ECONOMICS OF PUBLIC GOOD PROVISION: AUDITING, OUTSOURCING AND BRIBERY

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Abstract. The paper investigates the choice of government to audit or outsource the provision of a public good in the presence of a potential hidden bribe and information asymmetries. An audit mechanism is developed for characterizing the provision of the public good in the public sector given asymmetric information about the cost of production between the bureau (i.e., producer) and the department (funding source). Outsourcing is represented by Nash bargaining between the government department and a monopoly firm over the price of the public good, including a hidden bribe. The study also relaxes the assumption of a single-tier government underlying the Samuelson rule. The key findings are as follows: For small scale economies, bribery is predicted to be aimed at having an outsourcing contract persist even when public sector provision of the public good dominates (based on welfare and unit costs measurements). On the other hand, bribery in large scale economies is predicted to be aimed at motivating outsourcing when the provision of the public good through public sector production is in the public interest. The findings also predict that a reduction in the department’s corruptibility may result in increased prices as the firm acts to induce the department to maintain its choice of outsourcing over the public interest for auditing when the firm’s bargaining power is sufficiently low and the department’s preference towards bribes is relatively high.

Date: May 18, 2006.

Key words and phrases. Public good, audit, outsource, bribery

I would like to thank Neil Bruce for facilitating my sabbatical term at the University of Washington where much of this paper was written. I would also like to thank Fahad Khalil, Jacques Lawarree, and the other participants of the Economics Seminar Series at the University of Washington for their insightful comments. I am also grateful to Gregg Harbaugh for his assistance with Mathematica.
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1. Introduction

This paper investigates the economic implications of bribes on the choice of government to audit or outsource the provision of a public good. This study is motivated by the rising role of the private sector as an alternative to the provision of public goods and concerns about the economic cost of corruption involved in government outsourcing ventures. Samuelson’s Rule characterizes the optimal provision of a public good under assumptions of a benevolent provider who knows all the relevant information about the cost of production, the consumers’ willingness-to-pay, and a unified or single-tier governmental institutional arrangement (Mortimont, 2005). Fearon and Busch (2005) have presented an optimal mechanism for mediating informational rents given information asymmetries regarding the cost of production in a hierarchal bureaucracy. However, the analysis was not particular to a public good such that there remains scope for a more formal relaxation of the assumption of a single-tier governmental institution concerning the Samuelson Rule.

The economic implications of corruption have been notably highlighted by Shleifer and Vishny (1993). More recently, Dreher and Herzfeld (2005) have outlined the international effort aimed at addressing corruption, including the World Bank Anti-corruption Strategy of 1997, the OECD Convention on Bribery of Foreign Public Official in International Business in 1997, and the United Nations Anti-corruption Treaty in 2003. For instance, the World Bank has withheld the transfer of funding to a number of international projects and the Canadian 2006 federal election outcome has been reportedly influenced by concerns about corruption. Types of corruption have been categorized as petty corruption (e.g., theft of government funds, including kick-backs) and large scale corruption (e.g., establishing a monopoly with direct or indirect interests) by Rauch (2001); as grand corporation (bribes to a government official to gain improved business deal or position such as under conditions of privatization) by Glaeser (2001); and/or as grease money (e.g., bribery aimed at facilitating preferential access to government services) by Clarke and Xu (2004).
review of the transparency index suggests that corruption occurs at various scales of economic activity. Sheifer and Vishny (1994) suggest that the cost of corruption is directly related to the number of layers of government bribed within a country and the secrecy implicitly necessary in these activities. Khalil and Lawarree (2005) using a principal-agent framework suggest that auditing is only optimal if cheating occurs in equilibrium and cheating can be mitigated by increasing the costs of collusion through the utilization of internal and exterior auditors. The formal inclusion of auditing, outsourcing and bribery within a single economic framework concerning the provision of a public good remains a gap in the literature.

Rauch (2001) develops an hierarchal overlapping generation model consisting of a chief and a large number of deputies with monitoring, corruption, and internal promotional opportunities. The chief or head of the department is a Stakelberg leader and the deputies are followers. Internal promotional opportunities are shown to assist in mitigating corruption by having the decision making positions occupied by individuals with high integrity. In comparison to the Stakelberg model used by Rauch (2001), Fearon and Busch (2005) use an optimal auditing mechanism to characterize the institutional arrangement within the public sector and show that the government can reduce informational rents by switching between costly auditing of a public sector bureau and outsourcing to firms engaged in second price auction in the provision of a public sector good. However, the Fearon and Busch (2005) framework has yet to be applied to the provision of a public good.¹

The research questions addressed in this study are as follows: First, how is the Samuelson’s Rule modified by the presence of asymmetric information and auditing within a hierarchical government framework? Second, under what conditions can a monopoly firm be anticipated to offer a government department a bribe aimed at influencing the decision of outsourcing by government? Third, how does the monopoly firm bribing of a government department affect cost, output and the government’s choice whether or not to outsource the provision of the public good?

¹Fearon and Busch (2005) framework applies to a public sector good that may have private or mixed good characteristics as opposed to a public good. A public good is considered to be non-rivalous, non-excludable and, for pure-public goods, non-congestible.
These research questions are addressed through the development of a theoretical economic framework involving a government department, bureau and a monopoly firm engaged in the provision of a public good. The department is assumed to care about the public and private goods and, inversely proportional to its integrity, the department also cares about a hidden bribe. The department chooses to finance the public good produced in the public sector through an audit mechanism or in the private sector by a monopoly firm.

The key findings of the paper are as follows: First, a modified Samuelson Rule is characterized involving the marginal cost of auditing and the expected penalty from the bureau misreporting its costs. Bribery is found to distort the level of the public good provided and social welfare by inducing the department to choose outsourcing over auditing in opposition to the public interest. Second, a reduction in the department’s corruptibility may result in prices increasing as the firm acts to induce the department to maintain its choice of outsourcing over the public interest for auditing when the firm’s bargaining power is sufficiently low and the department’s preference towards bribes is relatively high. Correspondingly, an increase in the firm’s bargaining power may support its ability to resist the demands for bribes by the department or profit share allocated towards bribing the department resulting in the price of the public good declining in response to increased bargaining power of the firm. Third, a combination of reduction in the corruptibility, improvement in public sector governance (e.g., penalty for the bureau misreporting costs), and a reduction in the benefits from government-firm collusive behavior is predicted to be necessary to combat corruption. These results are consistent with the predictions of Khalil and Lawaree (2005).

The paper is organized into 3 sections. In section 1, the introduction is provided. In section 2, the theoretical model is described, the equilibrium outcomes characterized, and comparative statics as well as an application is provided. In section 3, the conclusions are presented.

2. THE MODEL

Consider an economy consisting of $N$ individuals with preferences over a public good, $G \in \mathbb{R}_+$, and a private good, $x_i \in \mathbb{R}^N_+$, represented by $u_i(x_i, G)$ given $i \in \{1, 2, \ldots, N\}$ and per capita income $I_i$. The government consists of a funding department and a bureau. The department provides
a transfer to the bureau that possesses the technology to produce the public good and covers the

cost associated with auditing the bureau’s reported costs. The bureau is assumed to have private

information about the cost of producing the public good. The department can choose to have the

public good produced in the public sector or outsourced to a private firm.

The department is assumed to have a quasi-linear utility function, caring about $G$ and $x_i$ in a

fashion consistent with a Benthamite social welfare function and caring about a bribe in quantity

$B$ such that preferences are represented as $\sum_{i=1}^{N} u_i(x_i, G) + \theta B$. $\theta$ can be considered to be the

sensitivity of the department to a bribe (i.e., a metric for corruptibility) and, inversely, the level of

integrity of the department. When $\theta = 0$, the department acts as a benevolent social planner in the
determination of the level of the public good and private goods in the economy. The price of each

$x_i$ is known and given by $p_x > 0$. The public good is produced either by a bureau or a firm utilizing

a constant marginal cost technology. It is assumed that the bureau’s marginal cost $c \in [c_l, c_r], c_r > 0,$
is distributed according to a density $f(c)$. The actual marginal cost is private information of the
bureau, while the department knows only the distribution of marginal cost. As in Fearon and
Busch (2006), the bureau is an extended Niskanen bureau that cares about its discretionary budget
in excess of its production costs. Given the reported cost $w$, let $t(w)$ denote the budget transferred
to the bureau by the department, $G$ be the actual level of production of the public good, and
the bureau’s preferences be represented by $v(w, G) = t(w) - cG$. The bureau’s reservation payoff is
normalized at zero. The bureau chooses the cost of the public good to report, $w$, given its private
observation of $c$ and transfer from the department, $t(w)$.

The department possesses an audit technology that provides an unbiased signal, $\hat{c}$, of the true
cost. The audit technology is costly resulting in the department facing a choice between the level
of the public good to provide and the audit level aimed at mitigating some of the informational
rents potentially earned by the bureau. A principal-agent model based on the mechanism design
approach of Fearon and Busch (2005) is used to represent the department and bureau’s problem.

The department can choose to have the public good provided by an outside firm (i.e., outsourcing)
as opposed to being produced by the bureau. The department and firm bargain over the price of
the public good given an anticipated demand schedule for the public good. The bargaining process is represented by a variable-threat Nash bargaining game (Fearon, 2001). It is assumed that the negotiation between the department and firm is subsequent to an un-modeled request-for-proposal (RFP) aimed at identifying the short-list of potential firms that possess the necessary technology and are willing to engage in the bargaining process for the provision of the good. It is assumed that the RFP process results in only a single firm being identified due to considerations such as: confidentiality, patent protection and/or strategic technology regulation (e.g., vaccine production, space exploration, and military or security application).

The monopoly firm is assumed to maximize profit with the marginal cost of production given by $c_f \in [c, \bar{c}]$ and density $f(c_f)$. $c_f$ is independent of the bureau’s costs $c$.\(^2\) During the bargaining process, the firm can choose to offer the department a bribe, $B \geq 0$ which is private information to the department and firm. The bribe is aimed at influencing the department’s decision in favour of outsourcing.

