Choice of Exchange Rate Regime in Developing Countries under Partial Labor Mobility

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VERY PRELIMINARY

Abstract
Beginning in the mid-1990s, a long string of economic crisis brought many emerging economies to their knees. A significant factor in many of these crisis – including Mexico (1994), Thailand and Indonesia (1997), and Argentina (2002) – was an unsustainable exchange rate regime characterized by limited to no flexibility. These crisis were all associated with significant suffering and social dislocation. In response to these crises, many economists examined ways in which developing economies could protect themselves from these fluctuations. Among the policies that attracted attention were those capable of reducing fluctuations in the real exchange rate (capital controls or currency inconvertibility as in China), or avoiding a buildup in imbalances that might lead to a crisis by introducing greater flexibility into the exchange rate regime. In this paper, we seek to use standard tools from the macroeconomics literature on the welfare costs of economic fluctuations first introduced by Lucas (1987) to gauge the welfare costs of terms-of-trade fluctuations with occasional crises. We then discuss how the choice of exchange rate regime might affect these costs.

Keywords: Underemployment; Subsistence; Terms-of-trade volatility; Partial labor mobility; Welfare costs

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1 Introduction

Beginning in the mid-1990s, a long string of economic crisis brought many emerging economies to their knees. A significant factor in many of these crises— including Mexico (1994), the Czech Republic (1997), the Asian crisis involving Thailand, Indonesia, Malaysia, the Philippines, and Korea (1997), Russia (1998), Brazil (1998), Turkey (2001), and Argentina (2002)— was an unsustainable exchange rate regime characterized by limited to no flexibility (see Corden, 2002). These crises were all associated with significant suffering and social dislocation. In response to these crises, many economists examined ways in which developing economies could protect themselves from these fluctuations. Among the policies that attracted attention were those capable of reducing fluctuations in the real exchange rate (capital controls or currency inconvertibility as in China), or avoiding a buildup in imbalances that might lead to a crisis by introducing greater flexibility into the exchange rate regime. In this paper, we seek to use standard tools from the macroeconomics literature on the welfare costs of economic fluctuations first introduced by Lucas (1987) to gauge the welfare costs of terms-of-trade fluctuations with occasional crises. We then discuss how the choice of exchange rate regime might affect these costs.

Previous work on fixed and flexible exchange rate regimes has found it difficult to generate any sizable difference in welfare costs stemming from choice of exchange rate regime, even with the introduction of sticky prices and other market imperfections. Our focus in this paper is on an entirely different mechanism: partial labor mobility. Using a
model of a developing economy, we show that when there are costs to relocating from the traditional to the modern sector the allocation of labor across sectors is distorted (that is, it differs from the allocation that prevails when there are no relocation costs). The degree of distortion varies with the choice of exchange rate regime.

There are several benefits to focusing on partial labor mobility. First, it allows us to introduce in a tractable way certain features of the structure of developing economies that are widely agreed to be essential, including sector misallocations of labor (underemployment) and subsistence consumption. It also lends itself to a tradable macroeconomic analysis of consumer welfare under uncertainty. Given that the model is highly stylized and does not include asset accumulation or well-defined institutions, we do not provide accurate estimates of the actual welfare costs faced by economies. Rather, we make use of a set of counterfactuals that we believe are useful in assessing the choice of exchange rate regime in developing countries.

Corden (2002) provides a recent overview of the long debate over exchange rate regimes, with careful attention to the evolution of thought in response to crises. In particular, the crises of the 1990s and early 2000s demonstrated that quasi-fixed exchange-rate regimes could function well for years while storing up seeds of destruction. The “sudden stop” phenomenon, in which a financial crisis emerges catastrophically, has received much attention as of late (see survey by Mendoza (2006)) and has led to many interesting insights into how different policy choices might render the economy vulnerable to different dangers. For example, Mendoza has been a chief proponent of the debt-deflation theory in
which access to vital foreign credit becomes restricted once indebtedness reaches a critical threshold. The creeping of the economy towards this threshold is often consistent with the appearance of stability and growth, yet the creeping is inevitable under a managed exchange rate regime. Once the threshold is hit, even a mild shock can trigger a crisis with a vicious negative feedback dynamic ensuing for both access to credit and exchange rate depreciation. This chain of causation is important for our story. Although still a topic of debate, consensus seems to now regard the choice of exchange rate as a fundamental determinant of financial crisis. Dedicating economic policy to pegging the exchange rate allows a host of imbalances within the financial and real sectors to develop until a crisis occurs.

We are interested in the welfare implications of the exchange-rate regime choice facing economies when such a sudden-stop dynamic is in place. We do not model the crisis mechanism, choosing instead to concentrate on the outcome for the distribution of resources, and the resulting implications for consumption volatility and welfare. While this approach makes our analysis tractable, we recognize that the severity of crises and the vulnerability of any given exchange rate regime to collapse is highly dependent on the specific mechanisms and institutions in place. We undertake our analysis using model-based simulations that employ parameters taken from the data and the literature. We find that plausible adjustments for subsistence and relocation costs lead to stark differences in the welfare outcomes for different choices of exchange rate regime.

Section 2 lays out some basic stylized facts of the exchange rate experiences of four
countries that have recently experienced crises. Section 3 describes our model, and Section 4 analyses the resulting migration dynamics. Section 5 describes the results of our simulations to determine the welfare costs of different choices of exchange rate regime, and Section 6 concludes.

2 Stylized Facts

As Corden (2002, p.18) writes, “A series of exchange rate crises in developing and transition countries began in 1994, each of the crises leading to an exit from a fixed-but-adjustable or crawling peg exchange rate regime.” In order to give a sense of the magnitudes of the crises that have been attributed to collapsing exchange rates, we have gathered select data in Tables 1 and 2 on four countries that experienced crises in which the exchange rate played a significant role. All of them moved from fixed/managed-peg arrangements prior to the crisis to a flexible arrangement afterwards. Table 1 describes changes in the current account/GDP ratio, industrial production, private consumption, and the unemployment rate. In Argentina, during the financial crisis beginning in January 2002, industrial production fell by 6.69 percent over two quarters, while private consumption fell by 6.71 percent. The open unemployment rate surged by 15.57 percent, a number widely believed to be vastly understated. In general, all four countries experienced a drop in industrial production of at least 6.5 percent, with the consequences for private consumption only slightly milder. Although unemployment data is hard to
come by, Thailand’s increase in its urban reported unemployment rate of 118 percent appears more in line with common perceptions. Clearly these crisis are deep, and the risks associated with them are (unsurprisingly) large.

In Table 2, we provide data on the exchange rate regimes in each of these countries. The first two columns show the trend and standard deviation of the stochastic process followed by the exchange rate during the fixed/managed-peg regime just prior to the crisis,

\[
\frac{de}{e} = \mu dt + \sigma dz.
\]

where \(e\) is the exchange rate measure, \(\mu\) is the (monthly) trend, \(\sigma\) is the (monthly) standard deviation, and \(dz\) is a standard Weiner increment. For each country, we would prefer to report both the nominal exchange rate and the (trade-weighted) real effect exchange rate (REER). Unfortunately, we do not yet have such a measure for Indonesia, and the closest we could come for Thailand is the terms-of-trade, whose interpretation is slightly different.

