A General Equilibrium Perspective on Outsourcing and Economic Growth

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Abstract

Faced with the empirical difficulty to demonstrate direct links to growth, several attempts have recently been made to isolate alternate channels through which inward outsourcing and foreign direct investment may promote economic growth in host countries. This paper offers a new macroeconomic perspective on this issue by shedding light on the positive effects on human and physical capital accumulation. It first makes the point that there need not be cross-border flows of resources for globalization to benefit an economy. Provided foreign entrepreneurs can undertake production activities in an economy - say, by raising resources locally as documented by Graham and Krugman (1991), Kindleberger (1969), and Lipsey (2003) - upward pressures on returns provide domestic agents with the necessary incentives to accumulate the relevant factors, thus spurring economic growth in a more “natural” way. An inter temporal general equilibrium framework in which outsourcing materializes as a positive demand shock to intermediate goods is developed and solved analytically. In addition to a higher consumption growth rate during the transition, outsourcing is shown to improve the net foreign asset position of the host country over the very long run while raising the capital intensity of tradeables.

Keywords: Outsourcing; Endogenous Growth; Foreign Asset Position.

JEL Classification: F12, F43, O11, O41.

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1 Introduction

The contribution of outsourcing and more generally of foreign direct investment - FDI - to the economic growth of host countries has been a recurrent theme of the international trade literature over the recent years. The theoretical strand of the literature has primarily focused on the channels through which FDI may foster economic growth in recipient countries. The process of technological diffusion associated to FDI and outsourcing is generally believed to be the main operating mechanism through spillover effects of knowledge and new capital goods.

In addition to the direct increase of capital formation of the recipient economy, FDI is often associated with the introduction of new technologies, such as new production processes and techniques, managerial skills, ideas, and new varieties of capital goods. Yet, in a recent survey of the literature, Kose et al. (2003) conclude that the empirical research does not find a robust significant effect of financial integration on growth.

Faced with the difficulty to demonstrate direct links to growth, several attempts have recently been made to isolate alternate channels through which FDI and outsourcing may promote economic growth in host countries.1 Gao (2005) develops a model of FDI and growth that closely follows Grossman and Helpman (1991) and studies the endogenous response of both features to economic integration. In his model, positive growth can be sustained through ceaseless expansion of product varieties thanks to knowledge spillovers. Likewise, Alfaro et al. (2006) uphold that backward linkages can come into play when more developed financial markets allow credit-constrained local entrepreneurs to borrow more easily to start a business and introduce new varieties of intermediate inputs, thus causing a positive spillover effect to the final goods sector and fostering economic growth. Similarly, Alfaro and Hammel (2007) provide evidence of their hypothesis that through access to foreign capital, international financial integration makes it possible for countries to reap the benefits embodied in capital goods and thus promotes economic growth. They find that stock market liberalizations are associated with a significant increase in the share of imports of machinery and equipment.

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1Forbes (2004) offers an overview of this emerging micro literature.
As the new growth literature also has emphasized the importance of technological change for economic growth (see, e.g. Grossman and Helpman, 1991; Romer, 1993; Barro and Sala-i-Martin, 1997), the debate has translated into the absorption capacity of technology spillovers in host countries. On the one hand, the adoption of new technologies and management skills requires a well-trained and educated labor force, hence the assertion that some threshold level of human capital needs crossing for benefits of cutting-edge technologies to unfold (see, e.g. Borensztein et al., 1998). On the other hand, the process of technological spillovers may be more efficient in the presence of well-functioning markets (see, e.g. Bhagwati, 1978; Ozawa, 1992; Balasubramanyam et al., 1996; Smarzynska, 1999).

In this paper we take a broad view on outsourcing and FDI whereby a firm may maintain headquarters and administration in one country while engaging in production elsewhere. We make the important point that there need not be cross-border flows of production resources for globalization to benefit an economy. Provided foreign entrepreneurs can undertake production activities in an economy - say, by raising resources locally as documented by Graham and Krugman (1991), Kindleberger (1969), and Lipsey (2003) - upward pressures on factor returns provide domestic agents with the necessary incentives for accumulation and thus spurs economic growth in a more “natural” way.

