms face such an intertemporal problem, sector-specific and economy-wide shocks have very different implications for the pricing behavior of producers, and argue that this is consistent with the empirical evidence on cost pass-through and the cyclicality of markups. We also find that the perceived shock persistence is an important determinant of the cost pass-through. On the basis of these results, we investigate what happens when firms are uncertain about the nature of the shocks they face. Finally, we briefly discuss a natural application of this model to the study of exchange rate pass-through.

*We would like to thank Martin Eichenbaum, Giorgio Primiceri and Sergio Rebelo for their valuable comments. All errors are our own.

†Kellogg School of Management, Northwestern University, email: ikleshchelski@kellogg.northwestern.edu

‡Economics Department, Northwestern University, email: n-vincent@northwestern.edu
1 Introduction

"The fight over market is one that never ends.," Spokesman for Mexican brewer Modelo, in an interview for Bloomberg, September 6, 2005

"...the company in the short term would continue to pursue market share over profitability.," Kevin Rollins, Dell Inc., February 16, 2001

One only has to open a business newspaper or magazine to notice that managers are continually concerned about their firm’s market share position and how it is affected by their actions and those of their competitors. Modern macro models, however, seldom incorporate a role for market share.\(^1\),\(^2\)

In standard imperfect competition models, firms sell to all consumers in the economy, and only the intensive margin plays a role in the firm’s pricing decision. Firms do not gain or lose customers, and their pricing decisions have an impact only on the \textit{quantity} each consumer purchases. Without introducing additional frictions (e.g. state or time dependent pricing), the problem of the firm is therefore static: charging a higher price today is inconsequential for the future. This appears to be at odds with managers’ concerns and business strategies.

In this paper, we develop a micro-founded yet tractable model in which firms are rationally concerned about their market share position.\(^3\) Our framework is closely related to the setting of Ball and Romer (1990), and the extension of Ireland (1998). However, the focus of these two studies is rather different than ours. The former constructs a static model to investigate the implications of the combination of real and nominal rigidities, while the latter investigates the impact of customer flows on the cyclical behavior of markups.

\(^1\)Throughout this paper, we define the firm’s “market share” as the measure of consumers in a particular market who are buying from this seller relative to the total measure of consumers who are buying from the sector to which the seller belongs. Also, we will use the terms "market share", "customer base", or "mass of customers" interchangeably.

\(^2\)Exceptions, for example, include Amano and Hendry (2003), Ireland (1998), and Rotemberg and Woodford (1991). The focus of these papers is different than ours. The first focuses on inflation persistence and the latter two focus on markup patterns over the business cycle. Our focus is on the pass-through of cost shocks to prices given the endogenous pattern of the firm’s market share.

\(^3\)By "rationally" we mean firms maximize present discounted profits, and, thus, the wealth of the households that own the firms. Therefore, market share serves as a mean to an end, and not the end itself.
Here, we focus instead on the effect of market share considerations on pricing decisions. In addition, our version is built upon the standard imperfect competition framework and therefore is easily implementable in modern macro models. As such, it collapses to the well-known Dixit-Stiglitz model under certain intuitive special cases.

The central mechanism in our setup is related to the customer market literature (e.g. Phelps and Winter (1970)), under imperfect information as discussed by Stiglitz (1979), Stiglitz (1984) and Woglom (1982). Consumers can buy each differentiated good from one of many sellers, and remain with the same seller until they optimally decide to switch based on the relative price charged by their current supplier. A typical firm therefore faces a choice between (1) increasing today’s profits by raising the price but at the expense of depleting its customer base, or (2) accepting a lower markup this period in order to invest in the market share to boost future profits. This intertemporal tradeoff between short term profitability and longer run objectives is essential to obtain any deviation from a standard imperfect competition model.\footnote{This is consistent with Ball and Romer (1990) who show that in their static version, real rigidities \textit{per se} do not alter the prediction of a model without such friction. Instead, real rigidities merely act as a catalyst for nominal rigidities. We will prove that this form of real rigidity has no effect in a static framework (i.e. when firms discount the future completely) but fundamentally changes the firm’s pricing decisions when the future is not completely discounted (i.e. whenever the discount factor is larger than zero).}

We find that the addition of a dynamic extensive margin to the firm’s problem alters the predictions of a standard model, and we give both intuitive and formal explanations for the results obtained. In particular, we show that sector-specific and economy-wide shocks have very different implications for the pricing behavior of producers: when only the direct competitors of a firm are hit by a negative productivity shock, the producer optimally decides to absorb part of the marginal cost increase into its markup in order to mitigate the impact of a price increase on future profits.\footnote{By direct and indirect competitors we mean sellers within the same sector and between sectors, respectively. The direct competition affects both the intensive and extensive margin, while the indirect competition affects only the intensive margin.} In fact, firms can even decide to sustain negative profits for a certain period of time in order to maintain their customer base. However, when all firms in the economy are hit by a common productivity shock, cost pass-through into prices is virtually complete, as would be predicted by the standard model. We argue that this is consistent with existing empirical evidence. For example, in

\begin{itemize}
  \item \textit{The central mechanism in our setup is related to the customer market literature (e.g. Phelps and Winter (1970)), under imperfect information as discussed by Stiglitz (1979), Stiglitz (1984) and Woglom (1982). Consumers can buy each differentiated good from one of many sellers, and remain with the same seller until they optimally decide to switch based on the relative price charged by their current supplier. A typical firm therefore faces a choice between (1) increasing today’s profits by raising the price but at the expense of depleting its customer base, or (2) accepting a lower markup this period in order to invest in the market share to boost future profits. This intertemporal tradeoff between short term profitability and longer run objectives is essential to obtain any deviation from a standard imperfect competition model.\footnote{This is consistent with Ball and Romer (1990) who show that in their static version, real rigidities \textit{per se} do not alter the prediction of a model without such friction. Instead, real rigidities merely act as a catalyst for nominal rigidities. We will prove that this form of real rigidity has no effect in a static framework (i.e. when firms discount the future completely) but fundamentally changes the firm’s pricing decisions when the future is not completely discounted (i.e. whenever the discount factor is larger than zero).}
  \item We find that the addition of a dynamic extensive margin to the firm’s problem alters the predictions of a standard model, and we give both intuitive and formal explanations for the results obtained. In particular, we show that sector-specific and economy-wide shocks have very different implications for the pricing behavior of producers: when only the direct competitors of a firm are hit by a negative productivity shock, the producer optimally decides to absorb part of the marginal cost increase into its markup in order to mitigate the impact of a price increase on future profits.\footnote{By direct and indirect competitors we mean sellers within the same sector and between sectors, respectively. The direct competition affects both the intensive and extensive margin, while the indirect competition affects only the intensive margin.} In fact, firms can even decide to sustain negative profits for a certain period of time in order to maintain their customer base. However, when all firms in the economy are hit by a common productivity shock, cost pass-through into prices is virtually complete, as would be predicted by the standard model. We argue that this is consistent with existing empirical evidence. For example, in
\end{itemize}
the context of the U.S automobile market, Gron and Swenson (2000) report that “measured pass-through is higher for cost shocks experienced by all products than for model-specific shocks”, a result also found in Ashenfelter et al. (1998) for a different industry. Another instance is Bloch and Olive (2001), where they document an “incomplete pass-through from cost into price, implying a negative relationship between cost and the markup.” More importantly, they also find that aggregate demand effects on the markup range from negative to not significantly different from zero depending on the industry. This is consistent with the results we obtain in our model for a range of parameterizations, and in line with the findings of Rotemberg and Woodford (1991) on countercyclical markups.

We also find that in our model, the perceived persistence of the shock is an important determinant of the pass-through of cost changes into prices. Intuitively, if firms believe that a rise in the marginal cost is temporary, they will find it optimal to take a bigger hit on their markups today in order to preserve their customer base. On the other hand, if the shock is permanent, there is no incentive for an intertemporal tradeoff, and prices will vary proportionately with marginal cost. On the basis of these results, we then investigate what happens when firms are uncertain about the nature of the shocks they are facing. We find that in an environment where temporary sector-specific shocks are predominant, producers will initially respond to a change in their marginal cost by only partially raising prices, and slowly revise their strategy as they update their priors.

Our model also has clear implications about exchange rate pass-through. In markets where both domestic and foreign firms compete, movements in the exchange rate create a wedge between their marginal costs expressed in a common currency. We analyze such scenario in the context of our model, and find that indeed foreign firms will not fully pass-through exchange rate changes to their prices as evidenced by international price data. Interestingly, what our results illustrate is that, in contrast to the claims of Ball and Romer (1990) and others, real rigidities can in fact deliver price stickiness even without the introduction of nominal rigidities. As we show in the paper, this is due to the fact that the specific real rigidity we are considering introduces an intertemporal dimension to the firm’s problem.

---

6In the context of exchange rate pass-through, this is related to the theory put forward by Froot and Klemperer (1989).

7See, for example, Campa and Goldberg (2002) and Frankel et al. (2005).
Indeed, when we take away the intertemporal motive of the firm, we find that cost shocks are fully passed-through into prices.

A potentially interesting interpretation of our model relates our mechanism to the one in Rotemberg (2005). There, firms are reluctant to raise prices if they fear that consumers will view the new price as “unfair”, an occurrence which in our model could be interpreted as switching to a different supplier. In fact, survey evidence on price stickiness often points at customer relations as the main reason behind the observed sluggishness of prices, as well as price adjustments by competitors.8

The outline of the paper is as follows: in Section 2, we describe the economic environment as well as the maximization problems both households and firms are facing, and describe the predictions of the model under the static case in Section 3. In Section 4, we present our findings for the dynamic environment. We start by considering the case of a sector-specific shock, before turning our attention to the effect of an aggregate productivity shock. We also analyze what happens when firms are uncertain about the persistence of the shock. In Section 5, we consider an application of our model to the study of exchange rate pass-through, and in Section 6 we conclude.

2 The Model

In this section we present the economic environment, which includes the households’ and firms’ dynamic problems. We carefully describe the decision of a consumer who each period must assess whether to stay with its current supplier or switch to a new seller, and how firms internalize this decision in their pricing problem.

2.1 Households

Time is discrete and infinite. There is a continuum of consumption goods indexed by $i \in [0, 1]$. Households buy a particular good $i$ from only one of an infinite number of sellers, each producing a distinct brand $k \in [0, 1]$.9 Households are infinitely lived and denoted by $j \in [0, 1] \times [0, 1]$. Each household

---

8See, for example, Blinder et al. (1998) and Apel et al. (2005).
9Throughout the paper we use the terms “supplier”, “producer”, “firm” and “seller” interchangeably. Also, we sometimes refer to a “sector” when talking about the set of firms which produce a similar good $i$. 

5
j derives disutility from labor $l^j$ and utility from a basket of goods $c^j$, and solves the following problem:

$$\max_{\{c^j_t, l^j_t\}_{t=t}^{\infty}} U^j_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u (\tilde{c}^j_{\tau}, t^j_{\tau})$$

$$u (\tilde{c}^j_{\tau}, t^j_{\tau}) = \frac{(\tilde{c}^j_{\tau})^{1-\sigma} - \eta (l^j_{\tau})^{1+\epsilon}}{1 - \sigma}$$

$$\tilde{c}^j_{\tau} = \left\{ \int_0^1 \left[ (\delta_{i\tau}^{ij}) - s_{i\tau}^{ij} c^j_{i\tau} \right]^{\gamma-1} \gamma \right\}^{\gamma-1}$$

subject to

$$\int_0^1 p^j_{i\tau} c^j_{i\tau} di + E_r r_{\tau+1} b^j_{\tau+1} = b^j_{\tau} + w^j_{i\tau} + \Pi^j_{\tau}$$

where $E_t$ is the expectation operator conditional on information up to, and including, time $t$. We do not model heterogeneity in information sets across households, and thus, do not denote the expectation with superscript $j$. $\gamma$ is the elasticity of substitution between varieties, and $\sigma$ is the inverse of the elasticity of intertemporal substitution, or risk aversion coefficient. $w_t$ is the nominal wage rate. Households also have access to complete contingent claims markets. The value of their portfolio is given by $E_t r_{t+1} b^j_{t+1}$, and the realized value of the portfolio from period $t - 1$ is $b^j_t$. Households own the firms in this economy and receive their share of the firms’ period $t$ profits, $\Pi^j_t$. To avoid future confusion, we denote with $\tilde{}$ prices and quantities which refer to the aggregate basket of goods.

