Hierarchies, intra-firm competition and mergers

by

Steffen Ziss

Wilfrid Laurier University
February, 2006

Abstract. This paper analyses the profit and welfare consequences of a merger between two hierarchical firms in a global oligopoly industry in which competition is described by a two-stage process. In the first stage product divisions design the incentive plans for local units. In the second stage local units engage in differentiated Cournot competition. After a merger the corporate centre must decide whether to merge the product divisions and/or the local units. The merging firms (insiders) can either credibly commit to an output contraction strategy by merging the product divisions and the local units thereby eliminating insider competition at both the incentive and output stages, or to an output expansion strategy by merging the local units only thereby eliminating output competition but retaining incentive competition between insiders. The output contraction and expansion strategies prove to be complementary in that the former is profitable in concentrated market settings whereas the latter is more profitable in unconcentrated market settings. Furthermore a merger can be welfare enhancing if it results in the insiders adopting the output expansion strategy. A final result is that if local units engage in price rather than output competition then mergers are always profitable and result in the insiders merging the local units and the product divisions.

JEL Classification Numbers. L10, L20, L40.

Keywords. Hierarchies, common agency, intra-firm competition, mergers

Address. School of Business & Economics, Wilfrid Laurier University, Waterloo, Ontario, Canada, N2L 3C5
tel: (519) 884-0710 (ext. 2776)      fax: (519) 888-1015      email: sziss@wlu.ca
1. Introduction

The purpose of this paper is to ask under what circumstances the merger of two large, multi-product global firms would result in the retention of some degree of intra-firm competition in the post-merger setting. The motivation for this study is the observation that multi-product firms are often organized as product divisions or company affiliates and that the acquisition of a rival often involves the later being given product division or company affiliate status\(^1\). One of the central features of a multi-divisional or multi-subsidiary organizational form is that a divisional or subsidiary manager is given sole responsibility for the operations of their business and that their pay is tied to performance of the business unit rather than to corporate performance. The potential implication of divisional autonomy and pay tied to divisional performance is intra-firm competition. The main proposition of this paper is that the organizational restructuring following a merger has implications for intra-firm competition and can lead to an overall increase or decrease in the level of competition within a market.

The proposition that organizational restructuring following a merger has strategic implications for the competition within markets rests on two main assumptions. The first main assumption is that organizational structure represents both a visible and credible commitment. Visibility is not an onerous requirement as rivals can readily observe the manner in which firms are organized and the responsibilities of each business unit\(^2\).

\(^1\) For example the 1998 “merger of equals” between Daimler-Benz AG and Chrysler Corporation resulted in Chrysler and Mercedes-Benz products being organized as separate ‘groups’ within the newly formed DaimlerChrysler corporation. Moreover the integration guidelines emphasized divisional autonomy by prohibiting the adoption of a common platform manufacturing strategy and prohibiting the establishment of combined dealerships (Thompson and Strickland 2001). Examples of wholly owned subsidiary status being given to acquired rivals is demonstrated by Volkswagen’s acquisition of Audi, Ford’s acquisition of Volvo and GM’s acquisition of Saab. More generally, Prechel, Boies and Woods (1999) show that the adoption of the multi-subsidiary organizational form is positively correlated with merger and acquisition activity.

\(^2\) Most of this information can be gleaned from company websites and company reports.
Credibility is assured by the fact that re-organizations are costly and by the fact that there may be non-strategic reasons for organizational structure\(^3\). A second main assumption of paper is that organizational structure is the only way to control the degree of intra-firm competition. This assumption can by justified by pointing out the limitations of altering intra-firm competition by alternate means such as the use of transfer prices or corporate wide incentives.

In theory the use of transfer prices to eliminate the degree of intra-firm competition is fairly straightforward and involves charging divisions a transfer price for corporate inputs in excess of marginal cost. The problem with this approach is that it provides no incentive for the selling division to minimise cost and thus passes on any cost inefficiencies to the buying division. A second problem is that transfer prices also serve to assess divisional performance. Transfer prices in excess of marginal cost distort the performance of the buying division and are thus objectionable on account of being unfair. A third problem, which arises in the context of international intra-firm transfers, is that transfer pricing may be motivated by desire to minimize tax. A final justification for not using transfer prices to control intra-firm competition is that the survey evidence suggests that the majority of firms either use a market price or a simple cost-plus rule to determine transfer prices. Neither of these pricing practices gives any regard to the degree of intra-firm competition\(^4\).

An alternate way to reduce intra-firm competition between business units is to

---

\(^3\) Non-strategic motivations for a strategic move serve to improve the credibility of a strategic move simply because the undoing of the move results in the loss of the non-strategic benefit. Non-strategic benefits of granting wholly-owned subsidiary status to acquired rivals include the creation of a more effective internal capital market, increased financial flexibility, tax advantages (in the US), and reduced reliance on external capital markets (Prechel, Boies and Woods 1999).

\(^4\) See Eccles (1985) ch. 2 and Riahi-Belkaoui (1995) ch. 6 for detailed discussions of the transfer pricing issues and survey evidence referred to above.
base a business unit manager’s compensation on both their own performance as well as on the performance of a competing business unit. In the model considered in this paper there would be no reason to preclude this possibility. However, in a more general agency setting in which managers face risk there would be a problem with basing managerial pay on the performance of other business units as it would create a cost by exposing risk averse managers to additional risk.\(^5\) Furthermore if managers are concerned about the prospects of either promotion or moving to another firm then they are interested in developing a reputation for being good managers. Presumably such a reputation would be based on the performance of the business unit directly within the control of the manager, and not on the performance of business units controlled by other managers. Finally one of advantages of the multi-divisional structure is that it allows the corporate centre to better allocate resources to high yield uses. This clearly implies that fund allocation be based on divisional performance. Managers who wish to grow their business must therefore focus on divisional performance in order to acquire the funds needed for growth\(^6\).

Notwithstanding the above arguments one does observe that firms offer corporate wide incentives such as stock options. The contention of this paper is that these measures may reduce but do not completely eliminate intra-firm competition due to the pre-eminence of the various business unit specific incentives mentioned above. Consequently the merger of two business units yields a discrete reduction in intra-firm competition, which is all that is needed for the results of this paper to hold.

\(^5\) Since demand shocks for substitute products are likely to be positively correlated then basing managerial compensation on the performance of business units producing substitute products will increase the risk faced by the business unit manager. If the manager is risk averse then the firm must increase the manager’s pay to compensate them for taking on the additional risk.

\(^6\) Most of the above arguments as to why managers are motivated to maximize divisional profits have been made by Baron and Besanko (2001) and others.
As a result of the agency issues listed above it will be assumed that managerial pay is tied exclusively to the performance of a manager’s business unit. A valid objection to this assumption is that it eliminates the incentive for divisions to co-operate so as to bring about cost synergies. In order to overcome these difficulties firms often set up set-up co-ordination committees or regional offices that help to share market intelligence about particular countries or regions. Alternatively firms may base managerial performance on the extent to which managers co-operate with the parent company and with company affiliates. These internal control devices allow firms to achieve cost synergies between divisions without altering the degree of intra-firm competition.

In order to explore the effect of mergers on intra-firm and inter-firm competition this paper follows Baron and Besanko (1998, 2001) in viewing the firm as a hierarchy consisting of local units, product divisions and a corporate centre. The corporate centre determines the degree of intra-firm competition by determining which products are included in each product division, and by determining whether two product divisions use separate local units to distribute their products or whether they use a single local unit to distribute both products. The centre delegates to product divisions, the responsibility for designing incentives and carrying out performance evaluation of local units, and

---

7 Statistical evidence that divisional CEO pay is tied to divisional performance is either weak or has measurement problems (Fisher and Govindarajan 1992, Lambert, Larcker and Weigelt 1993, Leonard 1990). Anecdotal evidence that the pay of divisional managers is tied exclusively to divisional performance is contained in Baye Crocker and Ju (1996) and Milkovich and Newman (2005:323-8).

