Partial privatization and entry of private firms

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Abstract

This paper investigates the relationship between partial privatization and the entry of private firms. The effects of foreign private firms’ entry on a public firm’s output, profit and on the home country’s welfare are different from well known results, if the public firm is partially privatized. Furthermore, the effects of partial privatization on market performance, as well as the socially optimal degree of privatization, crucially depend on whether the new entrant is a domestic or foreign one. These findings provide new policy implications for a government that is considering privatizing its public firm, while facing the pressure of open markets.

Keywords: Mixed oligopoly, Partial privatization, Entry of private firms

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1. Introduction

In recent years, the study of mixed oligopoly has developed dramatically. Following the trend of many countries privatizing their state-owned industries, one essential concern of the literature is the issue of privatization.\(^1\) At the same time, corresponding with economic globalization, another concern is the issue of competition with foreign firms.\(^2\) In line with these two concerns, a number of mixed oligopoly models have been developed to analyze privatization and foreign competition.

For example, De Fraja and Delbono (1989) suggest a public firm should be privatized and should maximize profit rather than welfare. Matsumura (1998) explicitly considers the possibility of partial privatization and shows that neither full privatization nor full nationalization is optimal. Lee and Hwang (2003) elaborate on the framework of Matsumura (1998) by allowing managerial inefficiency and show that partial privatization is a reasonable choice of government in a monopoly as well as a mixed duopoly. In each study, there are no foreign private firms in the market.

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\(^1\) Privatization, which is defined as the deliberate sale of a state-owned enterprise to the private sector, now appears as a legitimate roll for governments of more than 100 countries in Asia and Europe (Lee and Hwang, 2003). As current examples in the Middle East, Jordan decided to start to sell its share of Royal Jordanian, a fully (100\%) government-owned airline, from the end of 2007. Egypt Central Bank announced plans to sell an 80\% share of Cairo Band to the private sector before the first half of 2008.

\(^2\) In the mixed oligopoly literatures with foreign competition, two major policy concerns are market opening policy and international trade policy.
On the other hand, Fjell and Pal (1996) construct a mixed oligopoly model, in which a welfare-maximizing public firm competes with domestic and foreign private firms, to examine the effects of open markets (allowing entry of foreign private firms and foreign acquisition of domestic private firms). In their study, the issue of privatization is not addressed. Pal and White (1998) adopt Fjell and Pal’s model to investigate the consequence of full privatization and strategic trade policy taking the form of a domestic production subsidy or an import tariff. Fjell and Heywood (2002) extend Fjell and Pal’s (Cournot-Nash) model into a Stackelberg one, to examine the effects of an open door policy, foreign acquisitions and full privatization. These two works discuss the case of complete privatization only, and the possibility of partial privatization is neglected.³

One simple question may arise, here. For a government that is considering privatizing its public firm and is forced to open the corresponding domestic market at the same time, do policy suggestions based on the well known results of the literature still work well when both the aspects of partial privatization and foreign competition are taken into account? Considering, that it is anticipated that the speed of privatization of China’s state-owned enterprises will accelerate and more foreign firms will be encouraged to enter China’s huge markets (Chao and Yu, 2006), it is important to investigate this question.

³ Without the consideration of privatization, Matsumura (2003) also investigates a Stackelberg model where the welfare-maximizing public firm competes with a single foreign private firm.
Given this importance, works involving both the aspects of partial privatization and of foreign competition have recently appeared in the literature. Chang (2005) uses a mixed duopoly model with cost asymmetry to examine optimal trade policy and full/partial privatization policy. Chao and Yu (2006) construct a mixed oligopoly model to examine how partial privatization or foreign competition affects the optimal import tariff. Chang (2007) also examines optimal trade policy, industrial policy, and privatization policies under a mixed duopoly model with strategic managerial incentives. In each of these studies, the focus is on the international trade policy and domestic private firms are not included. While, focusing on the market opening policy, Han and Ogawa (2007) construct a model with one public firm and \( n \) identical private firms (each one of them is jointly owned by domestic residents and foreign investors in a certain share), to clarify how market opening affects the degree of privatization of the public firm. The market opening policy takes the form of raising the restricted share of foreign investments, as well as allowing new entry of joint ownership enterprises.\(^4\)

This paper attempts to introduce the framework of partial privatization by Matsumura (1998) into Fjell and Pal (1996)’s model to re-examine the effects of open markets. Though the direction of extension is straightforward in this present paper, the analytical results as compared

\(^4\) Recently, as an extension of the traditional framework of international mixed oligopoly, there are mixed oligopoly models with two countries each with public and private firms competing in a single international market. See Bárce-Ruiz and Garózn (2005), Dadpay and Heywood (2006).
with Fjell and Pal’s ones are different and perhaps interesting. Furthermore, this extension makes it possible to demonstrate how the effects of partial privatization on market performance changes, if new private firms (including domestic and foreign ones) enter the market. In addition, the relationship between the socially optimal degree of privatization and the new entry of private firms can be clarified as well. The policy implications based on these new results seem meaningful.

