Intertemporal Substitution in Macroeconomics: Evidence from a Two-Dimensional Labor Supply Model with Money*

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Abstract

The representative agent model of aggregate labor supply, and its cornerstone, the hypothesis of intertemporal substitution in labor supply, have a history of empirical failure when confronted with aggregate time-series data. We show that a model in which the representative agent has preferences defined in terms of consumption, leisure in the workweeks, leisure in non working weeks and real balances overcomes most difficulties previously encountered by empirical studies of intertemporal Euler equations. The overidentifying restrictions implied by the model are far from rejected. The estimated parameters of preferences are generally stable and meaningful. Furthermore, the estimated wage elasticities of labor supply are much higher than previously found in the literature.

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1. Introduction

Following the seminal work of Lucas and Rapping (1969), a large class of macroeconomic models have relied on the hypothesis of intertemporal elasticity of substitution to explain fluctuations in aggregate employment in postwar industrialized economies.\footnote{This includes real business cycle models, dynamic general equilibrium (DGE) models with sticky nominal prices, models of labor market search, and limited-participation models, among others. In these theoretical models, fluctuations in aggregate employment are modelled as movements along a labor supply curve.} This hypothesis, which is the cornerstone of representative agent models of aggregate labor supply, claims that the cyclical variations in employment result from the optimal decisions of a representative household that substitutes hours worked intertemporally in response to transitory movements in wage and interest rates. Thus, cyclical employment fluctuations are modelled as movements along a labor supply curve. Paradoxically, studies that use aggregate data to test the intertemporal-substitution-in-labor-supply (ISLS) hypothesis, find, in general, no supportive evidence, and often reach negative conclusions.\footnote{Card (1994) concludes that microeconomic evidence does not support the ISLS hypothesis either.} The evidence in Mankiw, Rotemberg, and Summers (1985), for example, is typical of the problems encountered with the representative agent model of aggregate supply over the years: the overidentifying restrictions implied by the theory are almost always rejected, the estimated parameters of preferences are highly unstable, and the utility function is often not concave, leading to elasticities of the wrong sign [see also Hall (1980), Altonji (1982), Killingsworth and Heckman (1986), Pencavel (1986), and Eichenbaum, Hansen and Singleton (1988)].

Alogoskoufis (1987) argues that one possible explanation for the apparent lack of success of representative agent models of aggregate labor supply is that most studies have measured movements in labor supply solely by variations in weekly hours or
intensive margin. Yet, fluctuations in employee hours should also reflect changes in the number of employees or extensive margin. Indeed, estimating an equation for log-linear labor supply in which work effort is measured by the number of workers, rather than by the hours worked per person, Alogoskoufis (1987) obtains findings which lend more credence to the ISLS hypothesis. However, no attempt is made to formally incorporate the optimal choices of employment at both margins into a single coherent framework or to test the overall plausibility of the model per se.

Our paper proposes a tractable framework which allows a formal test of the ISLS hypothesis in the context of a representative agent framework that explicitly accounts for the adjustment of employment at the intensive and extensive margins. Further, it explores the intertemporal labor-supply relationships implied by monetary economy models, in line with several recent microfounded models which account for money, by assuming that the representative agent derives utility from holding real balances. Based on U.S. postwar data, we show that a model with these features overcomes most problems previously encountered by empirical studies of intertemporal Euler equations.

Our model borrows some inspiration from the RBC literature, especially from the work of Bils and Cho (1994) and Cho and Cooley (1994). These authors develop RBC models which permit adjustment along both labor supply margins by realistically making hours per week and weeks per year of leisure imperfect substitutes. This feature is achieved by assuming that agents, who otherwise have identical preferences and opportunity sets, have a fixed cost associated with labor supply that depends on the fraction of days in a period that they will be employed.\(^3\) While our setting is broadly consistent with theirs, our main objective is not to study the business cycle properties of a calibrated version of the model as they do, but rather, to estimate the parameters

\(^3\)Choi and Rogerson (1988) also allow adjustment at both margins by introducing heterogeneity in the opportunity sets of household decision makers.
of preferences of the two-dimensional aggregate labor supply model and to assess the overall plausibility of the model.

The intertemporal Euler equations implied by our model, and some alternative models discussed below, are estimated with the Generalized Method of Moments. While estimating the intertemporal Euler equations, we assume that the representative household can hold different types of riskless and/or risky assets: namely, bonds, shares, or both. We also look at the sensitivity of our findings to different wage rates. Our main findings can be summarized as follows.

First, we find that the distinction between leisure in the workweeks and leisure in non-working weeks is strongly supported by aggregate data; each type of leisure contributes substantially to the representative household’s utility according to their estimated shares in the utility function. The inclusion of real balances in the utility function also finds empirical support. Importantly, the overidentifying restrictions implied by the two-dimensional ISLS models with money easily survive formal, statistical, tests. These findings are not too sensitive to specific measures of asset returns and wages.

Also, according to our estimates, the labor supply elasticity with respect to transitory movements in wages is higher for the weekly hours than for the weeks worked. Combining the two elasticities, our model delivers a wage elasticity of labor supply for total hours worked that, at the lowest, is 1.54 and can be as high as 2.15. In comparison, the highest wage elasticity of labor supply previously found with aggregate time-series data was around unity [see Alogoskoufis (1987) and Cho, Merrigan and Phaneuf (1998)]. We also find that asset returns have a significant independent influence on total hours worked.

