Hiring and Firing Costs: Which one is the Largest?
Evidence from French Firms

By Nicolas Roys*
University of Paris I

February 28, 2006

PRELIMINARY AND INCOMPLETE
PLEASE DON'T QUOTE

Abstract

This paper studies the dynamic of labor demand at the firm level. Recent studies emphasize the importance of non-convex components in the structure of hiring and firing costs in the form of either fixed or linear adjustment costs. The empirical literature based on panel data from several countries shows that labor adjustment is rather erratic at the plant level with periods of inactivity punctuated by large adjustments. We estimate an ordered probit model with correlated random effects on a large panel of more than 50,000 French firms for the period 1994-2000. Our results imply that it is important to depart from the standard specification of convex and symmetric adjustment costs: French firms do not continually adjust their employment level but when they do it, they fully resorb the accumulated disequilibrium without smoothing. Although some ambiguities exist about the nature of the asymmetry and contrary to existing empirical evidence, our results indicate that it is more costly to expand employment than to contract it but the difference is not significant. To us it indicates that adjustment costs are mainly implicit and that attempt to directly measure them is is probably time wasted.

Key Words: Labor Demand, Adjustment Costs, Panel-Data, Ordered Probit
JEL: J21, J23, E24

* Department of Economics, CES, University of Paris I - Panthéon - Sorbonne, Blvd. de l'Hôpital, 75013, Paris, France. nicolas.roys@malix.univ-paris1.fr
Acknowledgments: I thank Jean-Marc Robin, Patrick Sevestre, Pierre Cahuc, Jean-Olivier Hairault, Russell Cooper and Joeffrey Drouart for helpful comments. Usual disclaimers apply.
1 Introduction

The persistence of high unemployment, and the rise of non-employment for some groups such as less-skilled, in various European countries has reignited academic and political debate over the design of labor market regulation, especially employment protection and working time. On the subject, we focus on microeconomic flexibility, which is at the core of economic growth in modern market economies, by facilitating the ongoing process of creative-destruction. The main obstacle faced by microeconomic flexibility is adjustment costs. Some of these costs are purely technological, others are institutional. Chief among the latter is labor market regulation, in particular job security provisions. More precisely employment protection legislation is often pointed out as the most important source of rigidity on continental European labor markets. This paper studies the dynamic of labor demand at the firm level and provide a quantitative evaluation of adjustments costs on the French labor markets. In many contexts, actual factor demands clearly involve complicated dynamic elements absent in static demand theory. For example, empirical studies of the market demand for labor typically find that lags, either of demand or of the determinants of demand, contribute substantially to the explanation of employment determination. The most frequent rationalization of such lags is that individual plants face adjustment costs. The key is to recognize that the adjustment of many factors of production, including labor, is not costless. For example, due to the technological and organizational specificity of labor services firms incur hiring costs because they need to inform and instruct newly hired workers before they are as productive as the incumbent workers. Adjustment costs imposed on firms by job security legislation, in fact, are widely different across countries, sectors, and occupations, and the literature has given them a prominent role when comparing European and American labor market dynamics (see Bertola, 1999, for survey of theory and evidence). In most European countries, legislation imposes administrative and legal costs on employers wishing to shed redundant workers. Together with other institutional differences, this has been found to be an important factor in shaping the European experience of high unemployment in the last three decades of the twentieth century.

As recent studies emphasize the importance of non-convex components and the asymmetries in the

---

1 The creation and destruction of jobs (turnover) often entails costs for the workers too, not only because they may need to learn to perform new tasks, but also in terms of the opportunity cost of unemployment and the costs of moving. In fact, it is in order to protect workers against these costs of mobility that labor contracts and laws often impose firing costs, so that firms incur costs both at times when they expand and when they reduce their labor demand.
structure of hiring and firing costs, we model employment adjustment as a discrete decision. It allows us to explain the behavior of employment at firm level: labor adjustment is rather erratic with periods of inactivity punctuated by large adjustments. In the empirical section, we estimate an ordered probit model with correlated random effects on a large panel of more than 50,000 French firms observed over 7 years. Our results imply that it is important to depart from the standard specification of convex and symmetric adjustment costs. French firms adopt a similar pattern for employment adjustment: they don’t continually adjust their employment level but when they do it, they fully resorb the accumulated disequilibrium without smoothing. Although some ambiguities exist about the nature of the asymmetry and contrary to existing empirical evidence, our results indicate that it is more costly to expand employment than to contract it but the difference is not significant. To us it indicates that adjustment costs are mainly implicit and that attempt to directly measure them is probably time wasted.