The department’s production choice has two alternatives for the provision of the public good is denoted by $a$. It can either design an auditing mechanism for the bureau (denoted by $a = A$) or the department can outsource the provision of the public good (denoted by $a = C$). The bureau is assumed not to offer the department a bribe which implies that $B = 0$ when $a = A$. Furthermore, it is assumed that government regulation prohibits the department from making transfers to the bureau without oversight.\(^3\)

The government’s problem concerning the provision of the public good is represented as a sequential game. The game is as follows: At stage I, the department chooses to have the public good provided through an audit mechanism ($a = A$) or by outsourcing ($a = C$). If the department chooses $a = A$, then the department designs a Fearon-Busch audit mechanism for the provision of the public good. If $a = C$, then the department and the monopoly firm Nash bargain over the

\(^2\)An alternative cost distribution is presented in Garvie and Ware (1996) investigation of the regulation of a mixed (public and private) market oligopoly with private, but correlated, information about production costs.

\(^3\)The oversight assumption reflects contemporary constructs regarding public sector governance and the political fallout from the allocation of public funds in the absence of monitoring/auditing. Clearly, this assumption could be relaxed without materially altering the results.
price for the provision of the public good, denoted by $p_f$, and a hidden bribe to the department in quantity $B \geq 0$.

2.1. Public Good Provision. The institutional arrangement involves auditing the bureau or outsourcing the provision of the public good (i.e., $a \in \{A, C\}$). The subgame perfect equilibrium choices and expected payoffs for each of these strategies is presented below.

2.1.1. Auditing subgame. For auditing ($a = A$), the supervisor possesses an auditing technology that generates a signal of the true costs. This signal $\hat{c}$ is distributed on $[c, \pi]$ and is conditional on the actual value of the bureau’s cost realization by the density $h(\hat{c}|c)$. The signal is assumed to be unbiased such that $\int_c^\pi \hat{c} h(\hat{c}|c)d\hat{c} = c$. It is also assumed that the conditional distribution satisfies the monotone likelihood ratio property, or first order stochastic dominance. Let $H(\hat{c}|c)$ denote the (cumulative) distribution function and it is assumed that $H(\hat{c}|c) < H(\hat{c}|c') \forall c$ if $c > c'$. Let $q$ denote the probability of an audit occurring with costs defined by $m(q)$ and the fixed cost of auditing is $m(0) = m_0$, where: $m_0 > 0$. As in Fearon and Busch (2006), the following is assumed:

Assumption A1: Audit costs $m(q)$ satisfy: (i) $m(0) > 0$; (ii) $m'(q) > 0$, $m''(q) > 0$ for $q \in (0,1)$; (iii) $m'(0) = 0$; and (iv) $\lim_{q \to 1} m(q) = \lim_{q \to 1} m'(q) = \infty$.

Auditing is treated as a principal-agent problem and addressed in the usual mechanism design approach (Fearon and Busch, 2006). The department is the principal and receives a report from the bureau, denoted by $w$. Based on these reports, the principal chooses an audit contract consisting of a transfer from the department to the bureau, $t(w)$, the level of good provided by the bureau, $G(w)$, and an auditing intensity $q(w) \in [0,1]$ and the levels of the private good, $x_i(w)$, as functions of the bureau’s reported cost ($w$). Auditing returns a signal represented by $\hat{c}$. The bureau is penalized in the case where the audit returns a cost signal below reported cost. It is assumed that the penalty function is exogenous, proportional to the misreporting of costs, and does not reward the bureau. It follows that the bureau is subject to a penalty of $\max[0, p(w - \hat{c})]$ if assigned, where $\hat{c}$ is the cost signal, and $p$ is an exogenous parameter consistent with Fearon and Busch (2006). The penalty is assumed to be paid into the consolidated revenue fund of the government and is
not available to the department for altering the level of the public good or being reimbursed to individual consumers.

The bureau’s expected payoff from stating a cost of \( w \) when its true cost is \( c \) is

\[
v(w; c) = (t(w) - c \cdot G(w)) - q(w) \mathbb{E}_c[\max[0, p(w - c)]]|c|
\]

\[
= t(w) - c \cdot G(w) - q(w)p \int_{\xi}^c H(\hat{c}|c) d\hat{c}
\]

Both \( v(w; c) \) and \( \partial v(w; c)/\partial w \) are continuous at \( w = c \). By the revelation principle, attention can be restricted to truth-telling mechanisms (see appendix A). The participation constraint for the bureau is given by

\[
(1) \quad v(c, c) = t(c) - c \cdot G(c) - q(c)p \int_{\xi}^c H(\hat{c}|c) d\hat{c} \geq 0.
\]

Global incentive compatibility requires that

\[
(2) \quad v(c; c) \geq v(w; c) \quad \forall w, c \in [c, \pi].
\]

A local incentive compatibility approach is used, as shown in appendix A (Jewitt, 1988). This confines attention to the first derivative of the bureau’s value function \( v(c) \), denoted by \( \dot{v}(c) \), which is given by

\[
(3) \quad \dot{v}(c) = -G(c) - q(c)p \int_{\xi}^c \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c}.
\]

Specifically, the bureau’s utility \( v(\cdot) \) is considered to be the state variable, and the control variables are \( t(w), G(w), q(w), \) and \( x_i(w) \). Truth-telling together with (2) implies:

\[
(4) \quad t(c) = v(c) + c \cdot G(c) + q(c)p \int_{\xi}^c H(\hat{c}|c)d\hat{c}.
\]

The department has a balanced budget constraint given by \( \sum_{i=1}^N p_x x_i + t(w) + m(q) \leq \sum_{i=1}^N I_i \).

Given truth telling and substituting \( t(c) \), the constraint becomes \( \sum_{i=1}^N [I_i - p_x x_i] - m(q(c)) - v(c) - G(c)c - q(c)p \int_{\xi}^c H(\hat{c}|c)d\hat{c} \geq 0 \). Hence, the department maximizes its expected payoff by choosing a contract \( (G(c, p, I), x_i(c, p, I), q(c, p, I)) \) to offer to the bureau for any reported cost \( c \) given the bureau’s participation constraint (1) and the incentive compatibility constraint (3). Optimal control theory is used to solve this problem defined by the control variables \( (G(c, p, I), x_i(c, p, I), q(c, p, I)) \) and state
variable \{v(c)\}. The department’s problem is:

\[
\begin{align*}
\text{max}_{(G(\cdot), x_i(\cdot), q(\cdot))} & \left\{ \sum_{i=1}^{N} \left[ \int_{\xi}^{c} u_i(x_i(s), G(s)) f(s) ds \right] \right\} \\
\text{s.t.} & \quad (i) \quad \dot{v}(c) = -G(c) - q(c)p \int_{\xi}^{c} \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c} \\
& \quad \sum_{i=1}^{N} \left[ I_i - p_x x_i(c) \right] - v(c) - G(c)c - q(c)p \int_{\xi}^{c} H(\hat{c}|c)d\hat{c} - m(q(c)) \geq 0 \\
\end{align*}
\]

As appendix A shows, the optimal contract \((G(c, p, I), x_i(c, p, I), q(c, p, I))\) offered by the department to the bureau is characterized by:

\[
\begin{align*}
\sum_{i=1}^{N} \frac{\partial u_i(x_i(\cdot), G(\cdot))}{\partial x_i(\cdot)} & \frac{\partial G(\cdot)}{\partial x(\cdot)} = c + \frac{\left( m'(q(\cdot)) + \frac{p \int_{\xi}^{c} \partial H(\hat{c}|c)d\hat{c}}{\int_{\xi}^{c} \partial G(\cdot)|c} \right) \cdot \frac{1}{p_x}}{p} \\
m'(q(\cdot)) & = -p \left\{ \left( \frac{cF(c)}{c f(c) - F(c)} \right) \int_{\xi}^{c} \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c} - \int_{\xi}^{c} H(\hat{c}|c)d\hat{c} \right\} \\
\sum_{i=1}^{N} \left[ I_i - p_x x_i \right] - m(q(c, p, I)) - v(c, p, I) - G(c, p, I)c - q(c, p, I)p \int_{\xi}^{c} H(\hat{c}|c)d\hat{c} & = 0 \\
\text{where:} \quad MRS_i = \frac{\partial u_i(x_i(\cdot), G(\cdot))}{\partial x_i(\cdot)}, G(\cdot) \text{ / } \frac{\partial u_i(x_i(\cdot), G(\cdot))}{\partial x(\cdot)}. \text{ Let } I_i = I \text{ for simplicity in what follows. The audit level characterized in equation 7 is identical to the level determined in Fearon and Busch (2006).}
\end{align*}
\]

Using the Fearon and Bush (2006) approach, \(G(c)\) is solved using equation (8) and substituted into the dynamic transition equation \((i)\) resulting in \(\dot{v}(c) - v(c)\) since \(\partial [v(c)/c] \partial c = \dot{v}(c) - v(c)\) (see appendix A). It follows that

\[
\begin{align*}
v(c, p, I) & = cK(I) - c \int_{\xi}^{c} \frac{1}{s^2} \left\{ \sum_{i=1}^{N} \left[ I_i - p_x x_i \right] - m(q(c, p, I)) + q(c, p, I)p \int_{\xi}^{c} H(\hat{c}|c)d\hat{c} \right\} ds \\
\text{where:} \quad K(I) & = \int_{\xi}^{c} \frac{1}{s^2} \left\{ \sum_{i=1}^{N} \left[ I_i - p_x x_i \right] - m(\tau, p, I)) + q(\tau, p, I)p \int_{\xi}^{c} H(\hat{c}|c)d\hat{c} \right\} ds \quad \text{as } v(\tau, p, I) = 0 \\
\text{at } c = \tau. \text{ It can be noted that } v(c) \text{ is the bureau’s payoff relating to the informational rents}
\end{align*}
\]
earned through the exploration of the asymmetric information about its costs, \( c \). This informational rent is maximal when \( c = \zeta \), earning the bureau \( v(\zeta, p, I) = \zeta K(I) > 0 \) and at its minimum when \( c = \tau \) with \( v(\tau, p, I) = 0 \). The expected transfer from the department to the bureau is

\[
E_c[t(c, p, I)] = \int_\zeta^\tau \left\{ v(c, p, I) + c \cdot G(c, p, I) + q(c, p, I)p \int_\zeta^c H(\hat{c}|c)dc \right\} dc
\]

which includes a truth telling compensation given by \( v(c, p, I) \); the direct expenditures on the public good denoted by \( c \cdot G(c, p, I) \); and the expected penalty or budget loss that the bureau can be expected to pay from reporting \( w > c \) given by \( q(c, p, I)p \int_\zeta^c H(\hat{c}|c)dc \).\(^4\)

The department’s budget constraints can therefore be written as

\[
\sum_{i=1}^N p_i x_i(c, p, I) + cG(c, p, I) = \sum_{i=1}^N I - v(c, p, I) - q(c, p, I)p \int_\zeta^c H(\hat{c}|c)dc - m(q(c, p, I))
\]

It is clear that the public good could be financed by a lump-sum tax defined by \( E_c[T(c, p, I)]/N \), where: \( E_c[T(c, p, I)] = E_c[t(c, p, I)] + E_c[m(q(c, p, I))] \). Furthermore, \( G(c, p, I) = 0 \) if \( I \leq E_c[T(c, p, I)] \) under the audit mechanism given the assumption that government regulation requires transfers from the department to the bureau to be subject to oversight.