In an effort to better identify the key factors to our findings, we compare the results
of the two-dimensional ISLS models with real balances with those of alternative ISLS models, including one-dimensional ISLS models with and without money, and two-dimensional ISLS models without money. ISLS models that feature only one type of leisure, whether they include real balances or not, do not perform well; their problems range from non-concavity of the estimated utility functions to systematic rejection of the overidentifying restrictions implied by these models. The two-dimensional ISLS models that exclude real balances are also systematically rejected, even though they perform better than one-dimensional ISLS models.

In related work, Rogerson and Rupert (1991) use microeconomic data from the PSID for married males between the ages of 25 and 65 for the years 1976–82 to assess the household’s willingness to substitute work intertemporally. In their model, workers also choose weeks of work and hours of work per week. Focusing on the choice of weeks, they show that allowing some individuals to be at a corner solution for weeks worked, as a result of working year round, may substantially increase the intertemporal substitution elasticity of labor supply. Unlike Rogerson and Rupert (1991), we employ aggregate time series rather than microeconomic data, provide estimates of the preferences parameters, formally test the overidentifying restrictions implied by the theoretical models, and do not focus on individuals who are at a corner solution for weeks worked. Still, our evidence is consistent with theirs, suggesting that the representative agent model of labor supply is more plausible when labor supply decisions are made two-dimensional.

The rest of this paper is organized as follows. Section 2 constructs the two-dimensional ISLS model with real balances and the alternative ISLS models. Section 3 describes the econometric methods and data used to estimate the models. Section 4 reports the empirical results of the two-dimensional ISLS model with real balances.
and compares them with those of alternative ISLS models. Section 5 reports the intertemporal elasticities implied by the two-dimensional labor supply model with money. Section 6 offers concluding remarks.

2. The Models

2.1 A two-dimensional aggregate labor supply model with real balances

Actual movements in employment can be measured by the sum of changes in labor force participation and in the fraction of weeks that workers in the labor force are at work. Following Bils and Cho (1994) and Cho and Cooley (1994), we focus on the latter component. Hence, we do not consider how workers and nonworkers differ in compensation and consumption in a competitive equilibrium. Further, some evidence suggests that movements in employment rates for a given labor force are cyclically more important than movements in labor force participation. For instance, Bils and Cho (1994) report postwar U.S. evidence indicating that a 1 percent deviation from trend in employment has resulted, on average, from a 0.8 percent change in the weeks at work during the year for a given sized workforce and from a 0.2 percent change in the fraction of individuals who worked during the year.\footnote{In a similar vein, Hall and Lilien (1986) have shown that changes in the unemployment rate have contributed 70 percent of the variation in total employment, whereas changes in labor force participation have contributed 30 percent.} Also, using microeconomic data from the Michigan Panel Study of Income Dynamics (PSID), they show that the distinction between leisure time in workweeks and weeks off in the year seems to be empirically supported.

Hence, while our model does not seek to establish a distinction between people who are in or out of the labor force, it assumes that weekly hours worked and weeks worked
are imperfect substitutes. Specifically, let us consider an economy inhabited by a large number of identical households whose preferences are defined over expected streams of consumption, \(c_t\), leisure in the workweeks, \(l_{1t}\), leisure in the non-working weeks, \(l_{2t}\), and real balances, \(m_t\). The representative household maximizes the following lifetime expected discounted utility:

\[
E_t \sum_{t=0}^{\infty} \beta^t u \left( c_t, l_{1t}, l_{2t}, m_t \right),
\]

(1)

where \(u_c, u_{l_1}, u_{l_2}, u_m > 0\) and \(u_{cc}, u_{l_1l_1}, u_{l_2l_2}, u_{mm} < 0\), \(\beta\) is the rate of time discount and \(E_t\) denotes the mathematical expectation.

In this two-dimensional aggregate labor supply framework, money holding generates a positive utility. One possible interpretation is that money facilitates consumption and reduces the time allocated for shopping.\(^5\) Further, as we will see below, the presence of money in the utility function also implies an extra dynamic Euler equation associated with the optimal choice of real balances that can be used in estimating the model.

In each period, the representative household’s total time endowment is a product of the number of weeks, \(E\), and the hours available in the week, \(H\), with both \(E\) and \(H\) taken as given. The representative household allocates its time during the workweeks, \(e_t\), to work \(h_t\) (the weekly hours per worker), leisure \(l_{1t}\), and a fixed cost associated with labor supply \(\tau\). In non-working weeks, time is entirely devoted to leisure \(l_{2t}\). Hence, leisure in the workweeks is

\[
l_{1t} = (H - h_t - \tau) e_t,
\]

(2)

while leisure in non-working weeks is given by

\[
l_{2t} = (E - e_t) H.
\]

(3)

\(^5\)Feenstra (1986) establishes a functional equivalence between liquidity costs and the utility of money.
The representative household ranks alternative streams of consumption, leisure in
the workweeks, leisure in non-working weeks, and real balances, using the following
constant relative risk aversion (CRRA) utility function:

\[
E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t^{\alpha_c} l_{1t}^{\alpha_l} l_{2t}^{\alpha_l} m_t^{\alpha_m})^{1-\sigma} - 1}{1 - \sigma} \right],
\]  

(4)

where \( \alpha_c + \alpha_l1 + \alpha_l2 + \alpha_m = 1 \). Concavity requires that \( \alpha_c, \alpha_l1, \alpha_l2, \) and \( \alpha_m \) have positive
signs, and that the product of each of these exponents with \( (1 - \sigma) \) be less than unity.\(^6\)