Specifically, while Fjell and Pal showed that a foreign firm’s entry increases the public firm’s output, we show that if the public firm is a partially privatized one (with a degree of privatization more than 33%), then the entry reduces the privatized public firm’s output when the number of domestic private firms is sufficiently small. Fjell and Pal also showed that the entry reduces the profit of the public firm. We show that, given an appropriate degree of privatization, the entry increases the privatized public firm’s profit when the number of domestic private firms is sufficiently large and the number of foreign private firms is sufficiently small. Moreover, this paper shows that given a small degree of privatization, the entry increases welfare if and only if the number of domestic private firms is small relative to the number of foreign private firms (corresponding to the result of Fjell and Pal). However, given a large level of privatization, welfare increases even though the number of domestic private firms is larger than that of foreign private firms.

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Second, given appropriate numbers of private firms, we find that raising the degree of privatization reduces the output of the privatized public firm, increases the output of each private firm and increases the market price. Our further analysis shows that the decrease of the privatized public firm’s output becomes smaller, if a new domestic private firm enters the market, while it becomes larger if the new entrant is a foreign private one. The market price increase becomes smaller, if a new domestic private firm enters. However, if the entrant is a foreign private one, this effect becomes smaller (larger) when the given number of foreign private firms is sufficiently large (small).

Third, as results related with the relationship between the socially optimal degree of privatization and private firms’ entry, we finds that if a new domestic private firm enters, the socially optimal degree of privatization rises.\(^5\) However, if the new entrant is a foreign private one, then the optimal degree of privatization rises (falls), when the given number of foreign private firms is large (sufficiently small).

Finally, in the discussion of the foreign acquisition of a domestic private firm, Fjell and Pal showed that the acquisition decreases the profit of the state-owned public firm. In contrast, this paper with the consideration of partial privatization shows that the foreign acquisition increases the profit of the privatized public firm if the degree of privatization is sufficiently large.

\(^5\) The socially optimal degree is a degree that maximizes the social welfare of the home country.
Furthermore, we also find that if a domestic private firm is acquired by foreign nationals, the optimal degree of privatization falls.

The paper is organized as follows. In Section 2, the mixed oligopoly model with a two stage game is presented. In Section 3, the Cournot-Nash outcomes in the second stage are derived, and then the effects of the entry of foreign private firms are examined. In sub-section 4-1, we first examine the effects of partial privatization, given a fixed number of private firms. In sub-section 4-2, we demonstrate how these effects change if a new entry occurs. In Section 5, the socially optimal degree of privatization is derived, and then the relevance between this optimal degree and private firms’ entry is investigated. The issue of foreign acquisition of domestic firms is discussed in Section 6. Concluding remarks follow in Section 7.

2. The Model

We consider a mixed oligopoly model, originally constructed by Fjell and Pal (1996), in which foreign private firms exist. In our model, there is one partially privatized public firm (firm 0), $m$ domestic private firms, and $n$ foreign private firms. Each firm produces a homogenous commodity and has identical technologies, which can be represented by the cost function as $C(q) = f + (1/2)q^2$. That is, the marginal cost is increasing in outputs. $^6$

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$^6$ The assumption of the increasing marginal cost is crucial when the privatized public firm and the private firms have identical technologies. The justifications for this assumption have been provided by Fjell and Pal (1996), Pal and White (1998), among others.
Furthermore, the fixed cost \( f \) is assumed to be zero for simplicity.\(^7\)

Following Fjell and Pal (1996), we assume that the inverse demand function is linear and is given as \( p = a - (q_0 + \sum_{i=1}^m q_i + \sum_{j=1}^n q_j) \), where \( p \) is the market price, \( q_0 \) is the output of the privatized public firm, \( q_i \) is the output of the domestic private firm \( i \) \((i = 1, \ldots, m)\), and \( q_j \) is the output of the foreign private firm \( j \) \((j = 1, \ldots, n), \ i \neq j\). Consequently, the consumer surplus is given by \( CS = (1/2)(q_0 + \sum_{i=1}^m q_i + \sum_{j=1}^n q_j)^2 \). Each domestic private firm \( i \) maximizes its own profit denoted by \( \pi_i = pq_i - (1/2)q_i^2 \), and each foreign private firm \( j \) maximizes its profit denoted by \( \pi_j = pq_j - (1/2)q_j^2 \).

The partially privatized public firm is jointly owned by both the government and the private sector, and the government owns a share of \( (1 - \theta) \in [0,1] \) of the firm. Following Matsumura (1998), we assume that the partially privatized public firm maximizes the weighted average of social welfare and its profit, which is defined as \( V = \theta \pi_0 + (1 - \theta)W \), where \( \pi_0 = pq_0 - (1/2)q_0^2 \) denotes the profit of the privatized public firm, and \( W = CS + \pi_0 + \sum_{i=1}^m \pi_i \) denotes the social welfare of the home country.\(^8\) Note that \( \theta \) can be used to measure the degree of privatization. In the case where \( \theta = 0 \), firm 0 is a state-owned public firm which maximizes the

\(^7\) Fjell and Pal (1996) also assumed \( f = 0 \), then indicated the results change for \( f > 0 \) in footnote 3 and 5 in their study. These indications apply to our corresponding results, which are not explicitly presented in this paper, as well. As we can see below, our main results hold for \( f > 0 \).

\(^8\) Following Fjell and Pal (1996), the profits of foreign private firms are not included in the social welfare of the home country.
social welfare, while in the case where \( \theta = 1 \), firm 0 is a private firm which maximizes its own profits.

