This paper examines the profitability of mergers in polluting oligopolies where firms are charged a tax per unit of emissions. We determine the impact of the imposition of the emissions tax on the profitability of a merger.

We consider two scenarios. In the first scenario, the tax on pollution is considered given and, in particular, is not changed subsequent to a merger. In the second scenario, we compute the efficiency inducing tax designed by a regulator. This tax depends on the number of firms. We study the profitability of a merger when the emissions tax is optimally adjusted post-merger.

We find that mergers are more profitable when the tax adjusts optimally to the post-merger industry structure. Secondly, mergers are more profitable when the tax depends on the stock of pollution accumulated over time, rather than on the current emissions by the firms. Thirdly, we show that by allowing the tax to optimally adjust subsequent to a merger, rather than letting it remain unchanged, we increase current welfare at the expense of future welfare and cause greater damage to the environment.

1 Introduction

Stricter environmental regulation can have anti-competitive effects. At the proceedings of the roundtable held in 1995 on "Competition Policy and the Environment" by the Committee on Competition Law and Policy of the Organization for Economic Co-operation and Development (OECD) each of the members presented specific national examples of environmental regulations leading to anti-competitive behaviour. The secretariat of the OECD summarizes some of the possible effects as follows. If environmental regulation requires the adoption of capital-intensive abatement technology, this may increase the minimum efficient scale of the industry and consequently act as a barrier to entry. Furthermore, such regulations sometimes only apply to new entrants, excusing incumbents from compliance. Incumbents might claim that new or existing close substitutes are harmful for the environment, thereby reducing competition. Once the number of firms are reduced in the industry, collusion is facilitated. Taxes on emissions may act as a stabilizing influence on cartels by reducing the incentive of any individual firm to increase its output. Also, if dominant firms exist in

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the industry, they may outbid smaller firms or potential entrants when pollution permits are auctioned off or traded.

The type of anti-competitive behaviour we focus on in this paper is horizontal mergers among polluting firms. Do stricter environmental policies encourage such mergers?

Whilst international agreements are being forged for the protection of the environment, most notably the Kyoto Protocol (1997), the general spate of mergers has been on the rise. According to Gugler et al, during the period 1981-98 there were 70,000 mergers proposed across North America and Europe, each worth over $1 million, of which approximately 45,000 were realized, the average value of firms being $2 million. The implementation of stricter environmental policies across much of the developed world could account for some of these mergers. For example, it has been suggested that the "Black Gold Merger Mania", as the recent oil mergers have been labelled by the media, may be a response to the US Environmental Protection Agency’s federal fuel oxygenate requirements from the Clean Air Act of 1990 and those of the state level agencies. The $82 billion merger of Exxon and Mobil in 1999 was closely followed by that of British Petroleum Amoco and ARCO (Atlantic Richfield Company) and that of Chevron and Texaco in 2000. Acquiring additional refineries eases the production of the several different varieties of fuels necessary to satisfy the myriad environmental regulations (over 40 fuel formulation requirements exist at different times and places within the US) and recaptures some of the economies of scale lost due to the imposition of these regulations. Within Canada, the polluting industries to experience the most merger activity in 2005, according to The Globe and Mail newspaper, include consumable fuels (281 deals worth $40.2 billion altogether), metals and minerals (74 deals worth $21.9 billion), transport (58 deals worth $4.6 billion) and paper (24 deals worth $2.7 billion).

Such mergers, especially among large polluters, have raised welfare concerns. The media has vociferously objected to the US Fair Trade Commission’s decision to allow such mergers, revealing the millions allegedly paid through campaign contributions and lobbying by large polluting firms to thwart the ratification of the Kyoto Protocol in the US. Advocates for consumer protection, such as the litigation organization Public Citizen in Washington, DC, argue that mergers resulting in greater market shares and profits for the participants would make it easier to spend more on such lobbying geared towards the dismantling of environmental laws.

International organizations are likewise concerned. At the 2003 Ministerial Conference of the World Trade Organization (WTO) a session was held to discuss "Sustainable Competition Law". Although competition policy may support environmental protection indirectly, by stimulating innovation and product improvement should consumers demand goods that are more environmentally sound, the question addressed at this session was whether competition law should directly support sustainable development. It has been proposed that the WTO develop into an umbrella for international competition disciplines, and build mechanisms into the international competition treaties to ensure global environmental protection. Before this can be realized, according to Gehring
(2003), there is a need for further research to ascertain the linkage between competition and environmental policies.

There do exist theoretical models which analyze the effects of mergers on welfare after accounting for their effects on the environment (Ramirez and Kay-alica, 2001; Sandonis and Mariel, 2003). Instead of following this line of research, this paper reverses the perspective from which to view this issue. We analyze, not how mergers affect the environment, but how stricter environmental policies affect the incentives to merge; a necessary step towards extricating the complex relationship between competition and environmental policies.