The representative consumer’s allocations must satisfy the following sequence of
budget constraints:

\[ p_t c_t + p_t m_t + b_t + s_t \leq w_t h_t e_t + r_{bt} b_{t-1} + r_{st} s_{t-1} + p_{t-1} m_{t-1} + tr_t, \]

(5)

where \( p_t \) is the price of the consumption good in period \( t \), \( b_t \) and \( s_t \) are bonds and
shares (riskless and risky assets, respectively) purchased in period \( t \), \( w_t \) is the after-tax wage rate, \( r_{bt} \) is the after-tax riskless asset return, \( r_{st} = \frac{p_{st} + d_{st}}{p_{st-1}} \) is the after-tax risky asset return, \( p_{st} \) is the price of shares, \( d_{st} \) is the dividends earned by shares in period \( t \),
and \( tr_t \) is a lump-sum money transfer. The budget constraint (5) describes a situation
where the representative consumer holds bonds and shares. However, in deriving the
intertemporal Euler equations below, we consider that the representative consumer can
possibly hold bonds, shares, or both.

The representative household chooses \( c_t, h_t, e_t, m_t, b_t, \) and \( s_t \) that maximize expected
total discounted utility (4) subject to (5), while taking into account (2) and (3). The
\(^6\)Eichenbaum, Hansen, and Singleton (1988) describe in more detail the desirable characteristics
of the above form of utility function.
first-order conditions are:

\[ \alpha_c (c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}} m_t^{\alpha_m})^{1-\sigma} c_t^{-1} = \lambda_t p_t = 0, \]  
\[ \alpha_{l1} (c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}} m_t^{\alpha_m})^{1-\sigma} l_{1t}^{-1} = \lambda_t w_t = 0, \]  
\[ \alpha_{l2} (c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}} m_t^{\alpha_m})^{1-\sigma} l_{2t}^{-1} - \lambda_t w_t (1 - \frac{\tau}{H}) = 0, \]  
\[ \alpha_m (c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}} m_t^{\alpha_m})^{1-\sigma} (m_t p_t)^{-1} - \lambda_t + \beta E_t \lambda_{t+1} = 0, \]

where \( \lambda_t \) is the multiplier associated with the intertemporal budget constraint, which also represents the marginal utility of wealth at date \( t \).\(^7\)

Assuming that \( \lambda_t \) follows a martingale process (see MacCurdy 1985), we derive the following intertemporal Euler equations:

\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \times \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t r_{jt+1}}{p_{t+1}} - 1 \right\} = 0, \]  
\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \times \left( \frac{l_{1t+1}}{l_{1t}} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0, \]
\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \times \left( \frac{l_{2t+1}}{l_{2t}} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0, \]

\(^7\)From equation (9), we can derive the standard money-demand function in which real balances depend on consumption and interest rates.
\[
E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \times \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t}{p_{t+1}} + \frac{\alpha_m c_t}{\alpha_c m_t} - 1 \right\} = 0, \tag{15}
\]

with \( r_{jt+1}, j = b, s \) or both, denoting the assets the representative agent holds. With \( j = b \), the representative consumer holds only riskless assets, and hence, there are four Euler equations that can be estimated jointly to retrieve the preference parameters of the model. The same applies if \( j = s \), so that the representative consumer holds only risky assets. With \( j = b \) and \( s \), the representative consumer holds riskless and risky assets, which implies seven intertemporal Euler equations.

Conditions (12)–(14) state that the marginal cost of one unit of consumption or of an hour of both types of leisure in period \( t \) must equal the expected marginal benefit in period \( t + 1 \). Condition (15), which follows directly from the inclusion of real balances in the utility function, implies that the representative household compares its marginal utility of holding an additional dollar in period \( t \) with the marginal disutility of not consuming this dollar in the current period versus the expected discounted marginal utility of consumption in the future. Note also that real balances appear in each intertemporal Euler equation.

### 2.2 Alternative models

Our empirical investigation also entails a comparison of the two-dimensional aggregate labor supply model with money with some alternative models, namely, a standard model that features only one type of leisure and abstracts from money, the one-dimensional aggregate labor supply model with money, and the two-dimensional labor supply model without real balances. For simplicity, we provide only the intertemporal Euler equations associated with these models.
2.2.1 The standard aggregate labor supply model (no money)

The standard aggregate labor supply model does not rely on the distinction between leisure in the workweeks and leisure in non-working weeks. It uses instead the standard measure of leisure and does not account for real balances. In the standard model, the representative household’s preferences are described by,

$$E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^\alpha l_t^\alpha)^{1-\sigma} - 1}{1 - \sigma} \right),$$

(16)

where $\alpha_c + \alpha_l = 1$. Leisure is defined as $l_t = (E \times H) - n_t$, with $n_t$ representing the total hours worked during period $t$. The representative household faces the following budget constraint:

$$p_t c_t + b_t + s_t \leq w_t n_t + r_b b_{t-1} + r_s s_{t-1}.$$  

(17)

Optimal choices of $c_t, n_t, b_t,$ and $s_t$ are those that maximize expected total discounted utility (16) subject to (17). The intertemporal Euler equations corresponding to this optimization problem are:

$$E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{t+1}}{l_t} \right)^{\alpha_l} \right]^{1-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t r_{jt+1}}{p_{t+1}} - 1 \right\} = 0,$$

(18)

$$E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{t+1}}{l_t} \right)^{\alpha_l} \right]^{1-\sigma} \left( \frac{l_{t+1}}{l_t} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0,$$

(19)

for $r_{jt+1}, j = b, s$ or both. With $j = b$ or $s$, the model yields two intertemporal Euler equations, while with $j = b$ and $s$, it delivers four.

