Recent contributions on tax competition recognize the interaction between both horizontal and vertical tax externalities in a single federation. The possibility of dominant horizontal externalities leading to excessively low taxes or dominant vertical externalities associated with excessively high taxes has been subject to different empirical tests. In this paper, we extend the theoretical analysis to a framework with multiple federations (a Union). We show that the relative size of a federation in the Union determines not only the extent but also the direction of the tax inefficiency. The equilibrium state tax is set at a lower rate if the country is small relative to the Union but surprisingly, vertical externalities are more likely to dominate, i.e. for a relative small federation, the non-cooperative local tax rate will be lower than for a relative large federation but the tax rate is still higher than the one observed in the absence of tax competition. This result seems to contradict recent theoretical findings where a lower equilibrium state tax is followed by a dominant horizontal externality. (JEL-Codes: H21, H7, H3)
1 Introduction

Based on the results in Keen and Kotsogiannis (2002), recent empirical literature on tax externalities is trying to verify whether vertical or horizontal tax externalities dominate in a given federation. Different tax definitions and econometric approaches have been explored to answer this question in a number of countries (see, e.g., Brülhart and Jametti, 2006; Solé Ollé and Esteller Moré, 2002).

There is now a large literature on horizontal externalities. The basic idea, originally developed by Oates (1972) and formally modeled by Gordon (1983), Wilson (1986) and Zodrow and Mieszkowski (1986) is that when a region increases its capital tax rate, some amount of capital will be reallocated to other regions. This capital movement represents a positive externality, implying a tendency for taxes and public expenditures to be set inefficiently low in equilibrium. Vertical tax externalities on the other hand, have been more recently examined. They arise when different levels of government tax the same base. Each level of government neglects the adverse effect it has on the other by raising its tax rate, thereby causing the common tax base to shrink. This tax externality points towards excessively high state taxes. Recent literature include Johnson (1988), Boadway, Marchand and Vigneault (1998), Keen (1998), Wrede (2000) and Dahlby and Wilson (2003). Thus, horizontal and vertical tax externalities are likely to distort tax levels in opposite directions. A relevant question is then which one dominates the other and under which circumstances. Recent papers, most prominently Keen and Kotsogianis (2002), address this question.

\[\text{See also Wildasin(1989), and Wilson(1999) for a survey.}\]
\[\text{See also Boadway and Keen(1996) and Dahlby(1996) who analyze the implications of vertical externalities for the design of intergovernmental transfers.}\]
within a unified model featuring a single country. They show that which externality prevails hinges upon the balance between the interest responsiveness of the supply of savings and demand for capital, the extent to which factors are taxed by the states, and the strength of the preferences between federal and state expenditures. In particular, the horizontal tax externality dominates if the tax base is sufficiently mobile across states, whereas the vertical tax externality dominates if the aggregate tax base of the federation is sufficiently responsive to the state taxes. Hence, the balance of these effects seems actually to be an empirical issue.

Now, this finding is derived from a model assuming one single federation. Ignoring the outside world may be a too unrealistic assumption. As stated by Hayashi and Boadway (2001): "Ontario’s tax rate is not significantly influenced by any other provincial tax rate, but the Ontario tax rate has a significant positive effect on all the other provinces. This may be related to the unique position of Ontario in the Canadian federation: it is the largest and most populous province and has by far the biggest business tax base. Moreover, its major competitor for capital could be the United States". Considering other countries seems then to be meaningful and doing this might have important implications in terms of theoretical results and also in terms of how they are econometrically tested.

Formally, we extend the model of Keen and Kotsogianis (2002) to investigate the relative importance of tax externalities in a N-country setting, i.e., we consider multiple federations (a Union) There are two sources of horizontal externalities: between local governments within the federation and between countries in the economic area, while vertical externalities arise between local and federal government within each
federation. To analyze these externalities, we follow Keen and Kotsogianis in adopting the symmetric tax equilibrium in a given federation as a benchmark. This not only makes easy to compare results across models, but it also puts this benchmark, which is usually associated with a social optimum in this literature, into context.