The remainder of the paper is organised as follows. In the next section we survey the related literature on the dynamics of labor demand. In section 3 we present a simple theoretical model. In section 4 we deal with the French institutional setting and present our data. In section 5 we present our econometric strategy, while section 6 presents our results and compare them to the literature. Section 7 concludes.

2 Dynamics Of Labor Demand

The simplest (and widely used) model of dynamic labor demand assume convex and continuously differentiable adjustment costs. These models are analytically tractable as we can often estimate their parameters directly from first-order conditions. Researchers often specify a so-called « partial adjustment model » in which the firm is assumed to adjust the level of employment to a target. The assumed model of labor adjustment would be:

$$ e_t - e_{t-1} = \lambda (e_t^\ast - e_{t-1}) $$

So here the change in employment ($ e_t - e_{t-1} $) is proportional to the difference between the previous level of employment and a target ($ e_t^\ast - e_{t-1} $) where $ \lambda $ parameterizes how quickly the gap is closed. This model implies a smooth and continuous adjustment of employment to shocks. Adjustment costs dampen the response to changes in current wage and productivity and yield smooth, gradual changes in employment over time. As shown by Sargent (1978), the empirical partial adjustment model may be derived as the solution to a firm’s dynamic profit maximization problem under the
assumption that there are quadratic costs of adjusting the workforce.

But Nickell (1978) argues: “One point worth noting is that there seems little reason to suppose costs per worker associated with either hiring or firing increase with the rate at which employees flow in or out. Indeed, given the large fixed costs associated with personnel and legal departments, it may even be more reasonable to suppose that the average cost of adjusting the workforce diminishes rather than increases with the speed of adjustment.”

In part, the popularity of the quadratic adjustment cost structure reflects it tractability. But, the implications of this model conflict with evidence of inactivity and bursts at the firm level. In particular, they have implications of constant adjustment that are not consistent with microeconomic observations. Basically, adjustment at the plant level is discrete, occasional and asynchronous. Hamermesh (1989) examines monthly data on output and employment between 1983 and 1987 across seven manufacturing plants. For each plant, output fluctuates substantially over the sample. Employment exhibits long periods of constancy broken by infrequent and large jumps at times roughly coinciding with the largest output fluctuations. Hence, the plant data are not consistent with the smooth employment adjustment that would arise from convex adjustment costs. Caballero et al. (1997) show that labor adjustment is rather erratic at the plant level with periods of inactivity punctuated by large adjustments (we will discuss more on this paper later). So, employment change tends to be concentrated in a single-period, so that firms avoid paying adjustment costs too frequently. Moreover, firms only make those changes in the labor input which are justified by sufficiently large departures of desired employment from their most recent choice of the number of employees. The adjustment process is lumpy and intermittent: in the face of a shock, a firm may decide that it is optimal to maintain the same number of employees and to postpone adjustment to the future.

Also as discussed in Hamermesh and Pfann (1996), there is certainly evidence in favor of asymmetries in the adjustment costs. But depending on the country, time period and sample sometimes hiring is more expensive than firing or the other way around. As we have no hope of finding a result holding at the international level, we concentrate, in the following, our efforts on France.

**Empirical Evidence From France**

In our paper the dynamics of labor adjustment on a set of French firms is investigated. A number of

2 Nonetheless, the partial adjustment model continues to be a vehicle for applied work, essentially because it is a tractable way of capturing some important dynamic aspects of market demand. As pointed by King et Thomas (2005): « It is frequently thus employed in an apologetic manner, with the researcher suggesting that it is a description of market, rather than individual, factor demand ». 

earlier papers have estimated the adjustment costs for French firms. Goux et al. (2001) use a panel of 915 French manufacturing firms for which they can measure the number of hirings and firings for indefinite and fixed term contract workers separately. They estimate the costs of employment adjustment for these two types of workers using a dynamic labor demand model with quadratic convex adjustment costs. For indefinite contract workers they find firing cost to be much higher than hiring costs (around 40 times higher). Their estimates unfortunately only allow to make these comparative statements. They do not allow to measure the absolute amount of adjustment costs. They also find that it is practically costless to adjust workers on fixed time contracts. Using survey data on actual severance payments and actual costs (such as training hours, expenses on job advertising, etc.) upon hiring, Abowd and Kramarz (2003) provide an analysis of hiring and firing costs for French firms. They conclude that for permanent contracts, the cost of hiring are much lower than the cost of firing. However the highest cost of firing are associated with collective terminations, which are reductions in employment of 10 employees or more. These are mostly relevant for larger firms not for the firms in the dataset used here, where reductions in employment of 10 persons are very rare. Based on two cross-sections Kramarz and Michaud (2004) revisit the findings of Abowd and Kramarz (2003). Again they find that collective terminations have the highest costs. They also find hiring cost to be small, for instance they mention that in two surveys of French firms that hire workers respectively 49% (for 1992) and 62% (for 1996) of the firms declare a zero hiring cost.