According to Salant, Switzer and Reynolds (1983, SSR hereafter), in a Cournot oligopoly, a merger of identical firms (in the absence of fixed costs) is only profitable if the merging firms constitute at least 80% of the market.\(^3\)

There have been numerous modifications of this framework to explain merger activity. For example, in an international context, some have used it to examine the effects of trade liberalization on the incentives to merge (Horn and Levinsohn, 2001; Long and Vousden, 1995). We adapt the SSR framework in this paper to focus on the role of environmental policy in determining the profitability of merger.

We consider a single country with a polluting oligopoly. Firms have identical technologies and are charged an identical tax per unit of emissions. We determine the impact of this emissions tax on the profitability of a merger.

We begin by considering two scenarios within a static framework. In the first scenario, the tax on pollution is considered given and, in particular, is not changed after a merger takes place. This setting is equivalent to that used by SSR, so that we are able to generalize the main results of SSR to the context of a polluting oligopoly. In the second scenario, we compute the efficiency inducing tax designed by a regulator. This tax depends on the number of firms. We study the profitability of a merger when the per unit tax on emissions is optimally changed after a merger takes place. In this case, the optimal tax rate is shown to fall subsequent to a merger. This causes the merger participants to decrease output by a lower percentage than they would had the tax rate remained unchanged subsequent to the merger. Correspondingly, the percentage increase in the output of the non-participating firms is lower under the optimal tax than under a constant tax rate. Thus, the gain in market power subsequent to a merger of the merger participants is greater when the per unit tax on emissions is optimally changed after a merger takes place. The critical market share ensuring non-negative profitability of merger is, thus, shown to be lower in the second scenario than in the first. Implementation of the optimal emissions tax increases the profitability of horizontal mergers among the firms within a polluting oligopoly.

We then extend our analysis to a dynamic setting, similar to Benchekroun and Long (1998), where the pollution emissions accumulate into a stock of pollution and where the regulator implements an emissions tax rate that is increasing.

\(^3\)Davidson and Deneckere (1985) show that mergers are always profitable in models of price competition.
in the pollution stock. We again consider the same scenarios as in the static context. For both scenarios, we find that mergers are generally more profitable in the dynamic setting as compared to the static setting. As in the static case, mergers are more profitable in the second scenario. When comparing the two scenarios, we find that there arises an intertemporal tradeoff in terms of social welfare. Although total welfare is higher in the second scenario where the tax rate is optimally adjusted subsequent to a merger, than in the first scenario, we provide examples where this is achieved by increasing current welfare at the expense of future welfare.

Sections 2 and 3 present the analysis for the static and dynamic models respectively. Section 4 concludes.

2 The Static Model

Our model is that of a Cournot oligopoly with \( N \) identical firms that produce a homogenous good. Each firm has a constant marginal cost, \( c \geq 0 \) and produces an output of \( q_i \). Industry output is given by \( Q = \sum q_i \) and the inverse demand function is \( P = P(Q) \) with \( P' < 0 \) and \( P(0) > c \). The firm \( i \) is assumed to emit pollution of \( e_i = q_i \).

Firms are able to choose their own output levels. However, they have to pay a pollution tax. We consider the case where each firm’s tax bill depends on its level of output. Specifically, we consider tax rules which are linear in output. Since the firms are identical, we restrict our study to the case where all firms face the same tax rule. Thus, firm \( i \)’s tax bill is given by

\[ T_i = \tau q_i \]

Each firm maximizes its profits, given by:

\[ \pi_i = P(q_i + Q_{-i})q_i - (c + \tau)q_i \]

The first order condition is given by:

\[ P - c + P'q_i - \tau = 0 \quad (1) \]

We consider only symmetric equilibria where all firms behave identically. If \( \tau = 0 \) for all \( Q \), then \( Q = \bar{Q} \), where

\[ P(\bar{Q}) + (\bar{Q}/N)P'(\bar{Q}) = c \]

The level \( \bar{Q} \) is called the laissez-faire level of output. If \( \tau \) is positive then we have

\[ P - c + P' \frac{Q}{N} - \tau = 0 \quad (2) \]

We consider the following specific case. The demand function is linear: \( P(Q) = P(0) - bQ \), \( P(0) - c \equiv a > 0 \). The damage function is quadratic: \( D(Q) = (\gamma/2)Q^2 \).
In this context, we examine the profitability of a merger of size $M$. Pre-merger there exist $N \geq M > 1$ firms. The profitability of merger is given by:

$$G + F$$

where $F$ is a constant representing merger induced fixed effects. $F$ can be positive or negative. For example, if the merger leads to management discord within the merged entity $F$ is negative. If, on the other hand, subsequent ot a merger, inefficient managers are sacked or some fixed costs of production are saved, $F$ could be positive. In the following analysis, in order to make our results easily comparable to SSR, we set $F = 0$ unless otherwise mentioned.

$$G \equiv \pi_M - M\pi$$

where $\pi_M$ denotes the joint profits of the participating firms subsequent to the merger and $\pi$ denotes the individual profits of a single firm in the symmetric equilibrium prior to the merger.