We show that whether vertical or horizontal tax externalities dominate is not an empirical issue in the sense of Keen and Kotsogianis (2002), but it can be determined on the basis of the relative importance of a given federation in the corresponding economic area of competition. In particular, we find that the likelihood of a dominant vertical externality decreases with the size of the relative importance of a federation on the union. More specifically, the equilibrium state tax is set at a lower rate if the country is small relative to the Union but vertical externalities are more likely to dominate, i.e. for a relatively small federation, the non-cooperative local tax rate will be lower than for a relative large federation but the tax rate is still higher than the one observed in the absence of tax competition within the country. This model therefore helps explaining recent empirical findings in the literature. Thus, that vertical externalities seem to dominate Switzerland (Brülhart and Jametti, 2006) or that horizontal externalities seem to prevail in the United States () could be seen as a natural consequence of the relative sizes of the countries under question in their relevant areas of competition, namely, the European Union and NAFTA, respectively.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 examines the interplay of horizontal and vertical tax externalities when states in a Union set their taxes. Section 4 concludes.
2 The model

Assume a Union consisting of \( l > 1 \) countries (or federations) where each one consists of \( N_i \) identical jurisdictions (state governments) \( j \). The number of jurisdictions can differ from country to country, for example given two possible federations Germany (GE) and Switzerland (SW), the number of local governments in each federation may differ \( N_{GE} \geq N_{SW} \). In each jurisdiction \( j \), a single firm produces a private good according to a strictly concave production function \( F(k_{ij}) \), where capital \( k_{ij} \) is the only input. Capital is costlessly mobile not only across jurisdictions in a given federation but in the whole Union. Due to this feature, capital in the Union earns a unique post-tax return \( \rho \) in each jurisdiction.\(^5\) In a given federation \( L \), the central and state governments tax capital at a consolidated rate \( \tau_{Lj} = T_L + t_{Lj} \) where \( T_L \) denotes the federal tax rate on each unit of capital allocated in federation \( L \) and \( t_{Lj} \) denotes state’s tax rate on each unit of capital allocated in jurisdiction \( j \). Normalizing the price of the private good to one, the arbitrage condition \( F'(k_{ij}) = \rho + \tau_{ij} \), defines the demand for capital in each jurisdiction \( j \) as \( K_{ij} = K(\rho + \tau_{ij}) \), with \( K'(\rho + \tau_{ij}) = 1/F''(K_{ij}) < 0 \). Further, rents arising in a given jurisdiction \( \pi(\rho + \tau_{ij}) \) are defined as the difference between the value of production and the cost of capital:

\[
\pi(\rho + \tau_{ij}) = F(K_{ij}) - F'(K_{ij})K_{ij}\]

Tax collection at central and local level are spent exclusively in the provision of

\(^4\)Our model builds on Keen and Kotsogiannis (2002) one-federation’s framework. We use their notation where possible, in order to facilitate comparability.

\(^5\)Intuition how this could work: countries outside the union face a different cost of capital because the degree of capital mobility differs. Aca iria la posibilidad de que los paises enfrenten distintos \( \rho \) pero sobre todo la atencion esta puesta en los paises fuera de la union.

\(^6\)For simplicity and without loss of generality, we assume that rents are untaxed.
two distinct publicly provided goods, which are produced with constant returns. The budget constraint faced by each local government is defined by

$$ g_{lj} = t_{lj}K_{lj}(\rho + \tau_{lj}) $$

(1)

where $g_{lj}$ is a local publicly provided good. The central government in each of the countries in the union faces the following budget constraint:

$$ G_l = \frac{1}{N} \sum_{l=1}^{N_L} TK_{lj}(\rho + \tau_{lj}) $$

(2)

where $G_l$ is the amount of federal publicly provided good spent in each jurisdiction within the federation. We assume here that the federal government allocates its total tax receipts equally across states and recall jurisdictions are identical within a country. We assume further, that there are no intergovernmental transfers either central-local government transfers or between local governments.

As in Keen and Kotsogiannis (2002), there is a single consumer in each state $j$. She maximizes the intertemporal quasi-linear utility function $U_{lj}(C_1, C_2, g_{lj}, G_l) = u(C_1) + C_2 + \Gamma(g_{lj}, G_l)$, where $C_1$ and $C_2$ are private consumption in the first and second period respectively and $\Gamma(.)$ represents the utility she derives from the provision of local as well as federal public goods. Both $u(.)$ and $\Gamma(.)$ are strictly increasing concave functions. Each consumer is endowed with identical amount of income $e$ at the beginning of the first period and in the second receives principal and interest on her savings, which are defined as $S(\rho)$, with $S' \geq 0$.