As a corollary, while a host country’s absorptive capacity initially may be an issue, inward outsourcing has the potential to impulse a sustained growth dynamics through human and physical capital expansion. This channel clearly departs from the usual spillover-effect line of analysis which so far has found only mixed empirical support. Furthermore, as will be made clear later on, this mechanism enables us to connect different pieces of some recent empirical evidence showing that (1) developing countries’s share of outsourcing and FDI

\[ \text{Similarly, Prasad et al. (2007) show that countries with underdeveloped financial systems may not be able to use foreign capital to finance growth.} \]

\[ \text{That outsourcing is accompanied by capital inflows has no incidence on the conclusions one can derive from an inter temporal setting like ours. In fact, this initially may depress the return to physical capital while spurring human capital accumulation. As human capital expands, the marginal product of physical capital would increase, and thus trigger the same mechanism as the one highlighted in this paper.} \]
inflows has significantly increased over time; (2) developing countries’s contribution to FDI outflows has recently been on the rise; and (3) these countries’s exports increasingly are becoming skilled labor and capital intensive (see, UNCTAD - United Nations Conference on Trade and Development (2007), *World Investment Report*, p. 3; UNCTAD (2006), *World Investment Report*, p 108, and UNCTAD (2004), *Development and Globalization: Facts and Figures*, p. 93). These facts underly our theory of a trade-induced structural transformation of at least emerging countries.

Overall, our results carry major implications for living standards in developing countries, especially as we find that welfare improves faster after an initial drop in consumption, and more importantly, that countries that initially benefit from inward outsourcing and FDI are also likely to display the very same investment behavior later on in their development process. In fact, in the steady state of the economy we find that growth episodes initially pulled by outsourcing inflows have a positive impact on the net foreign asset position of the host country. This lays support to both the desirability and the sustainability of such a development strategy while rationalizing the second piece of evidence reported above.

Moreover, the third piece of evidence contrasts sharply with the standard Heckscher-Ohlin prediction of international trade patterns since the comparative advantage of labor-intensive developing countries should lie in primary goods and unskilled-labor, not in skill and technology-intensive activities. We rationalize the intuition that unprecedented inflows of outsourcing which some developing countries have witnessed over the recent decades may have triggered their structural transformation, especially through direct human and physical capital accumulation. Thus the paper also adds to the understanding of the process of structural transformations (see, e.g. Laitner, 2000; Gollin, Parente, Rogerson, 2002), and the potential role of human capital (see, e.g. Temple and Voth, 1998). In particular, our analysis contrasts well with the sectoral approach of this strand of the literature (Kongsamut et al., 2001; Echevarria, 2008; Ngai and Pissarides, 2007).

Exogenously-driven TFP growth differentials across sectors underly the main mechanism for structural transformation in Ngai and Pissarides (2007), whereas in Echevarria (2008)
the same feature combines with non homothetic preferences to explain the shift away in trade specialization from primary toward manufactured goods. While both papers offer compelling growth and structural change theories, they overlook the undeniable role of cross-borders arbitrage in production, a typical feature of the global economy era. Yet, that inward outsourcing, and more generally inward FDI, has played a crucial role in some recent economic development experiences is hardly debatable. This paper abstracts away from sectoral TFP growth differentials and emphasizes factor inputs accumulation along with the aforementioned feature.

In our model, final goods are produced by either foreign or domestic firms. In any event, this involves packaging together a continuum of non traded and horizontally differentiated intermediates goods for which firms claim on a competitive basis.

That intermediate goods are non traded is a common assumption in the literature (see, e.g. Grossman and Helpman, 1990; Markusen and Venables, 1999; Rodriguez-Clare, 1996). In that respect, our model shares Alfaro et al. (2006)’s assumption that foreign firms directly produce in the host country, say, by building a plant instead of simply exporting. This feature is best rationalized with reference to either the OLI framework - ownership localization, internalization advantage (see Dunning, 1981), or to domestic factors, such as local content requirements, opportunities to tap into local resources, access to low-cost inputs, low-wage labor, or to bypass tariffs that protect a market from imported goods. Clearly, that inward outsourcing takes place is not inconsistent with host countries closing their borders, thus our modeling strategy. This strategy is further motivated by the available evidence that investors often fail to bring all the capital with them, but rather tend to finance an important share of their investment in the local market (see, e.g. Graham and Krugman, 1991; Kindleberger, 1969; and Lipsey, 2003).