8 See International Business Corporation (1976) ch.7 for case study evidence of co-ordinating committees.


10 For detailed case study evidence that firms are organized in this ‘product line’ fashion see International Business Corporation (1976) ch.4 and ch. 7. For more recent survey evidence that decentralized product-based organizations are becoming more popular see Harrison (1995).

11 Product divisions are also responsible for R & D and organizing global manufacturing, assembly and parts production. The paper does not model these decisions but focuses rather on the monitoring role of the
delegates the responsibility for the distribution of output to local units\textsuperscript{12}. Managers of product divisions are responsible for the profits of their products generated by the various local units. In order to motivate local unit managers the product division managers use incentive contracts suggested in the strategic managerial contracting literature\textsuperscript{13} that base managerial pay on a convex combination of product revenues and profits. Local units are located in particular countries or regions and are responsible for distributing the output of one or more of the product divisions. Firms play a two-stage differentiated product oligopoly game in which the product divisions choose incentives for the local units in stage one and local units engage in output or price competition in stage two.

In the post-merger setting two of the $n$ firms (insiders) merge and must decide on whether or not to alter insider incentive competition between divisions and insider output or price competition between local units. If the local units engage in price competition then mergers are always profitable and result in the elimination of all intra-firm competition via the merger of both the product divisions and the local units. If the local units engage in output competition then the elimination of all intra-firm competition is the most profitable strategy if market concentration is sufficiently high or product differentiation is sufficiently low. This strategy allows the insiders to commit to an output contraction strategy which reduces welfare in the absence of cost savings gains.

In less concentrated settings the elimination of output competition combined with the preservation of incentive competition brought about by the merger of local units who then jointly distribute the output of two competing product divisions tends to be the most

\textsuperscript{12} Distribution includes importing, marketing and managing a network of retail outlets.
profitable strategy for intermediate degrees of product differentiation. This strategy allows insiders to commit to an output expansion strategy, which may result in the merger being welfare enhancing even in the absence of cost efficiency gains. The reason for the output expansion is that the merged local unit becomes more responsive to insider incentive competition, which then induces the product divisions to give incentives that are strong enough to overcome the stifling of output competition by the local unit.

The model assumes price or output competition to be controlled by country or regional distributors (i.e. local units) and it ignores the latter’s relationship with manufacturers and retailers. The justification for ignoring these relationships is that they may not necessarily result in rent shifting opportunities. For example the manufacturer-distributor relationship is often governed by a transfer price, but the latter has limited strategic value for reasons argued above. The distributor-retailer relationship on the other hand can involve an external transaction in which either a linear or a two-part tariff is used. If linear pricing is used then the model still applies provided the demand functions are interpreted as wholesale rather than retail demand functions. If a two-part tariff is used then the contract terms can be used to shift rents\(^{14}\) but the empirical evidence to support such a proposition is mixed.\(^{15}\) In contrast there is consistent evidence that wholly owned subsidiaries are evaluated strategically by virtue of the later being encouraged to pursue a wide variety of strategic goals including profits, revenues and market share.\(^{16}\)

The recent Chrysler/Daimler-Benz merger aptly illustrates the post-merger re-structuring issues addressed in this paper. Prior to the merger each firm was represented

---

14 See Bonanno and Vickers (1988) for example.
15 Slade (1998) finds evidence of strategic contracting in gasoline retailing. In contrast, LaFontaine and Shaw (1999) find no negative correlation, within firms, between up-front franchise fees and royalty rates and thus conclude that franchise fees are set so as to recover upfront costs rather than extract future rent.
16 See Riahi-Belkaoui (1995: 123-26) for survey evidence on how subsidiaries are evaluated.
by a parent company and various wholly owned subsidiaries. The subsidiaries were responsible for sales and marketing and/or manufacturing and assembly. After the merger the two parent companies were organized as separate divisions called the Chrysler Group and the Mercedes Benz Group within the newly formed DaimlerChrysler Corporation. In the context of the paper this would be interpreted as the preservation of incentive competition between the product divisions. In Canada, US and China\textsuperscript{17} the subsidiaries responsible for distribution (i.e. the local units) were kept separate thereby preserving output/price competition whereas in Australia, Argentina, Holland, Middle East and UK\textsuperscript{18} the subsidiaries responsible for distribution were merged thereby eliminating output/price competition. The predicted impact of these organizational changes is that the level of competition would remain the same in Canada, US and China and it would intensify in Australia, Argentina, Holland, Middle East and UK.

The rest of the paper is organized as follows. Section 2 sets up the model and analyses the output and incentive contracting results under output competition. The profit and welfare results for output competition are analysed in Section 3 and 4 respectively. Section 5 deals with price competition. Section 6 concludes and compares the results to those obtained in the mergers, vertical restraints and divisionalization literatures.

\textsuperscript{17} In Canada, US and China distribution is carried out by Daimler Chrysler Canada, Daimler Chrysler Motors and Beijing Jeep respectively for Chrysler products and by Mercedes-Benz Canada, Mercedes-Benz US and Mercedes-Benz China respectively for Mercedes Benz products. (www.daimlerchrysler.com)

\textsuperscript{18} In Australia Mercedes Benz and Chrysler operations were merged in 1999 to become DaimlerChrysler Australia. In Argentina Mercedes-Benz Argentina and Chrysler Argentina merged to become DaimlerChrysler Argentina in 2000. In Holland Mercedes-Benz Nederland and Chrysler Nederland merged to become DaimlerChrysler Nederland in 1999. In the UK Mercedes-Benz UK and Chrysler-Jeep Imports UK merged to become DaimlerChrysler UK in 2000. In the Middle East the regional offices of Mercedes Benz and Chrysler merged to form DaimlerChrysler Middle East in 1999. (www.daimlerchrysler.com).
2. The model and the comparative static effects of merger under output competition

The oligopoly model consider in this paper consists of \( n \) differentiated products that can each be produced at constant marginal cost \( c \). If industry output and the output of product \( i \) are denoted \( Q \) and \( q_i \) respectively and if \( Q_i = Q - q_i \) denotes the output of rival products then the inverted demand function for product \( i \) is assumed to be given by

\[
P_i(q_i, Q_{-i}) = a - q_i - \theta Q_i
\]

where \( 0 < \theta \leq 1 \) is the substitutability parameter. In particular if \( \theta = 0 \) then the products are completely unrelated whereas if \( \theta = 1 \) then the products are perfect substitutes. The profits generated by product \( i \) are thus given by

\[
\pi_i = (P_i(q_i, Q_{-i}) - c)q_i
\]

Each firm consists of local units, product divisions and a corporate centre. The corporate centre delegates to product divisions, the responsibility for designing incentives and carrying out performance evaluation of the local units, and delegates the responsibility for the distribution of output in particular countries or regions to local units. Since product divisions are further up the hierarchy than local units we shall refer to product divisions as being upstream and to local units as being downstream\(^{19} \).

Managers of product divisions are responsible for the profits of their product generated by the various local units in each country or region. In order to motivate local unit managers to distribute their product the manager of the product divisions base local unit manager’s pay on a convex combination of product revenues and profits. The pay-off (or

---

\(^{19}\) This is a slight abuse of terminology as the usual meaning of upstream and downstream businesses is that the former provides an input to the later whereas here we assume that the former monitors the latter. The true meaning of these words is partially restored by considering that the product divisions could also be providing product design and R and D services to the local units.
reward) of local unit manager for selling product $i$ is thus given by

$$ R_i = (P_i(q_i, Q_i) - w_i)q_i $$

$i = 1, \ldots, n$

where $w_i$ is referred to as the incentive variable and it is determined by the weight $\alpha_i$ that the product division assigns to product profits (i.e. if $\alpha_i = 1$ then $w_i = c$, if $\alpha_i < \text{(resp. >)} 1$ then $w_i < \text{(resp. >)} c$).\(^{20}\) Competition is modelled as a two-stage game in which the product divisions choose incentives for local units in the first stage and local units engage in output competition in the second stage.