In general, all four countries fixed their currencies to the US dollar (using a crawling peg in the case of Indonesia) prior to their crises. Argentina’s and Mexico’s REER measures indicate appreciation in real terms in these years (as would Thailand’s and Indonesia’s if we had the correct measure), a feature widely noted to be ultimately incompatible with the pegs that gave rise to it. What is especially notable is the tremendous increase in the standard deviation of the exchange rate measures in the flexible (post-crisis) period, as well as the percentage change in the exchange rate occurring right around the crisis itself. This number in the final column measures the widest change in the exchange rate measure
occurring in the six-month period straddling the crisis. For our two REER measures, the depreciation shocks ranged from 76 to an extraordinary 172 percent.

The differences across exchange rate regimes are evident. On the one hand, the consequences of the crises precipitated by fixed/managed regimes would seem to be significant. However, it is also the case that volatility in relative prices (as evidenced by fluctuations in the REER) is much higher for flexible rate regimes. To the extent that resources are fully mobile and can relocate across sectors costlessly, welfare consequences can be minimized. When resources (in our case, labor) are only partially mobile, the economy is more vulnerable to shocks and the welfare costs will be much higher. We turn now to developing the framework that we will use to evaluate the extent of welfare consequences when labor is only partially mobile.

3 The Model

The model is of a dual economy with modern and traditional sectors where both sectors produce distinct home country goods using different technologies. A single factor, labor, can move between both sectors. Migration to the modern sector is costly, which introduces an options value to the migration process. As a result, given that structural change favors the modern sector, a (fluctuating) wage premium in the modern sector will evolve endogenously. Both the domestic goods are consumed, but the modern sector good will be exported whereas traditional goods consumption is subject to a subsistence require-
The production of traditional goods requires only labor, but the modern sector good also requires imported inputs in addition to labor.\footnote{Mendoza (2006) argues that working capital financing, presumably to buy imported inputs, is an important part of the “sudden stop” phenomenon.} Exogenous changes in the relative price of the imported intermediate good (and hence the relative price of tradables) lead to uncertainty. We calculate the welfare costs of these shocks, and the extent to which they depend upon underemployment (or more precisely the distortion generating underemployment) and, separately but subject to interaction, subsistence. There are no asset markets to provide savings or insurance instruments, a point we discuss below.

\section{Preferences and Setup}

There is a continuum of workers, with the measure of the entire set of workers in the economy normalized to unity (with each worker of measure zero). The set of workers is fixed over time. Each worker inelastically supplies one unit of indivisible labor and earns wage income. Workers’ preferences are defined over a composite consumption good ($C$) and are represented by an expected utility function. All workers have identical preferences.

Each worker can be viewed as a worker-production unit.

Time is continuous, $t \in [0, \infty)$. Each worker starts at time 0, and lives infinitely. Workers make sectoral employment choices subject to institutional hiring constraints in the modern sector. Specifically, each individual maximizes the following expected utility
function:

\[
E \left[ \int_0^\infty e^{-\rho t} \ln(C_t) \, dt - \sum_j e^{-\rho t_j} C \right],
\]

subject to:

\[
C_t = \left( (C_{Tt} - \gamma)^{\frac{\varepsilon - 1}{\varepsilon}} + C_{Mt}^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}},
\]

\[
I_t \geq P_{Tt} C_{Tt} + P_{Mt} C_{Mt}.
\]

E is the expectations operator, \( C_T \) is the consumption of the traditional home good with price \( P_T \), and \( C_M \) and \( P_M \) are, respectively, the consumption and the unit price of the modern home good. Earnings \( I \) are expressed in nominal terms, and depend solely on the wage rate in the sector in which the worker is employed; either \( W_T \) in the traditional sector or \( W_M \) in the modern sector. E is the conditional expectations operator. The interpretation of the parameters is as follows: \( \varepsilon > 0 \) is the elasticity of substitution between traditional and modern varieties of the home good, and \( 1 > \gamma > 0 \) represents subsistence consumption of the traditional home good. Given that we assume below that maximum output of the traditional good is 1, \( \gamma \) must be less than 1. We will use the traditional variety of the home good as the numeraire, and set \( P_T = 1 \).

The subsistence parameter \( \gamma \) represents a measure of the nation’s poverty. Although the impact of subsistence should weaken over time as economic growth occurs, we are most interested in the medium term impact of subsistence and poverty on the choice of the exchange rate regimes. As we will show below, subsistence is represented as a fixed amount of traditional output which places a limit (by design) on the overall extent of development.
in the economy. In this way, we can control the degree of poverty and development in our examination of the choice of exchange rate (e.g., industrialized countries will correspond to a value of $\gamma = 0$).

The second term in the utility function captures the fixed (and sunk) cost $c > 0$ of relocating from the traditional to the modern sector. We think of this relocation cost as a psychic cost for tractability, however, this cost is not incurred for movements from the modern to the traditional sector (a point we return to below). Given that (one-way) reallocation costs are nontrivial, job switches to the modern sector take place in discrete instances denoted by $t_j$. Relocation out of the modern sector is continuous. The future is discounted at a subjective discount rate, $0 < \rho < 1$. In all equations below, we follow the expositional style of Dixit and Pindyck (1996) and drop the individual and time indices for simplicity.

### 3.2 Demand

The following first-order condition characterizes the optimal resource allocation by each worker:

$$\frac{C_T - \gamma}{C_M} = P_M^e.$$  \hspace{1cm} (2)
3.3 Supply

A fraction $L$ of the labor force is employed in the modern sector, leaving $1 - L$ employed in the traditional sector. Both modern and traditional domestic goods use constant returns to scale technology

$$Y_M = zL^\alpha M^{1-\alpha}, \quad \text{and} \quad Y_T = 1 - L. \quad (3)$$

where $Y_M$ is modern sector output, $z$ is a productivity parameter, $\alpha$ is the share contribution of labor in output, $M$ represents intermediate goods, and $Y_T$ is traditional sector output. We thus assume that labor productivity in the traditional sector normalized to 1. For simplicity, we initially assume that foreign goods are used as the sole intermediate input.