Finally, we consider a two-stage game. In the first stage, given the numbers of private firms \( m \) and \( n \), the government chooses the degree of privatization \( \theta \) to maximize the social welfare of the home country (the defined function \( W \)). In the second stage, given \( \theta \), \( m \) and \( n \), each private firm chooses its quantity to maximize its own profit, while the privatized public firm choose its quantity to maximize its objective function (the defined function \( V \)), in the standard Cournot-Nash fashion. We will proceed to derive the sub-game perfect equilibrium by backward induction.

3. **Cournot-Nash outcomes and effects of foreign private firms’ entry (2\(^{nd}\) stage)**

In the second stage, given the government’s policy variable \( \theta \), and the numbers of private firms \( m \) and \( n \), we first derive the Cournot-Nash outcomes. Then we examine the effects of market opening policy that allows foreign firms to enter the mixed oligopoly, by the comparative statics analysis. Solving the optimization problems defined in section 2, we have the Cournot-Nash outcomes as follows:

\[
q_i^* = q_j^* = a \cdot (1 + \theta)/[(4 + m + 2n) + \theta(2 + m)]
\]

\[
q_0^* = a \cdot [2 + (1 - \theta)n]/[(4 + m + 2n) + \theta(2 + m)]
\]

\[
p^* = 2q_i^* = 2a \cdot (1 + \theta)/[(4 + m + 2n) + \theta(2 + m)]
\]
\[ \pi^*_i = \pi^*_j = (3/2)q^*_i = (3/2)a^2 \cdot [(1 + \theta)/\Delta]^2, \text{ where } \Delta \equiv (4 + m + 2n) + \theta(2 + m) \]

\[ \pi^*_0 = a^2 \cdot [(2 + n) - \theta n][(2 - n) + \theta(4 + n)]/2\Delta^2 \]

\[ W^* = (a^2/2\Delta^2)(1 + \theta) \times [(8 + 7m + 8n + 4mn + m^2 + 3n^2) + \theta(3m - 4n + m^2 - n^2)] \]

The superscript " \( \ast \) " stands for the corresponding equilibrium values. Note that if \( \theta = 0 \) (i.e., the government has 100% ownership of firm 0), these outcomes are the same as Fjell and Pal’s ones.\footnote{Note that if the number of foreign private firms is too large (\( n > [(2 + 4\theta)/(1 - \theta)] \)), the public firm’s profits become negative. In this case, a lump-sum transfer to the firm may be conducted.}

Now, given \( \theta \) and the number of domestic private firms \( m \), the effects of a foreign private firm’s entry can be investigated by differentiation of the corresponding equilibrium values with respect to the number of foreign private firms \( n \). The results are as follows.

1. \( \partial q^*_0 / \partial n = (a/\Delta^2)(1 + \theta)[m(1 - \theta) - 2\theta] \)
2. \( \partial m\pi^*_i / \partial n = (a^2/\Delta^3)(-6m)(1 + \theta)^2 < 0 \)
3. \( \partial CS^* / \partial n = (a^2/\Delta^3)(1 + \theta)[(8 + 4m + 8n) + \theta(4m)] > 0 \)
4. \( \partial \pi^*_0 / \partial n = (a^2/\Delta^3)(-)(1 + \theta) \times 
   \quad [(8 + 4n + mn) - \theta(2m + 6n + 2mn) + \theta^2(2 + m)(2 + n)] \)
5. \( \partial W^* / \partial n = (a^2/\Delta^3)(-)(1 + \theta) \times 
   \quad [(2m - 4n + mn) - \theta(6n + 2mn) + \theta^2(2 + m)(2 + n)] \)

When a comparison is made with the results of Fjell and Pal (1996, proposition 2), we find
that if the public firm is partially privatized, the effects of the entry of a foreign private firm on the privatized public firm’s output, profit and the social welfare are different. The differences are perhaps interesting.

**Proposition 1.** If a foreign private firm enters the mixed oligopoly market, then

(a) given an appropriate $\theta$, the privatized public firm’s output increases (decreases) when the number of the domestic private firms is large (sufficiently small).

(b) given an appropriate $\theta$, the privatized public firm’s profit increases when the number of domestic private firms is sufficiently large and the number of foreign private firms is sufficiently small.

(c) given a small $\theta$, welfare increases if and only if the number of domestic firms is small relative to the number of foreign private firms. While given a large $\theta$, welfare increases even though the number of domestic firms is large relative to the number of foreign private firms.

First, while Fjell and Pal showed that independent of the number of domestic private firms $m$, a foreign private firm’s entry increases the public firm’s output. Our analysis result proposition 1-(a) shows that if the public firm is partially privatized (more than 33%), then the privatized public firm’s output decreases when $m$ is sufficiently small (see Eq. (1)). The intuition can be explained by reviewing the reaction functions approach provided by Fjell and Pal. In their
analysis where $\theta = 0$, the entry increases the public firm’s output since adding a new foreign firm not only shifts the reaction function of ‘all other firms’ outward, but also shifts the public firm’s reaction function outward.

In our study, a foreign private firm’s entry shifts the privatized public firm’s reaction function outward as well, and the amount of this shift depends on the degree of privatization $\theta \in [0, 1]$. A larger $\theta$ shifts the privatized public firm’s reaction function back, since a larger $\theta$ causes the privatized public firm to remove its concern from the consumer surplus with the profits of domestic private firms to its own profit, and thus causes the privatized public firm to reduce its output. When the given $\theta$ is small, the reaction function of the privatized firm shifts back slightly, and the result in Fjell and Pal still holds. However, when $\theta$ is sufficiently large, the privatized public firm’s reaction function shifts back greatly, and now the output of the privatized firm depends on the amount of the shift of the reaction function of ‘all other firms’.