### 2.1 Static Model: Scenario 1

In Scenario 1, the tax rate, $\tau$, is an exogenously given constant that is unaffected by the realization of mergers. In this case, the model is equivalent to that in SSR with each firm having an effective marginal cost of $c + \tau$.

Given the linear demand and quadratic damage functions, using 2 for an interior solution, the pre-merger equilibrium quantity produced by each firm is given by:

$$q_1 = \frac{a - \tau}{b(N + 1)}$$

It can be shown that pre-merger equilibrium profits are given by:

$$\pi = b(q_1)^2 = \frac{1}{b} \left( \frac{a - \tau}{(N + 1)} \right)^2$$

The post-merger quantity of each firm is given by:

$$q_{1M} = \frac{a - \tau}{b(N - M + 2)}$$

The post-merger profits of the merging firms is given by:

$$\pi_M = b(q_{1M})^2 = \frac{1}{b} \left( \frac{a - \tau}{(N - M + 2)} \right)^2$$
The profitability of merger is given by:

\[ G = \pi_M - M\pi \]

\[ = -\frac{1}{b}(a - \tau)^2 \left( \frac{1}{(N-M+2)^2} - \frac{M}{(N+1)^2} \right) \]

\[ = -\frac{1}{b}(a - \tau)^2 \frac{(2N - 3M - 2MN + M^2 + N^2 + 1)(M - 1)}{(N-M+2)^2(N+1)^2} \]

\[ = -\frac{1}{b}(a - \tau)^2 \frac{(M - M_{1SSR})(M - M_{2SSR})(M - 1)}{(N-M+2)^2(N+1)^2} \]

where \( M_{1SSR} = N - \frac{1}{2}\sqrt{4N + 5 + \frac{3}{2}} < N \) and \( M_{2SSR} = N + \frac{1}{2}\sqrt{4N + 5 + \frac{3}{2}} > N \). For any \( M < M_{1SSR} \) we have \( G < 0 \) and when \( M > M_{1SSR} \) we have \( G > 0 \). The critical number of merging firms required for a merger to be profitable is given by \( M_{1SSR} \). Let \( \hat{\alpha}_{SSR} = \frac{M_{1SSR}}{n} \), where \( \hat{\alpha}_{SSR} \) represents the ratio of merging firms to all the firms in the industry such that the merger causes neither losses nor gains. Thus \( \hat{\alpha}_{SSR} \) is given by:

\[ \hat{\alpha}_{SSR} = \frac{N - \frac{1}{2}\sqrt{4N + 5 + \frac{3}{2}}}{N} \]

### 2.2 Static Model: Scenario 2

In Scenario 2, the firms are faced with the optimal emissions tax. Social welfare is assumed to be \( W = U(Q) - cQ - D(Q) \) where \( U(Q) \) is the area under the demand curve and \( D(Q) \geq 0 \) is the damage function representing the harm caused by pollution. The damage function, \( D(Q) \), is assumed to be strictly convex with the following properties:

\[ D(0) = 0, D'(0) = 0, D'(Q) > 0 \text{ for } Q > 0, D''(Q) > 0 \]

The first best level of industry output solves the following problem:

\[ \max_Q U(Q) - cQ - D(Q) \]

Thus, the socially optimal \( Q \) satisfies the following first order condition:

\[ P(Q) - c - D'(Q) = 0 \quad (3) \]

Let \( \tilde{Q} \) represent the socially optimal level of industry output that satisfies 3. We want to find a tax rule, \( \tilde{\tau} \), such that the Nash equilibrium output level given by 2 is exactly the same as \( \tilde{Q} \). This implies that the following must hold:

\[ P - c + P' \frac{\tilde{Q}}{N} - \tilde{\tau} = 0 \quad (4) \]

where \( P = P(\tilde{Q}), P' = P'(\tilde{Q}) \). Thus, the optimal tax, \( \tilde{\tau} \), is given by:

\[ \tilde{\tau} = P - c + P' \frac{\tilde{Q}}{N} \quad (5) \]
Under the linear demand and quadratic damage specifications, we compute $\hat{Q}$, using 3.

$$\hat{Q} = \frac{a}{b + \gamma}$$

From 5 we have

$$\hat{\tau} = \frac{a (N\gamma - b)}{N (b + \gamma)} = \frac{\hat{Q} (N\gamma - b)}{N}$$

(6)

We note that the optimal tax rate is positive iff $N\gamma > b$. It can be shown that $\hat{\tau} > 0$ iff $\hat{Q} < Q^{\text{Cournot}} = \hat{Q}$. Also, the larger the marginal damage caused by emissions, $\gamma$, the greater is the optimal tax, as shown by 7.

$$\frac{\partial \hat{\tau}}{\partial \gamma} = \frac{b (1 + N)}{(b + \gamma) N} \frac{a}{b + \gamma} > 0$$

(7)

Proposition 1. In the static case, the pre-merger level of the optimal tax is higher than its post-merger level.