Our consumption aggregator (2.1) takes into account the switching decision of the household: we write $s_{zt}^j = 1$ if household $j$ switches seller for good $z$ at time $t$ and 0 otherwise. The parameter $\delta_{i\tau}^{ij}$ quantifies the utility implications for household $j$ of changing brand of good $i$ at time $t$. Throughout this paper we will refer to $\delta$ as a taste shock. More specifically, $\delta_{i\tau}^{ij}$ represents customer $j$’s taste of staying with his current supplier of good $i$ at time $t$.\footnotemark

\footnotetext{\footnotesize{This interpretation of the parameter $\delta$ is similar to the one in Ball and Romer (1990).}}

A crucial element is that $ceteris paribus$, a higher value $\delta_{i\tau}^{ij}$ reduces the incentive of the consumer to switch brand.
The timing of household $j$'s sequence of decisions during period $t$ is as follows. In $t - 1$, household $j$ bought good $i$ from one, and only one, supplier $k$, which we call his “home seller”. At time $t$, it draws a new independently and identically distributed idiosyncratic taste shock $\delta^j_{it} \in [\underline{\delta}, \overline{\delta}]$, $\underline{\delta} \geq \delta > 0$, $\forall i, j$ from a known time-invariant continuous distribution with a cumulative distribution function $F$, and probability density function $f$. The household then observes the price $p_{it}(k)$ set by his home seller as well as the distribution of prices of other brands of good $i$. The household can then decide to remain with his home seller and pay $p_{it}(k)$, in which case we denote its decision by $s^j_{it} = 0$. Conversely, it can opt to switch and be randomly assigned to a different seller ($s^j_{it} = 1$). A consumer can only switch once per period. Finally, she decides the quantity of good $i$ to buy, $c^j_{it}$.

Going back to the household’s problem, the optimality conditions with respect to $l^j_t$ and $b^j_{t+1}$ are standard

$$\eta \left( l^j_t \right)^\epsilon = \mu^j_t w_t \quad (2.2)$$

$$\mu^j_t E_t r^j_{t+1} = \beta E_{t+1} \mu^j_{t+1} \quad (2.3)$$

while the first-order condition with respect to good $i$ yields

$$\left( \frac{1}{\gamma} \frac{1}{\gamma} \right)^{1-\sigma} \left( \frac{\delta^j_{it}}{\gamma} \right) \frac{s^j_{it} (1-\gamma)}{\gamma} \left( c^j_{it} \right)^{-\frac{1}{\gamma}} = \mu^j_t p_{it}(k) \quad (2.4)$$

where $\mu^j_t$ is the multiplier on the household’s budget constraint.\footnote{The assumption that households draw a new taste shock every period will greatly simplify the solution of the dynamic program and allows us not to keep track of the distribution of tastes each firm is facing each period. With a new draw, each firm will face the same distribution of tastes among its customer base, regardless of the market share size and past evolution. Lastly, the taste shocks are also independent across goods. This assumption allows us a tractable derivation of the aggregate price index (see appendix ).} We can also rewrite the budget constraint as

$$p^j_t \tilde{c}^j_t + E_t r^j_{t+1} b^j_{t+1} = b^j_t + w_t l^j_t + \Pi^j_t$$

where, as we will see later, the price index for the basket of goods $\tilde{p}^j_t$ is household-specific. The first order condition with respect to $c^j_t$ then yields

$$\left( \frac{1}{\gamma} \right)^{-\sigma} = \mu^j_t \tilde{p}^j_t \quad (2.5)$$

\footnote{Note that we explicitly denote the price of good $i$ by $p_{it}(k)$, since we will later investigate what happens when firm $(i, k)$ charges a different price from the sector level price $p_{it}$.}
Using the optimality conditions (2.4) and (2.5), we get a general demand function of household \( j \) for good \( i \) as a function of the switching decision \( s_{it}^j \):

\[
c_i^j = \begin{cases} 
\left( \frac{p_{it}(k)}{p_{t}} \right)^{-\gamma} c_i^j & \text{if } s_{it}^j = 0 \\
\left( \delta_{it}^j \right)^{1-\gamma} \left( \frac{p_{it}}{p_{t}} \right)^{-\gamma} c_i^j & \text{if } s_{it}^j = 1 
\end{cases}
\]  

(2.6)

Notice that the demand in the case of switching \( (s_{it}^j = 1) \) does not include expectations over a distribution of prices. This is because as is usually the case in this type of environment, our analysis will focus on symmetric equilibria where all suppliers of a given good charge the same price. Hence, when the home seller deviates from the equilibrium, household \( j \) knows the price it will face if it decides to switch brand since the distribution of prices is degenerate. Also, in the Appendix we show that the expression for the price index \( \tilde{p}_t^j \) is

\[
\tilde{p}_t^j = \frac{1}{1 - F(1) + \int_0^1 \delta^{1-\gamma} dF(\delta)} \]  

(2.7)

where \( p_{it} = p_t, \forall i \) in a symmetric equilibrium.

Next, we characterize the switching decision. We start from a symmetric equilibrium where all suppliers in a particular sector \( i \) charge the same price \( p_{it} \). We focus on a producer \( k \) who considers deviating by charging a price sequence \( p_t^i(k) = \{p_{it+z}(k)\}_{z=0}^{\infty} \), while all the other suppliers are charging \( p_t^i = \{p_{it+z}\}_{z=0}^{\infty}, \) and we analyze the decision problem of household \( j \).

In order to facilitate the exposition, we consider a recursive representation of the household’s problem. Letting \( \Gamma_t = \{p_t^i, p_t^i(k)\}_{i \in [0,1]} \) and \( \Delta_t = \{\delta_{it}\}_{i \in [0,1]} \), we first denote by \( V_0^j \) the (maximized) value of staying with the home supplier for a particular variety \( i \) \( (s_{it}^j = 0) \):

\[
V_0^j [\Gamma_t] = U(c_t^j) + \beta E \max \{V_0^j [\Gamma_{t+1}], V_1^j [\Gamma_{t+1}, \Delta_{t+1}^j]\}
\]

13By *general* we mean that we have not yet derived an explicit expression for the reservation price, and thus, do not know yet how households choose \( s \).

14As already mentioned, households know the price charged by their seller and the distribution of prices charged by other sellers. However, since we are focusing on a symmetric equilibrium, the distribution is degenerate. Therefore, the relevant state in the household problem is \( \Gamma \).
while $V_i$ is the value of switching:

$$V_i^j [\Gamma_t, \Delta_t] = U [c_i^j] + \beta E \max \{ V_0^j [\Gamma_{t+1}], V_i^j [\Gamma_{t+1}, \Delta_{t+1}^j] \}$$

where $c_i^j, c_i^j_x$ are the optimal level of consumption flows as a function of the relevant state variables when household $j$ does not or does switch seller for good $i$, respectively. Theoretically, there is a continuum of state variables in this representation. However, since we are focusing on the decision to switch in one particular atomistic sector in isolation, we can omit all the other state variables $\{p_z\}_{z \neq i}; \{\delta_z\}_{z \neq i}, \forall t$.\(^{15}\) Also note that once the consumer decides to switch, the future prices charged by the initial home supplier, $p_{i+1}^t (k)$, are no longer relevant. In other words, even if the customer decides to switch again next period, the probability of ending up with supplier $k$ is zero. Hence, we can simplify the system of value functions.\(^{16}\)

$$\tilde{V}_0^j [p_i^t, p_i^t (k)] = \tilde{U} [c_i^j (p_{i+1}^t (k))] + \beta E \max \left\{ \begin{array}{l} \tilde{V}_0^j [p_i^{t+1}, p_i^{t+1} (k), \delta_{i+1}^j], \\ \tilde{V}_1^j [p_i^{t+1}, p_i^{t+1} (k), \delta_{i+1}^j] \end{array} \right\}$$

(2.8)

$$\tilde{V}_1^j [p_i^t, \delta_{i+1}^j] = \tilde{U} [c_i^j (p_{i+1}^t, \delta_{i+1}^j)] + \beta E \max \left\{ \begin{array}{l} \tilde{V}_0^j [p_i^{t+1}, \delta_{i+1}^j], \\ \tilde{V}_1^j [p_i^{t+1}, \delta_{i+1}^j] \end{array} \right\}$$

(2.9)

The threshold taste shock, denoted by $\tilde{\delta}_{i+1}$, is the one which makes the consumer indifferent between switching and staying:

$$\tilde{V}_0 [p_i^t, p_i^t (k)] = \tilde{V}_1 [p_i^t, \tilde{\delta}_{i+1}]$$

(2.10)

That is, all customers for which $\delta_{i+1} > \tilde{\delta}_{i+1}$ will remain with their home supplier of good $i$ while all those with $\delta_{i+1} \leq \tilde{\delta}_{i+1}$ will find it optimal to switch. As in Ireland (1998), this setup exhibits a time-consistency problem. Clearly, customers will be less inclined to switch away from supplier $k$ if the firm promises to charge low prices in the future. Hence, firms have an incentive at time $t$ to announce low future prices, but then renege on their promises later. To deal with this problem, we follow Bils (1989) and assume that firms cannot commit or send signals about future prices. Instead, all agents in the

\(^{15}\)We can do so since each good is atomistic and, thus, does not affect the consumption basket and the budget equation.

\(^{16}\)Note also that in this case, the consumer will always make the same decision in the future: she will switch every period if $\delta_{i+1} < 1$, and will never switch otherwise.
model forecast \( p_{i+1}^t(k) \) by solving the firm’s problem sequentially. In other words, the decision whether to switch will not directly depend on future prices.

Based on this assumption, a supplier will set today’s price \( p_{it}(k) \) taking the sequence \( p_{i+1}^t(k) \) as given, i.e. to be determined by future decisions. Therefore, when considering changing its price, supplier \( k \) is simply interested in knowing what direct impact the fluctuation in \( p_{it}(k) \) will have on the threshold customer, \( \delta_{it} \); this is so because future prices are not under its control today.

### 2.2 Firms

A firm in this environment is indexed by a pair \( g \in G \) indicating the good and the brand, where \( G \equiv \{ (i, k) : i \in [0, 1], k \in [0, 1] \} \). Thus, the set of producers of a certain good \( i' \) can be written as \( \{ (i', k) : i' \ fixed, k \in [0, 1] \} \subset G \). We also introduce a new variable, \( M_{it}(k) \), which denotes the mass of customers of firm \((i, k)\) at time \( t \).

Consider the problem of a seller \( k \) of variety \( i \) who comes into period \( t \) with a market share \( M_{it-1}(k) \). The firm observes the realization of a marginal cost shock at time \( t \) who is common to all producers of good \( i \) and ponders the possibility of changing its price. As discussed earlier, the supplier then sets the relative price of its brand to \( p_{it}(k)/p_{it} \) and current customers optimally decide between switching to another seller or not.\(^{17}\) Clearly, when changing its relative price, supplier \( k \) affects the threshold taste shock \( \delta_{it}(k) \). Notice that if firm \( k \) charges the same price as all the other suppliers, then \( \delta_{it}(k) = 1 \).