In the pre-merger setting each product is produced by a separate firm. In the post-merger setting two of the $n$ firms contemplate merger. Hereafter merging and non-merging firms shall be referred to as insiders and outsiders respectively. In the post-merger setting the insider corporate centre must decide whether or not to maintain competition at the incentive and output stages. For reasons given in the introduction it will be assumed that managerial pay can only be based on the performance of the manager’s own business unit and cannot be based on the performance of business units controlled by other managers. As a result the only way to eliminate intra-firm competition between business units is to merge them into a single business unit. The insider corporate centre is thus faced with four strategic options (or regimes): Non-merger (N) which preserves the pre-merger status quo, Upstream merger (U), Downstream merger (D) and Complete merger (C)\(^{21}\). Strategy U results in a single upstream division that monitors two competing local units. Strategy D results in a single

\(^{20}\) Since the incentive pay of the local unit manager that sells product $i$ is based on a convex combination of product $i$ profits ($\pi_i$) and revenues ($S_i$) then their pay-off is given by $R_i = \alpha_i \pi_i + (1-\alpha_i)S_i$. Now substitute $\pi_i = S_i - cq_i$ to obtain $R_i = S_i - \alpha_i cq_i$, which is the same as (3) except with $w_i = \alpha_i c$.

\(^{21}\) The decision to merge the product divisions is a global one whereas the decision to merge the local units can be done on a region-by-region basis thereby allowing insiders to pursue different strategies in each country. For example, in the DaimlerChrysler example cited in the introduction the insiders pursued strategy N in Canada, US and China and strategy D in Australia, Argentina, Holland, Middle East and UK.
local unit in each country that distributes the product of both product divisions. The product divisions jointly monitor the local unit and are each responsible for their product’s profits. Each product division thus provides performance incentives for the local unit to distribute their product. Strategy C involves expanding the pre-merger responsibilities of the product divisions and local units so as to include one more product.

It is assumed that firms 1 and 2 are the ones contemplating merger and thus to conduct the merger analysis it shall prove convenient to use

\[
\lambda_i = \begin{cases} 
\lambda & i = 1, 2 \\
0 & i = 3, \ldots, n
\end{cases}
\]

(4)

as the local unit merger parameter and

\[
\rho_i = \begin{cases} 
\rho & i = 1, 2 \\
0 & i = 3, \ldots, n
\end{cases}
\]

(5)

as the product division merger parameter, where \( \lambda \) and \( \rho \) are indicator functions which take on a value of 0 or 1 depending on whether or not merger of the business unit occurs.

**Output stage.** A general expression for the objective function of each local unit is

\[
R_i + \lambda_i R_j = 0, \quad i = 1, \ldots, n, \quad j = 1, 2, \quad j \neq i
\]

(6)

Substituting (1) and (3) into (6) and then differentiating with respect to \( q_i \) yields the following first order condition

\[
a - 2q_i - \theta Q_{ij} - \lambda_i \theta q_j = w_i, \quad i = 1, \ldots, n, \quad j = 1, 2, \quad j \neq i
\]

(7)

which indicates that the market power externality (i.e. \(-\theta q_j\)) internalised by the manager of a merged local unit is negative and proportional to output. In other words an increase in \( q_i \) lowers the price of product \( j \) and thus lowers the revenues from product \( j \) in proportion to \( q_j \).


**Seller responsiveness.** From hereon the local unit responsible for the distribution of product \( i \) shall be referred to as the seller of product \( i \). Re-arrange (7) to obtain

\[
q_i = \frac{1}{2} \left( a - w_i - \theta Q_{-i} - \lambda_i q_j \right) \quad i, j = 1, 2 \ j \neq i
\]

which implies that the seller of product \( i \) is more prepared to cede market share to product \( j \) if they also sell product \( j \) (\( \lambda_i = 1 \)) than if a rival seller sold product \( j \) (\( \lambda_i = 0 \)). The reason for this result is that an increase in \( q_j \) induces a merged insider seller to lower \( q_i \) not only because the marginal revenue of \( q_i \) has fallen but also because \( q_i \) now imposes a greater market externality on product 2 because \( q_2 \) is higher. This result implies that a merger of insider sellers will *increase* their responsiveness to *insider* incentive competition.

Sum (7a) over both insiders and substitute \( Q_{-i} = Q_o + q_j \), \( i, j = 1, 2 \ j \neq i \) and \( Q_i = q_1 + q_2 \) to obtain

\[
Q_i = \frac{a - \theta Q_o - \frac{1}{2} (w_i + w_j)}{1 + \frac{1}{2} (1 + \lambda_i)} \quad i = 1, 2
\]

which implies that the insider sellers are less willing to accommodate increases in outsider output if they have been merged (\( \lambda_i = 1 \)) than if they have not been merged (\( \lambda_i = 0 \)). The reason for this result is than an increase in outsider output lowers insider marginal revenue and thus induces the insider seller to reduce output. If the insider sellers are merged then the output reduction will be smaller because the reduction in output reduces the size of the market power externality. The implication of this result is that a merger of insider sellers will *decrease* their responsiveness to *outsider* incentive competition.

**Output stage comparative statics.** Substituting (4) into (7) and then solving yields solutions denoted \( q_i (\mathbf{w}, \lambda) \) where \( \mathbf{w} = (w_1, \ldots, w_n) \). Let \( Q_i = q_1 + q_2 \) and \( Q_o = Q - Q_i \) denote aggregate insider and outsider output respectively. In Appendix A equation (7) is
used to derive that the comparative static effects of $w_i$ are

(8a) \[ \frac{\partial Q_i}{\partial w_i} = \frac{(2 - \theta)(2 + \theta(n - 3))}{\nabla} \quad i = 1,2 \]

(8b) \[ \frac{\partial Q_o}{\partial w_i} = \frac{(2 - \theta)\theta(n - 2)}{\nabla} \quad i = 1,2 \]

(8c) \[ \frac{\partial q_i}{\partial w_i} = \frac{(2 - \theta)^2(2 + \theta(n - 2))}{(2 - \theta(1 + \lambda))\nabla} \quad i = 1,2 \]

(8d) \[ \frac{\partial Q_{-i}}{\partial w_i} = \frac{(2 - \theta)^2 \theta(n - 1 + \lambda)}{(2 - \theta(1 + \lambda))\nabla} \quad i = 1,2 \]

if $w_i$ is chosen by one of the insiders and by

(9a) \[ \frac{\partial q_i}{\partial w_i} = \frac{(2 + \theta(n - 2))(2 - \theta(1 - \lambda)) - 2\lambda \theta^2}{\nabla} \quad i = 3,\ldots,n \]

(9b) \[ \frac{\partial Q_{-i}}{\partial w_i} = \frac{\theta[(n - 1)(2 - \theta(1 - \lambda)) - 2\lambda \theta]}{\nabla} \quad i = 3,\ldots,n \]

if $w_i$ is chosen by an outsider, where $\nabla > 0$ is the determinant of the Jacobian matrix.