Proof. Subsequent to a merger of $M$ firms, the total number of firms in the industry falls to $N - M + 1$. This, together with 8, completes the proof.

$$\frac{\partial \hat{\tau}}{\partial N} = \frac{ab}{N^2 (b + \gamma)} > 0$$

(8)

The more firms there are, the greater the incentive for each firm to neglect the effect of its own output on the industry’s tax bill. To counter this effect, the optimal tax rate increases in response to an increase in the number of firms.

SSR (1983) have shown that a horizontal merger within an industry of identical firms competing in quantities is not profitable unless $\hat{\alpha}_{SSR}$ is at least 80%. In the face of an optimal tax of the form $\hat{\tau}$, is such a merger likely to be more or less profitable than in its absence (that is, the case presented by SSR)?

In Scenario 2, using 2 for an interior solution and 5, the pre-merger equilibrium quantity produced by each firm is given by:

$$q_2 = \frac{a}{(b + \gamma) N}$$

The pre-merger profit of each firm is given by:

$$\pi = b (q_2)^2 = \frac{Ma^2 b}{(b + \gamma)^2 N^2}$$

The post-merger quantity of each firm is given by:

$$q_{2M} = \frac{a}{(b + \gamma) (N - M + 1)}$$
The post-merger profit of the merged firm is given by:

\[ \pi_M = b (q_{2M})^2 = \frac{a^2 b}{(N - M + 1)^2 (b + \gamma)^2} \]

The profitability of merger is given by:

\[ G \equiv \pi_M - M \pi = \frac{a^2 b}{(N - M + 1)^2 (b + \gamma)^2} - \frac{Ma^2 b}{(b + \gamma)^2 N^2} \]

\[ = \frac{(M + 2MN - M^2 - N^2) (M - 1) a^2 b}{(N - M + 1)^2 (b + \gamma)^2 N^2} \]

\[ = \frac{- (M - M_1) (M - M_2) (M - 1) a^2 b}{(N - M + 1)^2 (b + \gamma)^2 N^2} \] (9)

where \( M_1 = N - \frac{1}{2} \sqrt{4N + 1} + \frac{1}{2} < N \) and \( M_2 = N + \frac{1}{2} \sqrt{4N + 1} + \frac{1}{2} > N \). For any \( M < M_1 \) we have \( G < 0 \) and when \( M > M_1 \) we have \( G > 0 \). Moreover, let \( \hat{\alpha} = \frac{M_{1\alpha}(n)}{n} \), where \( \hat{\alpha} \) represents the ratio of merging firms to all the firms in the industry such that the merger causes neither losses nor gains. Thus \( \hat{\alpha} \) is given by:

\[ \hat{\alpha} = \frac{n - \frac{1}{2} \sqrt{4n + 1} + \frac{1}{2}}{n} \]

2.3 Static Model: Scenario 1 versus Scenario 2

In Scenario 2, a merger results in the fall of the tax rate faced by each firm in the industry, whereas in Scenario 1, a merger leaves the tax rate unchanged. Does this necessarily mean that mergers become more profitable in Scenario 2 than in Scenario 1? The answer is not obvious since although the merger brings about the fall in the tax rate, the benefits of this lower tax are reaped by participants of the merger and non-participants alike. Thus any gain in the profitability of merger due to a lower tax rate may be offset by a simultaneous increase in the output of the non-participants.

The total quantity supplied falls subsequent to a merger in Scenario 1, whereas it remains unchanged in Scenario 2 due to the adjustment of the optimal tax rate. A merger can thus raise the market price in Scenario 1 but not in Scenario 2. Does this imply that the merging firms gain more market power in the first scenario? If so, mergers would be expected to be more profitable in Scenario 1 than in Scenario 2. Surprisingly, this is not the case.

In order to determine which scenario is more conducive to mergers, we compare the threshold values of \( M \) in Scenarios 1 and 2, \( M_{1\alpha} \) respectively, beyond which mergers become profitable.

**Proposition 2.** For all \( N \geq 2 \), we have \( M_{1\alpha} > M_1 \) and \( \hat{\alpha_{SSR}} > \hat{\alpha} \).
Proof.

\[ M_{ISSR} - M_1 \]
\[ = \left( -\frac{1}{2} \right) \left( \sqrt{4N + 5} - \sqrt{4N + 1} - 2 \right) \]
\[ > 0 \quad \text{for all } N \geq 2 \]

This implies \( \hat{\alpha}_{SSR} > \hat{\alpha} \).