By raising its price the producer knows that it will deplete its customer base. More specifically, starting from a symmetric equilibrium, the stock of customers of supplier \( k \) evolves according to:

\[
M_{it}(k) = M_{it-1}(k) \left[ \int_{\Theta} dF + F(1) \right], \quad \Theta = \left[ \delta_{it}(k), \bar{\sigma} \right]
\]

\[
= M_{it-1}(k) \left[ 1 - F \left( \delta_{it}(k) \right) + F(1) \right] \quad (2.11)
\]

The term in brackets has two components. The first expresses the mass of current customers who decide not to switch to a different supplier. The

\(^{17}\)By relative here we mean the price firm \((i, k)\) is charging at time \( t \), given by \( p_{it}(k) \), vis-à-vis the sector \( i \) price index, given by \( p_{it} \). In what follows it is very important to distinguish between \( p_{it} \) and \( \bar{p}_t \). The latter denotes the economy wide price index.
firm \((i, k)\) started with a mass of customers \(M_{it-1}(k)\), and a fraction \(\int_{\Theta} dF = 1 - F(\hat{\delta}_{it}(k))\) of them do not switch. Notice how the assumption of drawing a new taste shock every period is crucial here: if we did not impose this assumption, we would need to keep track of the distribution of current customers as indexed by their respective \(\delta\). Instead, in our setup, the mass of customers that the firm is keeping from one period to the next is simply distributed according to the time-invariant distribution \(F\). The second term represents the mass of households which left the other sellers in sector \(i\): since we are starting from a symmetric equilibrium, they are all charging a relative price of 1 and therefore lose all customers for which \(\delta < 1\).

We follow Ireland (1998) and assume that the rate at which a firm attracts new customers is proportional to its previous period’s market share, which explains why \(M_{it-1}(k)\) also multiplies the term \(F(1)\). In other words, a dominant producer, i.e. one with a large market share, will tend to both lose and gain more consumers in absolute terms compared with a small producer, given a relative price change.\(^{18}\) In Appendix B, we discuss some additional properties of \(M_{it}(k)\).

Aggregating over the mass of customers of firm \((i, k)\) and using (2.6), we obtain the following demand function

\[
c_{it}(k) = A \left( \hat{\delta}_{it}(k) \right) \left( \frac{p_{it}(k)}{\bar{p}_t} \right)^{-\gamma} \tilde{c}_t
\]  

(2.12)

where

\[
A \left( \hat{\delta}_{it}(k) \right) = M_{it-1}(k) \left[ 1 - F(\hat{\delta}_{it}(k)) + \int_{\hat{\delta}}^1 \delta^{1-\gamma} dF \right]
\]  

(2.13)

The structure of \(A(\hat{\delta}_{it}(k))\) is very similar to the one for \(M_{it}(k)\) we just described. A mass \(1 - F(\hat{\delta}_{it}(k))\) of customers do not switch and have a demand given by \(\left( \frac{p_{it}(k)}{\bar{p}_t} \right)^{-\gamma} \tilde{c}_t\), while those who switch from other sellers consume \((\delta_{it})^{1-\gamma} \left( \frac{p_{it}}{\bar{p}_t} \right)^{-\gamma} \tilde{c}_t\).

\(^{18}\)The results presented in this paper do not rely on this feature, though they are somewhat intuitively magnified under such assumption.
As a side note, one can show that a firm which charges \( p_{it}(k) > p_{it} \) permanently will eventually see its market share vanish.\(^{19}\) Therefore, the long-run elasticity faced by the firm is infinite, i.e. for any \( p_{it}(k) = p_{it} + \varepsilon \) where \( \varepsilon > 0 \), we have that \( \lim_{t \to \infty} c_{it}(k) = 0 \). The short-run elasticity of the demand for a single producer around the symmetric equilibrium is given by

\[
\varepsilon_{c_{it}(k), p_{it}(k)} \bigg|_{\frac{p_{it}(k)}{p_{it}}=1} = \gamma + \frac{f(1)}{A(1)} \tag{2.14}
\]

As we would expect, in this framework the elasticity faced by a particular seller is greater than in the Dixit-Stiglitz case, where it simply equals \( \gamma \): \( f(1) \) represents the marginal movement in the extensive margin due to the price change, and it is deflated by the size of the taste-adjusted mass of customers, \( A(1) \).

We now consider the dynamic problem of a supplier \( k \) of good \( i \) who wants to maximize discounted profits by choosing a sequence of prices, \( \{p_{it}(k)\} \). More specifically, the firm is solving the following problem:

\[
\hat{\Pi}_{i0}(k) = \max_{\{p_{it}(k)\}} \sum_{t=0}^{\infty} \beta^t E_0 \mu_t \left\{ c_{it}(k) \left[ p_{it}(k) - \frac{w_t}{z_{it}} \right] \right\} \tag{2.15}
\]

subject to (2.11), (2.12), (2.13), the linear production function \( c_{it}(k) \leq z_{it} l_{it}(k) \) and the initial condition \( M_{i0}(k) = 1 \). Note that the firm is discounting profits using the marginal value of a dollar to the households (and owners), \( \mu_t \), which will vary over time in the general equilibrium version of the model.

It is clear from the problem above that when the firm is setting its price \( p_{it}(k) \), it takes into account four effects: First, the impact on profit per unit sold, i.e. \( \left( p_{it}(k) - \frac{w_t}{z_{it}} \right) \); second, the effect on the intensive margin \( \left( \frac{p_{it}(k)}{p_{it}} \right)^{-\gamma} c_t \); third, the consequence on the extensive margin \( A \left( \hat{\delta}_{it}(k) \right) \) through the impact on \( \hat{\delta}_{it}(k) \); and fourth, the indirect effect on future market share \( M_{it+1}(k) \). In the case of a rise in the relative price, the first effect is positive (raising \( p_{it}(k) \) increases per-unit profit) while all the others are negative, as it will become obvious from the first-order conditions.

\(^{19}\)See Appendix B for a proof.
Once we rewrite the problem in a Lagrangean form, the first-order condition with respect to \( c_{it} (k) \) is given by

\[
p_{it} (k) - \frac{w_t}{z_{it}} = \lambda_{it} (k)
\]

(2.16)

where \( \lambda_{it} (k) \) is the Lagrange multiplier on the demand faced by the firm. Equation (2.16) simply equates the value for firm \((i, k)\) of selling one more unit of the good, \( \lambda_{it} (k) \), to the per-unit profit, \( p_{it} (k) - \frac{w_t}{z_{it}} \).

The optimality condition with respect to \( p_{it} (k) \) yields

\[
c_{it} (k) = \lambda_{it} (k) \gamma A \left( \frac{\tilde{\delta}_{it} (k)}{\tilde{p}_t} \right) \left( \frac{p_{it} (k)}{\tilde{p}_t} \right)^{-\gamma - 1} \tilde{c}_t
\]

\[
+ \nu_{it} (k) M_{it-1} f \left( \tilde{\delta}_{it} (k) \right) \frac{\partial \tilde{\delta}_{it} (k)}{\partial p_{it} (k)}
\]

(2.17)

where \( \nu_{it} (k) \) the Lagrange multiplier associated with the law of motion of the customer base (2.11). To gain intuition for(2.17), consider the case of an increase of one unit in the price \( p_{it} (k) \). The left-hand side gives the benefit of such action: it raises revenues by the quantity sold. The right-hand side defines the costs. First, raising the price means that each customer will consume less of the good (the intensive margin) multiplied by the number of consumers that actually buy from the firm, while the second term identifies the negative impact on the number of customers (the extensive margin): it multiplies the value of one more unit of customer base, \( \nu_{it} (k) \), with the change in market share following the price increase.

Finally, the derivative of the Lagrangian with respect to the market share \( M_{it} (k) \) is

\[
\nu_{it} (k) = \lambda_{it} (k) \left( \frac{p_{it} (k)}{\tilde{p}_t} \right)^{-\gamma} \tilde{c}_t
\]

\[
+ \beta E_t \left[ \frac{\mu_{t+1}}{\mu_t} \right] \left( \lambda_{it+1} (k) \int_0^1 (\tilde{\delta}_1)^{1-\gamma} dF - F (1) \right) \left( \frac{p_{it+1} (k)}{\tilde{p}_{t+1}} \right)^{-\gamma} \tilde{c}_{t+1}
\]

\[
+ \nu_{it+1} (k) \left[ 1 - F \left( \tilde{\delta}_{it+1} (k) + F (1) \right) \right]
\]

(2.18)
Equation (2.18) describes the composition of the marginal value of the market share. First, raising the customer base increases today’s sales: in our case this is expressed as the value to the firm of selling one more unit \( (\lambda_{it}(k)) \) multiplied by the amount of goods bought by one customer. Second, it will increase the sales next period by raising the customer base available to the firm at the beginning of \( t + 1 \).

Finally, we refer the reader to the Appendix for details on finding the derivative of the threshold customer with respect to price, and simply note that it is given by

\[
\frac{\partial \hat{\delta}_{it}(k)}{\partial p_{it}(k)} = \tilde{\delta}_{it}(k)^\gamma \left( \frac{p_{it}(k)}{p_{it}} \right)^{-\gamma} \frac{1}{p_{it}} \quad (2.19)
\]

In a symmetric equilibrium, \( p_{it}(k) = p_{it}, M_{it}(k) = 1, \nu_{it}(k) = \nu_{it} \) and \( \lambda_{it}(k) = \lambda_{it} \) for all \( k, t \).\(^{20}\) Also, \( \tilde{\delta}_{it}(k) = 1 \) since the consumer with \( \delta = 1 \) will be the only one indifferent between staying and switching, and (2.19) becomes

\[
\frac{\partial \hat{\delta}_{it}}{\partial p_{it}} = \frac{1}{p_{it}}
\]

### 3. Analytics under the static case

As is usually the case with this type of forward-looking model, we cannot derive closed-form solutions for the various endogenous variables. An exception is when the firm’s problem is static: when \( \beta = 0 \), firms do not care about the future, and hence consider only the impact of the pricing decisions on the current period’s mass of costumers and their level of consumption. Because we later argue that it is the dynamic elements arising from our model that deliver incomplete cost pass-through, we first need to prove that if the firm’s problem is static, the equilibrium is indeed one where the standard prediction of full pass-through holds.

Under the assumption that \( \beta = 0 \), notice that the value of the customer base (2.18) does not include a forward-looking component anymore

\[
\nu_{it}(k) = \lambda_{it}(k) \left( \frac{p_{it}(k)}{p_{i}} \right)^{-\gamma} \tilde{c}_{t}
\]

\(^{20}\)For now, we only consider symmetry across sellers within a particular sector \( i \). Later we will extend symmetry across all sectors.
Combined with (2.16), (2.17) and (2.12), and by imposing symmetry across producers $k$ we can express the price $p_{it}$ as only a function of some parameters and the marginal cost $mc_{it} = \frac{w_{it}}{z_{it}}$:

$$p_{it} = \left( \frac{\gamma A(1) + f(1)}{(\gamma - 1) A(1) + f(1)} \right) mc_{it}$$ \hspace{1cm} (3.1)

This expression has some important implications. First, it shows that the assumptions about the distribution of the taste shock have an impact on the level of the markup. In the special case where the distribution is such that $A(1) = 1$ and $f(1) = 0$, the markup is simply equal to $\frac{\gamma}{\gamma - 1}$, similar to the Dixit-Stiglitz case.

But most importantly, (3.1) implies that the cost pass-through is complete in the static version of our model: a rise of 5% in the marginal cost will translate into a 5% increase in prices. In the Appendix D, we also prove that such strategy is the unique symmetric equilibrium. Notice that this result is independent of our parametrization, and in particular holds for any distribution of $\delta$. It is, in fact, in line with the claim of Ball and Romer (1990) that “real rigidity does not imply nominal rigidity”. In their framework, however, the real rigidities do not distort the intertemporal problem of the firm. We will show in this paper that extending the mechanism described earlier to a dynamic setting can actually deliver incomplete pass-through of costs to prices, even without introducing any nominal rigidities.