A modified Samuelson’s Rule is characterized by equation 6, implying that the optimal combination of the public and private goods is determined by the sum of the marginal rate of substitution (\( MRS_i \)) being equated to imputed public good cost to private good price ratio. The imputed cost of the public good includes the direct production cost incurred by the bureau and the marginal cost of auditing the bureau incurred adjusted by the expected penalty imposed against the bureau for misreporting its cost given an audit intensity \( q(c) \).\(^5\) The imputed price of the public good under the audit mechanism is represented by \( p_b(c, p, I) \), where: \( p_b(c, p, I) = c + \left[ m'(q(\cdot)) + p \int_\zeta^c H(\hat{c}|c)dc \right] \left[ p \int_\zeta^c \frac{\partial H(\hat{c}|c)}{\partial c}dc \right]^{-1} \).

It can be observed from equation 6 that the level of the public good is inversely related to its costs of production as intuition would suggest. More particular, a rise in the marginal cost of

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\(^4\)Equation 8 implies that \( q(\cdot) \) is independent of income, \( I \). It is also helpful to note that \( cf(c) - F(c) < 0 \) for some values of \( c \). It is assumed that \( c' > \tau \) where \( c' \) is defined by \( c' = (cf(c) - F(c) = 0) \). The necessity of oversight assumption imply no audit contract offered were \( c \in [c', \tau] \).

\(^5\)The audit intensity is characterized by equation 7 and \( q(c, p) \), where: \( q(\zeta, p) = 0 \) and \( \partial q(c, p)/\partial c > 0 \) and \( \partial q(c, p)/\partial p > 0 \).
auditing and a decrease in the penalty \( p \) implies a corresponding increase in the overall cost of the public good and substitution away from this good.

For \( c = c_0 \), the audit intensity is zero as it can be easily shown that 
\[
m'(q(c_0, p, I)) = 0 = p \int_{c_0}^{c} H(\hat{c}|c) d\hat{c} = 0
\]
from equation 7 as well as \( v(c_0, p, I) > 0 \). It warrants noting that 
\[
m(q(c_0, p, I)) = m(0) > 0.
\]

The modified Samuelson’s Rule in 6 simplifies to the traditional Samuelson’s Rule, when \( c = c_0 \). The traditional Samuelson’s Rule is therefore a nested result of the auditing subgame for \( c = c_0 \). Yet, the fixed cost of auditing (i.e., \( m(0) > 0 \)) and the information rents of the bureau (i.e., \( v(c_0, p_x, I) > 0 \)) affect the department’s budget constraint. The hierarchical structure of this economic framework gives rise to allocations that are ex-ante efficient yet can be improved upon were the department able to directly and costlessly observe the bureau’s costs as under the traditional Samuelson’s Rule.

For \( c \in (c, \bar{c}] \), \( m'(q(c, p, I)) > 0 \) causes a reduction in the level of the public good relative to when \( c = c_0 \), assuming the public good is normal.\(^6\) It therefore follows that \( G(c, p, I) < G(c_0, p, I) \). If \( m'(q(c, p, I)) \) is sufficiently large and/or \( p \) is sufficiently small, then there exists a potential for a corner solution exhibited by 
\[
G(c, p, I) = 0 \quad \text{and} \quad X = \sum_{i=1}^{N} x_i(c, p, I) = \left[ \sum_{i=1}^{N} I - m(0) \right] / p_x.
\]
The inability to control government administrative (audit) cost or the absence of a sufficient deterrent against cost misreporting may therefore result in the curtailment of public provision of the public good through an audit mechanism. Additionally, if the expected lump-sum tax per capita, \( \{ E_c[T(c, p, I)] \} / N \), for financing the public good is sufficiently large relative to per capita income, then the public good will not be produced through the audit mechanism (i.e., \( q(c, p, I) = 0 \) for \( c \in (c, \bar{c}] \)) as it is assumed that interdepartmental transfers without oversight is prohibited in the public sector.

Payoff. The expected payoff of the department (i.e., expected indirect utility) from auditing (i.e., \( q(c, p, I) > 0 \)) is denoted by

\[
W(c, p, I : A) = \sum_{i=1}^{N} E_c[V_i(p_h(c, p, I), p_x, I - T(c, p, I)/N : A)]
\]

\(^6\)Gaube (2000) relaxes the assumption of public good being a normal good given a distortionary tax being used to finance the provision of public goods. Clearly, the use of a distortionary tax to finance the public good a direct effect (i.e., substitution and income effects) on the provision of the public good. An indirect effect is also implied by the affect associated with government budget available to fund auditing and the minization of informational rents.
2.1.2. **Outsourcing Subgame.** For the outsourcing subgame ($a = C$), the firm and the department engage in Nash bargaining over the price ($p_f$) and the bribe ($B$) at stage II. At stage III, the department chooses the level of the public good given the negotiated price $p_f$, represented by the Marshallian demand $G(p_f, p_x, I)$. A bargaining outcome that improves upon the threat point yields payoffs: (i) a profit function $M(p_f, c_f, B : C) = G(p_f, p_x, I)(p_f - c_f) - B$ for the firm, and (ii) an indirect utility function $\sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B$ for the department.

Assumption A2: It is assumed that $\frac{\partial M(p_f, B)}{\partial p_f} \geq 0$ at $p_f = \sigma$.\footnote{A2 is a simplifying assumption that could be relaxed without materially affecting the results. The absence of A2 complicates the analysis without any significant or transparent benefits.}

A2 and the prohibition of public sector provision of the public good without oversight imply that the department chooses outsourcing ($a = C$) and the firm sets the price $p_f = \sigma$ if auditing is not conducted (i.e., $q(c, p, I) = 0$). Correspondingly, the department’s expected payoff is given by $\sum_{i=1}^{N} E_c[V_i(\sigma, p_x, I : C)]$ for income, $I \in [0, I_0]$, and given $T(c, p, I)$. $I_0$ is defined by:

$$I_0 = \left\{ I : \sum_{i=1}^{N} E_c[V_i(p_b(c, p, I), p_x, I - T(c, p, I)/N : A)] - \sum_{i=1}^{N} E_{c_f}[V_i(\sigma, p_x, I : C)] = 0 \right\}$$

The failure to reach a bargaining agreement results in the threat point payoffs being realized. The firms’ threat point profit is given by $M_0 = G(\sigma, p_x, I)(\sigma - c_f)$ if $I \leq I_0$ and zero if otherwise. The department’s threat point payoff is denoted by $V_0 = 0$ for $I \in (0, I_0]$ and $V_0 = \sum_{i=1}^{N} [V(p_b(c, p, I), p_x, I : A)]$ for $I > I_0$. The bargaining power of the firm relative to the department is denoted by $\alpha \in (0, 1)$. The bargaining problem is therefore represented by a variable threat point Nash bargaining problem. In what follows, the outsourcing subgame perfect equilibrium is determined through backwards induction.

At stage III, the department chooses the allocation $(x_i(p_f, p_x, I), G(p_f, p_x, I))_{i=1,2,...,N}$ maximizes:

$$\sum_{i=1}^{N} u(x_i, G) + \theta B \text{ subject to } \sum_{i=1}^{N} p_x x_i + p_f G \leq \sum_{i=1}^{N} I.$$  

The equilibrium is characterized by $\sum_{i=1}^{N} MRS_i = p_f/p_x$ and $\sum_{i=1}^{N} p_x x_i + p_f G = \sum_{i=1}^{N} I$.

The equilibrium allocation does not necessarily satisfy the Samuelson Rule as the negotiated price of the public good, $p_f$, is unlikely to equal the actual cost of production given the Nash bargaining
process involved in determining this price. If \( p_f = c_f \), then the marginal rate of transformation is equal to the price ratio and the Samuelson Rule holds.

At stage II, the firm anticipates the department’s demand for the public good, \( G(p_f, p_x, I) \), and bargains with the department over the price \( (p_f) \) and hidden bribe \( (B) \). The bargaining outcome is defined by \( \{p_f(\alpha, \theta, c_f, I), B(\alpha, \theta, c_f, I)\} \) solving:

\[
\max_{\{p_f, B\}} \left\{ \left[ G(p_f, p_x, I)(p_f - c_f) - B - M_0 \right]^{\alpha} \left[ \sum_{i=1}^{N} V(p_f, p_x, I, C) + \theta B - V_0 \right] ^{1-\alpha} \right\}
\]

As shown in appendix C, the subgame equilibrium outcome \( (p_f(\alpha, \theta, c_f, I), B(\alpha, \theta, c_f, I)) \) is characterized by

\[
\frac{\partial G(p_f, p_x, I)}{\partial p_f} (p_f - c_f) + G(p_f, p_x, I) = \left[ \frac{-1}{\theta (1-\alpha)} \right] \left( \sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I, C)}{\partial p_f} \right)
\]

\[
B = (1-\alpha) [G(p_f, p_x, I)(p_f - c_f) - M_0] + \left( \frac{\alpha}{\theta} \right) [V_0 - V(p_f, p_x, I, C)]
\]

The subgame equilibrium bribe, \( B(\alpha, \theta, c_f, I) \) involves a profit sharing (or profit extraction) and a kick-back component reflective of the forms of corruption described by Rouch (2001), Glaeser (2001), and Clarke and Xu (2004). The department shares in the firm’s profits from the provision of the public good being outsourced in proportion to the bargaining power of the department relative to the firm (i.e., \( 1 - \alpha \)). On the other hand, the kick-back component represents a compensation for the department choosing outsourcing \( (a = C) \) when the public interest favors the audit mechanism (i.e., \( W(c, p : A) > \sum_{i=1}^{N} E_{c_f} [V_i(p_f, p_x, I, C)] \) for \( I \geq I_0 \)] Clearly, it is necessary that \( p_f(\alpha, \theta, c_f, I) > c_f \) for profits to support bribe \( B(\alpha, \theta, c_f, I) > 0 \).