The privatized firm’s output decreases, if the entry shifts the reaction function of ‘all other firms’ outward greatly. This large shift occurs when the output of the private firm is large, which corresponds to the case where $m$ is sufficiently small. Then, the result holds.

Second, when we look at the effects on the public firm’s profit, Fjell and Pal showed that the entry reduces the profit of the public firm. In our study, proposition 1-(b) shows that given an appropriate $\theta$, the entry may increase the partially privatized public firm’s profit, depending on
the numbers of private firms $m$ and $n$. According to Eq. (4), Figure 1 can be provided to show this result.\footnote{In Figure 1, the number $n$ (the degree of privatization $\theta$) is shown on the vertical (horizontal) axis. Given a large number of $m$ (e.g., $m=45$), the critical curve in which the profits of the privatized public firm is indifferent, can be drawn. Roughly given $\theta \in (0.5~0.85)$, a foreign firm’s entry increases the privatized public firm’s profits with the combinations of ($m=35$ to 45, $n=1$).} Intuitively, if the public firm is fully privatized, the entry reduces the firm’s profit. However, if the public firm is partially privatized, it is possible that the entry increases its profit, since the entry reduces the price but may increase its output. From proposition 1-(a) we know that entry increases its output if and only if $m$ is large. In addition, footnote 9 tells us that the privatized public firm has a positive profit if $n$ is small. This turns out to be the case that the privatized public firm’s profit increases when $m$ is sufficiently large and $n$ is sufficiently small.

Third, in the analysis without the consideration of partial privatization, Fjell and Pal showed that a foreign private firm’s entry increases the social welfare if and only if $m$ is small relative to $n$. In our study with the consideration of partial privatization, proposition 1-(c) shows that Fjell and Pal’s result still holds if the degree of privatization is small. However, if the degree is sufficiently large, then the entry increases the social welfare even when $m$ is large relative to $n$.

To demonstrate this result well, let us draw Figure 2 and 3 based on Eq. (5). In Figure 2, the numbers of private firms $m$ and $n$ are shown on the vertical axis. The horizontal line with $n = 8$, and the critical curve separate the analytical area into four regions. In both region III (where...
m < n) and region II (where m > n), a foreign firm’s entry increases the home country’s social welfare. Figure 3, with various numbers of n, shows us that if n is smaller than 4 (8), then Fjell and Pal’s result hold if θ < 0.15 (θ < 0.35).

####Figure 2 and Figure 3####about here

The intuition behind this is as follows. The social welfare includes the consumer surplus, the profit of the privatized public firm and the profits of domestic private firms. In the fully public firm case (θ = 0), the entry increases the consumer surplus, while reducing the profit of the public firm and the profits of all domestic private firms. Welfare increases if and only if the loss in domestic private firms’ profits is relatively small. This occurs when m is small relative to n. However, in the situation where the degree of privatization is appropriately large, the profit of the privatized public firm increases when m is sufficiently large and n is small. Moreover, when m is sufficiently large and n is small, the decrease in the profits of domestic private firms (shown in Eq.(2)) becomes small as the given θ increases.\(^{11}\) Thus, proposition 1-(c) holds.

4. **Partial privatization and entry of private firms**

4.1. **The effects of partial privatization without new entry**

In the first stage, the government chooses the degree of privatization θ to maximize the social welfare of its own country. Before deriving the socially optimal degree (section 5), let us

\(^{11}\) As we can see below (Eq. A.5 in Appendix), \(\partial \tilde{\pi}_i^* / \partial n \rangle / \partial \theta < 0\) when m is sufficiently large and n is small.
examine the effects of an additional $\theta$ on the allocation of production, market price and each part included in the social welfare, given the numbers of private firms $m$ and $n$ are fixed. By differentiating the corresponding equilibrium values in the second stage with respect to $\theta$, we have the following results.

(6) $\frac{\partial q_0^*}{\partial \theta} = \frac{a}{\Delta^2}(-2)(1 + n)(2 + m + n) < 0$

(7) $\frac{\partial q_i^*}{\partial \theta} = \frac{\partial q_j^*}{\partial \theta} = \frac{a}{\Delta^2}2(1 + n) > 0$

(8) $\frac{\partial Q_{m+n}^*}{\partial \theta} = \frac{a}{\Delta^2}2(1 + n)(m + n) > 0$, where $Q_{m+n}^* \equiv mq_i^* + nq_j^*$

(9) $\frac{\partial p^*}{\partial \theta} = 2[\frac{\partial q_i^*}{\partial \theta}] = \frac{a}{\Delta^2}4(1 + n) > 0$

(10) $\frac{\partial CS^*}{\partial \theta} = (\frac{a^2}{\Delta^3})2(1 + n)((2 + m + 2n) \theta(m)) < 0$

(11) $\frac{\partial m\pi_i^*}{\partial \theta} = (\frac{a^2}{\Delta^3})2(1 + n)(3m)(1 + \theta) > 0$

(12) $\frac{\partial \pi_0^*}{\partial \theta} = (\frac{a^2}{\Delta^3})2(1 + n) \times [(4 + 4n + mn + n^2) - \theta(4 + 2m + 6n + mn + n^2)]$

Lemma 1. Given the numbers of private firms $m$ and $n$, if the government raises the degree of privatization in the first stage, then in the second stage;

(a) The output of the privatized public firm decreases, the output of each private firm increases. The market price increases and correspondingly the consumer surplus decreases.