Proposition 2 is illustrated in Figures 1 and 2.

Since \( \hat{\alpha}_{SSR} > \hat{\alpha} \) (equivalently \( M_{ISSR} > M_1 \)) for all \( N \geq 2 \), we conclude that mergers are more likely to occur when \( \hat{\tau} \) is imposed, that is in Scenario 2, than in Scenario 1. In particular, a merger of two firms can be profitable in
Scenario 2, but never in Scenario 1. Therefore, the imposition of the optimal tax rate that depends on the number of firms may lead to monopolization of the industry, whereas in Scenario 1 with a constant tax rate and given our setting, the industry will not reach monopoly. In Scenario 2, all merger sizes can possibly be profitable: for any $M > 1$ there exists $N > M$ such that the merger of $M$ firms is profitable.

The intuition behind Proposition 2 is revealed by comparing the merger induced increase in market power of the merger participants in both the scenarios. In this context, there are essentially two effects that determine the merger induced increase in market power: the effect on the market price subsequent to the merger and the effect on the tax rate subsequent to the merger. In Scenario 1, the total quantity falls subsequent to a merger, raising market price, whereas in Scenario 2, total quantity remains unchanged at the social optimal level. Thus, the first effect is larger in Scenario 1 than in Scenario 2. In Scenario 1, however, the tax rate remains unchanged subsequent to a merger, whereas in Scenario 2, the tax rate falls. It can be shown that the second effect outweighs the first, so that market power increases more in the second scenario as a result of a merger.

The percentage increase in the merging firms’ market power subsequent to the merger is given by $(1 - M)^{(1-M)}(M-N-2) > 0$ in Scenario 1 and $(1 - M)^{(1-M)}(M-N-1) > (1 - M)^{(1-M)}(M-N-2) > 0$ in Scenario 2. This explains why mergers yield greater profits in Scenario 2 than in Scenario 1.

Before proceeding, we note that in Scenario 1, the marginal rate of damage from pollution, $\gamma$, does not affect the profitability of merger. However, in Scenario 2, $\gamma$ enters $G$ through the optimal tax, $\hat{\gamma}$. We determine the effect of changes in $\gamma$ on the profitability of merger.

Proposition 3. For $G > 0$ ($G < 0$), the profitability of merger is decreasing (increasing) in $\gamma$.

Proof: From 9, we have the following:

$$\frac{\partial G}{\partial \gamma} = \frac{-2}{(b + \gamma)}G$$

An implication of Proposition 3 is that changes in $\gamma$ cannot change the sign of the profitability of merger. However, if $F \neq 0$, we may have a change in $\gamma$ rendering a non-profitable merger profitable or vice versa. For $M < M_1$, and $F > |G| > 0$, we have a merger that is profitable and we have $\frac{\partial G}{\partial \gamma} > 0$. Thus, small mergers are the most profitable in the most damaging industries when the fixed effect of merger, $F$, is large and positive. For $M > M_1$, $F < 0$ and $|F| > G$, we have $G + F < 0$. Also, we have $\frac{\partial G}{\partial \gamma} < 0$. Thus large mergers are the least profitable in the most damaging industries.

Before proceeding, it is interesting to note that in both scenarios, $G$ is non-monotonic in $M$ with

$$\frac{\partial G(2, N)}{\partial M} < 0 \quad \text{for} \quad N \geq 5$$
and
\[
\frac{\partial G(N - 1, N)}{\partial M} > 0 \quad \text{for} \quad N \geq 2
\]

\section{The Dynamic Model}

We extend our analysis to a dynamic model by adopting a continuous time setting similar to that provided by Benchekroun and Long (1998).

In the dynamic model, pollution accumulates over time into a stock, $S$. The tax rate linearly increases in the stock (instead of in $Q$, as in the static case) and is given by:
\[
\tau(S) = \eta + \alpha S
\]

We study the Markov Perfect Nash equilibrium of this model, where the strategies of all firms are functions of the payoff relevant state variable, $S$.

Firm $i$’s optimal control problem is given by:
\[
\max \int_0^\infty e^{-rt}[P(Q_{-i}(S) + q_i)q_i - (c + \tau(S))q_i]dt
\]
subject to
\[
\dot{S} = Q_{-i}(S) + q_i - \delta S
\]
\[
q_i \geq 0
\]
\[
S(0) = S_0
\]
where $\delta$ represents the natural rate of decay of the stock of pollution.

In the dynamic context, the profitability of merger is given by:
\[
G + F = V^M(S) - MV_i(S) + F
\]
where $V^M(S)$ is the present discounted value of the future stream of profits of the merged firm and $V_i(S)$ is that of firm $i$ before the merger.