The indirect utility function can be written as:
\[ U_{ij}(\rho, \tau_{ij}, g_{ij}, G_t) \equiv u_{ij}(e - S(\rho)) + (1 + \rho)S(\rho) + \pi(\rho + \tau_{ij}) + \Gamma(g_{ij}, G_t) \] (3)

The post-tax rate of return \( \rho \) in the federation is determined by the market-clearing condition

\[ \sum_i N_i S(\rho) = \sum_i \sum_{j=1}^{N_i} K_{ij}(\rho + \tau_{ij}) \] (4)

Recall capital is costlessly mobile across countries within the union, therefore, savings provide the stock of capital for the productive sector within the Union\(^7\). A change in state \( l^j \)'s tax rate on \( \rho \) is defined by

\[ \frac{\partial \rho}{\partial \tau_{ij}} = \frac{K'_{ij}(\rho + \tau_{ij})}{\sum_i N_i S'(\rho) - \sum_i \sum_{j=1}^{N_i} K'_{ij}(\rho + \tau_{ij})} \] (5)

If we impose symmetry of state tax rates within the federation. All states in the federation set the same tax but these tax rates could differ from the ones set by all state governments in other federations within the Union\(^8\) \((\tau_{ij} = \tau_l, \forall j)\). In turn,

\[ p'(\tau_l) = \frac{\partial \rho}{\partial \tau_l} = \frac{N_i K'_{ij}(\rho + \tau_l)}{\sum_i N_i [S'(\rho) - K'_{ij}(\rho + \tau_l)]} = N_i \frac{\partial \rho}{\partial \tau_{ij}} \in [-1, 0] \] (6)

\(^7\)In Keen and Kotsogiannis’ one-federation set-up, savings provide the stock of capital only within the federation \( NS(\rho) = \sum_j K(\rho + \tau_j) \)

\(^8\)We investigate thereby the effect of uncoordinated vs coordinated state tax policy but in an international context, desregarding at this point the effects of possible international tax coordination.
State and federal governments are assumed to be perfectly benevolent. They maximize the welfare of their own inhabitants and do not take into account the effect of their actions on residents of other states or countries outside the federation. The strategic policy variable of each policy maker is the tax rate at their disposal.

3 Vertical vs Horizontal Tax Externalities

3.1 Inefficiency in the setting of state taxes in a Union

Given the two potential sources of tax externalities, state taxes could be set at a lower than efficient rate (if the horizontal externality dominates) or at a higher than efficient rate (if the vertical externality dominates). Write the welfare of the typical citizen in state \( l_j \) using the indirect utility function (#3) and the local and central government budget constraints (#1) and (#2),

\[
W_{lj} = u_{lj}(e-S(\rho))+(1+\rho)S(\rho)+\pi(\rho+\tau_{lj})+\Gamma[t_{lj}K_{lj}(\rho+\tau_{lj}), \frac{1}{N_{lj}}\sum_{j=1}^{N_l}TK_{lj}(\rho+\tau_{lj})] 
\]

The first order condition of the government in state \( l_j \), evaluated at the symmetric equilibrium, is

\[
\frac{\partial W_{lj}}{\partial t_{lj}} = -K_{lj} + \Gamma_g[K_{lj} + t^*K'_{lj}(1 + \frac{1}{N_l}p')] + \frac{1}{N_l}\Gamma_gT^*K'_{lj}(1 + p') = 0
\]

Condition (#8) defines the equilibrium state tax rate. Denoting welfare in a
symmetric equilibrium by $W_l$ where $(\tau_{lj} = \tau_l, \forall j)$, the effect of a coordinated increase in all state taxes within a single federation is

$$W_{tl} = -K_{lj} + \Gamma_g[K_{lj} + t^* K'_{lj}(1 + p')] + \Gamma_G T^* K'_{lj}(1 + p') \tag{9}$$

The non-cooperative Nash equilibrium for the state tax rate is defined by eq. (#8) and setting eq. (#9) to zero implicitly defines the socially optimal state tax rate for a given federal tax rate. The sign of $W_{tl}$ evaluated at the non-cooperative state tax equilibrium indicates which externality dominates. For a dominant horizontal externality, $W_{tl} > 0$ meaning that state taxes are too low. For a dominant vertical externality, $W_{tl} < 0$ meaning that a slight decrease in all state taxes would increase social welfare. To investigate which externality dominates substract (#8) from (#9),

$$W_{tl} = [\Gamma_g t^* p' + \Gamma_G T^*(1 + p')](1 - \frac{1}{N_l})K'_{lj} \tag{10}$$