**Incentive contracting stage.** Since product division managers are responsible for the profits generated by local units selling their products then the general expression for the objective function of product division managers is

(10) \[ \pi_i(q_i, Q_{-i}) + \rho_i \pi_j(q_j, Q_{-j}) \quad i = 1,\ldots,n \quad j = 1,2 \quad j \neq i \]

Now substitute the output stage solutions and then differentiate with respect to $w_i$ to obtain that the equilibrium choice of $w_i$ satisfies

(11) \[ \frac{\partial \pi_i}{\partial q_i} \frac{\partial q_i}{\partial w_i} + \frac{\partial \pi_i}{\partial Q_{-i}} \frac{\partial Q_{-i}}{\partial w_i} + \rho_i \left[ \frac{\partial \pi_j}{\partial q_j} \frac{\partial q_j}{\partial w_i} + \frac{\partial \pi_j}{\partial Q_{-j}} \frac{\partial Q_{-j}}{\partial w_i} \right] = 0 \quad i = 1,\ldots,n \quad j = 1,2 \quad j \neq i \]

which implies that $q_1 = q_2$ and $q_3 = \ldots = q_n$ hold in equilibrium and thus that
are satisfied in equilibrium. Now let

\[
\begin{align*}
(11a) \quad \frac{\partial \pi_i}{\partial q_1} &= \frac{\partial \pi_2}{\partial q_2} \quad \text{and} \quad \frac{\partial \pi_i}{\partial Q_{-1}} &= \frac{\partial \pi_2}{\partial Q_{-2}}
\end{align*}
\]

\[
(12) \quad v_i' = \left( \frac{\partial Q_{-i}}{\partial w_i} + \rho_i \frac{\partial Q_j}{\partial w_j} \right) \left( \frac{\partial q_i}{\partial w_i} + \rho_i \frac{\partial q_j}{\partial w_j} \right) \quad r \in \{N, U, D, C\} \quad i = 1, \ldots, n \quad j = 1, 2 \quad j \neq i
\]

denote the “effective conjectural variation (or ECV)” of firm \( i \) induced by regime \( r \), then divide (11) by \( \frac{\partial q_i}{\partial w_i} + \rho_i \frac{\partial q_j}{\partial w_j} \) and substitute (11a) and (12) into (11) to obtain

\[
(11)' \quad \frac{\partial \pi_i}{\partial q_i} + v_i' \frac{\partial \pi_i}{\partial Q_{-i}} = 0 \quad i = 1, \ldots, n \quad r \in \{N, U, D, C\}
\]

Since \( \frac{\partial \pi_i}{\partial Q_{-i}} \) is negative then (11)' indicates that a firm becomes more aggressive (i.e. the marginal profitability of output expansions rise) as the ECV becomes more negative.

The insider ECV for the two regimes which preserve incentive competition is found by substituting \( \rho_i = 0 \), (8c) and (8d) into (12) and then letting \( \lambda = 0 \) to obtain

\[
(13a) \quad v_i^N = -\frac{\theta(n-1)}{2 + \theta(n-2)}
\]

for the N regime or \( \lambda = 1 \) to get

\[
(13b) \quad v_i^D = -\frac{\theta n}{2 + \theta(n-2)}
\]

for the D regime. The insider ECV for the two regimes involving merger of the product divisions is obtained by substituting \( \rho_i = 1 \) into (12) and then simplifying to obtain

\[
(13c) \quad v_i^C = v_i' = 1 + 2 \left( \frac{\partial Q_0 / \partial w_i}{\partial Q_1 / \partial w_i} \right)
\]

which becomes

\[
(13c)' \quad v_i^C = v_i' = \frac{2 - \theta(n-1)}{2 + \theta(n-3)}
\]
after substituting (8a) and (8b). A comparison of (13a),(13b) and (13c) reveals that
\[(14)\quad v^D_i < v^N_i < (v^C_i = v^U_i, 0)\]
which implies that the D regime makes the insiders more aggressive than the N regime
which in turn makes the insiders more aggressive that the C or U regimes\(^{22}\).

The effective conjectures for the outsiders are obtained by substituting \(\rho_i = 0\), (9a)
and (9b) into (12) and then setting \(\lambda = 1\) to obtain
\[(15a)\quad v^C_O = v^D_O = \frac{-\theta(n-1-\theta)}{2 + \theta(n-2) - \theta^2}\]
for the C and D regimes or then setting \(\lambda = 0\) to obtain
\[(15b)\quad v^N_O = v^U_O = -\frac{\theta(n-1)}{2 + \theta(n-2)}\]
for the N and U regimes. Comparing (15a) and (15b) reveals that
\[(16)\quad v^N_O = v^U_O < v^D_O = v^C_O < 0\]
which implies that outsiders will be more aggressive if the insiders maintain intra-firm
competition between local units (as under the N or U regimes) than if the insiders were to
merge the local units (as under the C or D regimes).

Denote the symmetric equilibrium output in regime \(r\) as \(q^r_i\) for insiders and \(q^r_O\)
for outsiders. Substitute (1), (2), \(\alpha = a - c\) and \(v^r_i = v^r_i\), \(q_i = q^r_i\) for \(i = 1,2\) and \(v^r_i = v^U_i\),
\(q_i = q^r_O\) for \(i = 3,...,n\) into (11)' and re-arrange to obtain the following two equations
\[(17a)\quad \alpha - 2q^r_i - \theta(q^r_i + (n-2)q^r_O) - \theta v^r_i q^r_i = 0\quad r \in \{N,U,D,C\}\]
\[(17b)\quad \alpha - 2q^r_O - \theta(2q^r_i + (n-3)q^r_O) - \theta v^r_O q^r_O = 0\]

\(^{22}\) (14) states that \(v^D_i\) and \(v^N_i\) are both negative and are less than \(v^C_i\) and \(v^U_i\). It also states that \(v^C_i\) and \(v^U_i\)
could be positive or negative.
which can be solved to obtain

\[(18) \quad q_{ij}^r = \frac{\alpha(2 - \theta(1 - v_i^r))}{(2 + \theta(1 + v_i^r))(2 + \theta(n - 3 + v_i^r)) - 2(n - 2)\theta^2} \quad r \in \{N, U, D, C\} \quad i, j \in \{I, O\} \quad j \neq i\]

and which imply that the equilibrium price can be expressed as

\[(19) \quad P_i^r = c + q_{i}^r(1 + \theta v_i^r) \quad r \in \{N, U, D, C\} \quad i \in \{I, O\}\]

and thus that the equilibrium profits are given by

\[(20) \quad \pi_i^r = \left(1 + \theta v_i^r\right)^2 q_i^r \quad r \in \{N, U, D, C\} \quad i \in \{I, O\}\]

Furthermore by substituting (1) into (7) we obtain that \(w_i^r = P_i^r - q_i^r(1 + \lambda_i \theta)\) which when combined with (19) implies that the equilibrium incentive variable is

\[(21) \quad w_i^r = c + \theta q_i^r(v_i^r - \lambda_i) \quad r \in \{N, U, D, C\} \quad i \in \{I, O\}\]

Proposition 1: Under output competition the ranking of outputs and incentive variables is

\[(22) \quad q_i^D > (q_i^N, q_i^C) > q_i^U\]
\[(23) \quad w_i^D < (w_i^N, w_i^C) < w_i^U\]

for the insiders and by

\[(24) \quad q_i^D < (q_i^N, q_i^C) < q_i^U\]
\[(25) \quad w_i^D > (w_i^N, w_i^C) > w_i^U\]

for the outsiders.

Proof: See Appendix B.

Substitute (13) and (15) into (18) and then use simulation to obtain
Result 1: Under output competition the rankings of output for the N and C regimes are

\[ q_i^N > q_i^C \]  
\[ q_o^N < q_o^C \]

The comparative static effects of the various regimes are explained by the fact that the incentive contracting variables are used to shift rents. Since the sellers engage in output competition then rent shifting is achieved by allowing some degree of revenue maximization (i.e. set \( \alpha_i < 1 \) and thus \( w_i < c \)). This commits the seller to be more aggressive which then deters rival output and shifts rents away from rival firms. The extent to which a profit maximizing product division is prepared to distort local unit incentives away from profit maximization depends on: the number of rivals from whom the product division wishes to shift rents (upstream rivalry) and the reaction function response of each rival seller to expansions in output brought about by aggressive incentive competition (downstream responsiveness).

A merger that reduces rivalry by merging the product divisions serves to make the insiders less aggressive in their choice of incentives regardless of whether the merger alters the downstream structure. A merger that reduces insider downstream competition by merging the local units serves to make the insider seller less responsive to outsider incentive competition and more responsive to insider incentive competition. The former increases outsider incentive competition, whereas the latter reduces insider incentive competition if rivalry between insider product divisions is retained.