3 A Simple Theoretical Model

With adjustment costs, the simple conditions which states that the marginal productivity and the marginal costs of labor are equated in every period, is no longer efficient. The costs of hiring and firing require a firm to adopt a forward-looking employment policy. We assume that the firm maximizes the current discounted value of future cash flows. We consider the following dynamic programming problem:

$$V(A,e,h) = \max_{e} R(A,e,h) - w(e,h) - C(A,e,h) + \beta E[V(A',e)]$$

Here $A$ represents a shock to the profitability of the firm. This shock could reflect variations in product demands or variations in the productivity of inputs. $R(A,e,h)$ represents the revenues which depend on the hours worked ($h$), the number of workers ($e$) and the profitability shock. Others
factors of production, such as capital, are assumed to be rented and optimization over these inputs are incorporated into $R(A,e,h)$.

The function $weg(h)$ is the total cost of hiring $e$ workers when each supplies $h$ units of labor time. This general specification allows for overtime pay and other provisions. We assume that this compensation function is increasing and convex with respect to hours: $g'(h) > 0$, $g''(h) > 0$.

The function $C(e,e,i)$ is the cost of adjusting the number of workers. We consider linear and asymmetric adjustment costs. One of the criticisms of the quadratic adjustment cost specification is the implications of continuous adjustment. At the plant level, as mentioned earlier, there is evidence that adjustment is much more erratic than the pattern implied by the quadratic model. Piecewise linear adjustment costs produce inaction. It is relatively straightforward to introduce asymmetries into the model as they do not present any additional technical difficulties. And the simple proportionality between the cost and the amount of turnover allows a simple characterization of the optimal labor demand policies. We therefore assume that:

$$
C(e,e,i) =
\begin{cases}
S(e-e_{-1}) & \text{if } e > e_{-1} \\
0 & \text{if } e = e_{-1} \\
S(e_{-1}-e) & \text{if } e < e_{-1}
\end{cases}
$$

**Employment Adjustment**

Firms’ optimal actions are based on the shadow value of labor, defined as the marginal increase in the discounted cash flow of the firm if it hires one additional unit of labor. When a firm increases the employment level by hiring an infinitesimally small unit of labor while keeping the hiring and firing decisions unchanged, the objective function defined varies by an amount of

$$
\sum_{t=0}^{\infty} \beta^t \left( \frac{\partial R(A_{t,i},e_{t,i},h_{t,i})}{\partial e_{t,i}} - wg(h_{t,i}) \right)
$$

per unit of additional employment. If the employment levels $e_{t,i}$ on the right hand side of this equation are the optimal ones, measures the marginal contribution of an infinitesimally small labor input variation around the optimally chosen one. This follows from the envelope theorem which implies that infinitesimally small variations in the employment level do not have first order effects.

---

3 An alternative would have been to specify fixed adjustment costs
4 Unfortunately, this hypothesis is not per se sufficient to formulate a solution for the dynamic optimization problem. We need to impose functional form for the revenues function, the compensation function and the stochastic process of the profitability shocks. Only after, we could hope getting an analytical solution (at best) or a solution with numerical methods (at least).
on the value of the firm.

The optimal choices of the firm are obvious if we express them in terms of the shadow value of labor. First of all, the marginal value of labor cannot exceed the costs of hiring an additional unit of labor. Otherwise the firm could increase profits by choosing a higher employment level, contradicting the hypothesis that employment maximizes profits. Hence, given that the costs of a unit increase in employment are equal to $S$, while the marginal value of this additional unit is $\lambda$, we must have $\lambda \leq H$.