Since $(1 - \frac{1}{N_l})K'_{lj}$ is unambiguously negative and $p' \in [-1,0)$, the direction of inefficiency in the equilibrium state tax turns on the balance between the effects in the first bracketed term in (#10). Further, in symmetric equilibrium $G_l = T^* K(\rho + \tau_l)$ and $g_l = t_l K(\rho + \tau_l)$ and therefore the vertical externality dominates if and only if

$$|p'| < \frac{\Gamma_G G}{\Gamma_G G + \Gamma_G g} \tag{11}$$

Define $R \equiv N_L/ \sum_l N_l \leq 1$ as the relative importance of a given federation $L$ in the Union, measured as the number of identical state governments in federation $L$, that is $N_L$ over the number of identical state governments in the Union as a whole,
that is $\sum_i N_i$. In the one-federation set-up $\sum_i N_i = N_L$, what is equivalent to $R = 1$.

**Proposition 1** For a given federation $L$ with $N_L$ identical state governments, the non-cooperative equilibrium state tax rate (implicitly defined in eq(#8) as well as the socially optimal state tax rate increase with $R$.

**Proof.** [Sketch: see appendix] First note that in a symmetric equilibrium $\tau_l = \tau, \forall l$ and therefore eq.(#6) can be rewritten as $p'(\tau) = R \frac{K'(\rho+\tau)}{[S'(\rho)-K'(\rho+\tau)]}$. The non-cooperative equilibrium state tax rate defined implicitly in eq.(#8) can be rewrite as $\Gamma_g = \frac{K-N_L \Gamma_G T^* K'(1+p')}{[K+T^* K'(1+\frac{p'}{N_L})]} = C$. See first that the right hand side of the last expression is increasing in $p'(\tau)$, that is $\frac{\partial C}{\partial p'(\tau)} < 0$ for a given $T^*$. From eq(#6) in symmetric equilibrium we know that $\frac{\partial p'(\tau)}{\partial R} > 0$, and recall $\Gamma_g < 0$. Therefore an increase in $R$ is associated with a higher amount of locally provided public goods and thereby a higher equilibrium state tax rate, that is $\frac{\partial \tau}{\partial R} > 0$. The same proof applies for the socially optimal state tax rate $t_{SO}$ implicitly defined by $\Gamma_g = \frac{K-N_L \Gamma_G T^* K'(1+p')}{[K+T^* K'(1+\frac{p'}{N_L})]} = 0$. Following the same steps as above it is easy to show that $\frac{\partial t_{SO}}{\partial R} > 0$.  

The intuition go as follows: Even for a given number of state governments in a federation $N_L$, as the number of state governments that take part if the Union increase, the effect of both horizontal and vertical competition on state taxes in federation $L$ go in the same direction. Horizontal competition is now increased because capital is costlessly mobile across state governments in the Union and equilibrium state taxes decrease compared with the one-federation framework as $\sum_i N_i$ increases. Vertical competition in the context of a Union lead also to lower equilibrium state taxes compared with the one-federation framework. An increase in the state tax rate
in $jL$ only reduces federal expenditure in their state (as in all others) by $1/N_L$ but capital outflow outside federation $L$ given this increase in the state tax increases with $\sum_i N_i$.

We are interested now to see how the direction in the inefficiency in the setting of state taxes (vertical vs horizontal externalities) is affected by the relative importance of a federation $L$ in a Union:

**Proposition 2** For a given federation $L$ with $N_L$ identical state governments, there exist a critical value $\bar{R}$ such that: (a) If $R \equiv N_L/\sum_i N_i < \bar{R}$ the vertical tax externality dominates and (b) if $R > \bar{R}$ the horizontal externality dominates.

**Proof.** First look at the expression $[\Gamma_g t^* p' + \Gamma_G T^*(1 + p')]$ in eq.(#10) and recall in symmetric equilibrium $p'(\tau) = R\frac{K'(\rho+\tau)}{[S(\rho)-K'(\rho+\tau)]} \in [-1,0]$, with $\frac{\partial p'(\tau)}{\partial R} > 0$. Since from Proposition 1, $\frac{\partial^2 p'}{\partial R^2} < 0$, it follows that the expression between brackets is decreasing in $|p'(\tau)|$ for given values of $T^*, \Gamma_G, \Gamma_g$. Therefore if there exists $\bar{R} < 1$ such that $[\Gamma_g t^* p'(\bar{R}) + \Gamma_G T^*(1 + p'(\bar{R}))] = 0$, for all $R < \bar{R}$ it follows that $W_{tL} < 0$ and the vertical externality dominates and for all $R > \bar{R}$, it follows that $W_{tL} > 0$ and the horizontal externality dominates. Further, if for the given values of $T^*, \Gamma_G, \Gamma_g$, and $R = 1$, it follows than $[\Gamma_g t^* p' + \Gamma_G T^*(1 + p')] \geq 0$, then the vertical externality dominates for all possible $R$. ■