We first develop a closed-economy, endogenous growth model within an inter temporal general equilibrium framework where outsourcing materializes as a positive demand shock to intermediate goods, including human and physical capital goods. This is a well-documented feature of outsourcing and FDI in host countries (see, e.g. Markusen and Venables, 1997;
Feenstra and Hanson, 1997). However, while the available evidence shows that inward outsourcing unevenly spreads out across sectors and varies from one period to the next, we assume away any time-varying or sector-specific dynamics. In clear, we consider outsourcing and FDI inflows in average terms both over time and across sectors. This strategy greatly eases the exposition while preserving the fundamental mechanism we want to emphasize.4

Next, we investigate the long-run dynamics of the net asset position for a small open economy that initially benefited from inward outsourcing.

As firms’ motivations to outsource are brilliantly analyzed in numerous recent contributions to the literature (see, e.g. Nocke and Yeaple, 2008; Almazan et al., 2007; Grossman and Helpman, 2005), we elected to focus our attention on downstream outcomes.

With regards to human capital, outsourcing and FDI are often cited in explaining the raising wage premium for skilled workers in developing countries (see, e.g. Feenstra and Hanson, 1997). However, skill wage does not only capture the return on human capital investments (incentive for accumulation), but through trainers’ wages, also determines the cost of acquiring skills (constraint to human capital accumulation). Therefore, that skilled labor supply can be on the rise as a result of inward outsourcing is all but trivial. Clearly, one might witness a growing wage premium for skilled workers without any further accumulation so that inward outsourcing fails to benefit the host economy over the long run.

Similarly, capital inflows might be feared to lower the return to capital in host countries, and thus prevents any local accumulation. Yet, using data from 19 emerging economies, Kumar (2007) documents that FDI actually crowds in domestic investment and delivers a positive impact on savings, a puzzling finding we are able to rationalize in this paper. In our model, incoming resources raise the demand for complementary factors, which in turn raises the demand for capital needed to produce those very same factors. As we find that the return to capital is higher in the general equilibrium of the economy, this suggests that overall the demand for extra capital offsets the initial injection, and thus spurs local savings and investment.

4The sectoral aspect is best rationalized in relation to backward and forward linkages, whereas the temporal dimension is purely a technical simplification.
The production side of the economy consists of two stages of production, each dealing with two kinds of goods, “human capital goods” and physical goods. Imperfectly competitive firms à la Dixit-Stiglitz (1977) produce differentiated human capital and physical intermediate goods using human and physical capital composites. Composite goods are produced by perfectly competitive firms according to a CES technology, whereas intermediate goods are produced using a standard Cobb Douglas one. An important feature of this environment is that unlike the human capital composite, the physical composite (our numeraire) is tradeable. In addition, the former is either consumed or accumulated, while the latter only can be accumulated. Finally, intermediate goods, either physical or not, are non-tradeable.

The consumption side of the economy consists of a large number of homogenous households endowed with homothetic preferences defined over the physical composite only. To follow on Echevarria (2008), the representative consumer owns the factors of production and decides their distribution between alternative uses.

To summarize our findings, inward outsourcing:

1. raises the relative price of, and (even more so) the return to human and physical capital;
2. fosters human and physical capital accumulation, and as such, economic growth;
3. under plausible conditions, raises the capital intensity of tradeables;
4. is initially welfare-reducing but spurs consumption growth thereafter;
5. under mild conditions, improves the steady-state net foreign asset position of the host country.

\[5\text{In the appendix section, we show that the main results remain even if one considers different production functions for physical and human capital goods. Likewise, assuming different degrees of substitutability between intermediates goods makes formulas cumbersome without adding much to the results.}\]

\[6\text{One can still use this framework to investigate the effects of vertical integration of production across countries. Clearly, if foreign firms can enter the sectors of intermediate goods, say, with domestic affiliates producing for their parent enterprises abroad, both supply and demand factors would enter the analysis. Backward linkages would then imply an aggregate net positive demand shock to intermediate goods. In the current framework backward and forward linkages arise as producing intermediate goods involves composite human and physical goods.}\]
The remainder of this paper is structured as follows. Section 2 presents the benchmark model which we solve in section 3. Section 4 extends the model to the small open economy case and section 5 contains concluding remarks.