Similarly, if $\lambda < -F$ the firm could increase profits immediately by firing workers at the margin: the immediate cost of firing one unit of labor $-s$ would be more than compensated by an increase in the cash flow of the firm. Again, this contradicts the assumption that firms maximize profits. Hence, we must have:

$$-s \leq \lambda \leq S \quad \text{for each } t.$$

Moreover, either the first or the second inequality turns into an equality sign if $\Delta e, \neq 0$. Formally, at an interior optimum for the hiring and firing policies of a firm we have the following. Whenever the firm prefers to adjust the employment level rather than wait for better or worse circumstances, the marginal cost and benefit of that action need to equal each other. If the firm hires a worker we have:

$$\lambda = S,$$

which implies that the marginal benefit of an additional worker are equal to the hiring costs. Similarly, if a firm fires workers it must be true that:

$$\lambda = -s,$$

that is the negative marginal value of a redundant worker needs to be compensated exactly by the cost of firing this worker $g$. Notice also that the shadow value of the marginal worker can be negative only if the wage exceeds the value of marginal productivity.

**Hours Adjustment**

Conditionally of the firm having optimally chosen employment, the choice of hours is static. Formally,

$$\frac{\partial R(A_t, e_t, h_t)}{\partial h} = w e_t g'(h_t)$$

The firm weighs the gains to the increasing labor input against the marginal cost (assumed to be
increasing in hours) of increasing hours. This condition shows that if the firm does not adjust the number of workers following a positive shock, hours will increase to accommodate part of the shock. Conversely, if firms adjust the number of workers, she fully resorb the accumulated desequilibrium without smoothing. So firm will eventually let hours at their legal level to avoid paying a wage premium as overtime is remunerated at a premium rate.

Note that the conditions based on the shadow value are not in themselves sufficient to formulate a solution for the dynamic optimization problem in order to calculate $\lambda_t$ as in we need to know the distribution of $\{N_{t+i}, i = 0, 1, 2, \ldots\}$, and thus we already need to have solved the optimal demand for labor\(^5\).

---

4 Hiring and Separations: The French Labor Laws

French labor laws allow firms to hire workers on two types of regular employment contracts: indefinite-term contracts (Contrats à Durée Indéterminée, CDI) and fixed-duration contracts (Contrats à Durée Déterminée, CDD).

The current architecture of CDDs, introduced in 1979, dates back to an agreement signed in March 1990. Under this agreement, CDDs can be offered by firms for only very precise reasons: CDD cannot be used to fill a job that would exist under normal and permanent business conditions for a given firm (Article L.122). CDDs are subject to a very short trial period, typically one month. They have a fixed duration, they can only be renewed once and their length, including renewal, cannot exceed 18 months (24 months for youth employment programs). If the worker is kept, she must be hired on a regular contract. If the worker is not kept, she receives a 6% severance payment by law (10% since January 2002). Although their use is formally restricted, CDDs are the most common method of hiring: more than 2/3 of all hires are through CDD. On the other hand, during the 1990’s, more than 90% of the stock of employees in private for-profit or semi-public establishments were on CDIs. For those hired under CDD approximately one in three is eventually converted to CDI (Abowd, Corbel and Kramarz [1999]). Insofar as they have a fixed duration, termination of a CDD is not an issue.

Termination of CDIs is a more complex process, since these contracts are subject to employment protection. Employer-initiated termination of a permanent employee can take two broad forms: firing for “economic reasons”, in which case the firm must prove that it needs to reduce its

\(^5\) For a technical exposition and numerical solution, see Adda et Cooper (2003)
employment, or for “personal reasons”, in which case the firm has to show the worker cannot do the job he was hired for; and early or normal retirement, both of which are considered terminations under French Labor Laws (30 July 1987). For terminations (except firing for very serious misconduct) and for retirements, the employer must observe a mandatory waiting notice period and pay a severance payment. The notification period is the delay between reception by the worker of the formal letter announcing the termination and the actual end of the CDI. Workers with less than 6 months seniority are not given notice. For workers with 6 months to 2 years seniority, the notice period is 1 month. The notice period is 2 months for workers with more than two years of seniority. For engineers, professionals, and managers the notice period is 3 months. If the notice period is not respected, the worker must be fully compensated for the difference between the minimum notice period and the delay actually experienced in the termination. There are, however, no punitive damages. Severance payments are calculated as follows. Unless the sector collective bargaining agreement, the firm-level collective bargaining agreement, or the individual contract specify a more generous formula, the legal minimum severance payment must be paid to workers with at least two years of seniority. For every year of seniority at the firm, the employer must pay 20 hours if the worker is paid by the hour or 1/10th of the reference wage if the worker is paid by the month. The reference wage is computed as the average monthly wage over the last three months of service at the firm. Furthermore, for most workers, an additional 1/15 of a second monthly reference wage must be added for every year of service beyond 10. This second reference wage is the maximum of the first reference wage and the average wage over the last twelve months. Apparently, most workers are compensated well above their reference severance pay (Abowd and al. [1999]). It is worth noting that, in France, different rules apply to individual and collective terminations (the dismissal of at least 10 workers during a 30 days period). The August 2, 1989 law requires that firms with 50 or more employees formulate a “social plan” before implementing a collective termination. This social plan must place a limit on the total number of terminations and lay out solutions that facilitate reemployment of terminated workers. The plan may also offer a re-training program. When terminated workers are not entitled to receive a full-rate retirement pension, early retirement may be an option for the firm in case of terminations for economic reason, if the worker is old enough.