We restrict our study to the linear quadratic case, where demand and damage functions are given by 10 and 11 respectively.
\[
P(Q(t)) = P(0) - bQ(t)
\]
\[
D(S(t)) = (\gamma/2)(S(t))^2
\]

Benchekroun and Long (1998) show that for the closed loop equilibrium, the value function of each firm for the linear quadratic case is given by:
\[
V_i(S) = \frac{1}{2}AS^2 + BS + C
\]
where

\[
A = \frac{1}{4} N^{-2} \left( -\sqrt{\frac{2\alpha (N^2 + 1) + b (N + 1)^2 (r + 2\delta)}{(N + 1)^2 (br + 2\alpha + 2b\delta)} + \alpha (2N^2 - 4N + 2) + b (N + 1)^2 (r + 2\delta)} - b\right)
\]

\[
B = \frac{(a - \eta) ((N^2 + 1) A - 2\alpha)}{b(r + \delta) (N + 1)^2 + \alpha (1 + N^2) - 2N^2 A}
\]

\[
C = \frac{1}{-rb (N + 1)^2} (\eta - a - B) (a - \eta + BN^2)
\]

In our model, 12 represents the pre-merger value of each firm in the industry. The post-merger value is given by:

\[
V^M (S) = \frac{1}{2} A_M S^2 + B_M S + C_M
\]

where

\[
A_M = \frac{1}{4} (N-M+1)^{-2} \left( -\sqrt{\frac{2\alpha ((N-M+1)^2 + 1) + b ((N-M+1+1)^2 (r + 2\delta)}{(N-M+1+1)^2 (br + 2\alpha + 2b\delta)} + b ((N-M+1+1)^2 (r + 2\delta)} - b\right)
\]

\[
B_M = \frac{(a - \eta) (((N-M+1)^2 + 1) A_M - 2\alpha)}{b(r + \delta) ((N-M+1+1)^2 + \alpha (1 + (N-M+1+1)^2) - 2(N-M+1)^2 A_M}
\]

\[
C_M = \frac{1}{-rb ((N-M+1+1)^2} (\eta - a - B_M) (a - \eta + B_M (N-M+1)^2)
\]

In the static model, for any given \(a, b, M\) and \(N\), the only parameters to affect the profitability of merger were \(\tau\) and \(\gamma\). In the dynamic model, however, the profitability of merger is affected by changes in several other parameters such as \(S, \alpha, \delta, r,\) and \(\eta\). As we shall see, some of these parameters enter the optimal tax rate while others do not. Thus, any changes in these parameter values affect the profitability of merger differently in scenarios 1 and 2.

### 3.1 Dynamic Model: Scenario 1

For the dynamic case, in Scenario 1, \(\alpha\), and \(\eta\) are given constants that remain unchanged subsequent to a merger. Due to the complicated expressions for \(A, B, C, A_M, B_M,\) and \(C_M\) it is not possible to obtain meaningful results analytically. We, therefore, proceed with a numerical analysis. Unless otherwise mentioned the following parameter values are used: \(N = 101, S = 0, a = 10, b = 1,\)
The results summarized by Remarks 1-5 below can be obtained for a wide range of other parameter values.

**Remark 1.** $M_1$ is decreasing in $S$.

For the dynamic case, the threshold value of $M$ beyond which mergers become profitable, $M_1$, is decreasing in the stock of pollution. The more polluted the environment, the more profitable the merger within the polluting oligopoly. This is illustrated in Figure 3 below.

**Remark 2.** Small mergers are more profitable when the stock is large and large mergers are most profitable when the stock is low.

In Figure 4 above, we see the non-monotonicity of $G$ in $M$. For example, for...
$S = 0$, we have a merger of two firms being profitable whereas a merger of fifty firms being nonproftable.

**Remark 3.** *Small mergers are more profitable when $r$ is high and large mergers are more profitable when $r$ is low.*

Small mergers are likely to be undertaken by more myopic firms and large mergers by more forward-looking firms. This is illustrated in Figure 5 below where $F = 0$.

*Figure 5*

**Remark 4.** *Small mergers are more profitable for industries with a low $\delta$ and large mergers are more profitable when $\delta$ is high.*

In an industry with a low rate of decay of the pollution stock, small mergers are more likely to be realized. This is shown in Figure 6 for $F = 0$.

*Figure 6*
Remark 5. Small mergers are more profitable when $\alpha$ is high and large mergers are more profitable when $\alpha$ is low.

The parameter directly under the control of the social planner is the pollution tax rate. If the slope of the tax rate with respect to the stock of pollution, $\alpha$, is set high, small mergers are more likely to occur. If $\alpha$ is set low, large mergers are more likely to be realized. This is shown in Figure 7 for $F = 0$.