3.4 Factor Markets

Factors of production are paid their marginal products. Thus,

$$w_T = P_T = 1, \quad (4)$$

$$w_M = P_M \alpha z \left( \frac{M}{L} \right)^{1-\alpha}. \quad (5)$$

Maximization also implies that the ratio of inputs in the modern sector is given by:

$$\frac{M}{L} = \left[ (1 - \alpha)z \frac{P_M}{P_F} \right]^\frac{1}{z}. \quad (6)$$
3.4.1 Balance of Payments and Market Clearing

Although the foreign good is not purchased and consumed directly by households, intermediate inputs are purchased from abroad. Only modern sector goods are exportable. For our initial calculations, we assume that trade must be balanced, which implies:

\[ P_M(Y_M - C_M) = P_F M. \]  

(7)

Clearly this assumption is at odds with the sizable changes in the current account that take place during crises. Without a more sophisticated treatment

As an alternative, we later introduce an ad hoc term to capture inflows (or outflows) of resources, \( F \),

\[ P_M(Y_M - C_M) + F = P_F M. \]

By matching changes in \( F \) to stylized patterns observed in the countries on which we focus, we can obtain a better sense of the impact of exchange rate crises.

In addition, since traditional goods are not exportable, market clearing in this sector is given by

\[ P_T Y_T = P_T C_T. \]
3.5 Intratemporal Equilibrium

As we show in the technical appendix, we make use of equations (2), (3), (6), and (7) to obtain,

\[
p = P_M = \frac{P_M}{P_T} = \left\{ \alpha (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{L}{1 - L - \gamma} \right) \right\}^{\frac{1}{\alpha(1-\epsilon)-1}} z^{\frac{1}{\alpha(1-\epsilon)-1}} P_F^{\frac{\alpha-1}{\alpha}}.
\]  

(8)

Also note that the wages in each sector are

\[
w_T = 1,
\]

\[
w_M = \alpha (1 - \alpha)^{\frac{1}{\alpha}} P_M^{\frac{1}{\alpha}} z^{\frac{1}{\alpha}} P_F^{\frac{\alpha-1}{\alpha}}.
\]

The relative wage is therefore given by:

\[
\omega = \frac{w_M}{w_T} = A(L)z^{\beta_1} P_F^{\beta_2},
\]

(9)

where \( \beta_1 = \frac{\alpha (1-\epsilon)}{\alpha(1-\epsilon)-1} \), \( \beta_2 = \frac{(1-\epsilon)(\alpha-1)}{\alpha(1-\epsilon)-1} \), and \( A(L) = \left[ \alpha (1 - \alpha)^{\frac{1}{\alpha}} \right]^{\beta_1} \left( \frac{L}{1 - L - \gamma} \right)^{\frac{1}{\alpha(1-\epsilon)-1}} \).

If labor is able to relocate without cost, the relative wage will equal unity, and we obtain,

\[
L^* = \frac{1 - \gamma}{1 + P_F^{z^z(1-\epsilon)(1-\alpha)} z^{1-\epsilon} \Psi},
\]

where \( \Psi = \alpha^{(1-\epsilon) \left( 1 - \alpha \right) \left( 1 - \alpha \right)^{(1-\epsilon)}}. \)

Clearly, the response of \( L \), the share of labor in the modern sector, to changes in relative productivity \( (z) \) and the price of imported intermediate goods \( (P_F) \) depends on
the elasticity of substitution between traditional and modern sector goods ($\varepsilon$). If, as we argue, the elasticity is high ($\varepsilon > 1$), then the share of labor in the modern sector will rise with increases in $z$ and fall with increases in $P_F$.

3.6 The Structural Change Process

Structural change is primarily driven by relative productivity growth. We will assume that the growth rate is non-stochastic,

$$\frac{dz}{z} = \mu_z dt.$$

3.7 Uncertainty

The home country is a small country that has no influence on the relative price of its output in the world. The relative price of the foreign good follows a jump process (Dixit and Pindyck, pp. 85–87):

$$\frac{dP_F}{P_F} = \mu_F dt + \sigma_F dx + dq.$$

where $\mu_F$ is the mean proportional growth rate of $P_F$, $\sigma_F$ is the standard deviation of the proportional growth process, $dx$ is a standard Weiner increment, and $dq$ is a Poisson increment described below. Let $S$ be the proportional rise in the foreign good price resulting from an exchange rate crisis, such that $dq$ can be represented by

$$dq = \begin{cases} 
0 \text{ with probability } (1 - \lambda)dt, \\
S \text{ with probability } \lambda dt.
\end{cases}$$

where $\lambda$ is the mean arrival rate of an event.
3.8 Exchange Rate Regimes

The terms $\sigma_F$ and $\lambda$ represent our key means of differentiating between exchange rate regimes. Our key guideposts here are Corden (2002) and Mendoza (2005). There are, of course, many different types of exchange rate regime, and countries might choose to be at any point along a frontier of flexibility and fixity. The consensus seems to be, however, that fully-flexible exchange rate regimes do not lead to sudden stops. In his variance analysis of the Mexican economy, Mendoza (2005) shows that whereas relative tradables price volatility explains the vast majority of real exchange rate fluctuations, relative nontradables price volatility explain most real exchange rate fluctuations under managed exchange rate regimes. Given that the value of collateral or ability to repay debt is tied to the relative price of nontradables, managed economies are much more subject to sudden stops. In addition, it is well-known that the variability of the real exchange rate is higher under flexible exchange rate regimes (e.g., Mussa, 1986; Baxter and Stockman, 1989), so under a fully-flexible exchange-rate regime implies that $\sigma_F$ reaches its maximum, but that $\lambda = 0$. A fixed exchange-rate regime implies that $\sigma_F$ is at its minimum, but that $\lambda$ reaches its maximum. The exact numbers we use for $\sigma_F$ and $\lambda$ (based on the data presented in Table 2) are given in Table 3 and are described more fully below.

A key point that emerges from our analysis is that fixing the exchange rate has immediate consequences for migration if people perceive a consequence to be an increase in the likelihood of crisis (a rise in $\lambda$). This will exacerbate the misallocation of labor between
sectors and create welfare costs even before any crisis occurs.

3.9 The Composite Process

Recall that we defined a composite variable, \( Q = z^{\beta_1} P_F^{\beta_2} \). Since we know the dynamic processes for both \( z \) and \( P_F \), we can use Ito’s Lemma to obtain the dynamic process for \( Q \) itself (see Result 2 in appendix A.4). This process is:

\[
\frac{dQ}{Q} = \left[ \beta_1 \mu_z + \beta_2 \mu_F + \frac{\beta_2 (\beta_2 - 1)}{2} \sigma_F^2 \right] dt + \beta_2 \sigma_F dx + \beta_2 dq. \tag{10}
\]

In our formulation, \( \beta_1 > 0 \), and \( \beta_2 < 0 \) unambiguously given our assumption that \( \varepsilon > 1 \).

4 Migration Dynamics

For workers that face a cost of migration, a comparison must be made between the wages in both sectors and the utility associated with those wages. From the utility function, it is apparent that instantaneous utility is: \( \ln(C_t) = \ln(w/P) \), where \( P \) is the overall price index, \( P = \left[ (1 - \gamma) + P_M^{1-\varepsilon} \right]^{1/\varepsilon} \). This implies that the utility-based difference in wages can be written as,

\[
\Delta(L, Q) = \ln \left( \frac{w_M}{P} \right) - \ln \left( \frac{w_T}{P} \right) = \ln \left( \frac{w_M}{w_T} \right) = \ln A(L) + \ln(Q).
\]

An increase in this ratio raises the incentive to relocate to the modern sector, and vice versa.