Data

The data used are derived from the matching of two sources: the BRN (Real Normal Profits) and the DADS (Annual Declarations of Social Data). The BRN declarations are completed annually by firms with a turnover of more than 3.5 million francs (1992 threshold) liable for income tax in respect of BIC
(Industrial and Commercial Profits). The BIC correspond to the profits declared by firms whose commercial, industrial or craftwork activity is carried out for lucrative purposes (60% of the firms, 94% of the turnover). The DADS declarations are completed annually by any firm employing workers. They cover all employers and their employees with the exception of paid agricultural workers and civil servants. At present, the employees covered by the DADS represent almost 80% of dependent employment.

We restrict our analysis to firms between 20 and 100 employees in the manufacturing sector over the period 1994-2000. Our final sample contains 50 350 firms.

**TO BE COMPLETED**

### 5 Econometric Framework

« The Gap Approach »

In many aspects, our econometric framework is built from Caballero et al. (1997) and the so-called « gap approach ». The theme of creating an employment target to define an employment gap as a proxy for the current state is quite intuitive and powerful. As noted in our preceding discussion, when a firm is hit by a profitability shock, a gap naturally emerges between the current level of employment and the level of employment the firm would choose if there were no costs of adjustment. This gap should then be a good proxy for the gains to adjustment. These gains, of course, are then compared to the costs of adjustment which depend on the specification of the adjustment cost function.

Formally, define the gap as the difference between desired and actual employment levels (in logs):

\[ z_{it} = \ln e_{it}^* - \ln e_{it-1} \]

Here \( e_{it-1} \) is number of workers inherited from the previous period. So \( z_{it} \) measures the gap between the desired and actual levels of employment in period \( t \) prior to any adjustments but after the period \( t \) random variables are realized as these shocks are embedded in the target and thus the gap. The key of the empirical work is to estimate the function :

\[ e_{it} - e_{it-1} = \theta (\ln e_{it}^* - \ln e_{it-1}) \]

Unfortunately, this estimation is not feasible as the target and thus the gap are not observable. So, the basic theory must be augmented with a technique to measure the gap. Caballero et al. (1997)
postulate a second relationship between another (closely related) measure of the gap ($\tilde{z}_t$) and plant specific deviations in hours:

$$\tilde{z}_t = \theta(h_t - \bar{h}_t)$$

Here $\tilde{z}_t$ is the gap in period $t$ after adjustments in the level of $e$ have been made:

$$\tilde{z}_t = z_t - \Delta e_t$$

The argument in favor of this approach again returns to our discussion of the choice between employment and hours variation in the presence of linear adjustment costs. In that case we saw that the firm chose between these two forms of increasing output when profitability rose. Thus, if hours are measured to be above average, this will reflect a gap between actual and desired workers. If there was no cost of adjustment, the firm would choose to hire more workers. But, in the presence of these costs the firm maintains a positive gap and hours worked are above average.

The key to the preceding is how to estimate $\theta$. Since the left side of the preceding equation is also not observable, the analysis is further amended to generate an estimate of $\theta$. Caballero et al. (1997) estimate it from the following simple OLS regression:

$$\Delta e_t = \alpha - \theta \Delta h_t + \epsilon_t$$

where the error term ($\Delta e_t^*$) includes unobserved changes in the target level of employment. Caballero et al. (1997) note that this equation may have omitted variable bias as the change in the target may be correlated with changes in hours. From the discussion in Cooper and Willis (2004), these omitted variable bias can be quite important.