2 A benchmark model economy

The model is a direct extension to that in Lucas (1988) and Benassy (2003), and which we have adapted for the purpose of our study. We assume an environment in which economic activities extend over an infinite number of periods \( t = 0, 1, \ldots \). Foreign entrepreneurs may produce any of the composite goods and as such, boost the demand for intermediate goods. Our goal is therefore to investigate the implications for factors accumulation, the growth rate of the economy and welfare.

2.1 The production of composite goods

The physical composite, \( Y_t \), is produced by perfectly competitive firms according to a CES technology and consists of packaging together physical intermediates:

\[
Y_t = \left( \int_0^1 Y_{jt}^\theta dy \right)^{1/\theta}, \quad 0 < \theta < 1,
\]  

(1)

where \( Y_t \) denotes total output, \( Y_{jt} \) quantity used of intermediate good \( j \), and \( \theta \) a positive parameter depicting the substitutability between different goods and thus the market power of the corresponding suppliers, with \( j \in [0, 1] \). Since the physical composite is either consumed or accumulated as next period stock of physical capital, the identity follows:

\[
Y_t = C_t + I_t,
\]  

(2)

where \( C_t \) is aggregate consumption and \( I_t \) total investment.

Next, since \( Y_t \) is our numeraire, letting \( P_{jt} \) denote the relative price of intermediate good \( j \), profit maximization yields the following demand schedule:

\[
Y_{jt} = (P_{jt})^{-1/(1-\theta)} Y_t.
\]  

(3)
Similarly, human capital composite, \( X_t \), derives from a CES aggregation over human capital intermediates:

\[
X_t = \left( \int_0^1 X_{it}^\theta di \right)^{1/\theta}, \quad 0 < \theta < 1,
\]

(4)

where \( X_{it} \) denotes type-\( i \) “human capital good” with \( i \in [0, 1] \). Letting \( \rho_{it} \) denote its relative price, profit maximization in this sector yields the following demand schedule:

\[
X_{it} = \left( \frac{\rho_{it}}{\rho_t} \right)^{-1/(1-\theta)} X_t,
\]

(5)

where \( \rho_t \) is the price index associated to (4). Both (3) and (5) give the standard downward slopping demand curves facing the monopolists.

2.2 Introducing outsourcing

We first proxy the scope of outsourcing inflows using a generic index \( \varepsilon \in [0, 1] \), so that the higher the index, the larger outsourcing inflows. This section then models the latter as a demand shock to intermediate goods. Although firm or sector-specific shocks are intellectually more appealing, they tend to blur the analysis without adding much to the paper’s main mechanism. Thus, to ease the exposition it is assumed that on average and given aggregate demand, inward outsourcing raises the demand for intermediate goods by a factor of \( \delta \equiv (1 - \varepsilon)^{-1} \) which we assume time and sector invariant, hence the altered demand schedules:

\[
\tilde{Y}_{jt} = (1 - \varepsilon)^{-1} Y_{jt} = (1 - \varepsilon)^{-1} (P_{jt})^{-1/(1-\theta)} Y_t,
\]

(6)

\[
\tilde{X}_{it} = (1 - \varepsilon)^{-1} X_{it} = (1 - \varepsilon)^{-1} \left( \frac{\rho_{it}}{\rho_t} \right)^{-1/(1-\theta)} X_t.
\]

(7)

Given \( Y_t \) and \( X_t \), equations (6) and (7) state that overall demand for type \(- h, h = i, j\) intermediate good is higher the larger outsourcing inflows as measured by \( \varepsilon \).

The next section lays out the production environment for intermediate goods.
2.3 The intermediate-goods sector

Differentiated intermediate goods are produced by monopolistic competitive firms indexed by \( j \) (physical good) and \( i \) (“human capital goods”), with \( i, j \in [0, 1] \). Firms combine physical and human capital composites according to a standard Cobb Douglas production function:

\[
Y_{jt} = AK_{jt}^\gamma H_{jt}^{1-\gamma}, \quad 0 < \gamma < 1, \tag{8}
\]

\[
X_{it} = ZK_{it}^\gamma H_{it}^{1-\gamma}, \tag{9}
\]

where \( K_{ht} (H_{ht}) \) is physical (respectively, human) capital rented out by firm \( h = i, j \), \( A \) and \( Z \) are positive technological parameters which we assume time-invariant.