2.2.2 The one-dimensional aggregate labor supply model with real balances

In the one-dimensional aggregate labor supply model with real balances, preferences are given by
\[ E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t^{\alpha_c} l_{tt+1}^{\alpha_l} m_{tt+1}^{\alpha_m})^{1-\sigma} - 1}{1 - \sigma} \right], \tag{20} \]

where \( \alpha_c + \alpha_l + \alpha_m = 1 \). The representative agent has the following budget constraint:

\[ p_t c_t + p_t m_t + b_t + s_t \leq w_t n_t + r_t b_{t-1} + r_s s_{t-1} + p_{t-1} m_{t-1} + tr_t. \tag{21} \]

The representative agent’s optimization problem consists in choosing \( c_t, n_t, m_t, b_t, \) and \( s_t \) that maximize (20) subject to (21). The corresponding intertemporal Euler equations are:

\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{t+1}}{l_t} \right)^{\alpha_l} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \right]^{1-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t r_{jt+1}}{p_{t+1}} - 1 \right\} = 0, \tag{22} \]

\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{t+1}}{l_t} \right)^{\alpha_l} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \right]^{1-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0, \tag{23} \]

\[ E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{t+1}}{l_t} \right)^{\alpha_l} \left( \frac{m_{t+1}}{m_t} \right)^{\alpha_m} \right]^{1-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t}{p_{t+1}} + \frac{\alpha_m c_t}{\alpha_c m_t} - 1 \right\} = 0, \tag{24} \]

for \( r_{jt+1}, j = b, s \) or both. Hence, the one-dimensional labor supply model with real balances yields three Euler equations with \( j = b \) or \( s \), and five Euler equations with \( j = b \) and \( s \).

### 2.2.3 The two-dimensional aggregate labor supply model without real balances

The third model incorporates both types of leisure, \( l_{lt} \) and \( l_{lt} \), while omitting real balances. The representative agent’s preferences are described by,

\[ E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}})^{1-\sigma} - 1}{1 - \sigma} \right], \tag{25} \]

where \( \alpha_c + \alpha_{l1} + \alpha_{l2} = 1 \). The budget constraint is,

\[ p_t c_t + b_t + s_t \leq w_t h_t e_t + r_t b_{t-1} + r_s s_{t-1}. \tag{26} \]

\[ E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t^{\alpha_c} l_{1t}^{\alpha_{l1}} l_{2t}^{\alpha_{l2}})^{1-\sigma} - 1}{1 - \sigma} \right], \tag{25} \]

where \( \alpha_c + \alpha_{l1} + \alpha_{l2} = 1 \). The budget constraint is,

\[ p_t c_t + b_t + s_t \leq w_t h_t e_t + r_t b_{t-1} + r_s s_{t-1}. \tag{26} \]
The representative household chooses $c_t$, $h_t$, $e_t$, $b_t$, and $s_t$ that maximize expected discounted utility (25) subject to (26), while taking into account definitions (2) and (3). The intertemporal Euler equations corresponding to this optimization problem are:

$$E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{p_t r_{jt+1}}{p_{t+1}} - 1 \right\} = 0, (27)$$

$$E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0, (28)$$

$$E_t \left\{ \beta \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\alpha_c} \left( \frac{l_{1t+1}}{l_{1t}} \right)^{\alpha_{l1}} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{\alpha_{l2}} \right]^{1-\sigma} \left( \frac{l_{2t+1}}{l_{2t}} \right)^{-1} \frac{w_t r_{jt+1}}{w_{t+1}} - 1 \right\} = 0, (29)$$

for $r_{jt+1}$, $j = b$, $s$, or both. This model delivers three Euler equations with $j = b$ or $s$, and six with $j = b$ and $s$.

While, in principle, the intertemporal Euler equations can be estimated either individually or jointly, we focus on the joint estimation, given the large number of models considered.

### 3. Estimation Procedure and Data

This section describes the econometric procedure and data used to estimate the Euler equations of our ISLS models.

#### 3.1 Estimation method

The parameters of preferences of the alternative aggregate labor supply models are retrieved from the intertemporal Euler equations which are estimated using the generalized method of moments (GMM) procedure proposed by Hansen (1982) and Hansen and Singleton (1982). Let $E_t[q(y_{t+1}, \theta)]$ be a vector of $n$ Euler equations derived from
a particular model. The vector $y_{t+1}$ is composed of stationary variables dated $t$ and $t + 1$, while the vector $\theta$ is composed of the $l$ structural parameters that we seek to estimate. For a vector $z_t$ of $k$ instrumental variables included in the information set available in period $t$, we define a vector of $(n \times k)$ unconditional moment restrictions implied by the model,

$$Ef(y_{t+1}, z_t, \theta) = Ef[q(y_{t+1}, \theta) \otimes z_t],$$

(30)

where $\otimes$ is the Kronecker product. The sampling equivalence of equation (30) is given by,

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} f(y_{t+1}, z_t, \theta),$$

(31)

with $g_T(\theta)$ converging asymptotically towards zero under the null hypothesis that the structural model is well specified. The GMM estimator of the parameter vector ($\theta_T$) is the solution to the following problem:

$$\theta_T = \arg \min_{\theta} g_T(\theta)'W_Tg_T(\theta),$$

(32)

where $W_T$ is a non-negative symmetric weighting matrix. An optimal weighting matrix, $W^*_T$, is obtained as the inverse of the variance-covariance matrix of the moment conditions evaluated at a consistent first-step estimator; for example, by using the identity matrix as the weighting matrix at the first step. This optimal weighting matrix is consistently estimated with the procedure developed by Newey and West (1994).