4 Results

In this section we numerically solve for the equilibrium prices and quantities. First, we discuss what are the model predictions under a sector specific technology shock. In this section we also analyze what are the implications of the model when agents are faced with uncertainty concerning the persistence of the shock. Next, we present and discuss the implications of the model given an economy wide shock. The predictions of the model under each shock are remarkably different.

4.1 Sector-specific productivity shock

We start by focusing our attention on finding the optimal pricing behavior of a firm $k$ when only one sector $i$ is hit by a productivity shock. In
such environment, the atomistic nature of sector $i$ implies that the aggregate consumption and price levels, $\bar{c}_i$ and $\bar{p}_t$, the marginal utility $\mu_i$, as well as the wage rate $w_t$ are all time-invariant, and we therefore drop their time subscripts. In a symmetric equilibrium the relative price each producer is charging is $1$, and thus, we can omit the law of motion for the market share as $M_{it}(k) = 1 \forall t$. The variables we need to solve for are $p_{it}$, $c_{it}$, $\lambda_{it}$ and $\nu_{it}$. The equilibrium equations as well as the law of motion of productivity are given by

\begin{align*}
    p_{it} = w \frac{z_{it}}{z_{it}} = \lambda_{it} \\
    c_{it} = \lambda_{it} \gamma A(1) \left( \frac{p_{it}}{\bar{p}} \right)^{-\gamma-1} \bar{c} + \frac{\nu_{it} f(1)}{p_{it}} \\
    \nu_{it} = \lambda_{it} \left( \frac{p_{it}}{\bar{p}} \right)^{-\gamma} \bar{c} + \beta E_t \left\{ \lambda_{it+1} [A(1) - 1] \left( \frac{p_{it+1}}{\bar{p}_{it+1}} \right)^{-\gamma} \bar{c} + \nu_{it+1} \right\} \\
    c_{it} = A(1) \left( \frac{p_{it}}{\bar{p}} \right)^{-\gamma} \bar{c} \\
    \ln (z_{it}) = \rho z \ln (z_{it-1}) + \varepsilon_{zt}^z + \varepsilon_{it}^z
\end{align*}

where $\varepsilon_{zt}^z$ is an economy wide shock and $\varepsilon_{it}^z$ is a sector-specific shock. In this section, we set $\varepsilon_{zt}^z = 0$, $\forall t$.

In addition, setting $w = 1$ and normalizing $\bar{p} = p_i$, the steady state values of the control variables are

\begin{align*}
    p_i = \frac{f(1) - f(1)}{\beta - 1} - f(1) \left( 1 + f(1) A(1) - \gamma \right)^{-1} - f(1) A(1) - \gamma \\
    \lambda_i = p_i - 1 \\
    \bar{c}_i = A(1) \left( \frac{p_i}{\bar{p}} \right)^{-\gamma} \bar{c} \\
    v_i = \lambda_i \bar{c} \left( \frac{1 + \beta [A(1) - 1]}{1 - \beta} \right)
\end{align*}

\textsuperscript{21}It also makes the distinction between nominal and real profits and wages irrelevant, since we will keep the economy-wide price level $\bar{p}_i$ constant over time.
Despite the fact that long-run elasticity is infinite, notice that the steady state markup is larger than zero. This is due to the fact that firms discount future profits by $\beta < 1$, which means that in the limit they put no weight on the possibility that sales will eventually vanish. However, it is clear that $p_i \to 1$ as $\beta \to 1$. In most parameterizations, we find that the steady state markup is indeed very small because of the competition from other sellers of the same good. In addition, one can notice that the value of the market share in steady state, $v_i$, is a positive function of the discount factor $\beta$: in the limit, if sellers do not attach importance to the future ($\beta = 0$), then the only value attributed to the mass of customers comes from the sales it provides in the current period.

4.1.1 Simulations

In terms of distributional assumptions, we impose in our benchmark that the taste shock $\delta$ is distributed lognormal, with support over the interval $(0, \infty)$, mean $\mu_{\ln \delta} = 0.15$ (or $\mu_{\delta} \approx 1.16$) and variance $\sigma_{\ln \delta} = 0.1$. While this calibration is not backed by any particular evidence, our objective for now is mainly to illustrate the mechanics of the model. In addition, we believe that it is not an unreasonable parameterization: it implies that less than 7% of customers in a particular sector will want to switch supplier in a given period, and that about 87% of households have a taste parameter smaller or equal to 1.3.22 Finally, we set $\beta = 0.99$ and $\gamma = 5$, which is in line with the values estimated by Christiano et al. (2005), among others. We will show later the impact on our results of modifying those parameter values. We use Dynare to solve the model and compute the impulse responses, and start by considering a persistent negative productivity shock which raises the marginal cost of the $i$ producers, $w_{it}$, by 1% in the first period. As a benchmark, we set $\rho_z = 0.9$.

As is well known, in a standard Dixit-Stiglitz model such shock would imply a reaction of the price of good $i$, $p_{it}$, perfectly proportional to the movement in marginal cost. In other words, the markup would remain constant. However, Figure 4.1 makes it clear that the addition of intertemporal

---

22It is important to note that even in steady state, there are always some costumers who will switch seller of a certain good. This is due to the fact that there is a non-zero measure of households who have a distaste for staying with their current supplier (that is, $F_\delta(1) > 0$).
market share considerations to the standard model breaks this one-for-one relationship between prices and marginal cost.

In reaction to the 1% increase in their marginal cost, firms decide to raise their prices in order to mitigate the negative impact on their profit margin. However, unlike the Dixit-Stiglitz case, here the pass-through of marginal cost to prices is only about 80%, without the addition of any nominal rigidities to the model. Since the price of good  \( i \) rises relative to the price of other goods, \( c_{it} \) falls as expected.

In order to develop an intuition for our results, it is useful to look at the role of the discount factor. Figure 4.2 repeats the same simulation exercise, but for different values of \( \beta \). The picture we get is that the degree of pass-through is inversely proportional to the value of the discount factor: as firms value the future less and less (\( \beta \) falls), they are also more inclined to pass-through completely marginal cost changes into prices. In the extreme case where \( \beta = 0 \), we are back to the case we discussed in section 3: markups stay constant, and prices rise by 1% on impact.
Figure 4.2: Dynamic response under different values of $\beta$

To understand this result, first recall that the first-order condition with respect to the market share yielded

$$\nu_{it} = \lambda_{it} \left( \frac{p_{it}}{\bar{p}} \right)^{-\gamma} \tilde{c} + \beta E_{it} \left\{ \lambda_{it+1} [A(1) - 1] \left( \frac{p_{it+1}}{p_{t+1}} \right)^{-\gamma} \tilde{c} + \nu_{it+1} \right\}$$

Clearly, when $\beta = 0$, the forward-looking component of the value of the market share is equal to zero, which means that firms do not take into account intertemporal considerations when making their pricing decisions. Instead, they follow the optimal strategy of maintaining a constant markup. However, as their maximization problem becomes dynamic, in the sense that $\beta$ gets closer to 1, their actions are altered. Raising prices proportionately with marginal cost now implies that a given supplier is able to maintain its profit margin but only at the expense of losing customers today and in the future (recall that it takes time to get back lost market share). If firms care about the future, they will want to balance the advantage of maintaining the level of markup every period with the cost of inducing lower sales in the future.
As their discount factor rises, they attribute more and more importance to the intertemporal dimension of this tradeoff, and therefore decide to mute their price increases.

Based on this intuition, one might expect that firms would then react differently based on whether they expect the marginal cost shock to be more or less persistent: if the negative shock is seen as highly temporary, it might become more attractive for the producer to take a bigger hit on its markup today in order to maintain its customer base for tomorrow. Based on Figure 4.3, this intuition appears to be right: as the persistence of the shock is lower, so is the degree of pass-through of marginal cost to prices. For example, when the shock is expected to be purely temporary \( (\rho_z = 0) \), firms increase their prices by slightly less than 0.62%. Conversely, pass-through is complete when the shock is permanent \( (\rho_z = 1) \).

![Graphs showing dynamic response under different values of \( \rho \)](image)

Figure 4.3: Dynamic response under different values of \( \rho \)

Why is it so? Consider the case of a purely temporary shock at \( t = 1 \): because the model described by the system of equations is purely forward-looking (no lagged endogenous variable), we know that in the period following
the shock \((t = 2)\) the model will be back in steady state and the profit margin back to normal. However, if the negative productivity shock is persistent, the profit margin at \(t = 2\) is lower as the firm absorbs part of the shock into its markup, as we showed earlier. Therefore, the customer base in period 2 is worth less in the case of a persistent shock because less profit is made on each unit sold, and each customer buys less goods since she substitutes for products from other sectors following the price increase in sector \(i\). Consequently, the firm has more incentive to preserve its market share in the case of a temporary shock, and it will therefore pass-through less of the marginal cost shock into its price.

An interesting implication of our model is that firms may willingly and optimally decide to sustain negative profits, \(c_t [p_t - mc_t]\), for a certain period of time in order to maintain their customer base. It does not however imply that producers have an incentive to exit the market: their discounted profits in fact remain positive except in the case of extreme adverse shocks. In models à la Calvo, sellers can also under large shocks face the prospect of losses because they are not allowed to re-optimize and change their price, despite the fact that they would clearly have an incentive to do so. Alternatively, under the Kimball (1995) aggregator, the firm occasionally makes zero sales if its relative price deviates too far away from 1, due to the concavity of the demand function. In both instances the firm prefers to shut down for the period and re-enter later, since not making any sales today has no impact on future profits. In contrast, under our environment, exiting in a single period has severe consequences because firms lose their customers, who are difficult to recover.\(^{23}\) It clearly discourages sellers from shutting down operations temporarily, a feature which we believe is realistic and desirable.

In Figure 4.4, we illustrate this property of the model by plotting the responses of prices and profits to persistent increases \((\rho_z = 0.9)\) in the marginal cost. For small shocks, profits are reduced but remain positive. For larger shocks, however, the less than proportional rise in price means that the profit margin turns negative for a few periods.\(^{24}\) Also, we found that the reaction

\(^{23}\)In fact, under our assumption that the mass of switchers in a particular period is distributed across sellers in proportion to their market share, firms could never regain back their customer base after exiting. But even under a less extreme environment, the market share dynamics would create a strong disincentive to exit the market only for a few periods.

\(^{24}\)Clearly, a negative profit margin is the result of an interaction between the degree of pass-through and the level of the markup in steady state, which is very low in our model.
of profits is amplified when shocks are temporary (not shown), which is not surprising given our previous result that the degree of pass-through decreases with the persistence of the shock.

Figure 4.4: Reaction of profits to marginal cost shocks

This brings us to the role of the elasticity of substitution across varieties, \( \gamma \). What is clear from Figure 4.5 is that as goods become better substitutes, that is as \( \gamma \) is increasing, firms become more reluctant to raise prices. This is because when varieties are more substitutable, increasing the price (while other sectors are not) implies a larger drop in the quantity consumed by each customer. In turn, this amplifies the firm’s disincentive from changing too much their prices.

Finally, we complete the analysis of the mechanics of our model by taking a look at the impact of the distributional assumptions. The choice of the distribution for the taste shock \( \delta \) affects the model in two ways. First, it modifies the taste-adjusted demand parameter \( A(1) \). However, this channel does not appear to play much of a role in our setup. In addition, recall that any price change affects the mass of current customers who remain with the firm. This is why \( f(1) \) enters the system of first order conditions (4.1): it represents the first derivative of the mass of non-switchers, \( 1 - F(\delta_{it}(k)) \), in symmetric equilibrium. Now, consider that consumers’ taste shocks are distributed such that at the equilibrium relative price of 1, this first derivative is very large: a high value of \( f(1) \) means that the customer mass is very sensitive to changes in the relative price set by the firms. In equilibrium, the
high elasticity of the market share means that sellers enjoy very low market power and markup, and therefore attribute a low value to the market share. Accordingly, we find that the steady state value of $v_i$ is a decreasing function of $f(1)$. In other words, because customers are not as loyal and can easily switch to other suppliers, they are not valued as much by the firm. As we saw earlier during the discussion about $\beta$, if the seller attributes a lower value to its future market share, it means that the intertemporal dimension of its pricing decision becomes less important, and marginal cost pass-through will be closer to complete.