The first conclusion which follows from the above is thus that the standard effects of merger under output competition obtained under non-hierarchical regimes, namely that
insiders reduce output and outsiders expand output, arise under a hierarchical regime provided the corporate centre of merged entity decides to merge the product divisions. Moreover the standard comparative statics of merger are muted if the merger of the local units accompanies the merger of product divisions as the later serves to make outsiders less aggressive by virtue of making the insider sellers less responsive to outsider incentive competition.

The second conclusion is that the standard comparative static effects of merger are reversed if the corporate centre of the merged entity merges the local units but retains intra-firm competition between the product divisions. In this case there is no reduction in upstream rivalry but an increase in insider seller responsiveness to insider incentive competition, which makes insider product divisions more aggressive, and a reduced responsiveness of insider sellers to outsider incentive competition, which makes outsider product divisions less aggressive.

3. The profitability of merger

Proposition 2: Under output competition Strategy U is less profitable than strategy C.

*Proof:* See Appendix C.

The intuition behind Proposition 2 is straightforward. Strategy U is always less profitable than strategy C because both strategies have identical impacts on insider aggressiveness (i.e. both strategies eliminating incentive competition between insiders)
but the former results in more outsider incentive competition than the latter\textsuperscript{23} by virtue of making insider sellers more responsive to outsider incentive competition.

The remaining profit comparisons are derived via simulation by substituting the appropriate ECVs given in (13) and (15) into (18) and then into (20). The simulation results are reported in Table 1 and summarised in Results 2 and 3.

Table 1: Range of $\theta$ values for which N, D or C is the most profitable strategy for various levels of $n$ under output competition.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Number of products $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-4</td>
</tr>
<tr>
<td>N</td>
<td>No</td>
</tr>
<tr>
<td>D</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>0-1</td>
</tr>
</tbody>
</table>

“No” means that the strategy is not profitable for any value of $\theta$ between 0 and 1. $\sim 1$ means slightly less than 1. $I$ means a small interval from slightly less than 1 to 1. The above results hold for all values of $a$ and $c$.

Result 2: Under output competition the profitability of mergers results are as follows.

(i) In concentrated markets ($n \leq 4$) merger is always profitable.

(ii) For intermediate levels of concentration ($5 \leq n \leq 9$) merger is profitable provided products are sufficiently differentiated (i.e. $\theta < .86$).

(iii) In unconcentrated markets ($n \geq 10$) a merger is profitable for all levels of product differentiation except those approaching the perfect substitutes case.

\textsuperscript{23} The validity of this argument depends on the assumption of no demand uncertainty (Ziss 1999).
The main point of Result 2 is that merger is profitable for a wide variety of market environments in hierarchical settings in which products are differentiated. The increased profitability of merger in hierarchical regimes as compared with the standard non-hierarchical homogeneous product Cournot model is attributable to the enlarged set of complementary strategies that are available under hierarchical control but are not available in a Cournot regime, as well as the improved profitability of the output contraction strategy C when products are differentiated and strategic contracting is used.

Result 3: Under output competition the profitability of merger strategies are as follows.

(i) In concentrated settings \((n \leq 7)\) a profitable merger involves bringing about complete co-operation (strategy C).

(ii) In unconcentrated settings \((n \geq 8)\) a profitable merger involves bringing about either complete co-operation (strategy C) or downstream co-operation only (strategy D) depending on whether products are sufficiently poor or sufficiently close substitutes respectively.

The main point of Result 3 is as follows. Strategy D and C are complementary in that the former is an output expansion strategy which is more attractive in relatively competitive market environments (i.e. many products (high \(n\)) and close substitutes (high \(\theta\))), whereas the latter is an output contraction strategy which is relatively more profitable in relatively less competitive environments (i.e. few products (low \(n\)) and poor substitutes (low \(\theta\))). The intuition for this result is as follows. The profitability of credibly committing to an output expansion strategy is enhanced in more competitive market
settings because such settings result in a greater output contraction response of outsider firms, which then implies that the insider output expansion will have a muted deflationary impact on market price. Conversely the profitability of credibly committing to an output contraction strategy is enhanced in less competitive market settings because such settings result in a smaller output expansion response of outsider firms, which then implies that the insiders’ output contraction will have a greater inflationary impact on market price.

3. The welfare effects of merger

It is now assumed that the demand functions in (1) are derived from the utility maximizing behaviour of a representative consumer. A representative consumer utility function that is consistent with this assumption is

\[ U(q) = \sum_{i=1}^{n} q_i \left( a - \frac{1}{2} q_i + \theta Q_{-i} \right) \]

where \( q = (q_1, \ldots, q_n) \). Welfare is thus given by

\[ W(q) = U(q) - c \sum_{i=1}^{n} q_i \]

Substituting (27) into (28) then letting \( \alpha = a - c \) yields

\[ W(q) = \sum_{i=1}^{n} q_i \left( \alpha - \frac{1}{2} q_i + \theta Q_{-i} \right) \]

Substitute \( q_i = q_i^N = q_o^{N_i} = q^N \) and \( Q_{-i} = (n-1)q^N \) for \( i = 1, \ldots, n \) and then let \( nq^N = Q^N \) to obtain that the pre-merger level of welfare can be expressed as

\[ W^N = \alpha Q^N - \frac{q}{2} \left( Q^N \right)^2 - \left( \frac{1-q}{2} \right) Q^N q^N \]

Substitute \( q_i = q_i^r \) and \( Q_i = Q^r - q_i^r \) for \( i = 1, 2 \) and \( q_i = q_o^r \) and \( Q_i = Q^r - q_o^r \) for \( i = \ldots, r \).
Now let \(2q_i^r + (n-2)q_o^r = Q^r\) to get that the post-merger level of welfare for is

\[
W^r = \alpha Q^r - \frac{\theta}{2} \left( Q^r \right)^2 - \left( \frac{1-\theta}{2} \left( 2 \left( q_i^r \right)^2 + (n-2) \left( q_o^r \right)^2 \right) \right) \quad r \in \{ N, U, D, C \}
\]

Subtracting (30) from (31) yields that the change in welfare brought about by merger is

\[
\Delta W = \alpha (Q^r - Q^n) - \frac{\theta}{2} \left( Q^r \right)^2 - \left( \frac{1-\theta}{2} \left( 2 \left( q_i^r \right)^2 + (n-2) \left( q_o^r \right)^2 \right) - Q^n q^n \right) \quad r \in \{ N, U, D, C \}
\]

Substitute the ECVs given in (13) and (15) into the solution for output given in (18)\(^{24}\) and then use simulation to obtain the results reported in Table 2 and summarised in Result 4.

Table 2: Range of \(\theta\) values for which merger (i) has no effect on insider profit or welfare \((\Delta \pi = \Delta W = 0)\), (ii) raises insider profit and lowers welfare \((\uparrow \pi, \downarrow W)\) and (iii) raises both insider profit and welfare \((\uparrow \pi, \uparrow W)\), for various levels of \(n\), under output competition.

<table>
<thead>
<tr>
<th>Merger Outcome</th>
<th>Number of products (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-4</td>
</tr>
<tr>
<td>(\Delta \pi = \Delta W = 0)</td>
<td>No</td>
</tr>
<tr>
<td>(\uparrow \pi, \downarrow W)</td>
<td>0-1</td>
</tr>
<tr>
<td>(\uparrow \pi, \uparrow W)</td>
<td>No</td>
</tr>
</tbody>
</table>

“No” means that the outcome does not occur for any value of \(\theta\) between 0 and 1. \(~1\) means slightly less than 1. \(I\) means a small interval from slightly less than 1 to 1. The above results hold for all values of \(a\) and \(c\).

\(^{24}\) Note that \(q^n = q_i^N, Q^n = q_i^N, Q^N = q_o^N, n q_i^N, n q_o^N\) and \(Q^r = 2 q_i^r + (n - 2) q_o^r\).
Result 4: Under output competition the welfare effects of profitable mergers are that

(i) A profitable merger reduces welfare for all degrees of product differentiation in concentrated markets \((n \leq 11)\).