\(^2\)Note that the influence of subsistence is subsumed in the price index itself, which cancels out of our calculations.
When a migrant faces uncertainty (resulting in this case from stochastic fluctuations in the relative price of the foreign input) the migrant must weigh three things: the welfare gain of relocating, the (net) value of the option to wait before relocating, and the cost of relocating (expressed as a welfare cost). The welfare gain of relocating is

\[ U_\Delta(Q) = E \left[ \int_0^\infty e^{-\rho t} \Delta(L, Q) \, dt \right], \quad (11) \]

for an initial terms-of-trade, \( Q_0 \), and for a given \( L \) (since each worker takes this as a constant in competitive equilibrium). The dependence of \( U_\Delta \) on \( L \) is implicit.

The expected discounted value of \( \Delta \) is the solution \( U_\Delta(Q) \) to the following differential equation (see appendix):\(^3\)

\[
(r + \lambda)U_\Delta(Q) - \lambda U_\Delta \left[ (1 + S)^{\beta_2} Q \right] - \mu Q U_\Delta'(Q) - \frac{\sigma Q^2}{2} Q^2 U_\Delta''(Q) = \Delta(L, Q).
\]

Given an initial terms-of-trade \( (Q_0) \), this stochastic differential equation is solved analytically by the function \( U_\Delta \) (see appendix):

\[
U_\Delta(Q_0) = \frac{1}{\rho} \left[ \ln A(L) + \ln Q_0 \right] + \frac{1}{\rho^2} \left[ \mu Q - \frac{\sigma Q^2}{2} + \lambda \beta_2 \ln(1 + S) \right]. \quad (12)
\]

For a given value of \( L \), the expected discounted value of the wage gap between the manufacturing and traditional sectors increases in \( Q_0 \).

Second, consider the option value of waiting to relocate. Relocation means that the worker loses the option of postponing the move. Because there is a dynamic uncertainty

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\(^3\)This expression includes the term, \(-\lambda \{ U_\Delta(Q) - U_\Delta[(1 + S)^{\beta_2} Q] \}\), which is the probability of a sudden stop multiplied by the shock to utility caused by this event; in other words, the expected utility cost of a sudden stop.
about the wage gap, waiting has an option value. Thus, whenever a decision to relocate takes place, this option is lost. Of course, after relocating the worker acquires another option: that of returning to the original sector. The net value of these two options comprises a non-dividend paying “asset”, whose value depends purely on the revaluations (or “capital gains”) arising from the uncertain changes in $Q$.

We denote this asset’s utility value by $U_O(Q)$, which also depends on some fixed value of $L$. Over a time interval $dt$, the total expected return on the relocation opportunity ($\rho U_O$) is equal to the expected capital gain on the net options:

$$\rho U_O(Q) = \frac{1}{dt} E[dU_O(Q)].$$

Use Itô’s Lemma to expand the right hand side of this equation to obtain:

$$(\rho + \lambda)U_O(Q) - \lambda U_O \left[(1 + S)^{\beta_2}Q\right] - \mu_Q Q U'_O(Q) - \frac{\sigma_Q^2}{2} Q^2 U''_O(Q) = 0.$$

The solution to this equation is (Dixit and Pindyck 1996, pp.171):

$$U_O(Q) = K(L)Q^\epsilon$$

where $K$ is a constant to be determined, and $\epsilon > 0$ solves the nonlinear equation:

$$(\rho + \lambda) - \lambda(1 + S)^{\beta_2 \epsilon} - \mu_Q \epsilon - \frac{\sigma_Q^2}{2} \epsilon(\epsilon - 1) = 0. \quad (14)$$

We can solve for $\epsilon$ numerically using the equation above and the appropriate boundary conditions.\footnote{There is only one positive solution to $\epsilon$ for a plausible range of values based on our simulations.}
For workers in the traditional sector, increases in $Q$ will lead to an added incentive to relocate to the modern sector. In other words, the option “moves into the money” as its value increases. Thus, for the decision facing workers in the traditional sector, the positive solution, $\epsilon > 0$, is appropriate. Since there is no cost to relocating from the modern sector to the traditional sector, we do not need to worry about negative solutions for $\epsilon$.

Combining equations (12) and (13), we determine the total utility gains associated with relocation: $U_\Delta + U_O$:

$$U_\Delta + U_O = \frac{1}{\rho} \left[ \ln Q + \ln A(L) \right] + \frac{1}{\rho^2} \left[ \mu_Q - \frac{\sigma Q^2}{2} + \lambda \beta_2 \ln(1 + S) \right] + K(L)Q^\epsilon. \quad (15)$$

For any given $L$, the critical values of $Q$ that induce such a switch define relocation thresholds. These thresholds satisfy two optimality conditions known as “value-matching” and “smooth-pasting.” The value-matching condition suggests that the total benefits of relocating must exceed the cost, $c$, of doing so. The smooth-pasting condition prevents large discrete changes in the value of the net options (i.e., large capital gains) at the time of its exercise that rational agents would already have arbitrated away [cf. Dixit and Pindyck (1996)].

### 4.1 Relocation across Sectors

For traditional sector workers, the value-matching condition is that the value of relocation (including the net options value) must be at least equal to the cost of relocation:

$$\frac{1}{\rho} \left[ \ln Q + \ln A(L) \right] + \frac{1}{\rho^2} \left[ \mu_Q - \frac{\sigma Q^2}{2} + \lambda \beta_2 \ln(1 + S) \right] + K_1(L)Q^{\epsilon_1} = c. \quad (16)$$
The smooth-pasting condition, found by taking the derivative of the value-matching condition with respect to \( Q \), is:

\[
\frac{1}{\rho} \left[ \frac{1}{Q} \right] + \epsilon_1 K_1(L)Q^{(\epsilon_1 - 1)} = 0. \tag{17}
\]

Combining these two conditions, we solve for the threshold value of \( Q \) that triggers relocation from the traditional to the modern sector as:

\[
Q_M = \exp \left\{ \rho c + \frac{1}{\epsilon_1} - \ln A(L) - \frac{1}{\rho} \left[ \mu_Q - \frac{\sigma_Q^2}{2} + \lambda \beta_2 \ln(1 + S) \right] \right\}. \tag{18}
\]

Notice the role of sudden stops. Since \( \beta_2 \) is negative, a larger expected rise in the foreign good’s price, or increase in the probability of a sudden stop, will raise the threshold value of \( Q \) needed for relocation to the modern sector. In addition, the term \( A(L) \) contains the effect of subsistence. Higher absolute subsistence (or higher subsistence relative to per capita output in the traditional sector) reduces the term \( A(L) \), implying that the threshold value of \( Q \) rises with subsistence as well.