This is not the only force at play, however: for values of $f(1)$ very close to zero, the relationship between pass-through and $f(1)$ is in fact reversed. The result is intuitive: when $f(1)$ is very low, the incentive for the seller to deviate away from a complete pass-through symmetric equilibrium is almost nonexistent. This is because setting a relative price lower than 1, for example, will not allow the firm to preserve a larger share of its current customer base, as it is almost completely inelastic around $p_t(k)/p_t = 1$. In the extreme case
where the distributional assumptions are such that $f(1) = 0$, the market share does not react to changes in the relative price, the model collapses to the Dixit-Stiglitz basic framework and pass-through is complete. Figure 4.6 shows how pass-through is related to the value of $f(1)$, holding $A(1)$ constant.\footnote{The simulations use the same parameterization as in our benchmark. In particular, here the shock is persistent ($\rho_z = 0.9$).} One can clearly see the two forces just described at work.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{degree_of_passthrough.png}
\caption{Degree of passthrough as a function of f(1)}
\end{figure}

The role of the distribution of $\delta$ can also be illustrated using a concrete example. There is ample empirical evidence that service prices are significantly stickier (Bils and Klenow, 2004). Arguably, a particular characteristic of many services is that they entail more personal buyer-seller relationships: individuals value highly a close and continuous business relationship with their doctor, for example. This is consistent with the observation that consumers are much less willing to switch hair salon than gas station. The implications for pricing strategies are not immediately clear however. On
the one hand, hair salons might be more inclined to capitalize on this captive clientele by raising prices whenever they have an occasion to do so. On the other hand, however, they might realize that it will be very difficult to convince any lost customer to switch back in the future. Those two forces are akin to the ones we just described in the context of our model. To see this, consider that a service sector is one for which the average costumer is particularly reluctant to switch supplier, i.e. where the distribution of the disutility from switching ($\delta$) is shifted to the right. In other words, $f(1)$ is smaller for such goods. At first, having more switch-averse costumers means that hair salons are very wary of losing customers who will be difficult to regain later, and therefore price responses become more muted. However, as illustrated in figure (4.6), there is a point where the second force takes over: when customers almost never want to switch, there is no possibility for the hair salon to gain or lose clientele, and it will therefore adjust its price fully following changes in its marginal cost.

In this section, we considered a scenario where a subset of firms, or a single sector $i$, is hit by a shock to its marginal cost, while producers in other sectors $j \neq i$ are not affected. We have illustrated how the intertemporal nature of the firm’s problem in our model is central to obtaining an incomplete pass-through of marginal cost shocks to prices. We used simulations because finding closed-form solutions is not generally possible in this framework due to its dynamic nature. There is an exception however: in Appendix E we consider the case of a purely temporary shock ($\rho_z = 0$) and are able to show analytically that complete cost pass-through is not an equilibrium. In other words, we prove that if all producers follow such strategy, then a typical firm has an incentive to deviate by lowering its price.

Finally, we also showed how various parameters of the model affect our results. In particular, we argued that the perceived persistence of the shock has an impact on the pricing strategy of the firm. In the next section, we further analyze this result.

### 4.1.2 Persistence of the Shock and Learning

It is conceivable that when firms experience a productivity shock, they find it hard to recognize whether the nature of the shock is temporary or highly persistent. While in many models such distinction is inconsequential or very secondary, we saw earlier that in our setup the perceived persistence of the shock is important for the dynamics of the price response. In this section, we
briefly illustrate how uncertainty about the persistence of the productivity shock affects the optimal pricing decision of a firm.

One possible way to model such uncertainty would be to let the persistence parameter in the marginal cost process to be unknown, and allow Bayesian agents to learn about this parameter. Agents, endowed with prior beliefs about \( \rho \), would then use Bayes law to optimally update their beliefs and derive their posterior distribution for the persistence parameter. Here, however, we pursue a slightly different approach to capture uncertainty. We model the learning process of the firm by using a simple linear Kalman filtering framework. Agents do not observe the true marginal cost shock. Instead, they observe a noisy signal from which they try to infer what is the true state of the underlying marginal cost process. Using the Kalman filter algorithm, agents generate recursive forecasts of the true underlying state process.

The uncertainty in the model is captured explicitly in a rather standard way by the following two processes:

\[
mc_t = mc_t^* + \varepsilon_t, \varepsilon_t \sim N \left(0, \sigma^2_{\varepsilon}\right) \tag{4.4}
\]
\[
mc_t^* = \rho mc_{t-1}^* + v_t, v_t \sim N \left(0, \sigma^2_v\right) \tag{4.5}
\]

where \( mc \) is observable by agents, while \( mc^* \) represents the true unobservable state of the underlying marginal cost process. In other words, \( v \) is a fundamental persistent shock, while \( \varepsilon \) can be described as noise. Here, the structure of the model is known by agents. More explicitly, \( \rho \) is known by all agents.\(^{26}\) Equations (4.4) and (4.5) are the observation and measurement equations, respectively.\(^{27}\)

For this exercise we set \( \rho = 1 \). Hence, \( \varepsilon \) shocks are purely temporary while a \( v \) shock has a permanent effect on the marginal cost. However, agents cannot distinguish between the source of the disturbance. The important elements of the filtering process applied to (4.4) and (4.5) can be summarized

\[^{26}\text{We assume that all the standard Kalman filter assumptions hold. For a general discussion, see, for example, Hamilton (1994, section 13.1).}\]
\[^{27}\text{Equation (4.4) is actually only part of the observation system, since agents also observe other variables, aside from } mc, \text{ such as } c, l \text{ etc. This point is taken into account when solving for the optimal response in prices.}\]
as follows:

\[
mc^*_{t|t} = mc^*_{t|t-1} + K \left( mc_t - mc^*_{t-1|t-1} \right) \tag{4.6}
\]

\[
K = \frac{P}{P + \sigma^2 \varepsilon} \tag{4.7}
\]

\[
\sigma^2_v = \frac{P^2}{P + \sigma^2 \varepsilon} \tag{4.8}
\]

where equation (4.6) indicates that agents update their beliefs about \(mc^*_t\) after they observe \(mc_t\). As is common in the literature, in equation (4.7) we use the steady state level of the Kalman gain process, \(K\). \(P\) is the steady state level of the variance of \(mc^*\), which solves the Riccati equation given in (4.8).

We simulated our model under the maintained assumption of uncertainty about the nature of the shock. We use our benchmark parametrization described at the beginning of section 4.1.1, and analyze the responses under the following signal-to-noise ratio values: \(\frac{\sigma^2_v}{\bar{\sigma}_{\varepsilon}^2} = \{0.2, 0.02, 0.002\}\). We simulate a 5% positive fundamental (i.e. persistent) shock to \(mc^*\) and present the reaction of \(p_i\).

Under this setup, when the firm observes a jump in its marginal cost, it is unclear about its persistence. Because we assume that \(\frac{\sigma^2_v}{\bar{\sigma}_{\varepsilon}^2} < 1\), it initially puts a higher weight on the possibility that the shock is temporary, and accordingly only partially passes through the observed marginal cost increase into prices. However, as the firm continues period after period to observe a high level of \(mc\), it updates its belief and eventually converges to full pass-through as it becomes more convinced that the shock is permanent. Not surprisingly, the response of prices is the more delayed the higher the ratio of relative volatilities \(\frac{\sigma^2_v}{\bar{\sigma}_{\varepsilon}^2}\).

### 4.2 Economy-wide shock

While in the previous section we analyzed the case of a sector-specific disturbance, we now turn our attention to the response of our environment to an economy-wide productivity shock. Intuitively, we should expect to see important differences between the two scenarios: when a shock hits a single

\[^{28}\]The solution to (4.7) and (4.8) will depend solely on the signal-to-noise ratio parameter.
sector, any deviation of the sectoral price away from the general price level results in consumers substituting across product varieties. This deters firms from changing prices, and as we saw in section 4.1, the higher the elasticity of substitution $\gamma$, the more muted the price response. When all firms from all sectors are hit by a similar aggregate shock, however, this substitution across varieties does not occur in the symmetric equilibrium: the general price level moves in lockstep with the sector price level, and hence at a first approximation the demand faced by the firm remains constant in equilibrium (it might still fluctuate because of changes in aggregate consumption). Therefore, when a firm as well as all its competitors (both within its sector and across varieties) are facing the same shock, one would expect the pass-through behavior to be very different. To verify this intuition, we simulate the following system of equations, where the aggregate price level and
consumption are now endogenous:

\[
(c_t)^{-\sigma} = \mu_t \tilde{p}_t
\]

\[
\eta (l_t) = \mu_t w_t
\]

\[
\mu_t E_{t+1} = \beta E_{t+1} \mu_{t+1}
\]

\[
p_t - w_t = \lambda_t
\]

\[
c_t = \lambda_t \gamma A (1) \left( \frac{p_t}{\tilde{p}_t} \right)^{-\gamma-1} \tilde{c}_t + \frac{\nu_t(k) f (1)}{\tilde{p}_t} \]

\[
\nu_t = \lambda_t \left( \frac{p_t}{\tilde{p}_t} \right)^{-\gamma} c_t + \beta E_{t+1} \frac{\mu_{t+1}}{\mu_t} \left\{ \lambda_{t+1} \frac{[A (1) - 1]}{\tilde{c}_{t+1} + \nu_{t+1} (k)} \right\}
\]

\[
\tilde{p}_t = p_t A (1)^{\frac{1}{1-\gamma}}
\]

\[
c_t = A (1) \left( \frac{p_t}{\tilde{p}_t} \right)^{-\gamma} \tilde{c}_t
\]

\[
c_t = z_t l_t
\]

\[
\ln (z_t) = \rho_z \ln (z_{t-1}) + \varepsilon_{z,t}
\]

where all the variables have been defined earlier.

### 4.2.1 Simulation Results

We set the nominal wage as the numeraire \((w_t = 1 \ \forall t)\), and use the same parameterization as in the benchmark sector-specific case: \(\beta = 0.99, \gamma = 5, \rho_z = 0.9\), and the taste shocks are distributed lognormal with mean 0.15 and variance 0.1. Regarding the parameters specific to the general equilibrium framework, we use values which are standard in the literature: \(\sigma = 2\) and \(\epsilon = 1\). While the scaling parameter \(\eta\) is chosen such that labor \(l\) is equal to 1 in steady state, it has no impact on our results. We simulate a 1\% negative aggregate productivity shock.

Figure 4.8 plots the impulse responses for the main variables of the model.\(^{29}\) Consistent with our earlier intuition, the response of prices under the aggregate and idiosyncratic shocks are qualitatively very different:

\(^{29}\)We express the value of the market share and the profit margin in terms of units of consumption \((\mu_t \nu_t \text{ and } \mu_t \lambda_t)\). In the sector-specific case, such distinction was irrelevant since we abstracted from the consumer’s problem \((\mu_t \text{ was assumed to be constant})\).
Figure 4.8: Response to a 1% negative aggregate productivity shock
because all their competitors are hit by the same shock, firms do not anymore try to protect their future sales by absorbing part of the marginal cost increase into their profit margin. In other words, the complete synchronization across firms eliminates any movement in their intensive margins, which in turn renders the intertemporal market share motives previously discussed mostly irrelevant. Instead, we even witness a more than proportional rise in the price, though slight. To understand why we obtain a countercyclical behavior for the markup, recall that firms are owned by consumers who value one unit of profits by \( \mu_t \), the multiplier on their budget constraint. As one can see, the value of this multiplier is jumping up in the first period, before slowly going back towards its steady state. In other words, consumers value firm profits the most at the moment of the shock.\(^{30}\) For this reason, firms will initially increase their profit margins by raising prices by more than the rise in their marginal cost. Finally, labor increases because the wealth effect is larger than the substitution effect in our parameterization.