For \( I \in (0, I_0) \), A2 implies that \( p_f(\alpha, \theta, c_f, I) = \bar{c} \) and \( B(\alpha, \theta, c_f, I) = 0 \) so the threat point is \( M_0 = G(\bar{c}, p_x, I)(\bar{c} - c_f) \). For \( I > I_0 \), \( p_f(\alpha, \theta, c_f, I) \) and \( B(\alpha, \theta, c_f, I) \) are characterized by equations 13 and 14. The subgame equilibrium expected payoff schedules are stated below.

---

8Bargaining represents a discovery process that facilitates the firm learning about the demand schedule of the department and the department learning about the cost of production of the firm. Confidentiality agreements are generally practical realities of government-industry negotiations so often prohibit the release of proprietary information such as the firm’s costs. Hence, the nature of the bargaining process may cultivate the establishment of trust and the maintenance of secrecy that Shleifer and Vishny (1993) note are precursors and ingredients of bribery relationships.

9The affects of \( \theta \) and \( \alpha \) on the price and bribery levels are discussed in the comparative statics analysis to follow.
For the firm:

\[
M(\alpha, \theta, c_f, I : C) = \begin{cases} 
G(\pi, p_x, I) [\pi - c_f] & \text{for } I < I_0 \\
E_c f \{G(p_f(\alpha, \theta, c_f, I), p_x, I) [p_f(\alpha, \theta, c_f, I) - c_f]\} - E_c f \{B(\alpha, \theta, c_f, I)\} & \text{for } I \geq I_0
\end{cases}
\]

For the department:

\[
W(\alpha, \theta, c_f, I : C) = \begin{cases} 
\sum_{i=1}^{N} [V(\pi, p_x, I : C)] & \text{for } I < I_0 \\
\sum_{i=1}^{N} E_c f [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta E_c f [B(\alpha, \theta, c_f, I)] & \text{for } I \geq I_0
\end{cases}
\]

In what follows, it is helpful to refer to Figure 1.

\[\text{Figure 1. Department payoff and Income Levels with High Outsourcing Costs}\]

The equilibrium values for \(p_f(\alpha, \theta, c_f, I)\) and \(B(\alpha, \theta, c_f, I)\) in relationship to income level are now considered.
For \( I > I_0 \) and \( E_{cf}[p_I(\alpha, \theta, c_I, I)] \in [c, E_c[p_b(c, p, I)]] \), the department's expected payoff from outsourcing \((a = C)\) dominates the auditing mechanism \((a = A)\) and is expressed by \( \sum_{i=1}^{N} E_{cf} [V(\cdot : C)] > W(c, p, I : A) \) so no bribe is offered (i.e., \( B(\alpha, \theta, c_f, I) = 0 \)). For \( I > I_0 \) and \( E_{cf}[p_I(\alpha, \theta, c_I, I)] \in (E_c[p_b(c, p, I)], \overline{c}) \) and \( I \) being sufficiently large relative to \( E_c[T_c, p_x, I]/N \), it is clear that \( \sum_{i=1}^{N} E_{cf} [V(\cdot : C)] > W(c, p, I : A) \) for some values of \( I > I_0 \) so again the bribe is set at \( B(\alpha, \theta, c_f, I) = 0 \). Let this level of income be denoted by \( I_1 \) as follows:

\[
I_1 = \left\{ I : \sum_{i=1}^{N} E_{cf} [V(p_I(\alpha, \theta, c_f, I), p_x, I : C)] - W(c, p, I : A) = 0 \text{ given } E_{cf}[p_I(\cdot)] \in (E_c[p_b(\cdot)], \overline{c}) \right\}
\]

It therefore follows that a bribe is only paid for values of \( I > I_1 \) given \( E_{cf}[p_I(\alpha, \theta, c_I, I)] \in (E_c[p_b(c, p, I)], \overline{c}) \). The firm’s expected profits, \( E_{cf}[M(\alpha, \theta, c_f, I : C)] \), must be sufficiently large to support \( E_{cf}[B(\alpha, \theta, c_f, I)] > 0 \) and a payoff of the department given by

\[
\sum_{i=1}^{N} E_{cf} [V(p_I(\alpha, \theta, c_f, I), p_x, I_1 : C)] + \theta E_{cf} [B(\alpha, \theta, c_f, I_1)] > W(c, p, I : A).
\]

If so, a bribe offered by the firm (i.e., \( E_{cf}[B(\alpha, \theta, c_f, I)] > 0 \)) results in the department choosing to outsource the provision of the public good at a higher income than would be supported in the absence of a bribe. This level of income implied is denoted by \( I_2 \) satisfying:

\[
I_2 = \left\{ I : \sum_{i=1}^{N} E_{cf} [V(p_I(\alpha, \theta, c_f, I), p_x, I : C)] + \theta E_{cf} [B(\alpha, \theta, c_f, I)] - W(c, p, I : A) = 0 \text{ given } E_{cf}[p_I(\cdot)] > E_c[p_b(\cdot)] \right\}
\]

Clearly, \( B(\alpha, \theta, c_f, I_2) > 0 \) implies that the firm’s profit must be sufficient to support the bribe when \( I \in (I_1, I_2) \). On the other hand, no bribe is paid if profits are insufficient and \( I_2 \) vanishes from consideration.

The firm with fixed cost is now considered. Let the firm fixed costs be greater than \( I_0 \) and consider \( E_{cf}[p_I(\cdot)] < E_c[p_b(\cdot)] \). This implies that there exists an income level, say \( I_3 \), that leaves the department indifferent between outsourcing and auditing. This is shown in Figure 2. If a bribe is paid (i.e., \( B(\alpha, \theta, c_f, I) > 0 \)), then it follows that there is another income level, say \( I_4 \), satisfying \( I_4 < I_3 \). Here a bribe is paid to the support outsourcing at \( I_4 < I_3 \) even though the unit
cost from outsourcing is less the level achieved under the optimal audit mechanism. At high levels of per capita income, unit cost may therefore be an insufficient proxy for determining whether or not there is public sector corruption.

2.2. Department Choice. At stage I, the department chooses \( a \in \{A, C\} \). For \( I < I_0 \), the department does not audit the bureau such that the firm sets its price at \( p_f(\alpha, \theta, c_f, I) = \pi \) and \( B(\alpha, \theta, c_f, I) = 0 \). Hence, the department sets \( a = C \) for \( I < I_0 \). For \( I \geq I_0 \) and \( E_{cf}[p_f(\alpha, \theta, c_f, I)] \in [\underline{c}, E_c[p_b(c, p)]] \), the department’s expected payoff from outsourcing dominates the auditing mechanism when the firm has no fixed cost so the department chooses \( a = C \) without a bribe. For \( E_{cf}[p_f(\alpha, \theta, c_f, I)] \in (E_c[p_b(c, p)], \pi) \), the department chooses \( a = C \) without a bribe for \( I \in (I_0, I_1) \). For \( I > I_1 \), there is a potential for a bribe if it can be supported by the firm’s profits and it results in the department choosing outsourcing when the public interest favors auditing.
Definition 1. A bribery equilibrium is an allocation \( E_{cf} [B(a, \theta, c_f, I)] > 0 \) offered by the firm to the department to support outsourcing (i.e., \( a = C \)) over auditing (i.e., \( a = A \)) when \( \sum_{i=1}^{N} E_c [V_i(p_f(.), p_x, I) - T(c, p_x, I)/N] : A \).

The specific conditions under which a bribe will be supported is conditional on combinations of the bargaining power of the firm, \( \alpha \), and the sensitivity of the department to bribes (i.e., \( \theta \)). Let the sets \( \Gamma_D \) and \( \Gamma_F \) be defined by

\[
\Gamma_D = \left\{ (\theta, \alpha) : \sum_{i=1}^{N} E_{cf} [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C] + \theta E_{cf} [B(\alpha, \theta, c_f, I)] \geq \sum_{i=1}^{N} E_c [V_i(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A] \right\}
\]

\[\Gamma_F = \left\{ (\theta, \alpha) : E_{cf} [M(\alpha, \theta, c_f, I : C)] \geq M_0 \right\}\]

Proposition 1. If \( \Gamma_F \cap \Gamma_D \neq \emptyset \) and \( \sum_{i=1}^{N} E_{cf} [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C] < \sum_{i=1}^{N} E_c [V_i(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A] \), then there exists a combination \((\theta, \alpha)\) supporting a bribery equilibrium.

The existence of a bribery equilibrium implies that the department’s choice regarding the provision of the public good deviates away from the public interest.\(^{10}\) The combination \((\theta, \alpha)\) satisfying a bribery equilibrium is:

\[
\sum_{i=1}^{N} E_c [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C] + \theta E_{cf} [B(\alpha, \theta, c_f, I)] \geq \sum_{i=1}^{N} E_c [V_i(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A]
\]

So,

\[
\sum_{i=1}^{N} E_{cf} [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C] + \theta \left\{ (1 - \alpha) \left[ E_{cf} [M(\alpha, \theta, c_f, I : C)] - M_0 \right] + \left( \frac{\alpha}{\theta} \right) [V_0 - V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] \right\} \geq \sum_{i=1}^{N} E_c [V_i(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A]
\]

The proposition therefore holds if

\[
E_{cf} [M(\alpha, \theta, c_f, I : C)] \geq \sum_{i=1}^{N} E_c [V_i(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A] - \sum_{i=1}^{N} E_{cf} [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C]
\]

\(^{10}\) The public interest is defined by the welfare achieved from the equilibrium choices determined by the benevolent social planner (i.e., \( \theta = 0 \)).
given: \[ \sum_{i=1}^{N} E_c [V_i(p_i(c, p, I), p_x, I - T(c, p_x, I)/N) : A] > \sum_{i=1}^{N} E_{cf} [V_i(p_f(\alpha, \theta, c_f, I), p_x, I) : C] \] and, by definition, \( E_{cf} [M(\alpha, \theta, c_f, I : C)] \geq 0. \)

**Proposition 2.** If the firm offers the department a bribe, \( E_{cf} [B(\alpha, \theta, c_f, I)] > 0 \), then it necessarily expands the range of income (i.e., budget level) over which outsourcing is chosen over the audit mechanism.