Substituting equation (18) in equation (9), we find that workers will migrate from the traditional sector to the modern sector only when the modern sector wage reaches a fixed percentage premium over the traditional wage given by:

\[
\ln w_M - \ln w_T = T. \tag{19}
\]

No such premium is needed for migration back to the traditional sector. Rather, when the relative modern sector wage falls below one, reverse migration to the traditional sector
is instantly triggered implying the threshold,

$$\ln w_M - \ln w_T = 0. \quad (20)$$

We use equations (9), (10), (19) and (20) to solve for equilibrium labor shares and wage rates when labor is partly mobile. Each instantaneous equilibrium can be calculated for (current) values of $L$ and $Q$. Within the zone of inaction (which exists for relative modern sector wages above one but below $T$), adjustment to shocks occurs continuously through changes in relative prices, but labor remains immobile when faced with an added incentive to migrate to the modern sector (which is, in fact, optimal). Migration to the traditional sector depends upon the relative modern sector wage falling below one. When a random fluctuation in the real exchange rate pushes the economy outside the zone of inaction, workers relocate. Relocation continues until the wage gap is reduced to a level consistent with the threshold. Intuitively, those who relocate (if any) and those who stay must have identical expected lifetime utility given their actions. Movers incur a fixed cost, and earn a higher wage at the destination relative to those who stay. Not all workers can relocate. This would depress destination sector wages below the level of wages they would have received had they not migrated. Hence, a no-arbitrage condition ensures that relocation comes to a halt when the impulse associated with relocation gently brings the wage gap back within the maximum (or minimum) sustainable level.
4.2 Gauging Underemployment

We define underemployment as any situation in which labor is employed in a sector paying wages below the highest wage in the economy. Since workers do not differ in skills or experience, wages should be equalized across sectors for maximum efficiency. Although we do not have an explicit mechanism to generate underemployment, our setup does lead to seemingly inefficient allocation of labor (although not necessarily inefficient from the perspective of the workers) and to wage differentials in favor of the modern sector, key stylized features of developing economies. Given this definition, we measure underemployment as follows:

\[ N_t = L_t^* - L_t, \]

\[ = \left[ \frac{1 - \gamma}{1 + P_{F,t}^{(\varepsilon-1)(1-\alpha)} z_t^{1-\varepsilon} \Psi} - L_t \right]. \]

4.3 A Sample Economy Simulation

In order to give a sense of the magnitudes involved, we generate a set of diagrams in Figure 1 that correspond to a sample economy under both a fixed and a flexible exchange rate. Specifically, we generate paths for the relative price of the foreign good, the percentage underemployment, the log relative modern sector wage, and the modern sector share of labor.

The parameter values we use to generate the sample path are given in Table 3. We proxy the elasticity of substitution between the traditional and modern sector goods with...
Mendoza’s (1995) estimate of 1.3 for LDC’s elasticity of substitution between nontraded and traded goods (based on our association of nontraded goods with traditional goods and traded goods with modern sector goods). The discount rate is set to a standard value of 0.04. The subsistence parameter $\gamma$ is our measure of the overall poverty of the country, and we use a value of 0.2 to indicate that 20 percent of a traditional sector worker’s wage is devoted to subsistence.

For the stochastic processes, we assume that there is no trend in the foreign good’s price under either the fixed or flexible regime. However, based on the information presented in Table 2 for Argentina’s REER, we use a relative foreign price variance of 0.12 for the flexible rate regime, and 0.02 for the fixed rate regime. We set the mean arrival rate of a crisis under the fixed rate regime, $\lambda$, equal to 0.008 implying that the expected time until a crisis is 10.4 years. When a crisis occurs, the mean expected jump in the relative foreign price (RER) is set to (a modest) 60 percent. Evidence indicates that shocks can be much greater. In Table 2, Argentina experienced an 172 percent change in its RER as a consequence of its financial crisis in 2002.

The share of labor in the modern sector is set to 0.4, and the trend in relative modern sector productivity is set to a rapid 0.07. The cost of relocating from the traditional sector to the modern sector is fixed by choosing a maximum sustainable wage gap (based on empirical observations of wage gaps in LDCs) and calibrating the cost of relocation to match it. In this case, we choose a maximum wage gap in favor of the modern sector of 7 percent.
In Figure 1, all horizontal axes are measured in months with a total horizon of 20 years. In the simulated paths illustrated here, the fixed rate regime experiences a crisis just shy of 10 years from month 0. In the upper left panel, we see that the relative price of the foreign good (our measure of the real exchange rate) increases sharply. Whereas the volatility of the RER for the fixed rate regime was comparatively low until the crisis, its volatility is equivalent to that of the flexible rate regime afterwards. The upper right panel illustrates changes in the degree of underemployment resulting from these two different paths, and the lower left panel does the same for the percentage premium in the modern sector wage. Notice that fluctuations are bounded by the maximum premium of 7 percent above and a minimum of 0 in keeping with our earlier discussion. The final panel in the lower right illustrates the phenomenon of partial labor mobility. Workers relocate only at specific times (corresponding to a triggering threshold being reached in the lower left panel). While the fixed rate regime has initially allocated more labor to the modern sector, this outcome is reversed post-crisis.

This sample path is meant to give a sense of the underlying dynamics in this model economy. However, to make any conclusions regarding the desirability of one regime over another, we must calculate the expected (average) aggregate welfare under each regime for different sets of parameter values. We turn to that exercise in the following section.
5 Income and Consumption Risk and Choice of Regime

In order to shed light on the desirability of each exchange rate regime from an aggregate welfare standpoint, we use simulations to figure out the average (ex-post) welfare in each sector given realizations of the stochastic-jump process for $dP_F$. Given our preference structure, welfare depends upon the real wage earned,

$$U = E \int_0^\infty e^{-\rho t} \ln \varpi(t) dt,$$

where $\varpi$ is the average real wage, i.e.,

$$\varpi = L \times \varpi_M + (1 - L) \times \varpi_T.$$

where,

$$\varpi_M = \frac{w_M}{P} = \frac{A(L)Q [(1 - \gamma) + P_M^{1-\epsilon}]}{L^{1-\epsilon}},$$

$$\varpi_T = \frac{w_T}{P} = \frac{A(L)Q \left( (1 - \gamma) + A(L)Q \left( \frac{L}{1 - L - \gamma} \right) \right)^{1-\epsilon} L^{1-\epsilon}}{L^{1-\epsilon}}.$$

Given the stochastic nature of the underlying process for $Q$, our methodology is to simulate the economy under both a fixed and flexible regime (but using the same starting point) and then to take the ratio of $U_{\text{Flex}}$ to $U_{\text{Fix}}$. After performing 500 such paired simulations for each unique set of parameter values, we take the average $U$ ratio and report it in Table 4. Values greater than one imply that the flexible regime is superior, while values less than one favor the fixed regime.