**Econometric Model**

The specificity of our paper is to consider the decision of adjusting the number of employees as a discrete decision. In the theoretical model as well as in previous empirical studies (cf. supra), employment change tends to be concentrated in a single-period, so that firms avoid paying adjustment costs too frequently. Moreover, firms only make those changes in the labor input which are justified by sufficiently large departures of desired employment from their most recent choice of the number of employees. The adjustment process is lumpy and intermittent: in the face of a shock, a firm may decide that it is optimal to maintain the same number of employees and to postpone adjustment to the future; a type of behavior described as an (S,s) rule. In labor demand, a two-sided (S,s) rule may be defined as the following: if the number of employees is above (below) or equal to a critical threshold then the firm decides to reduce (increase) employment to its desired level, otherwise it leaves it unchanged. Hence there is a zone of non-adjustment delimited by two critical value.
Let i index a random draw from the cross section, and let t denote a particular time period. Z is an ordered variable which can take 3 different values. Z takes the value -1 (1) if employment fluctuations is negative (positive) in the plant i at time t ; and takes the value 0 if employment doesn't change from one year to another in plant i. We now need to specify the latent variable. It is based on the idea that linear costs of changing employment create an imbalance between actual and target behavior. Since adjusting employment is costly, firms adjust the number of their employees and pay the adjustment costs only when the gap between desired and actual employment is sufficiently large to exceed one of the threshold values. We consider the gap between desired and actual employment (Z*) as a latent variable, with a two sided (s,S) rule translating into an ordered probit panel model:

\[
Z_{it}^{*} = \begin{cases} 
-1 & \text{if } Z_{it}^{*} \in [-\infty, s] \\
0 & \text{if } Z_{it}^{*} \in [s, S] \\
1 & \text{if } Z_{it}^{*} \in [S, +\infty] 
\end{cases}
\]

The key point in empirically implementing the model is the specification of the gap. So we use a specification close to those of Caballero et al. (1997). We assume the gap depends of the deviation between actual and desired hours worked and production, and a time-varying residual term. Assumed that the gap (unobserved) is determined by:

\[
Z_{it}^{*} = a(h_{it}^{*} - \tilde{h}_{i}) + b(y_{it}^{*} - \tilde{y}_{i}) + \sum_{j=86}^{96} c_{j} e_{j} + \varepsilon_{i,t} + e_{j} \begin{cases} 
1 & \text{if } j = t \\
0 & \text{sinon} 
\end{cases}
\]

where the first (second) term represent deviation between actual and desired hours worked by the average worker (production) of firm i at time t. e; are temporal-dummies, \(\varepsilon_{i,t}\) is a residual term.

The cut points (or threshold parameters) are unknown and we want to estimate them. Note that we don't incorpore a constant in our model. The reason is because if we add one, we will have an identification problem : our cut-points will not be identified anymore. But there is no economic rationale for including a positive (or negative) constant in our model.

Correlated Random Effects

The problem with the preceding formulation is that \(\tilde{h}_{i}\) and \(\tilde{y}_{i}\) are inobservables. We can partially solve it by incorporating average hours worked and production over the observed periods.
The model can so be rewritten:

\[ Z^{*}_{i,t} = a(h_{i,j} - \bar{h}_i) + b(y_{i,j} - \bar{y}_i) + \sum_{j=1}^{N} c_j e_{ij} + d(h_j - \bar{h}_i) + b(y_j - \bar{y}_j) + \varepsilon_{i,t} \]

\[ Z^{*}_{i,t} = a(h_{i,j} - \bar{h}_i) + b(y_{i,j} - \bar{y}_i) + \sum_{j=1}^{N} c_j e_{ij} + u_i + \varepsilon_{i,t} \]

with \( u_i = a(h_j - \bar{h}_j) + b(y_j - \bar{y}_j) \) a random effects and

\[ \sum_{j=1}^{N} h_{i,j} = \bar{h}_i \quad \sum_{j=1}^{N} y_{i,j} = \bar{y}_i \]

As \( T \) is small (and even if \( N \) is large), the fixed effects model suffers from the incidental parameters problem. Indeed, as there does not exist a sufficient statistic allowing the fixed effects to be conditioned out of the likelihood, the fixed-effects estimator is biased. So we used a random effects estimator even if its properties are more dependent of the hypothesis we do.