Figure 7

3.2 Dynamic Model: Scenario 2

In Scenario 2, the optimal tax rate is imposed on the polluting industry. The method for computing the optimal tax rate for the dynamic model is similar to that for the static model, as presented in the previous section.

For industry size $N$, the optimal tax rate is given by $\tau^* (S) = \eta^* + \alpha^* S$, where $\alpha^*$ and $\eta^*$ are given by 14 and 15.

$$\alpha^* = \frac{b}{1 + \frac{N\delta - (N-1)\rho}{N(r+\delta)-(N-1)\rho}} \left(-\frac{\rho}{N} + \frac{\rho(N+1)(\rho - \delta) - \frac{N-1}{N} \rho^2}{N(r+\delta) - (N-1)\rho}\right) > 0 \quad (14)$$

$$\eta^* = a - \frac{(r + \delta)(\delta - \rho)}{\gamma + b\delta(r + \delta)} \left(b + \frac{b}{N}\right)$$

$$+ \alpha \frac{(r + \delta)(N - 1)}{\gamma + b\delta(r + \delta)} \frac{(\delta - \rho)}{N(r + \delta) - \rho(N - 1)}$$

$$+ ab \frac{r + \delta}{\gamma + b\delta(r + \delta)} \frac{(\delta - \rho)}{N(r + \delta) - \rho(N - 1)} \left(\rho(N+1) - \frac{1}{N} \rho (N-1)\right)$$

where

$$\rho = \frac{1}{2} r + \delta - \frac{1}{2} \sqrt{4\gamma + 4r\delta + r^2 + 4\delta^2} < 0 \quad \text{for all} \quad \gamma > 0$$
Before proceeding, it is interesting to note that $\eta^*$ and $\alpha^*$ are dependent on $\delta$, $r$, and $\gamma$. Thus, changes in the values of these parameters will affect the profitability of merger differently than in Scenario 1. However, $\eta^*$ and $\alpha^*$ are independent of $S$ so that any changes in $S$ affect the profitability of merger in the same way as in Scenario 1.

We note that $\eta^*$ can be negative or positive and that both $\eta^*$ and $\alpha^*$ depend on $N$ and therefore, will change as a result of a merger. The post-merger values of these parameters are denoted by $\eta^*_M$ and $\alpha^*_M$.

Proposition 4. We have $\alpha^* - \alpha^*_M < 0$.

Proof. From 14, we get 16.

$$\frac{\partial \alpha^*}{\partial N} = \frac{(2\rho - 2\delta - r)^2}{(2N\rho - 2\rho - 2N\delta - Nr)^2} \frac{bp}{b} < 0$$

Thus, the slope of the optimal tax rate with respect to the pollution stock increases subsequent to a merger. It is not, however, straightforward to determine the effect of a merger on $\eta^*$. We have

$$\frac{\partial \eta^*}{\partial N} = -\frac{(2Nr + 2N\delta + \rho \left(1 - N^2 - 2N\right)) \left(\delta - \rho\right) (r + \delta) \rho ba}{(Nr + \rho + N\delta - N\rho)^2 \left(\gamma + b\rho \delta + b\delta^2\right) N^2}$$

$$+ a \frac{(r + \delta) (N - 1)}{\gamma + b\delta (r + \delta)} \frac{\partial \alpha^*}{\partial N}$$

where the first term is positive but the second is negative, so that the sign of $\frac{\partial \eta^*}{\partial N}$ depends on the specific values taken by the parameters. For the parameter values used in the section above and using $N = 10$ and $\gamma = 0.5$, we have $\frac{\partial \eta^*}{\partial N} > 0$ for all $N > 1$. Subsequent to a merger of $M = 10$ firms, for example, $\alpha^*$ rises from 0.34255 to 0.76923 and $\eta^*$ falls from 4.6031 to −10. This implies that the tax rate falls subsequent to the merger for all $S \leq 135.1$ (that is, for all values of $S$ for which the quantity produced is positive). Having computed the optimal tax for several different sets of parameters values, we have yet to find a case the optimal tax rises subsequent to a merger. What does this imply for the profitability of merger in the dynamic model?

Figure 8 shows the joint profits of the participants of the merger. The black curve in Figure 8 shows the pre-merger joint profits evaluated at the optimal tax rate $(10V_i(S))$, the red curve shows the post-merger joint profits $(10V_i^M(S))$ in Scenario 1 (where, subsequent to the merger, the tax rate remains unchanged), and the dotted curve shows the post-merger joint profits in Scenario 2 (where, subsequent to the merger, the tax rate changes optimally). The profitability of merger is greater in Scenario 2 than in Scenario 1, as shown by Figure 8. In Figures 8-11 we show that case where the merger occurs at time $t = 0$, $N = M = 10$ and $S_0 = 0$. However, qualitatively, the results are very similar for $2 \leq M \leq 10$ and for higher values of $S_0$. 