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In constructing Table 4, we were most interested in the effect of poverty (as proxied by subsistence, \( \gamma \)), the incidence of crisis for fixed rate regimes (\( \lambda \)), the amount of friction behind partial labor mobility (the cost of relocation), the importance of imported inputs in modern sector production (\( 1 - \alpha \)), and the size of the shock if a crisis occurs (\( S \)).

In general, the following patterns emerge. Poorer countries, as indicated by the prevalence of subsistence, have more to lose from adopting fixed exchange rate regimes. For example, in the case of a maximum wage gap of 7 percent, with \( S \) set to 1.20 and \( \lambda \) set to 0.02, the relative aggregate welfare of the flexible exchange rate regime rises from 1.092 for wealthier countries (for which \( \gamma = 0.05 \)) to 1.426 for poorer countries (for which \( \gamma = 0.60 \)). This pattern is robust.

As expected, the relative aggregate welfare of the flexible exchange rate regime increases as the mean arrival rate of a crisis rises and as the size of any potential crisis rises (as proxied by \( S \)). Reducing the share of foreign inputs into modern sector production reduces the advantage of the flexible regime mainly by reducing the exposure of the economy to international shocks and hence reducing the cost of making a poor choice. Increasing the maximum wage gap (that is, raising the cost of relocation) has no clear effect on the outcome. At this stage, we believe that this is because the even the 7 percent threshold is unlikely to reached, and labor immobility does not decrease by much in the move to 25 percent. Some recalibration is required.
6 Conclusion

This paper uses a stylized model to address the welfare consequences of the choice of fixed versus flexible exchange rate regimes for developing countries. In particular, we investigate the roles of subsistence and frictions to labor mobility across sectors in a two sector model. Our results indicate that the choice of a flexible rate regime provides the highest level of aggregate welfare, and that these relative welfare gains increase with the level of subsistence (poverty).
References


Table 1:  
SELECTED CRISIS DATA FROM FOUR COUNTRIES

<table>
<thead>
<tr>
<th>Quarterly</th>
<th>Current account/ GDP ratio (%) Change</th>
<th>Industrial production (%) Change</th>
<th>Private consumption (%) Change</th>
<th>Unemployment Rate (%) Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina (01:4-02:2)</td>
<td>NA</td>
<td>−6.69</td>
<td>−6.71&lt;sup&gt;a&lt;/sup&gt;</td>
<td>15.57&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Indonesia (97:3-98:1)</td>
<td>NA</td>
<td>−13.35&lt;sup&gt;c&lt;/sup&gt;</td>
<td>−3.32&lt;sup&gt;c&lt;/sup&gt;</td>
<td>NA</td>
</tr>
<tr>
<td>Mexico (94:4-95:1)</td>
<td>5.24</td>
<td>−9.52</td>
<td>−6.44</td>
<td>NA</td>
</tr>
<tr>
<td>Thailand (97:2-97:4)</td>
<td>238.36</td>
<td>−6.62&lt;sup&gt;d&lt;/sup&gt;</td>
<td>−10.26</td>
<td>118.15&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Note: Data for Argentina were obtained from the Argentine Ministry of Economy and Production: Secretariat of Economy Policy (www.mecon.gob.ar/peconomica/baschome/infoeco_ing.html).  
<sup>a</sup> uses the Monthly Industrial Indicator measure.  
<sup>b</sup> uses Total Urban Economically Active Persons estimate.  
<sup>c</sup> Indonesian non oil-gas manufacturing product and private consumption expenditure are the difference between 1997 and 1998 (annual levels) from National Income of Indonesia 1996-1999, BPS. Thai data are mainly from Bank of Thailand: (www.bot.or.th/bothomepage/databank/EconData/EconFinance/tab89e.asp).  
<sup>d</sup> uses the Thai manufacturing production index.  
<sup>e</sup> based on data from LABORSTA database, ILO.
Table 2: Selected Exchange Rate Data from Four Countries

<table>
<thead>
<tr>
<th>Monthly</th>
<th>Fixed/Managed Exchange Rate</th>
<th>Flexible Exchange Rate</th>
<th>% Change Post-Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μ</td>
<td>σ</td>
<td>μ</td>
</tr>
<tr>
<td>Argentina</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal (w/USD)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>REER(^a)</td>
<td>0.003</td>
<td>0.021</td>
<td>0.022</td>
</tr>
<tr>
<td>Indonesia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal (w/USD)</td>
<td>0.003</td>
<td>0.002</td>
<td>0.021</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal (w/USD)</td>
<td>0.003</td>
<td>0.011</td>
<td>0.009</td>
</tr>
<tr>
<td>REER(^b)</td>
<td>0.003</td>
<td>0.019</td>
<td>0.000</td>
</tr>
<tr>
<td>Thailand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal (w/USD)</td>
<td>0.000</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>TOT(^c)</td>
<td>0.002</td>
<td>0.018</td>
<td>−0.006</td>
</tr>
</tbody>
</table>

Note: The underlying frequency of the parameters is monthly. Data for Argentina were obtained from the Banco Central de la Republica Argentina (www.bcra.gov.ar). \(^a\) uses their reported multilateral real effective exchange rate measure. Mexican data are from Banco de Mexico (www.banxico.org.mx/siteBanxicoINGLES). \(^b\) uses their reported trade weighed real exchange rate measure. Indonesian data are from Bank Indonesia (www.bi.go.id). Thai data are from Bank of Thailand (http://www.bot.or.th/bothomepage/databank/EconData/). \(^c\) uses their reported estimate of the terms-of-trade using imputed import and export unit prices.
### Table 3: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Mnemonic</th>
<th>Flexible</th>
<th>Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution between traditional and modern goods</td>
<td>$\varepsilon$</td>
<td>1.30</td>
<td>1.30</td>
</tr>
<tr>
<td>Share of intermediate goods in modern sector production</td>
<td>$1 - \alpha$</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Subsistence parameter</td>
<td>$\gamma$</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Drift of relative price of imported intermediate goods</td>
<td>$\mu_F$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Volatility of relative price of imported intermediate goods</td>
<td>$\sigma_F$</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>Mean arrival rate of a TOT crisis</td>
<td>$\lambda$</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Percentage change in $P_F$ if crisis occurs</td>
<td>$S$</td>
<td>0.00</td>
<td>0.60</td>
</tr>
<tr>
<td>Trend in relative modern sector productivity</td>
<td>$\mu_Z$</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\rho$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Note: Drift, volatility and discount rate parameters are annualized values. Column labeled “Flexible” corresponds to our flexible exchange rate regime parameters, and “Fixed” corresponds to the fixed exchange rate benchmarks. See text for further discussion. \(^a\); based on the elasticity of substitution between traded and notraded goods for LDCs in Mendoza (1995).
Table 4: Relative Aggregate Welfare of Flexible Exchange Rate Regime under Partial Labor Mobility