Also we know that the random effects model have desirable properties only if \( c_i \) is strictly exogenous. But, there are obvious reasons to believe that random individual effects are correlated with explanatory variables. So as we don't want to do such an hypothesis, we use Mundlak's formulation. Insofar we assume:

\[ u_i = \delta h_i + \gamma y_i + \eta_i \quad \text{with} \quad \eta_i \sim N(0, \sigma^2) \]

**Endogeneity Biais**

As the time-varying residual (\( \varepsilon \)) is positively correlated with the explanatory variable, estimating this model is likely to yield downward-biased estimates. Since hours are used to accommodate part of the frictionless shocks. When employment does not adjust, changes in hours and the component due to the shock are positively correlated. So we need to incorporate in our model a variable correlated with demand shocks. From our data, we use to two different procedures (but we will see later that they give the same result).

First, we assume that firms use simple AR(1) processes to forecast their sales. In order to construct a prediction for each firm at each time period, we estimate the following model:

\[ y_{i,t} = \epsilon y_{i,t-1} + w_{i,t} \]

Then we use the residual of this equation to construct the error of prediction. How to estimate this last equation? We use Arellano-Bond (1991) procedure as we know it gives consistent estimate of
this model. The results are reproduced in the following table\textsuperscript{6}.

| Coef.      | Std. Err. | z    | P>|z| |
|------------|-----------|------|------|
| \(\hat{\delta}\) | 0.4906352 | 0.0176767 | 27.76 | 0.000 |

Second, we simply differentiate the production \(y_t - y_{t-1}\) as we will see it does quite a good job as a proxy for the demand shock.

### Estimation Procedure

The likelihood can be written:

\[ L(\theta) = \prod_{i=1}^{N} f(z_{i,96}, \ldots, z_{i,00} / X_i) \]

with,

\[ f(z_{i,96}, \ldots, z_{i,00} / X_i, \theta) = \int_{s}^{s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{s}^{s} \left[ \prod_{i=1}^{N} \theta \left( \frac{\alpha_{i,t} - X_{i,t}\beta - u_i}{\sqrt{1 - \sigma_u^2}} \right) - \theta \left( \frac{\alpha_{i,t} - X_{i,t}\beta - u_i}{\sqrt{1 - \sigma_u^2}} \right) \right] \frac{1}{\sigma_u} \phi \left( \frac{u_i}{\sigma_u} \right) \, du_i \]

\[ a_{i,t} = \begin{cases} -\infty & \text{si } z_{i,t} = -1 \\ s & \text{si } z_{i,t} = 0 \\ S & \text{si } z_{i,t} = 1 \end{cases} \quad \text{and} \quad a''_{i,t} = \begin{cases} s & \text{si } z_{i,t} = -1 \\ S & \text{si } z_{i,t} = 0 \\ +\infty & \text{si } z_{i,t} = 1 \end{cases} \]

The SAS procedure NLMIXED fits the nonlinear mixed models and was used to construct the estimator. As starting values, we use the result of a simple ordered probit estimation without individual effects.

### 6 Results

As the results are similar with the two methods of estimating the demand shock, we only reproduce the one with the estimator in difference as it probably introduces less noise in the estimation. In fact several different specifications are considered. Some of them do not include an individual effects, an estimation of the demand shock and the mean of variables over the period of estimation. Taking

\textsuperscript{6} The usual tests reject first and second-order autocorrelation
into account the individual effects greatly improves the fits and change the signs of the majority of the coefficients; whereas taking into account the demand shock and using Mundlak's formulation give more realistic valued of the thresholds and greatly improve the usual criterions (AIC, AICC, BIC).

| Parameter | Estimate | S Standard Error | t Value | Pr > |t|
|-----------|----------|------------------|---------|------|
| $y_{it} - y_{it-1}$ | 0.000079 | 0.000015 | 5.28 | <.0001 |
| $h_{i} - \bar{h}_{i}$ | -0.00535 | 0.000213 | -25.09 | <.0001 |
| $y_{it} - \bar{y}_{i}$ | 0.000054 | 0.000016 | 3.44 | 0.0006 |
| D96 | -0.02251 | 0.03969 | -0.57 | 0.5708 |
| D97 | 0.09340 | 0.03924 | 2.38 | 0.0176 |
| D98 | 0.1359 | 0.03895 | 3.49 | 0.0005 |
| D99 | -0.01225 | 0.03936 | -0.31 | 0.7557 |
| $\sigma^2_n$ | 0.2091 | 0.02649 | 7.89 | <.0001 |
| $\sigma^2_n + \sigma^2_e$ | | | | |
| s: lower-bound | -0.4509 | 0.4060 | -1.11 | 0.2672 |
| S: upper-bound | 0.8415 | 0.4062 | 2.07 | 0.0387 |
| $\bar{h}_{i}$ | 0.005455 | 0.000333 | 16.39 | <.0001 |
| $\bar{y}_{i}$ | -0.00005 | 0.000017 | -3.14 | 0.0018 |