(b) The profit of each domestic private firm increases, and thus total profits of domestic private firms increase. The effect on the privatized public firm’s profit is ambiguous.
The result of lemma 1 can be explained as follows.\textsuperscript{12} First, given a certain degree of privatization $\theta$, if the privatized public firm increases its output, it obtains “marginal benefit” since its revenue increases and the consumer surplus increases. It also bears “marginal cost” since the product cost increases and the total profits of domestic private firms decrease.\textsuperscript{13} An additional $\theta$ (i.e., increasing privatization) reduces this “marginal benefit” since the privatized public firm has more concern for its own profits and less concern for the consumer surplus, and this causes the privatized public firm to reduce its output. On the other hand, an additional $\theta$ reduces the “marginal cost” since the firm now has less concern for the domestic private firm’s profits, and this causes the firm to increase its output. Due to an additional $\theta$, the privatized public firm’s output decreases since the decrease in the marginal benefit outweighs the decrease in the marginal cost, and then the output of its rival private firm increases. The market price rises (implying that the total output decreases) since the decrease of the privatized public firm’s output is larger than the increase of all private firms’ output. Lemma 1-(a) holds.

The result of lemma 1-(b) is perhaps interesting. The government raises the degree of privatization meaning that the privatized public firm behaves with less concern for the consumer

\textsuperscript{12} It should be noted here, that the effects of an additional degree of privatization are qualitatively the same as Han and Ogawa (2007). However, the economic intuitions for their results have not been provided under their framework.

\textsuperscript{13} Given appropriate $\theta$, $m$, and $n$, the marginal benefit ($MB_0$) and the marginal cost ($MC_0$) for firm 0 can be derived from the first order conditions for the optimization problem and written as $MB_0 = a - \theta(mq_i + nq_j) - (1 + \theta)q_0$. $MC_0 = q_0 + (1 - \theta)mq_i$. 

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surplus and for the profits of domestic private firms. However, as a result of the competition in
the second stage, the profits of domestic private firms increase. This result holds simply because
the output of each domestic private increases and the market price rises. On the other hand, the
effect on the privatized public firm’s profits is ambiguous since its output decreases and the
market price rises.

4.2. The effects of partial privatization with new entry

Now, let us further examine how the effects shown as lemma 1-(a) change, if the given number
of private firms changes. These additional examinations are helpful for understanding our main
argument on the relationship between the optimal degree of privatization and the entry of
private firms. The results of the corresponding differentiation are as follows.

\[
\frac{\partial q_i}{\partial \theta} = \frac{a^3}{3} - 2(1 + n)(m + \theta(2 + m + 2n)) < 0
\]
\[
\frac{\partial q_i}{\partial \theta} = \frac{a^3}{4} + 3m + 2n + \theta(1 + m + 2n) > 0
\]
\[
\frac{\partial^2 Q_m}{\partial \theta} = \frac{a^3}{2} - 8(1 + n)(1 + \theta) < 0
\]
\[
\frac{\partial^2 q_i}{\partial \theta} = \frac{a^3}{4} + (m - 2n) + \theta(2 + m) > 0
\]

Proposition 2.

(a) By Eqs. (13)-(14), the effect of an additional $\theta$ on the privatized public firm’s output
becomes smaller, if a domestic private firm enters the market, while it becomes larger if a foreign private firm enters.

(b) By Eqs. (17)-(18), the effect of an additional $\theta$ on the market price, as well as on each private firms’ output, becomes smaller if a domestic private firm enters. However, if a foreign private firm enters, this effect becomes smaller (larger) when the given number $n$ is sufficient large (small).

The economic intuition of proposition 2-(a) could be explained as follows. Recall that the increase of its output has “marginal benefit” and “marginal cost” for the privatized public firm, and these two marginal effects decrease if the given $\theta$ increases (see explanation for lemma 1-(a)). The effects of an additional $\theta$ depend on the given number of private firms as well. When a domestic private firm enters, the total outputs of all private firms becomes larger, which means “the decrease of the marginal benefit (caused by the additional $\theta$)” becomes larger. This entry also increases the total outputs of domestic private firms, which means the “the decrease of the marginal cost” becomes larger, too. The effect resulting from a domestic private firm’s entry holds, since the change in the marginal cost is much larger than that in the marginal benefit.\(^\text{14}\)

However, if the entry occurs by a foreign private firm, then “the decrease of the marginal cost”\(^\text{14}\) This relationship can be confirmed from the derived $MB_0$ and $MC_0$ in footnote 12. That is, $\frac{\partial mq_i^*}{\partial m} > \frac{\partial (mq_i^* + nq_f^*)}{\partial m} > 0$. 

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becomes smaller. This is because a foreign firm’s entry reduces each domestic private firm’s output. On the other hand, due to the foreign firm’s entry, “the decrease of the marginal benefit” may become larger or smaller, since the entry may increase or decrease the total output of all private firms. In the case where “the decrease of the marginal benefit” becomes larger, the effect resulting from a foreign firm’s entry holds. In the other case where “the decrease of the marginal benefit” becomes smaller, the result still holds since the change in the marginal cost is much stronger than the change in the marginal benefits.\footnote{Similarly, this relationship can be confirmed from the derived $M_{0}$ and $M_{0}$. That is, $|\partial q_{0}/\partial n| > |\partial (m_{0}q_{0} + nq_{j})/\partial n|$.}

Although the explanation for proposition 2-(b) is probably complex, the reaction functions approach is helpful for understanding the economic intuition.