The intuition goes as follows: Consider the case where only the horizontal externality is present like in Zodrow and Mietszkowski (1986). It is clear from eq.(#8) and eq.(#9) that as $R \to 0$, the socially optimal state tax rate converges to the non-cooperative equilibrium state tax rate, that is $t^{so} \to t^*$. The state tax rate under
cooperation within a federation converges to the non-cooperative one as the number of state governments outside the federation (but within the Union) increases. Consider now the case where only the vertical externality is present like for example in Dahlby et al (2000). See that the relation between the non-cooperative equilibrium and the one under cooperation keeps constant. For a given $N_L$, no matter the value of $R$, the source of the vertical externality is to be found within the federation between the federal government and the $N_L$ state governments in the federation.

In the one federation set-up with $R = 1$, we know from eq(#11) that the vertical externality dominates if and only if $|p'| < \frac{G_G G}{G_G + G_G g}$. In the context of the Union however, we know that $\frac{\partial p'(\tau)}{\partial \tau} > 0$ and therefore, the likelihood of a dominant vertical externality decreases with $R$, that is if the number of jurisdictions in the federation $N_L$ represent a small enough part of the Union $\sum_i N_i$, the vertical externality dominates. See that if the vertical externality dominates in the one federation set-up with $R = 1$, it will dominate in the context of the Union with $R < 1$. On the other hand, if the horizontal externality dominates in the one-federation set-up, it would be possible that actually vertical externalities dominate in this federation that takes part of a Union because $R$ is small enough or that the inefficiency associated with this dominant horizontal externality is less important and thereby local taxes are set closer to the social optimal rates.

**Corollary 1** For a given federation $L$ with $N_L$ identical state governments, the non-cooperative equilibrium state tax rate and the likelihood of a dominant horizontal externality increases with $R$. 

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Proof. It follows immediately from Proposition 1 and Proposition 2. □

This last result is of crucial importance for empirical work based on a given federation in isolation. See that although local taxes are lower in a federation that is relative small in the context of a Union \((R)\) is small enough), it is much more likely that indeed the vertical externality dominates, with local taxes set at a higher than the social optimal rate. A rather extreme example but intuitively helpful is the following: for a federation where the horizontal externality dominates considered in isolation, it could be that indeed there is no inefficiency in the setting of local taxes when considered in the context of the Union.

Appendix

Proof of Proposition 1

First take the derivative of eq.(#8) with respect to \(R\),

\[
\frac{\partial t}{\partial R} = \frac{-\frac{1}{N_L} K'_{Lj} (\Gamma_g t^* + \Gamma_G T^*) \frac{\partial p'}{\partial R}}{-K'_{Lj} (1 + \frac{1}{N_L} p')(1 - \Gamma_g) + \Gamma_{gg} [K_{Lj} + t^* K''_{Lj} (1 + \frac{1}{N_L} p')]^2 + K'_{Lj} (1 + \frac{1}{N_L} p') + t^* K''_{Lj} (1 + \frac{1}{N_L} p')^2 + \frac{1}{N_L} \Gamma_G T^* K''_{Lj} (1 + p')(1 + \frac{1}{N_L} p') + \frac{1}{N_L} \Gamma_G G T^* K''_{Lj} (1 + p')^2}
\]

with \(K'_{Lj} < 0; K''_{Lj} < 0; \Gamma_{gg} < 0; \Gamma_{GG} < 0\); then a sufficient condition for \(\frac{\partial t}{\partial R} > 0\) is that \((1 - \Gamma_g) \leq 0\) and \(\frac{\partial p'}{\partial R} < 0\).
\[
\frac{\partial p'}{\partial R} = \frac{\left(\frac{K'_{Lj}}{S'-K'_{Lj}}\right)}{1 - \frac{\partial t}{\partial p'} R[K''_{Lj}S'(1 + \frac{1}{N_L} p') - \frac{1}{N_L} K''_{Lj} S'']p'}
\]

with $S'' < 0$; then $\frac{\partial p'}{\partial R} < 0$ because $\frac{\partial t}{\partial p'} < 0$ for $(1 - \Gamma_g) \leq 0$ (sufficient condition)

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