The overidentifying restrictions implied by the different models can be tested formally using Hansen’s $J$-statistic if the dimension of the vector of moments is greater than the dimension of the vector of estimated parameters. This statistic is given by:

$$J = T \left\{ g_T(\theta)'W^*_Tg_T(\theta) \right\},$$

(33)
and asymptotically follows a $\chi^2$ distribution with $nk - l$ degrees of freedom.

The list of instruments used in the estimation of aggregate labor supply models has not been uniform across past studies. For example, Mankiw, Rotemberg, and Summers (1985) use different combinations, including lagged consumption, lagged interest rates, lagged leisure, lagged prices, and lagged wages. Eichenbaum, Hansen, and Singleton (1988) employ the current rates of change of consumption, leisure, and wages, and the current interest rate.

Following Tauchen (1986) and Kocherlakota (1990), we use only current and one-period lagged values of the instrumental variables to estimate the intertemporal Euler equations. Unlike the models that have been tested by Mankiw, Rotemberg, and Summers (1985) and by Eichenbaum, Hansen, and Singleton (1988), our model includes two different measures of leisure, money and different measures of asset returns. Hence, we use different subsets of the following instrumental variables: \( \{1, \frac{r_t}{R_t R_{pt}}, \frac{1}{R_t R_{pt}}, R_{ct}, R_{lt}, R_{l_1t}, R_{l_2t}, R_{lt}, R_{mt}\} \) (where $Rx_t = x_t/x_{t-1}$).

### 3.2 Data

The models are estimated with the help of quarterly (seasonally adjusted) data for the period 1960Q1–1993Q4. Aggregate real per capita consumption, $c_t$, is the sum of consumption expenditures on non-durable goods and services, converted into per capita terms after dividing by the total adult population (age sixteen and over). The aggregate price level, $p_t$, is the implicit price deflator corresponding to our measure of consumption expenditures. The rate of return on riskless assets, $r_{bt}$, is the 3-month

---

8 They occasionally use up to five-period lagged values of the variables.
9 We use $R_{l_1t}$ and $R_{l_2t}$ when estimating two-dimensional ISLS models, and $R_{lt}$ in the estimation of one-dimensional ISLS models.
10 Our choice of sample period is justified by the fact that we want to use hours worked from the Household Survey to be consistent with previous studies. This series is unfortunately unavailable after 1993Q4.
Treasury bill rate, expressed in real terms. The rate of return on risky assets, \( r_{st} \), is the value-weighted average of returns on the New York Stock Exchange, also expressed in real terms. The monetary aggregate used to calculate real money balances is M1.

We use two different wage measures. The first one, represented by \( w_1 \), is the average hourly compensation in non-agricultural employment. The second, represented by \( w_2 \), adheres more closely to the National Income and Product Accounts (NIPA) and is the sum of NIPA definitions of wages and salaries, other labor income, and proprietary income divided by total labor hours following the suggestions in Dutkowsky and Dunsky (1996).\(^{11}\) The wage rates and asset returns are both after-tax measures. The representative household’s average tax rate is based upon the taxability properties of the various components of disposable income.\(^{12}\)

The representative household’s quarterly time endowment is 1,456 hours (13 weeks × 112 hours).\(^{13}\) The weekly average hours worked, \( h_t \), is the hours series from the Household Survey. As in Alogoskoufis (1987), we approximate the working weeks, \( e_t \), by the product of the number of weeks in the quarter and the ratio of the civilian employment to the working population. The working-time cost is set at 6 hours per workweek.\(^{14}\)

\(^{11}\)Thus, proprietary income is included entirely within labour compensation, although part of this income could represent the return on capital.

\(^{12}\)Specifically, the average tax rate, \( \omega_t \), comes from the following disposable income equation:

\[
YD_t = (1 - \omega_t)(WS_t + INT_t) + OLY_t + TP_t,
\]

where \( YD \) is nominal disposable income; \( WS \) is the sum of nominal wages and salaries and proprietary income; \( INT \) is the sum of total interest, dividends, and rental income less interest paid on household debt; \( OLY \) denotes other labor income; and \( TP \) represents the nominal transfer payments. Other labor income (which consists primarily of labor benefits) and transfer payments are tax exempt, to be consistent with IRS tax laws. Solving the equation above for \( (1 - \omega_t) \) gives the average tax rate (see Dutkowsky and Dunsky 1996).

\(^{13}\)We adjust for sleeping time, so the representative household’s daily time endowment is 16 hours.

\(^{14}\)We have considered a range of 2 to 12 hours for the fixed time cost without any significant impact on the results.
When estimating one-dimensional ISLS models, we measure total hours worked during the quarter $n_t$ by $(\text{hours}_t \times \text{emp}_t \times 13)/\text{pop}_t$, with $\text{hours}_t$ representing the average weekly hours, $\text{emp}_t$ the weekly employees (total employed labour force), and $\text{pop}_t$ the total adult population.

4. Results

We begin by reporting the estimation results obtained from the two-dimensional aggregate labor supply model with real balances. We then compare these results with those of alternative models. Estimates of the parameters of preferences are recovered from the jointly estimated intertemporal Euler equations. Tables 1 and 2 report the results.