All parameter estimates (excepted times dummies) prove to be statistically significant and have the expected signs. The coefficient of the gap between actual and average hours is negative and significant. This is consistent with the negativity of the coefficient $\theta$ in Caballero et al. (1997). This is also consistent with the fact that the co-movement between hours per worker and the number of employees is negative at plant level-observation. We obtained a correlation of -0.5 in our data. Note that the sign of the correlation between hours and workers is positive at the aggregate level (for detailed analysis on US data see, Cooper et al. (2005)). This result support non-convex adjustment costs. Indeed, if they were convex, the sign would positive because after a positive shock both employment and hours per workers would increase. As Cooper et al. (2005) point out: «The quadratic adjustment cost model is unable to generate the negative co-movement of hours and employment growth at the plant level». This result imply that it is important to depart from the standard specification of convex and symmetric adjustment costs. Indeed, if costs were truely convex, we would obtain a positive coefficient as following a postive (negative) shock, hours and employment would both rise.

As expected, we obtained a positive and significant for the gap between actual and average production and for the estimated demand shock.
On the Estimation of The Thresholds

The estimated lower threshold is negative (as expected) and is lower (in absolute term) than the estimated upper threshold. So at first view, our results indicate that it is more costly to expand employment than to contract it. But this is not the end of the story as we need to take into account the standard errors of the parameter. And even if the lower threshold is negative, it is not statistically significant.

To get a better idea of the relative importance of the two thresholds we use two commonly test procedures: Wald test and Likelihood ratio test. These two tests are asymptotically equivalent under the null hypothesis. We consider the hypothesis that \(-s = S\). We can rewritten it under the general form:

\[ H_0: R\hat{\theta} = 0 \quad / \quad H_1: R\hat{\theta} \neq 0 \quad \text{with} \quad \theta = (\beta, \sigma_u, s, S) \quad \text{and} \quad R = (0_{\text{dim} \beta}, 0, 1, 1) \]

The intuition behind the likelihood ratio test is the following. If \(H_0\) is true, then imposing it should not lead to a large reduction in the log-likelihood function. Therefore, we base the test on the difference between the value of the likelihood function at the unconstrained value of \(\theta\) and the value of the likelihood function at the restricted estimate. The test statistic is:

\[ \hat{V} = -2 \times \ln \left( \frac{L_c(\theta)}{L(\hat{\theta})} \right) \]

where \(L_c(\theta)\) is the value of the likelihood function at the restricted estimate. Under regularity and under \(H_0\), the large sample distribution of \(V\) is chi-squared, with degrees of freedom equal to the number of restrictions imposed. So Under \(H_0\), we have \(\hat{V} \overset{\text{as}}{\rightarrow} \chi^2(1)\). It follows that the reject region \(Q\) is defined as:

\[ Q = \left\{ \hat{V} / \hat{V} \geq 3.84 \right\} \]

As we observe \(\hat{V}_{\text{obs}} = 0.3\), we accept \(H_0\) at 5%.

The intuition behind the Wald test is that if the restriction if valid, then \(R \hat{\theta}\) should be close to zero since the estimator of \(\theta\) is consistent. Therefore the test is based on \(R \hat{\theta}\). The Wald statistic is \(\hat{W} = (R \hat{\theta})' \left( V_{\text{as}}(R \hat{\theta}) \right)^{-1} (R \hat{\theta}) \). Under \(H_0\), \(\hat{W} \overset{\text{as}}{\rightarrow} \chi^2(1)\). Here \(Q\) is defined as:

\[ Q = \left\{ \hat{W} / \hat{W} \geq 3.84 \right\} \]

As we observe \(\hat{W}_{\text{obs}} = 0.24\), we accept \(H_0\) at a level of 5%.
So the conclusion of these two tests is that the difference (in absolute terms) between the two thresholds is not statistically significant.

7 Conclusion

TO BE WRITTEN
Bibliography


Adda, J. and R. Cooper [2003]: « Dynamic economics », MIT Press