Next, human and physical capital composites each can be accumulated for next period and fully depreciate after usage, hence the laws of motion:

\[
K_{t+1} = I_t, \tag{10}
\]

\[
H_{t+1} = X_t. \tag{11}
\]

Market clearing implies that:

\[
K_t = K_{Xt} + K_{Yt}, \quad H_t = H_{Xt} + H_{Yt}, \tag{12}
\]

with

\[
K_{Xt} = \int_0^1 K_{it} \, di, \quad K_{Yt} = \int_0^1 K_{jt} \, dj, \quad H_{Xt} = \int_0^1 H_{it} \, di, \quad H_{Yt} = \int_0^1 H_{jt} \, dj. \tag{13}
\]

In the appendix section we show that the first order conditions of the maximization problems can be rewritten as:

\[
(1 - \varepsilon)^{-1} \gamma \theta P_{jt} Y_{jt} = R_{jt} K_{jt}, \tag{14}
\]

\[
(1 - \varepsilon)^{-1} (1 - \gamma) \theta P_{jt} Y_{jt} = W_{jt} H_{jt}, \tag{15}
\]

\[
(1 - \varepsilon)^{-1} \gamma \theta P_{it} X_{it} = R_{it} K_{it}, \tag{16}
\]

\[
(1 - \varepsilon)^{-1} (1 - \gamma) \theta P_{it} X_{it} = W_{it} H_{it}. \tag{17}
\]

hence the result:
Lemma 1 (Part one) All else equal, imperfect competition (i.e. low $\theta$) lowers the return to human and physical capital.

(Part two) All else equal, inward outsourcing raises the return to human and physical capital.

Proof. In the appendix section we show that

$$R_t = \frac{\gamma \theta}{(1 - \varepsilon)} \left( Y_t + \rho_t X_t \right) K_t,$$

$$W_t = \frac{(1 - \gamma) \theta}{(1 - \varepsilon)} \left( Y_t + \rho_t X_t \right) H_t.$$  

We consider the consumption side of the economy next.

2.4 Preferences

A large number of homogenous households have preferences defined over expected streams of consumption, $C_t$. The representative household ranks alternative streams of consumption using the following expected utility function:

$$E_t \sum_{t=0}^{\infty} \beta^t \ln C_t,$$

where $0 < \beta < 1$ denotes the usual rate of time discounting and the operator $E_t$ denotes the mathematical expectation conditional on period $- t$ information.

The representative household seeks to maximize (20) subject to the sequence of its periodic budget constraints. Time $- t$ constraint is given by:

$$C_t + K_{t+1} + \rho_t H_{t+1} = R_t K_t + W_t H_t + \Pi_t,$$

where $K_{t+1}$ denotes investment, $K_t$ and $H_t$ are stocks of accumulated physical and human capital, $\rho_t$ is the relative price of human capital, $R_t$ is the monetary return on physical capital, $W_t$ is the nominal wage rate, and $\Pi_t$, the nominal profits accrued to the household. Equation (21) implies that (i) the physical composite is the numeraire, and (ii) physical capital fully depreciates after usage. Thus, the economy’s resource constraint satisfies

$$\psi_t \equiv Y_t + \rho_t X_t = C_t + K_{t+1} + \rho_t H_{t+1}.$$  

11
Furthermore, (1), (4), (8) and (9) imply that

$$\psi_t = BK_t^{\gamma}H_t^{1-\gamma},$$  \hspace{1cm} (23)

where total factor productivity, $B$, is some combination of $A$ and $Z$. 