<table>
<thead>
<tr>
<th>Degree of subsistence $\gamma$</th>
<th>Labor share $\alpha$</th>
<th>Prob. of Crisis $\lambda$</th>
<th>Max Wage Gap $(\ln w_M - \ln w_T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 0.05, \alpha = 0.4$</td>
<td>$\gamma = 0.30, \alpha = 0.4$</td>
<td>$\gamma = 0.60, \alpha = 0.4$</td>
</tr>
<tr>
<td>$\gamma = 0.07$</td>
<td>$S = 0.60$</td>
<td>$S = 1.20$</td>
<td>$S = 0.60$</td>
</tr>
<tr>
<td>$\lambda = 0.002$</td>
<td>1.021</td>
<td>1.030</td>
<td>1.023</td>
</tr>
<tr>
<td>$\lambda = 0.010$</td>
<td>1.049</td>
<td>1.079</td>
<td>1.048</td>
</tr>
<tr>
<td>$\lambda = 0.020$</td>
<td>1.055</td>
<td>1.092</td>
<td>1.056</td>
</tr>
<tr>
<td>$\gamma = 0.25$</td>
<td>$S = 0.60$</td>
<td>$S = 1.20$</td>
<td>$S = 0.60$</td>
</tr>
<tr>
<td>$\lambda = 0.002$</td>
<td>1.032</td>
<td>1.047</td>
<td>1.033</td>
</tr>
<tr>
<td>$\lambda = 0.010$</td>
<td>1.073</td>
<td>1.121</td>
<td>1.073</td>
</tr>
<tr>
<td>$\lambda = 0.020$</td>
<td>1.085</td>
<td>1.147</td>
<td>1.086</td>
</tr>
<tr>
<td>$\gamma = 0.3$</td>
<td>$\alpha = 0.7$</td>
<td>$\alpha = 0.7$</td>
<td>$\alpha = 0.7$</td>
</tr>
<tr>
<td>$\gamma = 0.07$</td>
<td>$S = 0.60$</td>
<td>$S = 1.20$</td>
<td>$S = 0.60$</td>
</tr>
<tr>
<td>$\lambda = 0.002$</td>
<td>1.015</td>
<td>1.022</td>
<td>1.014</td>
</tr>
<tr>
<td>$\lambda = 0.010$</td>
<td>1.033</td>
<td>1.054</td>
<td>1.032</td>
</tr>
<tr>
<td>$\lambda = 0.020$</td>
<td>1.040</td>
<td>1.065</td>
<td>1.039</td>
</tr>
</tbody>
</table>

Note: See text for discussion.
Figure 1: A Sample Path
Appendix

A Derivations

A.1 The manufacturing good’s price

To derive equation (8), we proceed as follows. Recall:

\[ C_M = [(1 - L) - \gamma] P_M^{-\alpha}, \]
\[ Y_M = z L^\alpha M^{1-\alpha}, \]
\[ M = \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{1/\alpha} L. \]

Insert these into equation (7):

\[ P_M (Y_M - C_M) = P_F M, \]
\[ P_M \left\{ z L^\alpha (1 - \alpha) z \frac{P_M}{P_F} \right\}^{(1-\alpha)/\alpha} L^{1-\alpha} - (1 - L - \gamma) P_M^{-\alpha} = P_F \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{1/\alpha} L, \]
\[ P_M \left\{ z L^\alpha \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{(1-\alpha)/\alpha} L^{1-\alpha} - (1 - L - \gamma) P_M^{-\alpha} \right\} = P_F \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{1/\alpha} L, \]
\[ P_M \left\{ z L^\alpha \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{(1-\alpha)/\alpha} L^{1-\alpha} - (1 - L - \gamma) P_M^{-\alpha} \right\} = \left[ z^{1/\alpha} P_F^{\alpha-1} \left( \frac{L}{1 - L - \gamma} \right) \right]^{\alpha/(1-\alpha)-1} \cdot \]

where: \( \Upsilon = \alpha(1 - \alpha)^{1-\alpha}. \)

A.2 The manufacturing wage

Use the following three equations:

\[ P_M = \left[ z^{1/\alpha} P_F^{\alpha-1} \left( \frac{L}{1 - L - \gamma} \right) \right]^{\alpha/(1-\alpha)-1}, \]
\[ w_M = P_M \alpha z \left( \frac{M}{L} \right)^{1-\alpha}, \]
\[ \frac{M}{L} = \left[ (1 - \alpha) z \frac{P_M}{P_F} \right]^{1/\alpha}. \]

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Substitute the third equation into the second to obtain:

\[ w_M = P_M(1 - \alpha)^{\frac{1-\alpha}{\alpha}} z^{\frac{1}{\alpha}} P_M^{\frac{1-\alpha}{\alpha}} P_{F, M}^{\frac{1}{\alpha}} \alpha z. \]

Now insert the manufacturing price:

\[ w_M = \left[ z^{\frac{1}{\alpha}} P_{F, M}^{\frac{1}{\alpha}} \left( \frac{L}{1 - L - \gamma} \right) \right]^{\frac{1}{\alpha}} z^{\frac{1}{\alpha}} P_{F, M}^{\frac{1}{\alpha}} \alpha z. \]

Let:

\[ \beta_1 = \frac{(1 - \varepsilon)}{\alpha(1 - \varepsilon) - 1}, \quad \text{and} \quad \beta_2 = \frac{(\alpha - 1)(1 - \varepsilon)}{\alpha(1 - \varepsilon) - 1}. \]

Thus, the manufacturing wage can be written as:

\[ w_M = \left( \frac{L}{1 - L - \gamma} \right)^{\beta_1 \alpha(1 - \varepsilon)} z^{\beta_2} P_{F, M}^{\beta_2}, \]

where \( A(L) = \left( \frac{L}{1 - L - \gamma} \right)^{\beta_1} \alpha^{\beta_1} (1 - \alpha)^{-\beta_2}. \)

A.3 Solving for \( U_\Delta \)

Note that the form of the differential equation is a hybrid of the discussion in Harrison and in Dixit and Pindyck (pp. 167–73; in particular, see their discussion of equation [39] in their text.). Our solution and its derivatives are given by:

\[ U_\Delta(Q) = \frac{1}{\rho} \left[ \ln A(L) + \ln Q \right] + \frac{1}{\rho^2} \left[ \mu - \frac{\sigma^2}{2} + \lambda \beta_2 \ln(1 + S) \right], \]

\[ U'_\Delta(Q) = \frac{1}{\rho Q}, \]

\[ U''_\Delta(Q) = -\frac{1}{\rho Q^2}; \]

\[ U_\Delta[(1 + S)^{\beta_2} Q] = \frac{1}{\rho} \left[ \ln A(L) + \beta_2 \ln(1 + S) + \ln Q \right] + \frac{1}{\rho^2} \left[ \mu - \frac{\sigma^2}{2} + \lambda \beta_2 \ln(1 - S) \right]. \]

Substituting these into the differential equation will deliver the desired result.
A.4 Conditional Covariances

In what follows, we make repeated use of the following two results.