###Figure 4, Figure 5## about here

In Figure 4, given appropriate $\theta$, $m$, and $n$, the reaction function curve of the privatized public firm and that of all other firms can be drawn as $R_{0}$ and $R_{m+n}$, respectively. The initial Cournot-Nash equilibrium is noted by point A. When $\theta$ increases, $R_{0}$ shifts inward since greater privatization gives the privatized public firm an incentive to reduce its output. Then, $R_{m+n}$ also shifts inward, since the output of each private firm increases due to the decrease of the privatized public firm’s output. The corresponding equilibrium moves to point B. From here, if a domestic private firm enters the market, proposition 2-(a) tells us that the effect of an
additional $\theta$ becomes smaller, implying that the privatized public firm has less incentive to reduce its output, and thus the increase of each private firm’s output becomes smaller. As a result, each of the reaction function curves shift back, and the equilibrium now moves to point C.

Then, comparing points A and B, we know that due to greater privatization, the output of the privatized public firm decreases and the total output of all other firms increases. In addition, the output decrease outweighs the increase, and thus the market price rises (coincides with Eqs (6)-(9)). Moreover, comparing points A, B and C, we know that if a domestic private firm enters, the decrease of total market output (caused by an additional $\theta$) becomes smaller, implying that the increase of the market price becomes smaller. The corresponding result holds.

Figure 5 is depicted to describe the situation where the entry occurred by a foreign private firm. Proposition 2-(a) tells us that if a foreign private firm enters, the privatized public firm has much incentive to reduce its output, and thus $R_0$ shifts inward much more from the mentioned point B. This turns into an increase in its rivals’ (each private firm’s) output. When the given number $n$ is sufficiently large, the effect of the entry on the increase of each private firm’s output is very small, and thus $R_{m+n}$ shifts quite slightly. Then, the equilibrium moves to point C in this case. In the opposite case where the given number $n$ is small, the effect of the entry is

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16 It can be easily shown by comparing the slope of the line linking point A and B, from that linking point A and C.
large, and thus $R_{men}$ shifts inward greatly. The equilibrium is now shown as point D. Comparing points A, B, C and D, we have the remaining results of proposition 2-(b).

5. **Optimal degree of privatization and entry of private firms (1st stage)**

In this section, we derive the optimal degree of privatization for the home country’s welfare.

Adding Eq. (10)-(12), we have the following results.

\[
\frac{\partial W^*}{\partial \theta} = \left(\frac{a^2}{\Delta^3}\right)2(1+n)[m(1+n) + n^2] - \theta[m(1+n) + (4+6n+n^2)]
\]

Setting Eq. (19) is equal to zero and solving the equation for $\theta$, we have the socially optimal degree of privatization shown as follows.\(^{17}\)

\[
\theta^* = \frac{m(1+n) + n^2}{m(1+n) + (4+6n+n^2)}
\]

This optimal degree of privatization $\theta^*$ is a function of the numbers of private firms. Differentiating this optimal value with respect to $m$ and $n$, respectively, we have the following results:

\[
\frac{\partial \theta^*}{\partial m} = \frac{(1+n)(4+6n)[m(1+n) + (4+6n+n^2)]^2}{m(1+n) + (4+6n+n^2)} > 0
\]

\[
\frac{\partial \theta^*}{\partial n} = \frac{2[n(4+3n) - m][m(1+n) + (4+6n+n^2)]^2}{m(1+n) + (4+6n+n^2)}
\]

*Proposition 3.*

(a) *The optimal degree of privatization $\theta^*$ is in $(0,1)$. This result holds not only for $m \geq 1$, $n \geq 1$, but also holds for either $m = 0$ or $n = 0.*

\(^{17}\) It can be confirmed that the second-order condition $\frac{\partial^2 W^*}{\partial \theta^2} < 0$ is satisfied.
(b) If a new domestic private firm enters the market, the optimal degree of privatization rises. However, if the new entrant is a foreign private one, then the optimal degree of privatization rises (falls), when the given number of n is large (sufficiently small).

Proposition 3-(a) describes that neither full nationalization nor full privatization is optimal in mixed oligopoly. This result holds not only for the market where a public firm competes with both domestic and foreign private firm(s), but also for the market where a public firm competes with only domestic or only foreign private firm(s). The optimal degree of privatization exists because accelerating privatization has marginal costs and marginal benefits on welfare. Specifically, an addition of \( \theta \) raises the price and thus reduces the consumer surplus (the marginal cost on welfare shown in Eq. (10)), while it increases the profits of both domestic private firms and the privatized public firm (two parts of the marginal benefits on welfare shown in Eq. (11)-(12)).\(^{18}\) The marginal cost on welfare is decreasing in \( \theta \), and the combined marginal benefits on welfare are mostly decreasing in \( \theta \).\(^{19}\) In addition, each of the marginal effects depends on the numbers of private firms as well.\(^{20}\)

\(^{18}\) It can be conformed that the profit function of the privatized public firm is concave in \( \theta \). Note that in the first stage, the government does not choose a pretty large \( \theta \) (close to 1) that causes the firm’s profit to decrease.