The proposition suggests that the decision to offer a bribe by the firm acts to prolong outsourcing when public sector provision through an audit mechanism would otherwise be preferred or to accelerate the move towards outsourcing when it would be otherwise preferred based on per unit cost of the public good (see Figure 1 and 2). This result therefore suggests that comparing private and public sector unit cost of providing the public good may not be an adequate indicator of the appropriateness to outsource nor does it necessarily indicate that bribery was not involved in the decision making process of the government department.

### 3. Comparative Statics: Corruptibility and Bargaining Power

The affect of corruptibility and bargaining power is material to the equilibrium choice of the agents within the economy. As before, \( \theta \) represents the corruptibility of the department and the relative bargaining power of the firm vis-a-vis the department is represented by \( \alpha \). Comparative statics are conducted using 13 and 14. The detail computations are shown in appendix D. For this analysis, it is useful to define two sets, namely, \( \Gamma_U \) and \( \Gamma_L \). \( \Gamma_U \) represents the set of \((\theta, \alpha)\) involving the firm having a relatively low bargaining power and the department having a relatively high corruptibility (i.e., high sensitivity to bribery). \( \Gamma_U = \{ (\theta, \alpha) : \alpha < \tilde{\alpha}(\theta) \} \), where:

\[ \tilde{\alpha}(\theta) = 1 - \left( - \sum_{i=1}^{N} \frac{\partial^2 V_i(p_f, p_x, I : C)}{\partial p_f \partial p_f} \left[ \frac{\partial^2 M(p_f, c, I : C)}{\partial p_f \partial p_f} \right] \right)^{-1} (\theta) \].

\( \Gamma_L \) is the set of \((\theta, \alpha)\) reflecting the firm having relatively high bargaining power and the department having low corruptibility. Let \( \Gamma_L = \{ (\theta, \alpha) : \alpha > \tilde{\alpha}(\theta) \} \).

Given the department chooses \( a = C \) and a bribery equilibrium exists (i.e., \( B(\alpha, \theta, c_f, I) > 0 \)), the affect of corruptibility on the equilibrium price and bribery is as follows:
The effect of bargaining power on the equilibrium price and bribe is as follows:

\[
\frac{dp_I}{d\alpha} = \frac{-\theta^2 \frac{\partial M(_\alpha)}{\partial p_I}}{|A|}
\]

11 It can be shown that \( dB/d\theta > 0 \) if \((\theta, \alpha) \in \Gamma_u \). However, it is also clear that a bribery equilibrium requires that \( B > (1 - \alpha)M(p_f, c_f, \beta) \) as the bribe includes the department’s share of profit, \((1 - \alpha)M(p_f, c_f, \beta)\), and compensation for the payoff discrepancy from choosing \( a = C \) over \( a = A \). Additionally, it is clear that were \( \theta \) to be zero then bribe would be zero so it reasonably follows that \( dB/d\theta > 0 \) for \((\theta, \alpha) \in \Gamma_u \). Consistent with the conclusion formulated from the comparative statics results.
The affect of bargaining power on the negotiated price of the public good is negative, \( \frac{dp_f}{d\alpha} < 0 \), when \((\theta, \alpha) \in \Gamma_u\), meaning a firm with relatively low bargaining power facilitates lower prices as its bargaining power increases since the firm is under less coercion to increase price in support of high bribes for the department. On the other hand, the affect of the firm’s bargaining power on the level of the bribe is ambiguous even though it seems to be negative for \((\theta, \alpha) \in \Gamma_u\).

The results suggest that the improvements in the integrity (i.e., reduction in corruptibility) of the department can be anticipated to reduce prices and the level of the bribe when the integrity of the department is relatively high and the firm’s bargaining power is relatively low (i.e., \((\theta, \alpha) \in \Gamma_u\)). However, when integrity is relatively low (i.e., \((\theta, \alpha) \in \Gamma_L\)), an increase in integrity of the department results in the firm increasing the price levels to support higher bribes aimed at maintaining the department’s decision to outsource when the public interest favors auditing. Furthermore, a firm with relatively small bargaining power will be more able to resist the department’s coercion for higher prices in support of bribes given corruption (i.e., a bribery equilibrium).

The affect of corruptibility and bargaining power on the equilibrium choice of the department is implied by equation 17. Clearly, the set of \((\theta, \alpha) \in \Gamma_F \cap \Gamma_D \neq \emptyset\) also satisfies equation 17. Additional examination of the affects of corruptibility and bargaining power on the equilibrium outcome using generalized functional forms is unlikely to yield more meaningful results than already derived. For this reason, an example is presented in the application below using an assumed utility function for the department.

3.1. Application. In what follows, the department’s payoff schedule is assumed to be \( \sum_{i=1}^{N} x_i^{1-\beta} G^\beta + \theta B \). It can be shown that \( G(p_f, p_x, I) = \beta \{NI/p_f\} \) and \( x_i(p_f, p_x, I) = (1-\beta)\{I/p_x\} \) so \( p_f(\alpha, \theta, c_f, I) = \beta \bar{c}_f (1-\beta)^{-1} N \bar{c}_f [\theta(1-\alpha)]^{\frac{1}{1-\beta}} \bar{c}_f^{\frac{1}{1-\beta}} \) (as shown in appendix E). Clearly, \( \partial p_f(\alpha, \theta, c_f, I)/\partial \theta > 0 \) and \( \partial p_f(\alpha, \theta, c_f, I)/\partial \alpha < 0 \) which implies the Cobb-Douglas based utility sets a restriction denoted by \((\theta, \alpha) \in \Gamma_u\). Public good output, \( G(p_f, p_x, I) \), is therefore decreasing in the level of corruptibility
(i.e., θ) and the bargaining power of the department (1 − α). In this respect, the department’s bargaining power can be considered to be the level of coercion that can be levied against the firm to facilitate the extraction of bribes that is transmitted through to consumers by a high underlying price of the public good.

Appendix E shows the firm’s expected margin (i.e., revenues less production costs excluding the bribe) for I ≥ I₀ can be written as

\[ E_{cf} [\bar{M}(\alpha, \theta, c_f, I : C)] = \beta NI \left\{ 1 - (1 - \beta)^{\frac{\beta}{1 - \alpha}} \left\{ \theta[1 - \alpha] \right\} \right\} - \frac{1}{N} \int \frac{(c_f)^{\frac{\beta}{1 - \alpha}} f(c_f) dc_f}{\bar{f}} \]

The department’s expected payoff is

\[ W(\alpha, \theta, c_f, I : C) = \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta E_{cf} [B(\alpha, \theta, c_f, I)] \]

where:

\[ \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] = \left\{ (1 - \beta)^{\frac{\beta}{1 - \alpha}} N \frac{\beta}{1 - \alpha} p_x^{-1} \int \frac{\theta}{\bar{f}} (c_f)^{\frac{\beta}{1 - \alpha}} f(c_f) dc_f \right\} \]

The department’s expected payoff is linear in income (I) as depicted in Figures 1 - 3.

A bribery equilibrium is defined by \( \Gamma_F \cap \Gamma_D \neq \emptyset \) and \( \sum_{i=1}^{N} E_{cf} [V_{i}(p_f(\alpha, \theta, c_f, I), p_x, I - T(c, p_x, I)/N) : A] < \sum_{i=1}^{N} E_{c} [V_{i}(p_b(c, p, I), p_x, I - T(c, p_x, I)/N) : A] \). Using equations 18 and 19, the boundary of \( \Gamma_F \) can be characterized by

\[ g_F(\alpha) = \left\{ \alpha : E_{cf} [\bar{M}(\alpha, \theta, c_f, I : C)] = 0 \right\} \]

and the boundary of \( \Gamma_D \) can be characterized by

\[ g_D(\alpha) = \left\{ \alpha : W(\alpha, \theta, c_f, I : C) - W(c, p, I : A) = 0 \right\} \].

Figure 3 shows the relationship between \( g_F(\alpha) \) and \( g_D(\alpha) \) in (\( \alpha, \theta \))-space based on the properties implied by \( \tilde{g}_F(\alpha) \) and \( \tilde{g}_D(\alpha) \) and the numeric analysis shown in Appendix F.\(^{13}\)

\[ ^{12}\text{The firm’s expected profit (i.e., expected margin less the bribe) is:} \]

\[ E_{cf} [M(\alpha, \theta, c_f, \beta)] = E_{cf} [\bar{M}(\alpha, \theta, c_f, \beta)] - E_{cf} [B(\alpha, \theta, c_f, I)] \]

so

\[ E_{cf} [M(\alpha, \theta, c_f, \beta)] = \alpha E_{cf} [\bar{M}(\alpha, \theta, c_f, \beta)] - \frac{(\alpha/\theta)}{V_0 - \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)]]}. \]

\[ ^{13}\text{It may helpful to write } E_{cf} [\bar{M}(\alpha, \theta, c_f, I : C)] = \beta N I \{1 - d_1 [\theta[1 - \alpha]]^{-\beta/(1 - \alpha)} \} \text{ which implies that } E_{cf} [\bar{M}(\alpha, \theta, c_f, I : C)] = 0 \text{ when } \tilde{g}_F(\theta) = \alpha. \]

It is clear that \( \partial \tilde{g}_F(\theta)/\partial \theta > 0 \), \( \partial^2 \tilde{g}_F(\theta)/\partial \theta^2 < 0 \) and \( \tilde{g}_F(\theta) \) approaches 1 as \( \theta \) approaches infinity. The firm’s expected margin is positive when \( \tilde{g}_F(\theta) < \alpha \), meaning the bargaining power of the firm is sufficiently large to facilitate a positive expected margin for the firm that is a prerequisite to paying a bribe.