In Figure 4.9, we illustrate the change in the price response under alternative parameterizations, in addition to the benchmark results and the marginal cost. Similar to the sector-specific shock case, marginal cost pass-through is complete when \( \beta = 0 \) due to the absence of any intertemporal substitution. The case of a purely temporary shock deserves more attention, however: when \( \rho_z = 0 \), we actually observe a more pronounced response of the price level. Once again, the reason lies in the dynamics of \( \mu_t \) (not shown): relative to our benchmark, the marginal value of wealth rises to about the same level in the period of the shock, before going back to the steady state level in the next period. This implies that with a highly temporary shock, consumers find the firms’ profits even more valuable today relative to tomorrow than in our benchmark case, which explains why firms raise their profit margins by more when \( \rho_z = 0 \).

To complete the illustration of the mechanism just described, consider the case of a higher value for the parameter \( \sigma \). As the degree of intertemporal substitution, \( 1/\sigma \), decreases, the response of \( \mu_t \) becomes more important, which exacerbates the incentive of the firm to raise prices today in order to boost profits when it is the most needed. Inversely, with log utility (\( \sigma = 1 \)) there is no movement in \( \mu_t \) and the pass-through is complete.

Once again, we refer the reader to Appendix E for an analytical derivation under the special case of a purely transitory shock. We show that unlike the

---

\(^{30}\)This is due to the decline in consumption which raises the marginal utility.
case of a sector-specific shock, when the whole economy is hit by a fall in productivity, sellers do not have an incentive to deviate from a complete pass-through equilibrium by lowering their price. This confirms the stark contrast between these two cases we uncovered through our simulation exercises.

5 An Application: Market Share and Exchange Rate Pass-Through

“Although the dollar’s exchange rate has been declining since early 2002, increasingly tight competitive conditions in the United States, as elsewhere, in 2002 and 2003 apparently induced exporters to the United States to hold dollar prices to competitive levels to ensure their market share and foothold in the world’s largest economy” Alan Greenspan, Advancing Enterprise 2005 Conference, London, February 4, 2005.
In this section, we turn our attention to what we believe is a natural application for our framework. Despite hedging and other measures, exporters’ revenues are highly affected by fluctuations in the exchange rate, particularly when their sales abroad are denominated in the destination currency.\textsuperscript{31} But even when firms are pricing in their own currency, they might be unwilling to let the relative price of their product fluctuate excessively in the foreign market because of competitive pressure from native firms.\textsuperscript{32} Such concern appears to be a plausible mechanism behind the well-documented evidence that exporters do not fully pass-through exchange rate fluctuations into their prices.

For this application, we make abstraction of many side issues and consider a small open economy $H$ who produces and exports goods, as well as imports from abroad. In particular, home consumers import a certain number of varieties from country $F$. We assume that the size of the $H$ market is small relative to total sales by $F$ firms, which means that profit fluctuations from this particular market will not influence significantly total $F$ profits. Similarly, we assume that exports of $F$ firms to country $H$ are very small relative to the size of the home market: this ensures that any change in import prices from country $F$ will not affect the general price level in $H$.

To simplify the exposition, let’s define only two kinds of varieties available in country $H$: those which are sold by $F$ firms and those produced by sellers from the rest of the world, including home producers. We will denote the second group simply as $H$. Therefore, for the purpose of our analysis, a household $j$ from country $H$ derives utility from a consumption basket $c^j_t$, which is a combination of $H$ ($c^j_{H,t}$) and $F$ goods ($c^j_{F,t}$). The relevant aggregators are standard in the literature, except for the switching decision which is specific to our framework.

\textsuperscript{31}See, for example, Dominguez and Tesar (2006) for evidence that the firm value of exporters is affected by exchange rate fluctuations.

\textsuperscript{32}See Mair (2005) for an analysis of surveys of exporters conducted by the Bank of Canada. Its main focus is to study the reaction of exporters to the recent appreciation of the Canadian dollars.
The parameter $\gamma$ denotes the elasticity of substitution across varieties produced by a particular country, while $\omega$ is the elasticity of substitution between $H$ and $F$ varieties. The weights $\kappa_H$ and $\kappa_F$ will dictate the relative importance of $H$ and $F$ goods in the consumption basket of a typical home household. Consistent with our earlier model, each $F$ variety is sold by a continuum of unit mass of foreign sellers producing homogenous brands, and similarly for $H$ varieties. The decision problem of household $j$ in country $H$ is very similar to the one from Section 2:

$$\max U_j = \sum_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c^j_\tau, l^j_\tau\right)$$

subject to

$$u\left(c^j_\tau\right) = \frac{1}{1 - \sigma} \left(c^j_\tau\right)^{1-\sigma} - \eta \left(l^j_\tau\right)^{1+\epsilon}$$

$$\int_0^1 p^j_{ht}(k)c^j_{ht}dh + \int_0^1 p^j_{ft}(k)c^j_{ft}df + E_t r_{t+1} b^j_{t+1} = b^j_t + w_t l^j_t + \Pi^j_t$$

The first-order conditions of the household problem are similar to the ones we derived in Section 2.1, and we will not repeat them here. Aggregating across customers, the demand function faced by foreign seller $k$ producing good $f$ is given by

$$c_{ft}(k) = \kappa_{FA} \left(\delta_{ft}(k)\right) \left(\frac{p_{ft}(k)}{\overline{p}_{Ft}}\right)^{-\gamma} \left(\frac{\overline{p}_{Ft}}{\overline{p}_t}\right)^{-\omega} c^j_t$$

33 This assumption is necessary since our environment requires symmetry across suppliers within a particular sector.
where

\[
A \left( \delta_{ft} (k) \right) = M_{ft-1} (k) \left[ \int_{\frac{1}{2}}^{1} \delta^{1-\gamma} dF + 1 - F \left( \delta_{ft} (k) \right) \right]
\]

\(\bar{p}_{Ft}\) is the price index for the basket of \(F\) goods, while \(\bar{p}_t\) and \(\bar{c}_t\) are the aggregate price and consumption levels in country \(H\) respectively. The expressions for the price indexes in the symmetric equilibrium are

\[
\begin{align*}
\bar{p}_{Ft} &= A \left( 1 \right)^{\frac{1}{\gamma}} p_{ft} \\
\bar{p}_{Ht} &= A \left( 1 \right)^{\frac{1}{\gamma}} p_{ht} \\
\bar{p}_t &= \left[ \kappa_H (\bar{p}_{Ht})^{1-\omega} + \kappa_F (\bar{p}_{Ft})^{1-\omega} \right]^{\frac{1}{1-\omega}}
\end{align*}
\]

where the derivations for \(\bar{p}_{Ft}\) and \(\bar{p}_{Ht}\) are similar to those in Appendix A, and \(\bar{p}_t\) follows directly from our aggregator.

Consider the decision problem of a typical \(F\) firm. It sells domestically and engages in exports activities, and we assume that profits from these activities are separable. This means that we can focus solely on the firms’ export decision to country \(H\), since only the bilateral exchange rate between \(F\) and \(H\) will be affected in our simulations. The decision problem of a foreign firm \((f, k)\) selling in the home market is therefore\(^{34}\)

\[
\max_{\{p_{ft}(k)\}} \sum_{t=0}^{\infty} \beta^t E_0 \mu_t^* \left\{ c_{ft} (k) \left[ e_t p_{ft} (k) - \frac{w_t^*}{z_{ft}} \right] \right\}
\]

subject to the demand schedule (5.3) and the law of motion for the market share. \(p_{ft} (k)\) is expressed in the currency of the home market, and \(e_t\) is the exchange rate expressed as units of foreign currency per home currency. The law of motion of the exchange rate is given by \(\ln (e_t) = \rho \ln (e_{t-1}) + \varepsilon_t\). The

\(^{34}\)Once again, notice that we abstract from the pricing decision in the \(F\) market, simply because the firm’s return function is additive across markets. Given the nature of our assumptions, that dimension of the producer’s problem will not be affected in our simulations.
other variables have been described in Section 2, and asterisks denote their foreign counterparts.

The experiment consists in an exogenous temporary depreciation of the home currency (a negative realization of $\varepsilon_t$). We impose that $\kappa_F$ is very small which, in conjunction with our earlier assumptions about the relative market sizes, ensures that the aggregate economies of $H$ and $F$ are, at a first approximation, unaffected by shocks to the bilateral exchange rate, and that fluctuations in $\tilde{p}_{Ft}$ and $\tilde{c}_{Ft}$ have no significant impact on $\tilde{p}_t$ and $\tilde{c}_t$. Consequently, the only variables which will not be constant over time in our simulations are $e_t$, $p_{ft}$, $c_{ft}$, $\tilde{p}_{Ft}$ and $\tilde{c}_{Ft}$. In particular, the foreign variables $w^*_t$, $\mu^*_t$ and $z^*_t$ will be held constant throughout the exercise. This allows us to focus on the problem faced by the foreign exporter following the exchange rate shock.

Clearly, the first-order conditions to the firm’s problem are very similar to the ones we derived in Section 2, and we will not repeat them for the sake of conciseness. What is important to realize is that the exchange rate $e_t$ will enter our system of equations and affect the pricing decision.

Figure 5.1 shows what happens to the foreign exporters following a 5% temporary depreciation of the home currency.\footnote{For this simulation, we use the following parameter values: $\beta = 0.99$, $\gamma = 5$, $\omega = 3$, $\mu_{ln,\delta} = 0.15$, $\sigma_{ln,\delta} = 0.1$, $\rho = 0.8$.} Remember that these firms are facing competition from home sellers producing competing varieties. In a standard framework without customer flows and constant elasticity of demand, the optimal strategy of the foreign exporter would be to raise export prices proportionately in order to maintain its profit margin. As we have already seen earlier, in our setup this is not the best strategy: when faced by negative fluctuations in the exchange rate, the exporter is reluctant to fully raise prices. The reason is intuitive: while increasing the price allows to improve the markup, it also involves the risk of losing customers who will be hard to regain later, exactly when the exchange rate will have realigned itself and revenues per customer will be higher.

Our customer flow mechanism therefore delivers incomplete exchange rate pass-through, in line with the extensive empirical evidence on the subject. While the literature generally imposes nominal rigidities or demand functions with non-constant elasticity to achieve this result (e.g. Devereux, 2003, and Burstein et al., 2005), our model delivers incomplete pass-through as an
endogenous outcome: exporters are reluctant to pass-through completely exchange rate changes to their prices because they care about their market share position.\footnote{Atkeson and Burstein (2005) obtain a similar result in a game-theoretic, partial equilibrium framework.}

If firms live in an environment where the exchange rate process is very noisy, they are likely to initially put a high weight on the possibility that an exchange rate shock is temporary and only slowly update their beliefs about the nature of the shock. Similar to the results from Section 4.1.2, our model would then deliver a degree of pass-through which is incomplete in the short run, but complete at longer horizons. This would be consistent with the evidence from international prices (see Campa and Goldberg, 2002), and related to the intuition of Froot and Klemperer (1989). Finally, our model can easily be embedded in a general equilibrium open-economy model, as it builds on the standard framework used in the literature.
6 Conclusion

In their day to day operations, firms appear to be highly concerned about their market share position, yet this is not an issue which has received much attention in the macroeconomic literature. In this paper, we presented a general equilibrium model in which customer flow dynamics add an intertemporal dimension to the firms’ problem. Our model is micro-founded in the sense that consumers make optimal decisions about switching supplier of a certain good. Without introducing any nominal rigidities, we showed that this real rigidity gives rise to a muted response of prices to sector-specific marginal cost shocks, even if under the static case pass-through is complete. However, under economy-wide shocks, we found that the markup is either constant or slightly countercyclical.