**Result 1:** Let $dQ$ be given by equation (10), and $F(Q) = (a + bQ^{g})^{h}$. Then, using Itô’s Lemma, we have

$$\frac{dF}{F} = \mu_{F}(Q) dt + \sigma_{F}(Q) dz,$$

where

$$\mu_{F}(Q) = \Gamma(Q) \left\{ \mu + \frac{\sigma^{2}}{2} \left[ (g - 1) + \Gamma(Q) \left( 1 - \frac{1}{h} \right) \right] \right\},$$

$$\sigma_{F}(Q) = \Gamma(Q) \sigma,$$

$$\Gamma(Q) = \left( \frac{bQ^{g}}{a + bQ^{g}} \right) \cdot g \cdot h.$$

The instantaneous variance rate of the changes in $F$ is $[\sigma_{F}F]^{2}$, or alternatively the instantaneous variance of the growth rate of $F$ is $\sigma_{F}^{2}$.

**Result 2:** Let $dQ$ be given by equation (10), and $F(Q) = G(Q) \cdot H(Q)$, where $G$ and $H$ are both Itô processes with differentials

$$dG = \mu_{G}G dt + \sigma_{G}G dz,$$

$$dH = \mu_{H}H dt + \sigma_{H}H dz,$$

where both are built from a common standard Brownian motion $z$. Thus, using the “chain rule” (Harrison 1985, p.72), we have

$$dF = GdH + HdG + (dG)(dH),$$

where $(dG)(dH) = \sigma_{G}\sigma_{H}F dt$. After substituting the expressions for $dG$ and $dH$, we obtain

$$\frac{dF}{F} = [\mu_{G} + \mu_{H} + \sigma_{G}\sigma_{H}] dt + [\sigma_{G} + \sigma_{H}] dz.$$

We now turn to the calculation of instantaneous variances and covariances.

A.4.1 Overall price level

The overall price level index is given by:

$$P = \left[ 1 + \rho_{M}^{1-\varepsilon} \right]^{1/(1-\varepsilon)},$$

$$= \left[ 1 + \Omega Q \right]^{1/(1-\varepsilon)}.$$
where $\Omega = XX$. Making use of Result 1 above, we find that the stochastic process for $P$ is given by,

$$\frac{dP}{P} = \mu_P dt + \sigma_P dz,$$

$$\mu_P = \Gamma_P(Q) \left[ \mu_Q + \varepsilon \frac{\sigma_Q^2}{2} \Gamma_P(Q) \right],$$

$$\sigma_P = \Gamma_P(Q) \sigma_Q,$$

$$\Gamma_P(Q) = \left[ \frac{1}{1 - \varepsilon} \right] \left[ \frac{\Omega Q}{1 + \Omega Q} \right].$$

### A.4.2 Traditional sector real wage

The traditional sector real wage is given by,

$$\varpi_T = \frac{1}{P} = \left[ 1 + \Omega Q \right]^{1/(\varepsilon - 1)}.$$  

Using Result 1, we find:

$$\frac{d\varpi_T}{\varpi_T} = \mu_{\varpi_T} dt + \sigma_{\varpi_T} dz,$$

$$\mu_{\varpi_T} = \Gamma_{\varpi_T}(Q) \left[ \mu_Q + (2 - \varepsilon) \frac{\sigma_Q^2}{2} \Gamma_{\varpi_T}(Q) \right],$$

$$\sigma_{\varpi_T} = \Gamma_{\varpi_T}(Q) \sigma_Q,$$

$$\Gamma_{\varpi_T}(Q) = \left[ \frac{1}{\varepsilon - 1} \right] \left[ \frac{\Omega Q}{1 + \Omega Q} \right].$$

### A.4.3 Modern sector real wage

The modern sector real wage is given by:

$$\frac{w_M}{P} = P_M \left[ 1 + P_M^{1-\varepsilon} \right]^{1/(\varepsilon - 1)},$$

$$= \left[ 1 + P_M^{\varepsilon-1} \right]^{1/(\varepsilon - 1)},$$

$$= \left[ 1 + \Omega^{-1} Q^{-1} \right]^{1/(\varepsilon - 1)}.$$

### A.4.4 Real exchange rate

The real exchange rate

### B Calculating the labor supply at the threshold

Whenever the relative modern sector wage exceeds the percentage threshold that triggers relocation to the modern sector, relocation to the modern sector will take place in such
a way that the relative wage returns back just inside of the threshold. We can calculate exactly what the labor supply in the modern sector will be just after such an event. Note that our threshold condition is:

\[ \ln w_M - \ln w_T = T, \]
\[ \ln w_M = T. \]

where the last line follows from the fact that \( w_T \) is normalized to one. Note that this implies:

\[ \ln A(L) + \ln Q = T, \]
\[ \ln A(L) = T - \ln Q, \]
\[ A(L) = \exp(T - \ln Q), \]
\[ \left[ \frac{L}{1 - L - \gamma} \right]^{1/(\alpha(1-\varepsilon)-1)} = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{-\beta_1} \exp(T - \ln Q), \]
\[ \frac{L}{1 - L - \gamma} = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{\beta_1(1-\alpha(1-\varepsilon))} [\exp(T - \ln Q)]^{\alpha(1-\varepsilon)-1}. \]

Set \( \Lambda = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{\beta_1(1-\alpha(1-\varepsilon))} [\exp(T - \ln Q)]^{\alpha(1-\varepsilon)-1} \). This allows us to write:

\[ \frac{L}{1 - L - \gamma} = \Lambda, \]
\[ L(1 + \Lambda) = (1 - \gamma)\Lambda, \]
\[ L = (1 - \gamma) \left[ \frac{\Lambda}{1 + \Lambda} \right]. \]

**B.1 Reverse Migration to the Traditional Sector**

The relative modern sector wage is essentially bounded below by one. Any fall in the relative wage below one will trigger instantaneous migration out of the modern sector and into the traditional sector. This implies that the threshold condition for reverse migration is:

\[ \ln w_M - \ln w_T = 0, \]
\[ \ln w_M = 0, \]
\[ \ln A(L) = -\ln Q, \]
\[ A(L) = \exp(-\ln Q), \]
\[ \left[ \frac{L}{1 - L - \gamma} \right]^{1/(\alpha(1-\varepsilon)-1)} = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{-\beta_1} \exp(-\ln Q), \]
\[ \frac{L}{1 - L - \gamma} = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{\beta_1(1-\alpha(1-\varepsilon))} [\exp(-\ln Q)]^{\alpha(1-\varepsilon)-1}. \]
Setting $\Theta = [\alpha(1 - \alpha)^{(1-\alpha)/\alpha}]^{\beta(1-\alpha(1-\epsilon))} [\exp(-\ln Q)]^{(1-\epsilon)-1}$, we obtain:

$$L = (1 - \gamma) \left[ \frac{\Theta}{1 + \Theta} \right].$$