It is helpful to note that

\[ \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, I) \geq V_0 \]

implying \( \tilde{g}_D(\theta) = \left\{ \theta : \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] - V_0 = 0 \right\} \). Specifically, \( \tilde{g}_D(\theta) = 1 - \{1/\theta\} V_0/d_2 \right\} ^{\frac{\beta}{1 - \alpha}} \text{ and } \tilde{g}_D(\theta) > \alpha \) implies that

\[ \sum_{i=1}^{N} E_{cf} [V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, I) > V_0. \]

Clearly, \( \tilde{g}_D(\theta) \) has a form similar to \( \tilde{g}_F(\theta) \) as the expected payoff of the department is weighted by the expected margin of the firm.
Figure 3. Department and Firm’s Indifference Contours

The contour defined by $g_D(\alpha)$ represents the locus of points for which the department is indifferent between outsourcing (upon accepting a bribe) and having the public good produced within the public sector using an optimal audit mechanism. Outsourcing (upon accepting a bribe) dominates auditing for values of corruptibility, $\theta$, higher than the contour $g_D(\alpha)$. On the other hand, the contour defined by $g_F(\alpha)$ represents the expected profit (expected margin less the bribe) of the firm is zero. The firm therefore only makes a bribe for values of the department’s corruptibility higher than the level defined by the contour $g_F(\alpha)$. The intersection of the two contours is given by $g_D(\alpha) = g_F(\alpha) = \theta_0$ and let $\alpha_0 = g_F^{-1}(\theta_0) = g_F^{-1}(\theta_0)$. For simplification, it is assumed that there is only a single intersection.

A rise in the threat point payoff given from the optimal audit mechanism implies that the contour defined by $g_D(\alpha)$ shifts upward and towards the left. Under these conditions, a higher corruptibility
for the department and a low bargaining power for the firm would be needed to support even an indifference between outsourcing, with a bribe, and the optimal audit mechanism.

Proposition 3. If \( \theta > \theta_0 \) and \( g_D^{-1}(\alpha) > \alpha > g_F^{-1}(\alpha) \), then a bribe offered by the firm is accepted by the department choosing to outsource the public good even though it is not consistent with the interest of the public. Otherwise, no bribe is offered as it would not be accepted.

The proposition suggests that simply improving the integrity or reducing the corruptibility of government officials (i.e., departments) does not necessarily curtail bribery. Rather, the combination of increasing the department’s integrity and reducing the feasibility of firms to offer bribes may be needed. It therefore suggests that public sector initiatives aimed at improving governance and integrity in government is only one component potentially needed to deter corruption. This puts further weight to Khalil and Lawarree (2005) findings that corruption can be deterred by increasing the cost or penalties associated with collusion. The proposition further stresses the importance of the transparency of public accounts to deter corruption. This transparency must include information on service levels, quality and unit costs. The public must also have access to data regarding program delivery, administration and auditing of public goods produced in-house or through outsourcing.

4. Conclusion

The paper investigates the choice of government to audit or outsource the provision of a public good in the presence of a potential hidden bribe and information asymmetries. An audit mechanism is developed for characterizing the provision of the public good in the public sector given asymmetric information about the cost of production between the bureau (i.e., producer) and the department (funding source). Outsourcing is represented by Nash bargaining between the government department and a monopoly firm over the price of the public good. During the bargaining process, the firm may choose to offer the department a bribe aimed at influencing the department’s decision in favor of outsourcing when it is in opposition to the public interest (otherwise, a bribe is not offered provided). The study also relaxes the assumption of a single-tier institutional structure
within government that is considered to hold for the Samuelson Rule for the optimal provision of the public good.

The key findings of the study are as follows: First, bribery is predicted to distort the level of the public good provided and social welfare by inducing the department to choose outsourcing over auditing in opposition to the public interest. Second, an increase in the department’s integrity may result in increased prices as the firm acts to induce the department to maintain its choice of outsourcing over the public interest for auditing when the firm’s bargaining power is sufficiently low and the department’s preference towards bribes is relatively high. Correspondingly, an increase in the firm’s bargaining power may support its ability to resist the demands for bribes by the department resulting in the price of the public good declining in response to increased bargaining power of the firm. Third, a combination of improvements in the integrity and public sector governance (e.g., penalty for the bureau misreporting costs) and a reduction in the benefits from government-firm collusive behavior is predicted to be necessary to combat corruption. These results are consistent with the predictions of Khalil and Lawaree (2005). The results also suggest that bribery for small and large scale economies in terms of per capita income is welfare reducing. For small scale economies, bribery is predicted to be aimed at having an outsourcing contract persist even when public sector provision of the public good dominates (based on welfare and unit costs measurements). On the other hand, bribery in large scale economies is predicted to be aimed at motivating outsourcing when the provision of the public good through public sector production is in the public interest.

The study contributes to the literature by relaxing the assumption of a single-tier government underlying the Samuelson rule and by characterizing a bribery equilibrium within a economic framework that endogenize the institutional choice between outsourcing and an audit mechanism for the public sector provision of a public good. The scope for further study include empirical estimation of the relationships predicted and application of numeric methods aimed at verifying the conditions for the predictions to hold.
5. Appendix A

Bureau

The bureau’s expected utility from stating a cost of \( w \) when its true cost is \( c \) then is

\[
v(w; c) = (t(w) - c \cdot G(w)) - q(w)E[c \max[0, p(w - \hat{c})]]
\]

\[
= (t(w) - c \cdot G(w)) - q(w)p \int_{\xi}^{w} (w - \hat{c})h(\hat{c}|c) d\hat{c}
\]

\[
= t(w) - c \cdot G(w) - q(w)p \int_{\xi}^{w} H(\hat{c}|c) d\hat{c}
\]

The penalties both \( v(w; c) \) and \( \partial v(w; c)/\partial w \) are continuous at \( w = c \). By the revelation principle, attention can be restricted to truth-telling mechanisms. The participation constraint for the bureau is given by

\[
v(c, c) = t(c) - c \cdot G(c) - q(c)p \int_{\xi}^{c} H(\hat{c}|c) d\hat{c} \geq 0.
\]

Global incentive compatibility requires that

\[
v(c, c) \geq v(w, c) \quad \forall w, c \in [\xi, \overline{c}].
\]

Know

\[
\frac{\partial v(w; c)}{\partial w} = t'(w) - c \cdot G'(w) - q'(w)p \int_{\xi}^{w} H(\hat{c}|c) d\hat{c} - q'(w)pH(\hat{c}|c)
\]

This restricts the derivative of the bureau’s value function \( v(c) \), denoted by \( \dot{v}(c) \), and is given by

\[
\dot{v}(c) = -G(c) - q(c)p \int_{\xi}^{c} \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c}.
\]

Department

The department’s problem then is to maximize its expected payoff by choosing a contract \((G(w), x(w), q(w))\) to offer to the bureau for any reported cost \( w \).

\[\dot{v}(c) = \partial v(c, c)/\partial c \text{ after substituting } \partial v(w; c)/\partial w = t'(w) - c \cdot G'(w) - q'(w)p \int_{\xi}^{w} H(\hat{c}|c) d\hat{c} - q'(w)pH(\hat{c}|c) \text{ and setting } \partial v(w; c)/\partial w = 0 \text{ with truth telling so } w = c.\]
The department faces a budget constraint given by $\sum_{i=1}^N p_x x_i + t(c) \leq I_i$. After substituting $t(c)$, it can be shown that the budget constraint is:

\[(A6) \quad \sum_{i=1}^N [I_i - p_x x_i] - v(c) - c \cdot G(c) - q(c)p \int_{c}^{c'} H(\hat{c}|c) \, d\hat{c} - m(q(c)) - B \geq 0\]

where: $t(c) = v(c) + c \cdot G(c) + q(c)p \int_{c}^{c'} H(\hat{c}|c) \, d\hat{c}$.

The department offers the bureau a contract $\{G(c, I), x(c, I), q(c, I)\}$ solving the following optimal control problem defined by the state variable $v(c)$ and the contract consists of the control variables. The department’s problem is:

\[
\max_{\{q(\cdot), x(\cdot), G(\cdot)\}} \left\{ \sum_{i=1}^N \left[ \int_{c}^{c'} u_i(x_i(s), G(s))f(s) \, ds \right] \right\} \tag{A7}
\]

subject to:

(i) $\dot{v}(c) = -G(c) - q(c)p \int_{c}^{c'} \frac{\partial H(\hat{c}|c)}{\partial c} \, d\hat{c}$

(ii) $\sum_{i=1}^N [I_i - p_x x_i(c)] - v(c) - G(c)c - q(c)p \int_{c}^{c'} H(\hat{c}|c) \, d\hat{c} - m(q(c)) - B \geq 0$

(iv) $v(c) \geq 0$

(iv) $v(\bar{c}) = 0$

Let $\lambda(c)$ denote the (differentiable) multiplier on the transition equation (i) and let $\mu(c)$ denote the (differentiable) multiplier on (ii). The first order condition for the optimal control problem satisfies:\[See Kamien and Schwartz (1981) for an outline of the techniques used to solve such programming problems.\]

\[It \ is \ helpful \ to \ note \ that \ \frac{\partial}{\partial x} \left[ \int_{g(x)}^{h(x)} f(z) \, dz \right] = f(h(x))h'(x) - f(g(x))g'(x).\]
\[\sum_{i=1}^{N} [I_i - p_x x_i] - v(c) - G(c)c - q(c)p \int_{c}^{\hat{c}} H(\hat{c}|c) d\hat{c} - m(q(c)) - B \geq 0 \quad \mu(c) \geq 0 \quad (A8)\]

\[\dot{v}(c) = -G(c) - q(c) p \int_{c}^{\hat{c}} \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c} \quad (A9)\]

\[v^{(\pi)} = 0 \quad (A10)\]

\[\lambda^{(c)} = -\mu(c) \quad (A10)\]

\[\lambda^{(c)} = 0 \quad (A11)\]

\[\sum_{i=1}^{N} \left[ \frac{\partial u_i(x_i(c), G(c))}{\partial G(c)} \right] f(c) - \lambda(c) - c\mu(c) = 0 \quad (A12)\]

\[\left[ \frac{\partial u_i(x_i(c), G(c))}{\partial x_i(c)} \right] f(c) - p_x \mu(c) = 0 \quad (A13)\]

\[-\lambda(c)p \int_{c}^{\hat{c}} \frac{\partial H(\hat{c}|c)}{\partial c} d\hat{c} - \mu(c)p \int_{c}^{\hat{c}} H(\hat{c}|c) d\hat{c} - \mu(c)m'(q(c)) = 0 \quad (A14)\]