Solving for the optimal values of $C_t$, $H_{t+1}$ and $K_{t+1}$, it can be shown that

$$\frac{\rho_t}{C_t} = \beta E_t \left( \frac{W_{t+1}}{C_{t+1}} \right),$$ \hspace{1cm} (24)

and

$$\frac{1}{C_t} = \beta E_t \left( \frac{R_{t+1}}{C_{t+1}} \right).$$ \hspace{1cm} (25)

Combining (24) and (25) yields the current relative price of the human capital composite as equal to its expected relative return:

$$\rho_t = E_t \left( \frac{W_{t+1}}{R_{t+1}} \right),$$ \hspace{1cm} (26)

hence the result:

**Lemma 2** The current relative price of human capital is higher, the higher the expected supply (stock) of physical capital relative to human capital.

**Proof.** We prove the result by substituting (18) and (19) into (26):

$$\rho_t = (1 - \gamma) \gamma^{-1} E_t \left( \frac{K_{t+1}}{H_{t+1}} \right).$$ \hspace{1cm} (27)

This ends the proof. ■

Lemma 2 illustrates the important point that anticipations of human capital shortage relative to physical capital cause upward pressures on the current price of human capital goods. This is because forward-looking agents would typically react to such prospects by raising their demand for skill-enhancing programs in the current period.

The next section solves for the general equilibrium of the economy.
3 Equilibrium analysis

The following definition introduces the equilibrium concept which underlies our study:

**Definition 1 (Equilibrium)** An intertemporal general equilibrium for this economy is a sequence of prices, \( \{\rho_t, \rho_{it}, P_{jt}, R_{it}, R_{jt}, R_{yt}, W_{it}, W_{jt}, W_{xt}, W_{yt}\}_{t=0}^{\infty} \), a sequence of allocations of resources across firms, \( \{K_{ht}, K_{ht}, H_{ht}, H_{ht}\}_{t=0}^{\infty} \), \( h = i, j \), and a sequence of consumption \( \{C_t\}_{t=0}^{\infty} \) and investment levels \( \{I_t, X_t\}_{t=0}^{\infty} \) such that, for all \( t \):

(i) \( W_{jt} = W_{it} = W_{xt} = W_{yt} = W^*_{t} \) and \( R_{jt} = R_{it} = R_{xt} = R_{yt} = R^*_{t} \);

(ii) given \( (\rho_t, \rho_{it}, P_{jt}, R_{t}, W_t) \), \( h = i, j \), \( C_t, H_{t+1} \) and \( K_{t+1} \) solve (20) - (25), \( I_t = K_{t+1}, \) and \( X_t = H_{t+1} \);

(iii) given \( Y_t, K_{ht}, K_{ht}, H_{ht}, H_{ht}, W_t \) and \( R_t, \ h = i, j \), solve (6) - (17);

(iv) all markets clear.

The next definition introduces the conceptual framework of our analysis.

**Definition 2 (Balanced growth path)** Along a balanced growth path the ratio of (physical) capital to total output is constant.

This definition underlies our first and only theorem. Let

\[ \delta \theta \beta < 1, \]  

(28)

then:

**Theorem.** There exists a balanced growth path for this economy.\(^7\)

**Proof.** From equation (23) \( K_t/\psi_t = B^{-1} (K_t/H_t)^{1-\gamma} \). In the appendix section we also prove that \( K_t/H_t = (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta \). Thus,

\[ \frac{K_t}{\psi_t} = \frac{1}{B} \left[(1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta\right]^{1-\gamma}. \]  

(29)

This ends the proof. \( \blacksquare \)

\(^7\)Unbalanced growth may arise in the event of sector-specific shocks.
Equation (29) also carries the important implication that the ratio of physical capital to total output is higher, the larger inward outsourcing. While it may seem natural to relate this feature to the fact that outsourcing generally involves cross-border transfers of resources, this would be a simplistic analysis of its real underlying mechanism as we show below.

Before featuring the dynamics of this economy, we first highlight some important results in relation to factor prices and returns.

**Proposition 1** Let condition (28) hold. Then: (1) The return to physical capital is higher, the larger inward outsourcing. (2) The latter also commands higher returns on human capital investments, and a higher relative price for “human capital goods.”

**Proof.** Provided in the appendix section. ■

As both the cost of, and the return on human capital accumulation rise under the influence of inward waves of foreign investment, whether the net effect is positive or not is worth investigating, thus the next proposition:

**Proposition 2** Let condition (28) hold. Then, in the long run outsourcing inflows foster both human and physical capital accumulation.