\(^{19}\) Note that if \( n \) is sufficiently large relative to \( m \), then only the marginal benefit from the increases of domestic private firms’ profits, is increasing in \( \theta \).

\(^{20}\) An appendix is provided to show how a new private firm’s entry affects the effects of an additional degree of privatization on each part of social welfare.
Now, the economic intuition for proposition 3-(b) can be explained. From Appendix, we know that a private firm’s entry (either a domestic or a foreign one) may increase or decrease the mentioned combined marginal benefits on welfare, which give the government an incentive to raise or lower the degree of privatization. On the other hand, the private firm’s entry has comparatively stronger effects on the mentioned marginal cost on welfare. Proposition 2-(b) has shown us that a domestic private firm’s entry reduces the marginal cost on welfare, which gives the government an incentive to raise the degree of privatization. Thus, the optimal degree of privatization rises due to the domestic firm’s entry. However, if the entry occurs by a foreign private firm, it increases the marginal cost on welfare when the given number of foreign private firms is small, and thus causes the government to lower the degree of privatization. When the given number of foreign private firms is sufficiently small, so that the marginal cost on welfare increases hugely, then the optimal degree of privatization falls. Proposition 3-(b) holds.

6. Partial privatization and foreign acquisition of domestic firms

In this section, let us first examine the economic effects of a foreign acquisition of a domestic private firm, and then discuss the issue of partial privatization and the foreign acquisition. Given the degree of privatization $\theta$, we compare the equilibrium outcomes before and after a domestic firm is acquired by foreign nationals. By these comparisons we find that the effects on outputs,
all private firms’ profits, consumer surplus and social welfare are exactly the same as Fjell and Pal (1996, proposition 3), except the effect on the profit of the privatized public firm.\footnote{All the comparisons have been done by the author. These presentations are not included in this paper for brevity.}

Here we have the following lemma:

**Lemma 3.** Given appropriate numbers of domestic and foreign private firms, foreign acquisition of a domestic private firm increases the privatized public firm’s profits if the degree of privatization is sufficiently large.

**Proof:**

Denote the profit of the privatized public firm after the acquisition as $\pi_0^A$, then we have

$$
\pi_0^A - \pi_0^* = [(1 - \theta)(1 + \theta)a^2] \alpha + \beta \theta + \gamma \theta^2 / \{2\Delta^2[\Delta + (1 - \theta)]^2\},
$$

where

$$
\alpha = (-)(52 + 16m + 64n + 20mn + m^2 + 27n^2 + 6mn^2 + 2m^2n + 4n^3)
$$

$$
\beta = (+)(32 + 20m + 68n + 20mn + 4m^2 + 32n^2 + 4mn^2 + 4n^3)
$$

$$
\gamma = (+)(20 + 20m + 20n + 16mn + 5m^2 + 3n^2 + 2mn^2 + 2m^2n)
$$

While this comparison result cannot be unambiguously signed, simulation makes the pattern clear. The simulation results are shown in Table 1. Q.E.D.

####Table 1#### about here

Fjell and Pal (1996) have shown that a foreign acquisition of a domestic private firm decreases the profit of the state-owned public firm. However, in our study with the
consideration of partial privatization, the foreign acquisition increases the profit of the privatized public firm if the degree of privatization is large enough. The economic intuition is as follows. When \( \theta = 0 \), reducing a domestic private firm increases the public firm’s profit, while adding a new foreign private firm reduces its profits. The foreign acquisition reduces the public firm’s profit since the decrease is larger than the increase in the profit. However, when \( \theta > 0 \), reducing a domestic private firm increases the privatized public firm’s profit. Furthermore, as shown in proposition 1-(b), adding a new foreign private firm may increase its profits too.²² Thus, the result of lemma 3 emerges.

Now, let us discuss the relationship between the optimal degree of privatization and the foreign acquisition. The optimal degree of privatization when a domestic firm is acquired by a foreign national (denoted by \( \theta^A \)) can be derived by replacing \( m \) with \((m-1)\) and \( n \) with \((n+1)\) in Eq. (20). Then it can be easily shown that \( \theta^A < \theta^* \). Here we have the following proposition:

Proposition 4. The optimal degree of privatization falls, if a domestic private firm is acquired by foreign nationals.

The economic intuition can be provided based on the explanation for proposition 3-(b). Recall that an additional \( \theta \) has marginal costs and marginal benefits on welfare. Due to the

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²² Note that in Table 1, the larger \( m \) is (as well as the smaller \( n \) is), the specified range of \( \theta \) where a foreign acquisition increases the privatized public firm’s profit becomes wider. This corresponds with the result of proposition 1-(b).
foreign acquisition, one of the domestic private firms disappears, while one new foreign private firm appears. These two changes may increase or decrease the combined marginal benefits on welfare, however these changes increase the marginal cost on welfare largely, and thus the optimal degree of privatization falls.

7. Conclusions

This paper introduced the analytical framework of partial privatization by Matsumura (1998) into the mixed oligopoly model by Fjell and Pal (1996) where foreign private firms exist, to re-examine the economic effects of private firms’ entry and foreign acquisition of domestic firms. Taking the aspect of partial privatization of a stated-owned firm into account, the analytical results obtained in this paper, as compared with the corresponding results provided by Fjell and Pal, are different and perhaps interesting. Furthermore, relying on the straightforward extension, this paper has found that the effects of partial privatization on market performance, as well as the socially optimal degree of privatization, crucially depends on new entry of private firms. In addition, different/opposite results could be achieved depending on whether the new entrant is a domestic or foreign private firm. As far as we know, these findings have not been contributed in the previous mixed oligopoly studies, and may provide some new policy implications.