For \(\mu(c) \geq 0\) and \(\lambda(c) \geq 0\), it can be easily shown that \(\{G(c, I), x(c, I), q(c, I)\}\) satisfies

\[\sum_{i=1}^{N} \frac{\partial u_i(x_i(c), G(c))}{\partial G(c)} = \frac{c + \lambda(c)/\mu(c)}{p_x} \quad (A15)\]

\[\sum_{i=1}^{N} p_x x_i(c) + v(c) + G(c)c + q(c)p \int_{c}^{\hat{c}} H(\hat{c}|c) d\hat{c} + m(q(c)) + B = NI_i \quad (A16)\]

For \(c = \xi\), Samuelson’s Rule applies directly as \(\lambda(\xi) = 0\) and \(m'(q(\xi)) = 0\). The level of \(G(c, I)\) and \(x(c, I)\) at \(c = \xi\) is defined also by the department’s budget constraint as follows: \(\sum_{i=1}^{N} p_x x_i(\xi) + v(\xi) + G(\xi)\xi + m_0 = NI_i\), meaning the transfer payments to the bureau, \(v(\xi)\), must still be covered by the department even when the bureau reports a cost \(c = \xi\). It is now useful to solve for \(\lambda(c)\) by using A14 as follows:
\[-\lambda(c)p \int_{\xi}^{c} \frac{\partial H(\hat{c}|c)}{\partial c} \hat{c} dc = \mu(c)p \int_{\xi}^{c} H(\hat{c}|c) \hat{c} dc - \mu(c)m'(q(c)) = 0\] (A17)

\[-\lambda(c)p \int_{\xi}^{c} \frac{\partial H(\hat{c}|c)}{\partial c} \hat{c} dc - \lambda'(c) \left[ p \int_{\xi}^{c} H(\hat{c}|c) \hat{c} dc + m'(q(c)) \right] = 0\]

Using A14 and A10:

\[
\sum_{i=1}^{N} \left[ \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right] f(c) - \lambda(c) = c\mu(c) = 0 \quad \text{(A18)}
\]

\[
\lambda(c) + c\lambda'(c) = \sum_{i=1}^{N} \left[ \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right] f(c)
\]

Known \(\frac{\partial [c\lambda(c)]}{\partial c} = \lambda(c) + c\lambda'(c)\) so \(\lambda(c) = \frac{1}{c} \left\{ \frac{[\lambda(c) + c\lambda'(c)]}{f(c)} \right\} \) which implies:

\[
\lambda(c) = \frac{1}{c} \left[ \sum_{i=1}^{N} \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc \right]
\] (A19)

By B6, \(\mu(c) = \lambda'(c)\) so

\[
\frac{\lambda(c)}{\mu(c)} = \frac{\lambda(c)}{\lambda'(c)} \quad \text{(A20)}
\]

\[
= \frac{1}{c} \left[ \sum_{i=1}^{N} \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc \right]
\]

\[
= \frac{1}{c} \sum_{i=1}^{N} \left[ \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc \right] - \frac{1}{c} \sum_{i=1}^{N} \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc
\]

\[
= \frac{e}{c} \sum_{i=1}^{N} \left[ \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc \right] - \sum_{i=1}^{N} \int_{\xi}^{c} \left( \frac{\partial u_i(x_i(c),G(c))}{\partial G(c)} \right) f(c) dc
\]

For \(m'(q(c))\):
Using the Fearon and Busch (2005) method, it is known that
\[ m'(q(c)) = -\frac{\lambda(c)}{\mu(c)} \int_{\xi}^{\xi_c} \frac{\partial H(\hat{c}|c)}{\partial c} \, d\hat{c} - p \int_{\xi}^{\xi_c} H(\hat{c}|c) \, d\hat{c} \] (A21)

This equation can be expressed as \( m'(q(c)) = -\gamma'(p, q(c)) \) so \( m(q(c)) = -\int_{\xi}^{\xi_c} \gamma'(p, q(c)) \, dc \) and let \( m(q(c)) = m_0 + \varphi(p, c) \). At \( c = \xi \), it is known that \( m(q(c)) = m_0 \) as \( q(c) = 0 \) and clearly for \( p = 0 \) it would not be rational to monitor so implying \( \partial \varphi(p, c)/\partial p > 0 \) and \( \partial \varphi(p, c)/\partial c > 0 \). This result is

\[ \sum_{i=1}^{N} \frac{\partial u_i(x_i(c), G(c))}{\partial x_i(c)} = \frac{c + \lambda(c)/\mu(c)}{p_x} \] (A22)

But from (A9) we can solve for \( x(c) \) and substitute into (A8). This yields

\[ \sum_{i=1}^{N} [I_i - p_x x_i(c, p_x, I_i)] - m(q(c), p) - v(c) - cG(c)m_0 - q(c, p) \int_{\xi}^{\xi_c} H(\hat{c}|c) \, d\hat{c} = 0 \] (B1)

\[ \dot{v}(c) = -G(c) - q(c) p \int_{\xi}^{\xi_c} \frac{\partial H(\hat{c}|c)}{\partial c} \, d\hat{c} \] (B2)

Solving for \( G(c) \) and substituting yields

\[ \dot{v}(c) = \sum_{i=1}^{N} [I_i - p_x x_i(c, p_x, I_i)] + q(c, p) p \int_{\xi}^{\xi_c} H(\hat{c}|c) \, d\hat{c} - m(q(c, p)) \] (B3)

Using the Fearon and Busch (2005) method, it is known that \( \partial[v(c)/\partial c] = [\dot{v}(c) - v(c)]/c^2 \) so
\(v(c) = cK(I_i) - \int_{s}^{c} \frac{1}{s^2} \left\{ \sum_{i=1}^{N} [I_i - p_x x_i(c, p_x, I_i)] + q(c, p) \int_{s}^{c} H(\hat{c}|\hat{d} - m(q(c, p))] \right\} ds \)

where:

\( K(I_i) = \int_{s}^{c} \frac{1}{s^2} \left\{ \sum_{i=1}^{N} [I_i - p_x x_i(\hat{c}, p_x, I_i)] + q(\hat{c}, p) \int_{s}^{c} H(\hat{c}|\hat{d} - m(q(\hat{c}, p))] \right\} ds \)

as \( v(\hat{c}) = 0 \) and, therefore, \( v(c) \) reaches a maximum at \( c = \hat{c} \).

Example using a Cobb-Douglas utility function

In what follows, it will be useful analysis for a more closed form to assume a simplifying utility function such as a Cobb-Douglas form. The expected payoff of the department is given by a Cobb-Douglas utility function (noting \( B = 0 \) for \( a = A \)) can be therefore expressed as:

\[
\sum_{i=1}^{N} E_c V_i(p_b(c, p), p_x, I_i; A) = \begin{cases} 
\sum_{i=1}^{N} \left\{ \beta^\beta (1 - \beta) \frac{\beta p_x^{(1 - \beta)} N^\beta I_i}{[I_i]} \right\} & \text{for } I \leq I_0 \\
\sum_{i=1}^{N} E_c \left\{ \beta^\beta (1 - \beta) \frac{p_0(c, p)^{-\beta} p_x^{(1 - \beta)} N^\beta I_i}{[I_i]} \right\} & \text{for } I > I_0
\end{cases}
\]

The expected payoff of the department with the Cobb-Douglas utility function is linear in per capita income and inversely related to price as expected.

7. Appendix C

D. Firm-Supervisor Bargaining

The firm-supervisor bargaining problem is defined by the choice of \((p^*_g, B^*)\) to solve the following:

\[
\max_{\{p_g, B\}} \left\{ [G(p_f, p_x, I)](p_f - c_f) - B - M_0 \right\}^{\frac{1}{1-\alpha}}
\]

where:

\[
V_0 = \max \left\{ \sum_{i=1}^{N} V(\hat{c}, p_x, I : E), \sum_{i=1}^{N} V(p_b(c, p), p_x, I : A) \right\}
\]

and \( M_0 = G(\hat{c}, p_x, I)(\hat{c} - c_f) \) for \( I < I_1 \) and zero otherwise.

The equilibrium outcome \((p_f(\alpha, \theta, c_f, I), B(\alpha, \theta, c_f, I))\) for the Nash bargaining problem is characterized by

For \( p_f(\alpha, c_f, I) \):
\[ \alpha \left[ G(p_f, p_x, I)(p_f - c_f) - B - M_0 \right]^{\alpha-1} \left[ \sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0 \right]^{-\alpha} \left\{ \frac{\partial G(p_f, p_x, I)}{\partial p_f} (p_f - c_f) + G(p_f, p_x, I) \right\} \tag{C2} \]

\[ + (1 - \alpha) \left[ G(p_f, p_x, I)(p_f - c_f) - B - M_0 \right]^{\alpha-1} \left[ \sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0 \right]^{-\alpha} \left[ \theta \sum_{i=1}^{N} V(p_f, p_x, I : C) \right] = 0 \]

For \( B(\alpha, c_f, I) \):

\[ \alpha \left[ G(p_f, p_x, I)(p_f - c_f) - B - M_0 \right]^{\alpha-1} (-1) \left[ \sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0 \right]^{-\alpha} \left[ (G(p_f, p_x, I)(p_f - c_f) - B - M_0) \right] + (1 - \alpha) \theta \left[ G(p_f, p_x, I)(p_f - c_f) - B - M_0 \right]^{\alpha} \left[ \sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0 \right]^{-\alpha} = 0 \]

Using C2 and C3, it follows that

\[ \frac{\partial G(p_f, p_x, I)}{\partial p_f} (p_f - c_f) + G(p_f, p_x, I) = \left( \frac{1}{\alpha} \right) \left( -\sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I : C)}{\partial p_f} \right) \left[ \sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0 \right] \]

\[ \frac{G(p_f, p_x, I)(p_f - c_f) - B - M_0}{\sum_{i=1}^{N} V(p_f, p_x, I : C) + \theta B - V_0} = \frac{\alpha}{\theta (1 - \alpha)} \]