We analyzed further the mechanics of our framework and found that firms are more likely to absorb the shock in their profit margins the more temporary is the disturbance, and investigated an environment where firms are uncertain about the nature of the shock. Also, we provided intuition behind the role of the distribution of taste parameters in affecting our results. We showed that in our model firms can rationally decide to sustain a negative profit margin following an adverse shock, yet without having an incentive to exit the market. Finally, we embedded our mechanism within a simple small open economy model, and showed that the framework predicts an incomplete pass-through of exchange rate fluctuations into prices, in line with a large body of empirical evidence on the subject.

Many possible research avenues emerge in the wake of our findings. What we showed in this paper is that real rigidities which add a dynamic component to the firm’s problem, such as customer flows, can in fact generate price stickiness under certain contexts. It is therefore conceivable that a mechanism like ours could act as an additional source of amplification for nominal rigidities and deliver plausible degrees of price stickiness. Using a richer environment, one could then attempt to match observed moments of business cycle variables.

Also, an interesting question would be to study the implications of introducing market share considerations in an environment where firms are uncertain about the scope of the observed shock. One of the difficulties in studying such an extension is the fact that prices can be used as a channel through which information is revealed. Since each firm observes the aggregate price level, it can immediately infer whether a shock is sector-specific or
economy-wide. Introducing noise into the economy could be one simple way of preventing prices from being fully revealing.

Moreover, we only briefly explored the implications of our model within an open economy framework. The market share story we put forward to explain the observed incomplete exchange rate pass-through is one which, we believe, appears to be both promising and appealing and deserves further investigation. It remains, however, that the degree of pass-through is quite high in this version of the model, and we intend to find ways to lower it further.
References


A Price Index

Based on our results from section 2.1, we can derive an expression for the price index $\tilde{p}_t^j$.

$$\widetilde{p}_t^j c_t^j = \int_0^1 p_{it} c_{it}^j di = \int_0^1 p_{it} \left[ (\delta_{it}^j) s_{it}^j \right]^{1-\gamma} \left( \frac{p_{it}}{\overline{p}_t} \right)^{-\gamma} c_t^j di$$

$$\tilde{p}_t^j = \left[ \int_0^1 \left( (\delta_{it}^j) s_{it}^j p_{it} \right)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

$$= \left[ \int_{i:s_{it}^j=0} p_{it}^{1-\gamma} di + \int_{i:s_{it}^j=1} (\delta_{it}^j p_{it})^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

The above expression that the price index $\tilde{p}_t^j$ is a function of the endogenous switching decision and is household-specific. For future reference, consider a symmetric equilibrium where $p_{it} = p_t$, $\forall i$. In this case

$$\widetilde{p}_t^j = \left[ \int_{i:s_{it}^j=0} p_{it}^{1-\gamma} di + \int_{i:s_{it}^j=1} (\delta_{it}^j p_{it})^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

$$= p_t \left[ \int_{i:s_{it}^j=0} di + \int_{i:s_{it}^j=1} (\delta_{it}^j)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

$$= p_t \left[ \int_1^\infty dF (\delta) + \int_0^1 \delta^{1-\gamma} dF (\delta) \right]^{\frac{1}{1-\gamma}}$$

where the last line obtains from our distributional assumptions and the fact that in equilibrium, all the varieties $i$ for which the consumer switches ($i$ such that $s_{it}^j = 1$) are those for which $\delta_i < 1$, and he remains with his home seller every time $\delta_i \geq 1$. Clearly, in a symmetric equilibrium the price index
is the same for every consumer \( j \), that is \( \tilde{p}_t^j = \tilde{p}_t \) where

\[
\tilde{p}_t = p_t \left[ 1 - F(1) + \int_0^1 \delta^{1-\gamma} dF(\delta) \right]^{1/\gamma}
\]  

(A.1)

B Properties of the Market Share

In this section we discuss some properties of the customer flow equation (2.11). Since we are starting from a symmetric steady state in which \( M_{it-1}(k) = 1 \), if firm \( k \) charges a relative price of 1, then the market share will remain unchanged.

\[
M_{it}(k) / M_{it-1}(k) = 1
\]

By charging a higher price than the other producers, a seller raises the threshold parameter \( \delta_{it}(k) \) above 1. Intuitively, we would expect the market share to fall. For simplicity, let’s assume that supplier \( k \) starts with a market share of \( M_{it-1}(k) = 1 \) and that \( \delta_{it}(k) = \delta_{it+1}(k) = \delta^* > 1 \). Then

\[
M_{it}(k) / M_{it-1}(k) < 1
\]

which follows from the fact that \( F(\tilde{\delta}_{it}(k)) > F(1) \) if \( \tilde{\delta}_{it}(k) > 1 \). That is, the market share is falling when the firm keeps its relative price above 1. Also, in the limit, the market share can vanish: iterating over the growth rate of the market share and taking the limit

\[
\lim_{z \to \infty} [M_{it+z}(k) / M_{it-1}(k)] = \lim_{z \to \infty} [1 - F(\delta^*) + F(1)]^{z+1} = 0
\]

since \( F(1) < F(\delta^*) \) under the assumption that \( \delta^* > 1 \). A related result is that when a seller loses a portion of his market share, he can only regain what is lost by charging a relative price lower than 1. The gain will accumulate slowly, depending on how aggressive the producer is when cutting its price. To prove this property, let a firm start with a mass of customers below 1 \( (M_{it-1} < 1) \), and assume that the firm charges a relative price of 1 \( (p_{it}(k) / p_{it} = 1) \), meaning that \( \tilde{\delta}_{it}(k) = 1 \). Then

\[
M_{it}(k) / M_{it-1}(k) = 1
\]

Hence, the market share will not grow until the producer decides to lower the relative price it charges below 1. Clearly, the analysis is analogous in the case of \( M_{it-1} > 1 \).
C Switching Rule

Before applying a symmetric equilibrium, we derive the object \( \partial \tilde{\delta}_{it} (k) / \partial p_{it} (k) \). We mentioned earlier that the threshold switching cost \( \tilde{\delta}_{it} (k) \) is implicitly defined by equating the value of remaining with the current supplier, \( \tilde{V}_{0} \), and the value of switching, \( \tilde{V}_{1} \):

\[
\tilde{V}_{0} \left[ p_{i}^{t}, p_{i}^{t} (k), \tilde{\delta}_{it} (k) \right] = \tilde{V}_{1} \left[ p_{i}^{t}, \tilde{\delta}_{it} (k) \right]
\]

Also, recall that we assumed that firms cannot commit to a sequence of future prices. Instead, agents infer future prices by solving the problem that the firm will be facing in the future, and the firms take future prices as being determined later. Therefore, the forward-looking parts of the value functions above are not directly affected by a change in the price of supplier \( k \), \( p_{i}^{t} (k) \).

By defining a function \( G = \tilde{V}_{0} - \tilde{V}_{1} \), we can use the implicit function theorem to find \( \partial \tilde{\delta}_{it} (k) / \partial p_{it} (k) \):

\[
\frac{\partial \tilde{\delta}_{it} (k)}{\partial p_{it} (k)} = -\frac{\partial G / \partial p_{it} (k)}{\partial G / \partial \tilde{\delta}_{it} (k)}
\]

Notice that only \( \tilde{V}_{0} \) depends on \( p_{it} (k) \). Denoting by \( \tilde{c}_{0it}, \tilde{c}_{1it} \) the optimal demand of the threshold consumer in case she is either not switching or switching, respectively, we have:

\[
\frac{\partial G / \partial p_{it} (k)}{\partial G / \partial \tilde{\delta}_{it} (k)} = \frac{\partial U \left( \tilde{c}_{0it} \right)}{\partial \tilde{c}_{0it}} \frac{\partial \tilde{c}_{0it}}{\partial p_{it} (k)}
\]

\[
= \left( \tilde{c}_{t} \right)^{\frac{1}{\sigma} - \sigma} \left( \tilde{c}_{0it} \right)^{-\frac{1}{\sigma}} \left( -\frac{\gamma \tilde{c}_{0it}}{p_{it} (k)} \right)
\]

\[
= -\gamma \tilde{c}_{t}^{\sigma} \frac{p_{it} (k)}{\tilde{p}_{t}} \frac{\tilde{c}_{0it}}{p_{it} (k)}
\]

\[
= -\gamma \tilde{c}_{t}^{\sigma} \frac{\tilde{c}_{0it}}{\tilde{p}_{t}}
\]

Where we used our earlier result that \( \left( \tilde{c}_{t} \right)^{\frac{1}{\sigma}} \left( \tilde{c}_{0it} \right)^{-\frac{1}{\sigma}} = p_{it} (k) / \tilde{p}_{t} \). Also,
only $\tilde{V}_1$ depends on $\tilde{\delta}_{it}(k)$. Hence
\[
\frac{\partial G}{\partial \tilde{\delta}_{it}(k)} = -\frac{\partial V_1}{\partial \tilde{\delta}_{it}(k)} = -\frac{\partial U(\tilde{c}_{1it})}{\partial \tilde{\delta}_{it}(k)} = \frac{\gamma (\tilde{c}_t)^{\frac{1}{\gamma} - \sigma} (\tilde{c}_{1it})^{1-\frac{1}{\gamma}} \tilde{\delta}_{it}(k)^{\frac{1}{\gamma} - 2}}{\delta_{it}(k) \frac{1}{\tilde{p}_t}}.
\]
where we used $(\tilde{c}_t)^{\frac{1}{\gamma}} (\tilde{c}_{1it})^{-\frac{1}{\gamma}} \tilde{\delta}_{it}(k)^{\frac{1}{\gamma} - 1} = \tilde{p}_t/\tilde{p}_t$. Knowing that
\[
\frac{\tilde{c}_{0it}}{\tilde{c}_{1it}} = \tilde{\delta}_{it}(k)^{\gamma - 1} \left( \frac{p_{it}(k)}{p_{it}} \right)^{-\gamma}
\]
we obtain
\[
\frac{\partial \tilde{\delta}_{it}(k)}{\partial p_{it}(k)} = \frac{\tilde{c}_{0it}\tilde{\delta}_{it}(k)}{\tilde{c}_{1it}p_{it}} = \tilde{\delta}_{it}(k)^{\gamma - 1} \left( \frac{p_{it}(k)}{p_{it}} \right)^{-\gamma} \tilde{\delta}_{it}(k)^{1} p_{it}
\]
\[
\frac{\partial \tilde{\delta}_{it}(k)}{\partial p_{it}(k)} = \tilde{\delta}_{it}(k)^{\gamma} \left( \frac{p_{it}(k)}{p_{it}} \right)^{-\gamma} \frac{1}{p_{it}}.
\]

We have now derived all the necessary optimality conditions for both households and firms. Next, we can apply a symmetric equilibrium where $p_{it}(k) = \tilde{p}_t$ and $M_{it}(k) = 1$ for all $t$. This simply means that $\tilde{\delta}_{it} = 1$: clearly, the consumer with $\delta_{it} = 1$ will be the only one indifferent between staying and switching. Then, in the symmetric equilibrium where all the suppliers within sector $i$ charge the same price, $\frac{\partial \tilde{\delta}_{it}(k)}{\partial p_{it}(k)}$ simply collapses to
\[
\frac{\partial \tilde{\delta}_{it}}{\partial p_{it}} = \frac{1}{p_{it}}.
\]

### D Equilibrium in the static case

In this Appendix we will prove analytically that even under a model with customer flows, the unique symmetric equilibrium is one where the optimal pricing strategy is to fully pass-through marginal cost shocks to prices.\(^{37}\)

\(^{37}\)We have already shown that such equilibrium exists by deriving (3.1).
Consider a sector $i$ which is initially in steady state at $t - 1$ with a marginal cost of 1. Let’s denote the new exogenous marginal cost at time $t$ by $mc_{it} = \kappa$, and by $p_{it} = \kappa'p_i$ the new price charged by all competitors in sector $i$, where $p_i$ is the steady state value of the price. For example, in the case where marginal cost goes up ($\kappa > 1$), pass-through is incomplete if $\kappa' < \kappa$ and complete if $\kappa' = \kappa$.