4.1 The two-dimensional aggregate labor supply model with real balances

The preference parameters of two-dimensional ISLS models with money are retrieved from the estimated intertemporal Euler equations (12)–(15). Table 1 reports the estimates. The instrumental variables used in estimating the models are listed at the bottom of the table. Each of the three panels in this table reports findings that correspond to a particular assumption about the holding of assets. Also listed within each panel are the results obtained with the two wage measures.

The overidentifying restrictions implied by the two-dimensional aggregate labor supply models with real balances are generally not rejected at a conventional confidence level. The estimates of $\beta$, $\alpha_c$, $\alpha_{12}$, $\alpha_m$, and $\sigma$ always imply that the concavity conditions are satisfied. Note that the estimates are highly stable with different measures of asset returns and wages. In contrast, Mankiw, Rotemberg, and Summers (1985) find...
that the overidentifying restrictions implied by standard one-dimensional aggregate labor supply models are systematically rejected and that the estimated parameters of preferences are highly unstable.

Panel A of Table 1 reports the estimated structural parameters with the representative consumer holding only risky assets, $s_t$. The estimates of the discount factor, $\beta$, are below unity, a finding that is interesting in itself, since most previous empirical studies of intertemporal Euler equations have found estimates of $\beta$ that are above unity.

Our estimates confirm that the two dimensions of leisure are statistically significant, each type of leisure contributing quite substantially to the household’s preferences, as shown by their estimated shares in preferences. The estimated share of leisure in working weeks is 0.34 when the wage rate is measured by $w_1$, and 0.36 when it is $w_2$. The share of leisure in non-working weeks is 0.33 with both $w_1$ and $w_2$. The estimates of $\alpha_c$ are 0.33 and 0.31, respectively. The estimated share of real balances, while small at 0.002, is nonetheless statistically significant. The estimated preference parameter, $\sigma$, which is also statistically significant, is 3.88 with wage rate $w_1$, and 5.89 with $w_2$.

Panel B of Table 1 reports the results obtained with the assumption that the representative household holds only riskless assets, $b_t$. While the results are not as good as those with the risky assets, they are still encouraging. The overidentifying restrictions are not statistically rejected at a conventional confidence level, but their non-rejection is not as strong as previously. The discount factor, $\beta$, is now slightly above unity, as in most empirical studies of intertemporal Euler equations.

With $r_{bt}$, the wage measure seems to have a greater impact on the estimated preference shares of consumption and leisure in working weeks. Estimates of $\alpha_c$ are 0.44 with $w_1$, and 0.31 with $w_2$. The estimates of $\alpha_{l1}$ are 0.22 and 0.35, respectively. In comparison, the estimated share of leisure in non-working weeks, $\alpha_{l2}$, is stable when
we change the wage rate, with estimates of 0.34 and 0.33, respectively. The preference share of real balances is 0.0018 or 0.0012, depending on the wage measure, and is statistically significant. The estimates of $\sigma$ are broadly similar to those obtained with the risky assets, at 3.90 with wage rate $w_1$ and 5.94 with $w_2$.

Panel C of Table 1 reports the results with the representative household holding riskless and risky assets. By far, the overidentifying restrictions implied by the model are not refuted. The estimates of $\alpha_c$ and $\alpha_{l1}$ are quite stable when the wage rate is changed from $w_1$ to $w_2$, with 0.46 and 0.39 for $\alpha_c$, and 0.22 and 0.29 for $\alpha_{l1}$. Estimates of $\alpha_{l2}$ are virtually unchanged at 0.32. The estimated share of real balances, $\alpha_m$, is also stable at 0.0013 and 0.0016. Estimates of $\sigma$ are 4.04 with $w_1$ and 5.45 with $w_2$.\footnote{When $\beta$ was found to be slightly above unity, the corresponding Euler equations were re-estimated, constraining the parameter $\beta$ to be below unity, with almost no changes in the estimates of the other structural parameters of the model.}

The findings reported in this subsection suggest that allowing the representative consumer to have preferences defined in terms of consumption, leisure in the workweeks, leisure in non-working weeks, and real balances results in a better overall fit of U.S. aggregate time-series data by the representative agent model of aggregate labor supply. In particular, the overidentifying restrictions implied by the model cannot be rejected and the estimated parameters of preferences are highly stable even when we use different measures of asset returns and wages.

4.2 Alternative aggregate labor supply models with and without money

It is also possible to assess indirectly the empirical success of the two-dimensional aggregate labor supply model with money by estimating alternative versions of the model that feature less theoretical ingredients than in the more general framework.
Table 2 reports the results of one-dimensional labor supply models, with and without real balances, and of two-dimensional labor supply models without money.\(^{16}\)

Panel A of Table 2 reports the estimates of one-dimensional labor supply models that abstract from money (or standard models). The estimated intertemporal Euler equations are (18) and (19). As in Mankiw, Rotemberg, and Summers (1985), Hansen’s J-test strongly rejects the overidentifying restrictions implied by the standard models. Assuming that the representative consumer holds riskless and risky assets, the estimate of \(\sigma\) is \(-2.56\), which violates the concavity conditions.

Panel B of Table 2 reports the results of one-dimensional labor supply models with real balances. The estimated structural parameters are recovered from the Euler equations (22)–(24). The results are still very negative, with overidentifying restrictions that are strongly rejected. The concavity conditions are not satisfied when the representative consumer holds only risky assets or a combination of riskless and risky assets, the estimates of \(\sigma\) being \(-2.73\) and \(-6.98\), respectively. Note also that these estimates are highly unstable. While the estimate of \(\alpha_c\) is \(0.32\) with the riskless assets, it drops to \(0.044\) with the risky assets. Altogether, these negative results constitute strong evidence against one-dimensional aggregate labor supply models.