(ii) In unconcentrated markets \((n > 11)\) profitable merger enhances welfare for an intermediate range of product differentiation and reduces welfare otherwise.

(iii) Merger lowers welfare if in the post-merger setting the insiders adopt strategy C and may raise or lower welfare if they adopt strategy D.

Result 4 states that merger is welfare reducing if insiders commit to an output contraction strategy (i.e. strategy C) in the post-merger setting. If insiders commit to an output expansion strategy (i.e. strategy D) then welfare rises for an intermediate range of product differentiation and falls for all remaining cases. The adoption of the output expansion strategy does not necessarily imply an increase in welfare because products are differentiated which implies that the output expansion of the insiders does not necessarily compensate consumers for the output contraction of outsiders.

5. Price competition

If \(p_i\) denotes the price of product \(i\) and \(P_{-i} = \sum_{j \neq i} p_j\) then the demand function for product \(i\) is assumed to be given by

\[
q_i(p_i, P_{-i}) = a - bp_i + \theta P_{-i}
\]

where \(b = 1 + \gamma(1 - \frac{1}{n})\) and \(\theta = \frac{\gamma}{n}\ i = 1, \ldots, n\)

and where \(0 \leq \gamma < \infty\) denotes the substitutability parameter which is zero if goods are

---

25 The demand formulation is the same as Deneckere and Davidson (1985).
unrelated and approaches infinity when goods are perfect substitutes. The above specification implies that \( b > (n - 1)\theta \). The profits generated by profit \( i \) are thus given by

\[
\pi_i = (p_i - c)q_i(p_i, P_i) \quad i = 1, \ldots, n
\]

A local unit manager’s pay-off for selling product \( i \) is given by

\[
R_i = (p_i - w_i)q_i(p_i, P_i) \quad i = 1, \ldots, n
\]

**Price stage.** A general expression for the objective function of each local unit is given by (6). Substituting (33) and (35) into (6) and then differentiating with respect to \( p_i \) yields the following first order condition

\[
a - 2bp_i + \theta P_i + bw_i + \lambda_i(p_j - w_j) = 0 \quad i = 1, \ldots, n
\]

which indicates that the market power externality internalised by the managers of a merged local unit is proportional to the mark-up of the products involved in the merger. In other words an increase \( p_i \) raises the demand for product \( j \) and thus raises the profits for product \( j \) in proportion to the latter’s mark-up.

**Incentive stage.** Following the output competition analysis yields that the prices for both insiders and outsiders are highest when the insiders merge both the local units and the product divisions. The result occurs because the merger of the product divisions softens insider incentive competition by removing upstream rivalry whereas the merger of the local units softens outsider competition by making the insiders more accommodating to outsider price increases. The increased insider responsiveness arises because outsider induced increases in insider prices raises the market power externality and results in insider local units further raising price, provided they are merged. The increased accommodation of price increases by insiders results in the outsider product divisions inducing their local units to be less aggressive.
Proposition 3: Under price competition the ranking of prices and incentive variables are

\[
\begin{align*}
(37a) \quad & p_i^C > p_i^U > p_i^N \quad \text{and} \quad p_i^C > p_i^D \\
(37b) \quad & p_o^C > p_o^U > p_o^N \quad \text{and} \quad p_o^C > p_o^D \\
(37c) \quad & w_o^C > (w_o^U, w_o^N, w_o^D)
\end{align*}
\]

*Proof*: See Appendix D.

**Profit analysis.** The insider profit is highest when both local units and product divisions are merged because each of these mergers results in a profit increase. The merger of product divisions results in the internalisation of incentive competition which then implies higher insider profits arising from the joint profit maximizing strategic response to the reaction functions of the outsider local units. The merger of insider local units yields a further rise in insider profits as it induces outsiders to be less aggressive.

Proposition 4: Merger is always profitable and involves the adoption of the C strategy.

*Proof*: See Appendix E.

6. Conclusions and relevant literature

The main conclusion of this paper is that under output competition merger in a hierarchical setting allows insiders to commit to either an output expansion or to an output contraction strategy. The former is more profitable in market settings involving low concentration *and* intermediate degrees of product differentiation and is brought about by retaining competition in the incentive contracting stage and eliminating
competition at the output stage. The latter is more profitable if market concentration is sufficiently high or the degree of product differentiation is sufficiently low and involves merger at both the incentive contracting and output stages. The complementarity of the output contraction and expansion strategies imply that merger is more profitable in hierarchical settings than in the standard Cournot setting analysed by most of the papers in the mergers literature, in which firms only have access to an output contraction strategy. Reduced profitability of merger in hierarchical settings also occurs because product differentiation and strategic contracting considerably enhance the profitability of the output contraction strategy. The ability of firms to commit to an output expansion strategy also overturns the standard Cournot and differentiated Bertrand result that merger reduces welfare in the absence of cost efficiency gains. In particular the current paper shows that merger can enhance welfare in unconcentrated market settings, albeit for a small range of product differentiation.

A second conclusion of this paper is that mergers of hierarchical firms engaged in incentive contracting and differentiated price competition are always profitable and

---

26 In particular it has been shown that if marginal cost is constant then a merger which does not create cost efficiencies is profitable in a Cournot setting only if it involves more than 50 percent of the firms in the industry (see Levin 1990, Gaudet and Salant 1991 or Cheung 1992). Moreover a 50 percent market share is a necessary but not sufficient condition for profitable merger. For example, if demand is linear then Salant, Switzer and Reynolds (1983) have shown that profitable merger requires the merging parties’ market share to be at least 80 percent. The main reason behind the dismal profitability results is that the output contraction of the merging firms is met with an output expansion of the non-merging firms. Merger profitability is enhanced if the response of non-merging firms is muted by virtue of rising marginal cost (Perry and Porter 1985) or products being differentiated (Deneckere and Davidson 1983).

27 González-Maestre and López-Cuñat (2001) and Ziss (2001) consider the effect of strategic contracting on the profitability of merger when products are identical and intra-firm competition is absent. The current paper extends the results in these papers by considering both intra-firm competition and product differentiation and shows that these features result in a further expansion of the profitability of merger under a strategic contracting regime.

28 Output contraction strategies have also been shown to be more profitable in Stackelberg settings by Huck, Konrad and Müller (2001) who show that a merger between a leader and a follower in which the follower is shutdown is always profitable.

29 Output expansion strategies have also been analysed by Daughety (1990) who considers a Stackelberg setting in which two followers become a leader.
involve the elimination of all intra-firm competition. This result extends the results obtained by Deneckere and Davidson (1985) for the case in which there is no strategic contracting, and shows that mergers are always profitable and anti-competitive regardless of whether or not firms are engaged in strategic incentive contracting. The above results will now be compared to those in the intra-firm competition and vertical restraints literatures.

The idea of insiders maintaining some degree of intra-firm competition in the post-merger setting has been explored by Huck, Konrad and Müller (2004) and Creane and Davidson (2004) who show that insiders can commit to an output expansion strategy by organizing the timing of production so that competing insiders gain first mover advantages over other insiders but not over outsiders. It is shown that such a strategy may be profitable under output competition but is never profitable under price competition. A second group of papers look at maintaining intra-firm competition in order to improve the bargaining position of an insider that attempts to monopolize the industry via acquisition. The main result in this literature is that monopolization is either the exception (Kamien and Zang 1990) or the norm (Tombak 2002) depending on the nature of the bargaining game and on whether or not there are cost efficiency gains. The current paper differs from all of the above papers in that it allows for incentive competition whereas the above

---

30 Deneckere and Davidson (1985) show that their results hold provided reaction functions are upward sloping and provided merger shifts out the insider reaction function. These conditions hold for fairly general demand functions. In addition to the above assumptions the price competition results of the current paper also require that merger softens rival incentive competition. The intuition as to why merger softens rival incentive competition is presented in Section 5 of the text and seems fairly general, although the demand restrictions required for this result to hold would be fairly complex and thus place a potentially important caveat on the results.