First, without the consideration of privatization of a public firm, Fjell and Pal (1996) found
that a new entry of foreign private firms increases the home country’s welfare if and only if the number of its domestic private firms is smaller than that of foreign private firms. In contrast, this present paper, with consideration of partial privatization, found that Fjell and Pal’s results still hold when the given degree of privatization is small. However, if the given degree of privatization is large enough, then the foreign entry may increase the home country’s welfare even though the number of its domestic private firms is larger than that of foreign private firms. This finding provides a new policy guideline for a government who is considering allowing new entry by foreign private firms.

Second, the finding on the socially optimal degree of privatization and the findings on the relationship between the optimal degree and private firms’ entry may provide policy implications as follows. For a government that is considering privatizing its public firm, “full privatization” is not optimal, irrespective of whether its public firm competes with only domestic or only foreign private firm(s), or a mixture of both. For a government that is considering privatizing its public firm and also opening the market to private firms, if the government is allowing entry of its own domestic firms, it should accelerate the privatization. However, if the government is allowing entry of foreign firms, then it should accelerate (restrict) privatization when the given number of foreign private firms is large (sufficiently small), relative to a certain number of domestic private firms. On the other hand, if the
government is allowing foreign acquisition of its domestic private firms, it should restrict the
degree of privatization.

Finally, we discuss the limitations for this paper and suggest some directions for future
research. First, similar with many previous literatures, the linear inverse demand function and
identical cost functions are assumed for tractability. On the other hand, there is discussion on the
cost asymmetry between the public firm and private firms in the literature (for example, Lee and
Hwang, 2003, Matsumura and Matsushima 2004). The extension for the specification on
demand function as well the symmetry on cost function remains for future research. Second, in
the present paper firms were exogenously assumed to take Cournot-Nash behaviors in the
second stage. The examination of a desirable role (either Stackelberg leader or follower) for a
partially privatized public firm, and furthermore the investigation of endogenous order of moves,
would be interesting directions for future study.

Appendix

In this appendix, we differentiate Eq. (10)-(12) with respect to the numbers of private firms $m$
and $n$, respectively, and obtain the results as follows.

(A.1) $\frac{\partial \text{ECST}^*}{\partial \theta} / \partial m = \frac{a^2}{\Delta^4}(-8)(1+n)(1+\theta)\left[(m + 2n + m\theta) + (1 - \theta)\right] < 0$

(A.2) $\frac{\partial^2 m\pi_i^*}{\partial \theta \partial m} = \frac{\partial^2 m\left[(3/2) q_i^*\right]}{\partial \theta \partial m}\

= \frac{(a^2/\Delta^4)12(1+n)(1+\theta)[2 + n + \theta - m(1 + \theta)]}{\partial \theta \partial m}$
\begin{align*}
(A.3) & \quad \frac{\partial^2 \pi_0}{\partial \theta \partial m} = (a^2/\Delta^4)2(1 + n) \times \\
& \quad \left[(-)(12 + 8n + 2mn + n^2) - \theta(8 - 4m + 2n^2) + \theta^2(8 + 4m + 16n + 2mn + 3n^2)\right] \\
(A.4) & \quad \frac{\partial \left[\pi_{CS}^*\right]}{\partial n} = (a^2/\Delta^4)4 \times \\
& \quad \left[(4 + 2m + m^2 - 4n^2) + \theta(8 + 4m + 8n + 2m^2) + \theta^2m(2 + m)\right] \\
(A.5) & \quad \frac{\partial^2 \pi_i^*}{\partial \theta \partial n} = \frac{\partial^2 \pi_i^*}{\partial \theta \partial n} = \frac{\partial^2 m}{\partial \theta \partial n} = (a^2/\Delta^4)[(3/2)q_i^*^2]/\partial \theta \partial n \\
& \quad = (a^2/\Delta^4)(6m)(1 + \theta)[(m - 4n - 2) + \theta(2 + m)] \\
(A.6) & \quad \frac{\partial^2 \pi_0^*}{\partial \theta \partial n} = (2 a^2/\Delta^4) \times \{x + y\theta + z\theta^2\}, \text{ where} \\
& \quad x = (+)(8 + 12m + 8n + 14mn + m^2 + 2n^2 + 2m^2n + mn^2) \\
& \quad y = (+)(4n + 4mn - 2m^2 + 8n^2 + 2mn^2) \\
& \quad z = (-)(20 + 16m + 28n + 18mn + 3m^2 + 6n^2 + 2m^2n + 3mn^2) \\

References:


Figure 1.

Regions where a foreign entry increases the partially privatized public firm’s profits

\[ \frac{\partial \pi_0^*}{\partial n} < 0, \text{ outside this curve} \]
\[ \frac{\partial \pi_0^*}{\partial n} > 0, \text{ inside this curve} \]
Figure 2.

Regions where a foreign entry increases the home country’s social welfare
(examples for n=8)

Figure 3.

Regions where a foreign entry increases the home country’s social welfare.
(examples for various n)
Figure 4.

Figure 5.
Table 1. The effect of a foreign acquisition on the privatized public firm’s profit

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Note: + foreign acquisition increases the privatized public firm’s profit within the ranges specified.