Since $\beta = 0$, the problem of the firm is purely static, and we can easily express the impact on a firm’s profit if it decides to deviate by changing its price.

$$\frac{\partial \Pi_{it}(k)}{\partial p_{it}(k)} = \frac{\partial c_{it}(k)}{\partial p_{it}(k)} [p_{it}(k) - m_{c_{it}}] + \frac{\partial [p_{it}(k) - m_{c_{it}}]}{\partial p_{it}(k)} c_{it}(k)$$

We evaluate this expression around the current state where all firms charge the same price. Hence, we set $p_{it}(k) = p_{it}$.

$$\frac{\partial \Pi_{it}(k)}{\partial p_{it}(k)}\bigg|_{p_{it}(k)=1} = A(1) \left( \frac{p_{it}}{p_{it}} \right)^{-\gamma} \tilde{c}_t - \left( \frac{p_{it}}{p_{it}} \right)^{-\gamma - 1} \tilde{c}_t \left[ \gamma A(1) + f(1) \right] [p_{it} - m_{c_{it}}]$$

Next, we use our initial definitions that $mc_{it} = \kappa$ and $p_{it} = \kappa'p_i$, and introduce $\zeta = \frac{\kappa'}{\kappa}$. 

$$\frac{\partial \Pi_{it}(k)}{\partial p_{it}(k)}\bigg|_{p_{it}(k)=1} = \left( \frac{\kappa'p_i}{p_{it}} \right)^{-\gamma} \tilde{c}_t \left[ A(1) - \frac{1}{\kappa'p_i} \left[ \gamma A(1) + f(1) \right] [\kappa'p_i - \kappa] \right]$$

Finally, if we use the expression for the steady state price in our model,

$$p_i = \frac{\gamma A(1) + f(1)}{(\gamma - 1) A(1) + f(1)}$$

and simplify, we obtain

$$\frac{\partial \Pi_{it}(k)}{\partial p_{it}(k)}\bigg|_{p_{it}(k)=1} = \left( \frac{\kappa'p_i}{p_{it}} \right)^{-\gamma} \tilde{c}_t \left[ (\gamma - 1) A(1) + f(1) \right] (\zeta - 1) \quad (D.1)$$

47
We can now analyze this expression to prove that there is a unique, stable symmetric equilibrium under the static case. For example, consider that following an increase of the marginal cost at time $t$ from 1 to $\kappa$, all sellers are setting $p_{it} = \kappa' p_i$ where $\kappa' < \kappa$. In this case, there is incomplete pass-through, as the rise in the price is proportionately less than the increase in the marginal cost.\footnote{In other words, the markup falls.} The expression in (D.1) tells us that since $\zeta = \frac{\kappa'}{\kappa} > 1$, $\frac{\partial \text{profits}_i(k)}{\partial p_{it}(k)} > 0$ and therefore a particular seller $k$ has an incentive to deviate and increase its price. Conversely, if $\kappa' > \kappa$ (firms overshoot following the rise in marginal cost), then our derivation above shows that any producer will raise its profits by lowering its price. The same analysis can be applied to a fall in the marginal cost ($\kappa < 1$).

Most importantly, (D.1) shows that the unique symmetric equilibrium is where $\zeta = 1$, that is where cost pass-through is complete: only in this particular case is there no incentive for sellers to deviate. In any other instances where firms behave symmetrically but do not practice complete pass-through, there is incentive to price in order to maintain a constant markup. This confirms that if firms face real rigidities in the context of non-dynamic profit-maximization problem, the unique and stable symmetric equilibrium is one in which producers fully pass-through cost shocks.

### E Equilibrium in a dynamic setting

We now study the incentive of a seller $k$ to deviate from a symmetric equilibrium where all firms pass-through fully the marginal cost shock into their prices. Here we focus on the case of a purely transitory shock ($\rho_z = 0$) because it is analytically tractable, while relying on the simulations for other values of $\rho_z$.

Recall that the discounted sum of profits $\hat{\Pi}_{i0}(k)$ of the $(i,k)$ seller is given by (2.15). Its derivative with respect to the price at $t = 0$ around the symmetric equilibrium is:

$$\left.\frac{\partial \hat{\Pi}_{i0}(k)}{\partial p_{io}(k)}\right|_{\rho_z(k) = 1} = \mu_0 c_{i0} + \sum \beta^t \mu_1 \left.\frac{\partial c_{it}}{\partial p_{io}(k)}\right|_{\rho_z(k) = 1} [p_{it} - mc_{it}]$$

Since we are starting from a complete pass-through equilibrium, let’s denote the new exogenous marginal cost at time 0 by $mc_{i0} = \kappa$, and by...
\( p_{i0} = \kappa p_i \) the new price charged by all sellers in sector \( i \), where \( p_i \) is the steady state value of the price. Notice that because we use the nominal wage \( w \) as numeraire and normalize it to 1, \( mc_{i0} = \kappa \) can also be interpreted as \( z_{i0} = 1/\kappa \). For \( t > 0 \), we assume that \( mc_{it} = 1 \) (temporary shock), and since the model is purely forward-looking in equilibrium, it can be shown that \( p_{it} = p_i \), that is the model is back to steady state starting from period \( t = 1 \).

Based on the demand function (2.12), we find that

\[
\frac{\partial c_{i0} (k)}{\partial p_{i0} (k)} \bigg|_{\frac{p_{i0}}{\bar{p}_{i0}}=1} = - \left( \frac{p_{i0}}{\bar{p}_0} \right)^{1-\gamma} \frac{\tilde{c}_0}{\bar{p}_0} [\gamma A (1) + f (1)]
\]

The derivative of future consumption with respect to \( p_{i0} (k) \) identifies an effect only through the extensive margin. This is because a change in price today will impact your market share in the future, but not the per-customer level of consumption (the intensive margin).

\[
\frac{\partial c_{it} (k)}{\partial p_{i0} (k)} \bigg|_{\frac{p_{i0}}{\bar{p}_{i0}}=1} = - \frac{f (1)}{p_{i0}} A (1) \left( \frac{p_{it}}{\bar{p}_t} \right)^{-\gamma} \tilde{c}_t
\]

Recall that \( \bar{p} = p_i A (1)^{1/\gamma} \), which yields

\[
\frac{\partial c_{it} (k)}{\partial p_{i0} (k)} \bigg|_{\frac{p_{i0}}{\bar{p}_{i0}}=1} = - \frac{f (1)}{p_{i0}} A (1)^{1/\gamma} \tilde{c}
\]

Plugging into our initial expression and using \( mc_{i0} = \kappa \) and \( p_{i0} = \kappa p_i \), we get

\[
\frac{\partial \Pi_{i0} (k)}{\partial p_{i0} (k)} \bigg|_{\frac{p_{i0}}{\bar{p}_{i0}}=1} = \mu_o A (1) \left( \frac{\kappa p_i}{\bar{p}_0} \right)^{-\gamma} \tilde{c}_0 - \mu_o \left( \frac{\kappa p_i}{\bar{p}_0} \right)^{-1} \tilde{c}_0 \left[ \gamma A (1) + f (1) \right] - \sum \beta^t \mu_t \frac{f (1)}{p_{i0}} A (1)^{1/\gamma} \tilde{c} [p_i - 1] = \left( \frac{\kappa p_i}{\bar{p}_0} \right)^{-\gamma} \mu_o \tilde{c}_0 \left[ A (1) - \frac{1}{\kappa p_i} [\gamma A (1) + f (1)] [\kappa p_i - \kappa] \right] - \sum \beta^t \mu_t \frac{f (1)}{k p_i} A (1)^{1/\gamma} \tilde{c} [p_i - 1]
\]

49
Next, we use the steady state expression for the price $p_i$ and re-arrange in order to simplify the equation.

\[
\frac{\partial \tilde{\Pi}_{i0}(k)}{\partial p_{i0}(k)}\bigg|_{p_{i0}(k)=\tilde{p}} = \left(\frac{\kappa p_i}{\tilde{p}_0}\right)^{-\gamma} \mu_0 \tilde{c}_0 \left[ A(1) - [\gamma A(1) + f(1)] [1 - p_i^{-1}] \right]
\]

\[- \sum_{t=1}^{\beta^t} \frac{\rho_{i0}(k)}{p_i} f(1) A(1)^{1+\gamma} \frac{\tilde{c}}{\kappa} [1 - p_i^{-1}] \]

\[- \beta \frac{\rho_{i0}(k)}{1 - \beta} \frac{\tilde{c}^{1-\sigma}}{\tilde{p}} f(1) A(1)^{1+\gamma} \frac{1}{\kappa} \left[ \frac{1}{\gamma + \frac{\beta f(1)}{1-\beta} + \frac{f(1)}{A(1)}} \right] \]

\[= \left[ \frac{\beta f(1) A(1)}{\gamma + \frac{\beta f(1)}{1-\beta} + \frac{f(1)}{A(1)}} \right] \left[ \left(\frac{\kappa p_i}{\tilde{p}_0}\right)^{-\gamma} \frac{\tilde{c}^{1-\sigma}}{\tilde{p}_0} - \frac{\kappa p_i A(1)}{\kappa^{\gamma-1}} \right] \]

where the last line made use of the definition of the aggregate price index $\tilde{p}$.

We now analyze some specific cases which are considered in the text. First, notice that if firms and households do not care about the future ($\beta = 0$), the derivative of the profit function with respect to the price around a full pass-through equilibrium is simply equal to 0. In other words, when the agents are not forward-looking, full pass-through is a sustainable symmetric equilibrium, in line with our previous results.

Second, if the shock at $t = 0$ is sector specific, the price index $\tilde{p}_0$ and the aggregate consumption remain constant as sectors are atomistic. By setting $\tilde{p}_0 = p_i A(1)^{1+\gamma}$ and $\tilde{c}_0 = \tilde{c}$ we obtain

\[
\frac{\partial \tilde{\Pi}_{i0}(k)}{\partial p_{i0}(k)}\bigg|_{p_{i0}(k)=\tilde{p}} = \left[ \frac{\beta f(1) A(1)}{\gamma + \frac{\beta f(1)}{1-\beta} + \frac{f(1)}{A(1)}} \right] \left[ \left(\frac{\kappa p_i}{p_i A(1)^{1+\gamma}}\right)^{-\gamma} \frac{\tilde{c}^{1-\sigma}}{p_i A(1)^{1+\gamma}} - \frac{\kappa p_i A(1)}{\kappa^{\gamma-1}} - 1 \right] \]

In this case, there is an incentive to deviate for any non-zero shock to the marginal cost ($\kappa \neq 1$). For example, if the marginal cost increases in period...
0 (\(\kappa > 1\)), the term in the last bracket becomes negative, indicating that a seller has an incentive to deviate from the full pass-through equilibrium by lowering its price. Therefore, under the scenario of a sector-specific shock, the symmetric equilibrium will be one where firms do not fully pass-through changes in their marginal cost.

Finally, we consider the case of an economy-wide shock hitting all sectors simultaneously. We know from our previous results that the aggregate price index can be replaced by 

\[ \tilde{p}_0 = p_{i0} A (1)^{1/\gamma} = \kappa p_i A (1)^{1/\gamma} \]

The implications for the symmetric equilibrium are clear. Only in the cases of log utility (\(\sigma = 1\)) and is there no incentive to deviate from a full pass-through equilibrium. However, when \(\sigma > 1\), there is a tendency to overshoot in the most likely case that aggregate consumption is a positive function of the productivity level. For example, a negative shock to productivity which induces \(\tilde{c}_0\) to fall below its steady state value \(\tilde{c}\) implies that the last bracket is positive. That is, despite a rise in the marginal cost faced by the firm, it actually wants to raise its price more than proportionately in order to maximize profits. In other words, there is more than complete pass-through.