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Having established that mergers are more profitable in Scenario 2, we compare the effects of mergers on the environment and on social welfare in the two scenarios. We begin by examining the path followed by the output of each firm over time, as shown in Figure 9, which is given by:

\[ q_i(S) = \frac{1}{N} \delta \left( \frac{N (a - \eta + B)}{-b (N + 1) \left( -\delta + \frac{N(A-\alpha)}{(N+1)b} \right)} + \frac{1}{N} \left( \frac{N (A - \alpha)}{(N + 1)b} \right) (S - \frac{N (a - \eta + B)}{-b (N + 1) \left( -\delta + \frac{N(A-\alpha)}{(N+1)b} \right)}) \right) \]

where \( S \) represents the time path of the stock of pollution.
Figure 10 plots the time path of the stock of pollution, given by:

\[ S(t) = -\frac{(\eta - a - B) N}{b \delta (N + 1) + N(\alpha - A)} + \left( S_0 + \frac{(\eta - a - B) N}{b \delta (N + 1) + N(\alpha - A)} \right) e^{t(-\delta + \frac{N(A - \alpha)}{N + 1})} \]

Figure 10

In Figure 10, the black curve is the pre-merger stock path. The red curve is the post-merger stock path in Scenario 1. The dotted curve is the post-merger stock path in Scenario 2. As expected, in the second scenario, the tax rate adjusts
itself so that subsequent to the merger the optimal stock path is maintained. Here we see a divergence of interest between environmentalists and the social planner. Scenario 1 turns out to be better for the environment, even though the social welfare is maximized by Scenario 2.

The time path of social welfare is shown by Figure 11, where welfare is given by:

\[ W(t) = aQ(t) - \frac{b}{2}(Q(t))^2 - \frac{\gamma}{2}(S(t))^2 \]

Figure 11

Figure 11 shows that implementing the optimal tax leads to an intertemporal tradeoff. For \(2 \leq M \leq 10\), it can be shown that, as shown in Figure 11, there is a transfer of welfare away from the future towards the present.

Now we revert to further discussion about the profitability of merger. In general, we find that for both scenarios, mergers are more profitable in the dynamic setting than in the static setting. Also, just like the static case, in the dynamic case mergers are more profitable in Scenario 2. These features of our model are illustrated in Figure 12 below, for the parameter values given by \(S = 4.5, \ a = 10, \ b = 1, \ \delta = 0.5, \ r = 0.3, \ F = 0\) and \(\gamma = 1.9\). We note that, unlike in the static case where even in Scenario 2, a merger of two firms is only profitable if \(N \leq 3\), it is possible that a merger of two firms is profitable for \(N > 3\). Specifically, Figure 12 shows that a merger of two firms is profitable in an industry of four firms for Scenario 2 within the dynamic setting.
4 Conclusion

In this paper we determined the impact of an emissions tax on the profitability of a merger, in the context of a polluting oligopoly.

We considered two scenarios, first within a static framework and then within a dynamic framework. In the first scenario, the tax on pollution was considered given and, in particular, is not changed after a merger was realized. In the second scenario, we computed the efficiency inducing tax designed by a regulator. This tax was shown to depend on the number of firms. We studied the profitability of a merger when the per unit tax on emissions was optimally changed subsequent to a merger. This efficiency inducing tax rate was shown to fall as a result of a merger. This induced the merger participants to decrease output by a lower percentage than they would had the tax rate remained unchanged subsequent to the merger. Correspondingly, the percentage increase in the output of the non-participating firms was lower under the optimal tax than under a constant tax rate. This explained our finding that mergers are more likely to be profitable under the second scenario than under the first.

For both scenarios, we found that mergers are generally more profitable in the dynamic setting as compared to the static setting. In our dynamic setting, we considered the case where pollution emissions accumulate into a stock of pollution and where the regulator implements an emissions tax rate that is increasing in the pollution stock. In this context, we provided examples to
illustrate the following. When a constant tax rate, independent of the industry size, is imposed on the polluting oligopoly, we showed that mergers are more likely to be realized in industries with higher pollution stocks. Small mergers are likely to be undertaken by more myopic firms and large mergers by more forward-looking firms. In an industry with a low rate of decay of the pollution stock, small mergers are more likely to be realized. If the slope of the tax rate with respect to the stock of pollution, $\alpha$, is set high, small mergers are more likely to occur. If $\alpha$ is set low, large mergers are more likely to be realized. When comparing the two scenarios, we showed that there can arise an intertemporal tradeoff in terms of social welfare. Although total welfare is higher in the second scenario where the tax rate is optimally adjusted subsequent to a merger, than in the first scenario, we provided examples where this is achieved by increasing current welfare at the expense of future welfare.

References


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