31 Another related group of papers are those in the divisionalization literature that assume that firms can internally create autonomous divisions. The current paper differs from this literature in that it assumes that divisions cannot be created, but can only be acquired. The basic divisionalization model considers identical firms and homogeneous products (Baye et al. 1986a,b, Corchon 1991, Polasky 1990) and has been extended to consider product differentiation (Ziss 1998) and incentive contracting (Ziss 1999) and has been applied to consider entry deterrence (Veendorp 1991, Schwartz and Thompson 1986).
papers assume that each autonomous division maximizes division profits. Secondly, the current paper introduces the idea of a firm delegating incentive contracting responsibilities to managers of a product division so as to create intra-firm competition at the contracting stage. A third conclusion of this paper is intra-firm competition at the contracting stage has strategic merit if firms engage in output competition but not if they engage in price competition.

The results of the current paper can be compared to those obtained in the vertical restraints literature because the manufacturer-retailer setting considered in the latter exactly parallels the ‘product divisions-local units’ setting in this paper. Previous papers in this literature have analysed the merits of eliminating output competition by using non-exclusive dealing (or common agency) as opposed to exclusive dealing in a duopoly context, and have argued that common agency is more profitable than exclusive dealing if and only if it facilitates collusion between manufacturers (Lin 1990, Bernheim and Whinston 1985). A fourth conclusion of the paper is thus that the above result continues to hold in non-duopoly contexts if firms engage in price competition or they engage in output competition and markets are sufficiently concentrated. If firms choose output, markets are unconcentrated and there exists an intermediate degree of product differentiation then common agency which maintains rivalry between insider manufacturers is more profitable than both exclusive dealing and common agency which eliminates rivalry between manufacturers.

A final conclusion is that if firms engage in output competition then merger is a better way of eliminating output competition than the adoption of common agency by independent firms using arms length contracting. The hierarchical control afforded by a
merger is an advantage over arms-length contracting in that it: reduces transactions costs and bargaining\textsuperscript{32}; removes some of the strategic ambiguity associated with arm’s length contracting and; expands the set of plausible strategic options. In particular the adoption of common agency by independent manufacturers may or may not facilitate collusion amongst the manufacturers depending on assumptions of the contracting game\textsuperscript{33}. As a result common agency via contract between independent manufacturers allows for either the elimination of upstream and downstream competition between insiders, or the elimination of downstream competition only, but \textit{not both}. On the other hand merger in a hierarchical setting affords a firm the opportunity to credibly commit to both strategies.

\textsuperscript{32} In particular O’Brien and Shaffer (1993) assume that the common retailer can acquire countervailing bargaining power by refusing to sell the product of one of the manufacturers.

\textsuperscript{33} Bernheim and Whinston (1985) assume that manufacturers who adopt common agency set the fixed fee component of the two part tariff for each product so as to satisfy the common retailer’s \textit{overall} break-even constraint. The latter assumption then implies that the wholesale price component of the two-part tariff for each product is set so as to maximize the sum of manufacturing and retail profit associated with both products and thus \textit{facilitates collusion} between manufacturers. On the other hand Lin (1990) shows that if the fixed fee component of the two part tariff for each product is set so as to satisfy the \textit{individual} break even constraint for \textit{each} product sold by the common retailer then common agency \textit{facilitates competition} between manufacturers.
References


Appendix A: Derivation of output stage comparative statics

Substitute \( \lambda_i = 0 \) and \( Q_i = Q - q_i \) into (7) to get (A1). Substitute \( \lambda_i = \lambda \) into (7), sum (7) over all \( i = 1, \ldots, n \) to get (A2), and over all \( i = 1, 2 \) to get (A3). Note that

\[
\sum_{i=1}^{n} Q_i = (n-1)Q
\]

and \( Q_f = q_1 + q_2 \)

\[
\text{(A1)} \quad a - (2 - \theta q_i - \theta Q) = w_i \quad \text{for} \quad i = 3, \ldots, n
\]

\[
\text{(A2)} \quad na - (2 + \theta(n-1))Q - \lambda \theta Q_f = \sum_{k=1}^{n} w_k
\]

\[
\text{(A3)} \quad 2a - 2 \theta Q - (2 - \theta(1 - \lambda))Q_f = w_1 + w_2
\]

Totally differentiate any one of the \( n-2 \) equations given in (A1) to obtain (A4) and then totally differentiate (A2) and (A3) to obtain (A5) and (A6) respectively

\[
\text{(A4)} \quad \begin{bmatrix} \theta & 0 & 2-\theta \\ 2 + \theta(n-1) & \lambda \theta & 0 \\ 2\theta & 2 - \theta(1 - \lambda) & 0 \end{bmatrix} \begin{bmatrix} dQ \\ dQ_f \\ dq_i \end{bmatrix} = \begin{bmatrix} dw_i \\ \sum_{k=1}^{n} dw_k \\ dw_1 + dw_2 \end{bmatrix} \quad i \in \{3, \ldots, n\}
\]

The above system can be solved to obtain (8a) and (9a) as well as

\[
\text{(A7)} \quad \frac{\partial Q}{\partial w_i} = -\frac{(2 - \theta)^2}{\nabla} \quad i = 1, 2
\]

\[
\text{(A8)} \quad \frac{\partial Q}{\partial w_i} = -\frac{(2 - \theta)(2 - \theta(1 - \lambda))}{\nabla} \quad i = 3, \ldots n
\]

Subtract (8a) from (A7) to get (8b) and (9a) from (A8) to get (9b). Substitute \( Q_i = Q - q_i \), \( q_j = Q_f - q_i \) and \( \lambda_i = \lambda \) into (7) to get

\[
\text{(A9)} \quad a - (2 - \theta(1 + \lambda))q_i - \theta Q - \lambda \theta Q_f = w_i \quad i = 1, 2
\]

Implicit differentiation yields

\[
\text{(A10)} \quad \frac{\partial q_i}{\partial w_i} = -\left(\frac{1}{2 - \theta(1 + \lambda)}\right)\left(\theta \left(\frac{\partial Q}{\partial w_i} + \lambda \frac{\partial Q_f}{\partial w_i}\right) + 1\right) \quad i = 1, 2
\]

Substitute (8a) and (A7) into (A10) to get (8c). Subtract (8c) from (A7) to get (8d).
Appendix B: Proof of Proposition 1

Equilibrium outputs are obtained by solving (17a) and (17b). These two equations are rearranged below to obtain what shall be referred to as the “effective reaction functions (ERFs)” of insiders and outsiders respectively.

(B1) \[ \text{ERF}_I : q_I^r = \frac{\alpha - \theta(n - 2)q_O^r}{2 + \theta(1 + v_I^r)} \quad r \in \{N, U, D, C\} \]

(B2) \[ \text{ERF}_O : q_O^r = \frac{\alpha - 2\theta q_I^r}{2 + \theta(n - 3 + v_O^r)} \quad r \in \{N, U, D, C\} \]

The insider ERF (denoted $\text{ERF}_I$) measures the insider output response to outsider output that is induced by the insider’s choice of incentive variable. A similar definition applies to the outsider ERF (denoted $\text{ERF}_O$). The values of ECVs given in (13) and (15) imply that: both insider and outsider ERFs have a negative slope under all regimes and for all allowable parameter values, the $q_O$ intercept of the outsider ERF lies below its insider counterpart for all regimes and for all allowable parameter values indicating that insiders always produce positive output and, the $q_I$ intercept for the insider ERF lies below its outsider counterpart for all allowable parameter values and for all but the D regime indicating that outsiders produce positive output for all allowable parameter values for all but the D regime. Under the D regime the $q_I$ intercept for the insider ERF is less than or equal to its outsider counterpart depending on whether $\theta < 1$ or $\theta = 1$. The above implies that outsiders produce positive or zero output depending on whether $\theta < 1$ or $\theta = 1$.