Panel C of Table 2 reports estimates of two-dimensional ISLS models that abstract from real balances, based on the intertemporal Euler equations (27)–(29). As before, the overidentifying restrictions of the model are strongly rejected. Note also that the omission of money from the two-dimensional aggregate labor supply model greatly affects the estimates of \(\sigma\), which now becomes unstable. Indeed, estimates of \(\sigma\) are \(51.58\) with the risky assets, \(4.11\) with the riskless assets, and \(19.66\) with riskless and

\(^{16}\)Since the results of one-dimensional ISLS models and two-dimensional ISLS models without money are essentially negative, we report only those obtained with wage rate \(w_1\). The alternative models have been estimated using the same set of instruments as in the more general model or different subsets of instruments with the same negative results.
risky assets held jointly.

The findings presented in this subsection thus provide indirect empirical support to the claim that the two dimensions of the aggregate labor supply decisions and the insertion of real balances in the utility function are key factors contributing to the empirical success of the two-dimensional aggregate labor supply model with money.

5. Intertemporal Elasticities

Using the estimates reported in Table 1 of the two-dimensional aggregate labor supply models with real balances, we can derive the short-run elasticities of consumption, weekly hours worked, weeks worked, and real balances with respect to transitory movements in wage rates, asset returns, and prices. To compute these elasticities, we assume that transitory changes in wages, asset returns, and prices have only one-period effects. MacCurdy (1981) assumes that an unanticipated transitory change in the wage rate triggers a wealth effect. However, under the assumption that the wealth effect is negligible and has no long-run effect, McLaughlin (1995) shows that the constant elasticity of marginal wealth ($\lambda$-constant elasticity) is equivalent to the short-run, compensated elasticity of substitution.\footnote{With transitory shocks, infinite lifetime, and a low discount rate, which are features of our model, the wealth effect is indeed negligible.}

Assuming a deterministic environment and a small wealth effect, the elasticities can be derived as in Mankiw, Rotemberg, and Summers (1985) and McLaughlin (1995).\footnote{With this assumption, there is certainty equivalence in the Euler equations, which allows us to ignore the expectation operator (Mankiw, Rotemberg, and Summers 1985; Alogoskoufis 1987).} The intertemporal Euler equations are first transformed into log-linear systems, and are then differentiated with respect to the log of endogenous and exogenous variables.
This gives the following short-run, intertemporal substitution matrix:

\[
\left( \frac{\partial \ln y_t}{\partial \ln x_t} \right) = \left( \frac{\partial \ln q_t}{\partial \ln y_t} \right)^{-1} \left( \frac{\partial \ln q_t}{\partial \ln x_t} \right),
\]

where \( y_t \) is a vector of endogenous variables such that \( y_t = (c_t, m_t, h_t, e_t)' \); vector \( x_t = (w_t, r_{jt+1}, p_t)' \) is composed of exogenous variables, and vector \( q_t \), of the Euler equations.

Table 3 reports the intertemporal elasticities that are based on the estimates of the two-dimensional aggregate labor supply models with money. The estimated short-run elasticities are generally lower than unity, except for the weekly hours of work and the total hours worked. The signs of the elasticities are what the theory predicts.

A transitory rise in wages generates an increase in consumption, weekly hours worked, weeks worked, and real balances. The wage elasticity of weekly hours worked is in the range of 1.45–1.72 with wage rate \( w_1 \), and 1.24–1.44 with \( w_2 \). The wage elasticity of working weeks is between 0.36 and 0.43 with \( w_1 \), and 0.31 and 0.36 with \( w_2 \).

Combining the two elasticities for the weekly hours worked and the working weeks, the wage elasticity of total hours worked falls in the range of 1.81–2.15 with \( w_1 \), and 1.54–1.81 with \( w_2 \). These elasticities are much higher than those previously reported in the literature using aggregate time-series data.

A transitory increase in asset returns raises labor supply, while reducing both consumption expenditures and real money balances. The asset-return elasticity of total hours worked lies between 0.48 and 0.63 with \( w_1 \), and between 0.27 and 0.29 with \( w_2 \). These elasticities are consistent with those obtained by Alogoskoufis (1987).
6. Conclusion

The representative agent model of aggregate labor supply, and its cornerstone, the hypothesis of intertemporal substitution in labor supply, have not been very successful when confronted with aggregate time-series data. The findings reported here suggest that some credibility in this general framework may be restored if the representative agent has preferences defined in terms of consumption, leisure in workweeks, leisure in non working weeks and real balances.

Future work should allow for non-separability of preferences that would accommodate either intertemporal substitution or complementarity of leisure using a two-dimensional aggregate labor supply framework of the kind we have proposed in this paper.
References