Hence,

\[ \frac{\partial G(p_f, p_x, I)}{\partial p_f} (p_f - c_f) + G(p_f, p_x, I) = \left( \frac{1}{\theta (1 - \alpha)} \right) \left( -\sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I : C)}{\partial p_f} \right) \]

\[ B = (1 - \alpha) G(p_f, p_x, I)(p_f - c_f) - M_0) + \frac{\alpha}{\theta} \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I : C) \right) \right] \]

8. APPENDIX D

Comparative Statics:

Note \( M_0 = 0 \) for \( I > I_0 \) and \( \partial M(\alpha, \theta, c_f, \beta)/\partial p_f = \{ \partial G(p_f, p_x, I)/\partial p_f \} (p_f - c_f) + G(p_f, p_x, I) \) so:

\[ (1 - \alpha) \frac{\partial M(\cdot)}{\partial p_f} d\theta - \frac{\theta \partial M(\cdot)}{\partial p_f} d\alpha + \theta (1 - \alpha) \frac{\partial^2 M(\cdot)}{\partial p_f^2} dp_f + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f \partial p_f} dp_f = 0 \]
\[-M(\alpha, \theta, c_f, \beta)\theta d\alpha + (1 - \alpha)M(\alpha, \theta, c_f, \beta)d\theta + (1 - \alpha)\theta \frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f}dp_f + \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I : C) \right) \right] \]

\[\alpha \sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I : C)}{\partial p_f}dp_f - Bd\theta - \theta dB = 0\]

For \(dp_f/d\theta\) and \(dB/d\theta\):

\[(D3) \quad \begin{bmatrix} (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} \quad 0 \end{bmatrix} \begin{bmatrix} \frac{dp_f}{d\theta} \\ -\theta \end{bmatrix} = \begin{bmatrix} -(1 - \alpha)\frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f} \end{bmatrix} \]

\[|A| = -\theta \left[ (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} \right], \text{ where: } |A| > 0 \text{ if } (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} < 0. \text{ This implies that } (1 - \alpha)\theta < -\left[ \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} \right] \left[ \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} \right]^{-1}. \text{ Otherwise, } |A| < 0.\]

\[(D4) \quad \frac{dp_f}{d\theta} = \frac{\theta(1 - \alpha)\frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f}}{|A|}\]

\[\frac{dB}{d\theta} = \frac{-\theta(1 - \alpha)\frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f}}{|A|} \left\{ -(1 - \alpha)M(\alpha, \theta, c_f, \beta) + B \right\} \left\{ (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} \right\} \]

\[+ \frac{(1 - \alpha)\frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f}}{|A|} \left\{ (1 - \alpha)\theta \frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f} - \alpha \sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I : C)}{\partial p_f} \right\} \]

For \(dp_f/d\alpha\) and \(dB/d\alpha\):

\[(D6) \quad \begin{bmatrix} (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial p_f^2} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I : C)}{\partial p_f^2} \quad 0 \end{bmatrix} \begin{bmatrix} \frac{dp_f}{d\alpha} \\ -\theta \end{bmatrix} = \begin{bmatrix} -M(\alpha, \theta, c_f, \beta)\theta d\alpha + \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I : C) \right) \right] \end{bmatrix} \]

\[(D7) \quad \frac{dp_f}{d\alpha} = \frac{-\theta^2 \frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial p_f}}{|A|}\]
\frac{dB}{d\alpha} = \left[ (1 - \alpha)\theta \frac{\partial^2 M(\alpha, \theta, c_f, \beta)}{\partial \alpha \partial \theta} + \sum_{i=1}^{N} \frac{\partial^2 V(p_f, p_x, I; C)}{\partial \alpha \partial \theta} \right] \left\{ -M(\alpha, \theta, c_f, \beta) \theta d\alpha + \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I; C) \right) \right] \right\} \\
\left( \frac{dB}{d\alpha} \right) \left( \frac{dM(\alpha, \theta, c_f, \beta)}{\partial \alpha} \right) = \left\{ (1 - \alpha)\theta \frac{\partial M(\alpha, \theta, c_f, \beta)}{\partial \alpha} - \alpha \sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I; C)}{\partial \alpha} \right\}
\left\{ A \right\}

\left(9. \text{APPENDIX E}\right)

Outsourcing Example using a Cobb-Douglas Function

Assuming a Cobb-Douglas utility function (i.e., \( W(x, G, B) = \sum_{i=1}^{N} x_i^{1-\beta} G^\beta + \theta B \), where: \( x = (x_1, x_2, ..., x_N) \), it can be shown that \( G(p_f, p_x, I) = \beta \{NI/p_f\} \) and \( x_i(p_f, p_x, I) = (1 - \beta)\{I/p_x \} \) so

\[ \frac{\partial G(p_f, p_x, I)}{\partial p_f} (p_f - c_f) + G(p_f, p_x, I) = \beta \left( \frac{NI}{p_f^2} \right) c_f \]

and

\[ \sum_{i=1}^{N} \frac{\partial V(p_f, p_x, I; C)}{\partial p_f} = -\beta \sum_{i=1}^{N} (1 - \beta)^{1-\beta} \beta \beta N^\beta p_x^{-(1-\beta)} p_f^{-\beta-1} I \]

Using C4, E1 and E2, it can be therefore shown that \( p_f(\alpha, \theta, c_f, I) = \beta \frac{\beta}{\theta} (1 - \beta)^{-1} N^{-\beta} p_x \theta(1 - \alpha) \right]^{\frac{1}{\beta - 1}} c_f^{\frac{1}{\beta - 1}} \). Clearly, \( \partial p_f(\alpha, \theta, c_f, I)/\partial \theta > 0 \) and \( \partial p_f(\alpha, \theta, c_f, I)/\partial \alpha < 0 \).

The firm’s payoff is:

\[ M(\alpha, \theta, c_f, \beta) = \left\{ \begin{array}{ll}
\beta \left( \frac{NI}{p_f^2} \right) (\sigma - c_f) \\
\alpha [G(p_f, p_x, I)(p_f - c_f) - M_0] - \frac{\theta}{\theta} \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I; C) \right) \right] & \text{for } I < I_0
\end{array} \right. 
\]

For \( I \geq I_0 \):

\[ E_{c_f}[\tilde{M}(\alpha, \theta, c_f, \beta)] = G(p_f, p_x, I)(p_f - c_f) \]

\[ = \beta NI \left\{ 1 - (1 - \beta)^{\frac{\beta}{\theta}} \left\{ \theta [1 - \alpha] \right\}^{-\frac{1}{\beta - 1}} N^{-\beta} p_x^{1-\beta} \int (c_f)^{\frac{\beta}{\theta}} f(c_f) dc_f \right\} \]
It is helpful to write \( E_{c_f}[\hat{M}(\alpha, \theta, c_f, \beta)] = \beta N I \left[ 1 - d_1 \{\theta[1 - \alpha]\} \right] \) and \( E_{c_f}[\hat{M}(\alpha, \theta, c_f, \beta)] \geq 0 \) if \( 1 - d_1 \{1/\theta\}^{(1-\beta)} \leq \alpha \). Let \( g_F(\theta) = 1 - d_1 \{1/\theta\}^{(1-\beta)} \) and it is clear that \( \partial g_F(\theta)/\partial \theta > 0 \) and \( \partial^2 g_F(\theta)/\partial \theta^2 \theta < 0 \) with \( g_F(\theta) \) approaching 1 as \( \theta \) approaches infinity.

The department’s expected payoff is:

\[
\sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta B(\alpha, c_f, I)
\]

\[
= \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta \left\{ (1 - \alpha) [M(\alpha, \theta, c_f, \beta) - M_0] + \frac{\alpha}{\theta} \left[ V_0 - \left( \sum_{i=1}^{N} V(p_f, p_x, I : C) \right) \right] \right\}
\]

where: \( \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] = \left\{ (1 - \beta) \beta^{\frac{\alpha}{\theta}} N^{\frac{\alpha}{\theta}} p_x^{-1} I \right\} \left\{ \int (c_f)^{\frac{\alpha}{\theta}} f(c_f) dc_f \right\} [\theta(1 - \alpha)]^{\frac{\alpha}{\theta}} I \) (E5)

It follows that \( \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta B(\alpha, c_f, I) \geq V_0 \) implies \( (1 - \alpha) \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, \beta) \geq (1 - \alpha)V_0 \) and \( \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, \beta) \geq V_0 \).

For \( \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, \beta) \):

\[
\sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] + \theta M(\alpha, \theta, c_f, \beta) = \left\{ (1 - \beta) \beta^{\frac{\alpha}{\theta}} N^{\frac{\alpha}{\theta}} p_x^{-1} I \right\} \left\{ \int (c_f)^{\frac{\alpha}{\theta}} f(c_f) dc_f \right\} [\theta(1 - \alpha)]^{\frac{\alpha}{\theta}} I \) (E6)

\[
+ \theta \beta N I \left\{ 1 - (1 - \beta) \beta^{\frac{\alpha}{\theta}} \{\theta[1 - \alpha]\}^{\frac{\alpha}{\theta}} N^{\frac{\alpha}{\theta}} p_x^{-1} \right\} \left\{ \int (c_f)^{\frac{\alpha}{\theta}} f(c_f) dc_f \right\}
\]

It follows that we can write \( \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] = d_2 \{\theta[1 - \alpha]\}^{\frac{\alpha}{\theta}} \sum_{i=1}^{N} E_{c_f}[V(p_f(\alpha, \theta, c_f, I), p_x, I : C)] \geq V_0 \) implies \( 1 - d_1 \{1/\theta\}^{(1-\beta)} \geq \alpha \). Let \( \hat{g}_D(\theta) = 1 - d_1 \{1/\theta\}^{(1-\beta)} \) and it is clear that \( \partial g_F(\theta)/\partial \theta > 0 \) and \( \partial^2 g_F(\theta)/\partial \theta^2 \theta < 0 \) with \( g_F(\theta) \) approaching 1 as \( \theta \) approaches infinity.

10. APPENDIX F

Using Mathematica, numeric analysis was conducted for the Cobb-Douglas based utility function.

Parameter values include setting \( \beta = 0.2 \); \( E_{c_f}[c_f] = 25 \); \( p_x = 1 \); \( N = 100 \); and \( I = 20000 \).
Figure 4. Department’s expected payoff including bribe

Figure 5. Firm’s expected profit adjusted for bribe
11. References


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