From (B1) and (B2) it follows that reductions in a firm’s ECV will induce the firm’s ERF to rotate outward. The comparison of insider ECVs in (14) implies that the insider ERF rotates outward in going from an N to a D regime and rotates inward in going from an N to a C or U regime. Similarly, a comparison of outsider regime ECVs
given in (16) indicates that outsider ERF rotates outward in going from a C or D regime to an N or U regime. The above analysis implies that the relative positions of the ERFs are as indicated in Figure 1 which then implies the output ranking given in (22) and (24).

Substituting $\lambda_i = 0$ into (21) yields that outsider incentives becomes more aggressive if outsider output rises and/or the outsider ECV becomes more negative, otherwise it becomes less aggressive. The ranking of outsider output and ECVs given in (24) and (16) respectively reveal that at least one of the conditions for more (resp. less) aggressive incentive contracting is satisfied in going from an N or C regime to a U (resp. D) regime thereby implying the outsider incentive variable ranking given in (25).

From (21) it follows that insider incentives becomes more aggressive if insider output rises and/or the insider ECV becomes more negative and/or the insiders go from a competing local units regime ($\lambda_i = 0$) to a single local unit regime ($\lambda_i = 1$), otherwise it becomes less aggressive. The ranking of insider output and ECVs given in (22) and (14) respectively reveal that at least two of the conditions for more (resp. less) aggressive incentive contracting is satisfied in going from an N or C regime to a D (resp. U) regime thereby implying the insider incentive variable ranking given in (23). Q.E.D.

Appendix C: Proof of Proposition 2

The C and U strategies have the same EVC (see (13c)) but the C strategy yields higher equilibrium output than the U strategy (Proposition 1) which then implies, from (20), that Strategy C is the more profitable of the two strategies. Q.E.D.
Appendix D : Proof of Proposition 3

Let \( P = \sum_{i=1}^{n} p_i \) and \( P_I = p_1 + p_2 \) and apply the analysis in Appendix A to (36) to obtain price versions of (8) and (9). Use the price version of (12), (8) and (9) and the procedure outlined in the text between equations (13a) – (15b) to obtain the following ECVs

(D1) \[ v_i^N = \frac{\theta(n-1)}{2b - \theta(n-2)} \]

(D2) \[ v_i^D = \frac{\theta(n-2)(2b-\theta)}{(2b-\theta)(2b-\theta(n-2)) - 2\theta^2} \]

(D3) \[ v_i^C = v_i^U = \frac{2b + \theta(n-1)}{2b - \theta(n-3)} \]

(D4) \[ v_o^C = v_o^D = \frac{\theta(b(n-1)+\theta)}{2b^2 - \theta(b(n-2) - \theta^2} \]

(D5) \[ v_o^N = v_o^U = \frac{\theta(n-1)}{2b - \theta(n-2)} \]

which can be ranked using \( b > \theta(n-1) \) to get

(D6) \[ v_i^C = v_i^U > 1 > v_i^N > v_i^D > 0 \]

(D7) \[ v_o^D = v_o^C > v_o^N = v_o^U > 0 \]

It will prove convenient to solve for the retail mark-up rather than for retail prices. The mark-up version of insider and outsider ERFs which follow from (11)’ are given by

(D8) \[ ERF_I: p_i^r - c = \frac{\alpha + \theta(n-2)(p_o^r - c)}{2b - \theta(1 + v_i^r)} \quad r \in \{N, U, D, C\} \]

(D9) \[ ERF_O: p_o^r - c = \frac{\alpha + 2\theta(p_i^r - c)}{2b - \theta(n-3 + v_o^r)} \quad r \in \{N, U, D, C\} \]

respectively, where \( \alpha = a - c(b - \theta(n-1)) > 0 \) in order for \( q_i > 0 \) when \( p_i = c \) for all \( i \). \( p_i^r \) > \( c \) for all \( i \) and \( r \) because (D1) – (D9) imply that the ERFs have positive intercepts and
slopes between 0 and 1. The relative positioning of the ERFs in figure 2 follow from (D6) – (D9) and imply (37a) and (37b). The price version of (11)' yields

\[(D10) \quad q_i' = (b - \theta v_i')(p_i' - c) \quad r \in \{N, U, D, C\} \quad i \in \{I, O\}\]

Since \(p_i' - c > 0\) and given that (D1) – (D5) yield \(b > \theta v_i'\) then (D10) implies that \(q_i' > 0\) for \(i = I, O\) and \(r \in \{N, U, D, C\}\). (33), (36) and (D10) yield

\[(D11) \quad w_i' = c + \frac{\theta}{b - \theta \theta} (p_i' - c)(v_i' - \lambda_i) \quad r \in \{N, U, D, C\} \quad i \in \{I, O\}\]

(37c) follows from substituting \(\lambda_i = 0\) into (D11) and then using (37b) and (D7).

Appendix E: Proof of Proposition 4

Substitute \(p_i = p_O\) and \(P_i = 2p_I + (n - 3)p_O\) into (36) and then re-arrange to obtain that the slope of the outsider local unit reaction function\(^{34}\) (referred to as \(RF_O'\)) is

\[(E1) \quad \left. \frac{\partial p_0}{\partial p_I} \right|_{RF_O'} = \frac{2\theta}{2b - \theta(n - 3)} \quad r \in \{N, U, D, C\}\]

Totally differentiate \(\pi_1 + \pi_2\), set \(dp_i = dp_I\) for \(i = 1, 2\) and \(dp_i = dp_O\) for \(i = 3, \ldots, n\), substitute (11)' (i.e \(\frac{\partial \pi_i}{\partial p_i} = -v_i' \frac{\partial \pi_i}{\partial P_i}\) for \(i = 1, 2\)) and \(\frac{\partial \pi_1}{\partial P_1} = \frac{\partial \pi_2}{\partial P_2}\) (symmetry), set the total derivative to zero to obtain that the equilibrium slope of the insider isoprofit contour is

\[(E2) \quad \left. \frac{\partial p_0}{\partial p_I} \right|_{\pi_i' + \pi_2'} = \frac{v_i' - 1}{n - 2} \quad r \in \{N, U, D, C\}\]

(E2) and (D7) imply (E3). Substituting (D3) into (E2) yields (E4)

\(^{34}\) Note that \(RF_O'\) is drawn for given \(w_O'\) whereas \(ERF_O'\) takes into account changes in \(w_O'\). Furthermore \(RF_O'\) and \(ERF_O'\) go through the same equilibrium points but have different slopes. An increase in \(v_O'\) causes an increase in \(w_O'\) which causes a parallel upward shift of \(RF_O'\). An increase in \(v_O'\) causes \(ERF_O'\) to shift up and become steeper.
(E3) \[ \frac{\partial p_0}{\partial p_1}_{\pi'_i + \pi'_j} < 0 < \frac{\partial p_0}{\partial p_1}_{RF^O} \quad r \in \{N, D\} \]

(E4) \[ \frac{\partial p_0}{\partial p_1}_{\pi'_i + \pi'_j} = \frac{\partial p_0}{\partial p_1}_{RF^O} \quad r \in \{U, C\} \]

(E3) and (E4) state that the insider isoprofit and outsider local unit reaction function are tangent if insiders adopt strategy C or U but not if they adopt strategy N or D. Insider profits rise when moving from the strategy N, D or U to strategy C because outsiders increase \( w_O \) (Proposition 3) which shifts \( RF^O \) upwards, and because insiders choose a tangency point along the new \( RF^O \). The increase in profit from strategy N to strategy C is illustrated in Figure 3 by a movement to a higher isoprofit contour. The diagrams illustrating the increase in profits from strategy D or U\(^{35}\) to strategy C are similar.

\(^{35}\) One slight difference is that the isoprofit will be tangent to \( RF^O \) at the strategy U equilibrium point.
Figure 1: Output comparisons using ERFs
Figure 2: Price comparisons using ERFs
Figure 3: Profit comparisons using RFs and isoprofit contours