Table 1:
Estimates of the Two-Dimensional ISLS Models with Money

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\alpha_c$</th>
<th>$\alpha_{l1}$</th>
<th>$\alpha_{l2}$</th>
<th>$\sigma$</th>
<th>$J$-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>A. Euler equations with risky asset return ($r_{st}$)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$w_1$</td>
<td>0.9971</td>
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<td>0.3380</td>
<td>0.3289</td>
<td>0.0023</td>
<td>3.8798</td>
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<td></td>
<td>(0.005)</td>
<td>(0.171)</td>
<td>(0.138)</td>
<td>(0.054)</td>
<td>(0.0012)</td>
<td>(1.778)</td>
</tr>
<tr>
<td>$w_2$</td>
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<td>0.3100</td>
<td>0.3576</td>
<td>0.3304</td>
<td>0.0021</td>
<td>5.8916</td>
</tr>
<tr>
<td></td>
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<td>(0.104)</td>
<td>(0.085)</td>
<td>(0.033)</td>
<td>(0.0007)</td>
<td>(1.916)</td>
</tr>
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<td>B. Euler equations with riskless asset return ($r_{bt}$)</td>
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<tr>
<td>$w_1$</td>
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<td></td>
<td>(0.002)</td>
<td>(0.221)</td>
<td>(0.196)</td>
<td>(0.055)</td>
<td>(0.0009)</td>
<td>(1.807)</td>
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<tr>
<td>$w_2$</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.093)</td>
<td>(0.079)</td>
<td>(0.0320)</td>
<td>(0.0003)</td>
<td>(1.927)</td>
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<td>C. Euler equations with both asset returns ($r_{st}$ and $r_{bt}$)</td>
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<tr>
<td>$w_1$</td>
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<td>0.4651</td>
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<td>0.3176</td>
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<tr>
<td></td>
<td>(0.001)</td>
<td>(0.203)</td>
<td>(0.185)</td>
<td>(0.037)</td>
<td>(0.0004)</td>
<td>(1.560)</td>
</tr>
<tr>
<td>$w_2$</td>
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<td>0.2922</td>
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<td>(0.110)</td>
<td>(0.095)</td>
<td>(0.028)</td>
<td>(0.0005)</td>
<td>(1.532)</td>
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</table>

Notes: Standard errors are in parentheses. $P$-values are in parentheses under $J$-statistics; the wage rates $w_1$ and $w_2$ are defined in the text. The vectors of instrumental variables, $z_t$, used to estimate the models are: in A, $z_t = (1, R_{ct}, R_{mt}, R_{l1t}, R_{l2t}, \frac{r_{st}}{R_{ct}}, \frac{1}{R_{ct}})^{'}$; in B, $z_t = (1, R_{ct}, R_{mt}, R_{l1t}, R_{l2t}, \frac{r_{bt}}{R_{ct}}, \frac{1}{R_{ct}})^{'}$; in C, $z_t = (1, \frac{1}{R_{ct}}, R_{ct}, R_{mt}, R_{l1t}, R_{l2t})^{'}$. 

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Table 2:
Estimates of Alternative ISLS Models

<table>
<thead>
<tr>
<th></th>
<th>( \beta )</th>
<th>( \alpha_c )</th>
<th>( \alpha_{l1} )</th>
<th>( \alpha_{l2} )</th>
<th>( \alpha_m )</th>
<th>( \alpha_l )</th>
<th>( \sigma )</th>
<th>( J)-stat.</th>
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<td><strong>A. Standard ISLS model</strong></td>
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<td></td>
<td></td>
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<td>( r_s )</td>
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<td>(0.201)</td>
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<td>(0.0001)</td>
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<tr>
<td>( r_b )</td>
<td>1.0129</td>
<td>0.2272</td>
<td>-</td>
<td>-</td>
<td>0.7728</td>
<td>10.156</td>
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<td>(0.054)</td>
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<td>( r_{s,b} )</td>
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<td>-</td>
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<td>(0.0008)</td>
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<td><strong>B. One-dimensional ISLS model with money</strong></td>
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<tr>
<td>( r_s )</td>
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<td>( r_s )</td>
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<td>-</td>
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<td>(0.029)</td>
<td>(7.948)</td>
<td>(0.060)</td>
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</table>

Notes: Standard errors are in parentheses. P-values are in parentheses under \( J\)-statistics; the wage rate is \( w_1 \), defined in the text. The instrumental variables used to estimate the alternative models are taken from the following set of variables: \( \{1, \frac{r_{jt}}{R_{ct}}R_{pt}, \frac{1}{R_{ct}}R_{pt}, R_{ct}, R_{lt}, R_{lt1}, R_{lt2}, R_{mt}\} \), \( j = b, s \), and \( Rx_t = x_t/x_{t-1} \).
Table 3:
Elasticities Implied by the Two-Dimensional ISLS Models with Money

<table>
<thead>
<tr>
<th></th>
<th>A. Risky return, $r_{st}$</th>
<th>B. Riskless return, $r_{bt}$</th>
<th>C. Both return rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_t$</td>
<td>$r_{t+1}$</td>
<td>$p_t$</td>
</tr>
<tr>
<td>$c_t$</td>
<td>$w_1$</td>
<td>0.495</td>
<td>-0.176</td>
</tr>
<tr>
<td></td>
<td>$w_2$</td>
<td>0.571</td>
<td>-0.080</td>
</tr>
<tr>
<td>$h_t$</td>
<td>$w_1$</td>
<td>1.454</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>$w_2$</td>
<td>1.235</td>
<td>0.231</td>
</tr>
<tr>
<td>$e_t$</td>
<td>$w_1$</td>
<td>0.360</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>$w_2$</td>
<td>0.305</td>
<td>0.057</td>
</tr>
<tr>
<td>$e_t h_t$</td>
<td>$w_1$</td>
<td>1.814</td>
<td>0.634</td>
</tr>
<tr>
<td></td>
<td>$w_2$</td>
<td>1.540</td>
<td>0.288</td>
</tr>
</tbody>
</table>

Notes: The elasticities are calculated using the estimates of the two-dimensional ISLS model with money reported in Table 1; the wage rates $w_1$ and $w_2$ are defined in the text.