**Proof.** Provided in the appendix section. ■

Proposition 2 is important in that it suggests that inward outsourcing fuels economic growth through its fundamentals, *i.e.* the accumulation of factor inputs. This result arises as the net benefit of accumulation - which we define in natural logarithmic terms as the difference between, say, the return to human capital accumulation, $W_t$, and its cost, $\rho_t$ - increases with inward outsourcing. In fact, it can be shown that

$$\frac{W_t}{\rho_t} = (1 - \varepsilon)^{-\gamma} (1 - \gamma)^{-(1-\gamma)} \beta^{-(1-\gamma)} \gamma^\gamma A.$$

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8Using data from 19 emerging economies, Kumar (2007) documents that FDI actually crowds in domestic investment and delivers a positive impact on savings. He finds that one percentage point rise in the ratio of FDI to GDP leads to an increase of a half percentage point in domestic investment and three-fourths percentage point in domestic savings.
Likewise, since the physical composite is our numeraire, its higher net return follows directly from the fact that the return to capital is higher. A natural step to take is therefore to formally rationalize the connection between outsourcing and economic growth:

**Proposition 2 - Corollary:** Let condition (28) hold. Then, the growth rate of the economy is higher, the larger outsourcing inflows.

**Proof.** The result follows from (23) and proposition 2, which together imply that $g_c = g_k = g_h$.

This ends the proof. □

Furthermore, relaxing the simplifying assumption that production functions are identical for intermediate goods provides more insight into the growth process of this economy. The next proposition highlights the dynamics of factor contents in the light of the available evidence which shows that developing countries’s exports increasingly are becoming capital intensive.

**Proposition 3** Let condition (28) hold. Provided the capital-elasticity of output is higher for physical goods relative to human capital goods, in the long run inward outsourcing raises capital intensity in the former sector.

**Proof.** To prove the result, we consider the following production functions:

\[
Y_{jt} = AK_{jt}^\gamma H_{jt}^{1-\gamma}, \quad 0 < \gamma < 1; \\
X_{it} = ZK_{it}^\alpha H_{it}^{1-\alpha}, \quad 0 < \alpha < 1.
\]

In the appendix section, while generalizing Proposition 2 we also show that

\[
\frac{K_{yt}/K_t}{H_{yt}/H_t} \equiv \frac{K_{yt}/H_{yt}}{K_t/H_t} \equiv \kappa_y = \frac{(1 - \varepsilon) \gamma}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta}.
\]

Differentiating the above with respect to $\varepsilon$ then yields the result:

\[
\frac{d\kappa_y}{d\varepsilon} = \frac{(\gamma - \alpha) \gamma \theta \beta}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta}^2.
\]
This ends the proof. ■

The intuition of this result is as follows. Assume the capital-elasticity of output is higher for physical goods. Then, in order to boost output when facing a demand shock, physical goods producers put relatively more emphasis on capital compared to human capital goods producers. This reaction stems from the fact that output in the former sector is relatively more sensitive to capital (by assumption). The result above then suggests that over time this strategy leads capital goods to become more capital intensive, even after controlling for the capital intensity of the economy as a whole.

We discuss some welfare implications next - under the maintained and simplifying assumption that $\gamma = \alpha$. Proposition 4 below summarizes our findings:

**Proposition 4** Let condition (28) hold. Inward outsourcing initially is welfare-reducing but induces a higher consumption growth rate thereafter.

**Proof.** As shown in the appendix section, consumption’s share of national output is constant and given by

$$\frac{C_t}{\psi_t} = 1 - \frac{\theta\beta}{1 - \varepsilon}. \quad (30)$$

The first claim then follows by way of differentiation with respect to $\varepsilon$.

The second claim arises from the fact that the growth rate of output is higher the larger outsourcing inflows. That $C_t/\psi_t$ is constant then implies that consumption growth keeps pace with economic growth.

This ends the proof. ■

The intuition of proposition 4 is as follows. As inward outsourcing leads the typical household to accumulate more of both types of capital, proposition 4 points to a crowding-out effect of both investments on consumption. However, inward outsourcing also spurs consumption growth. Thus, one can expect the initial welfare loss to be offset over time so that overall, living standards improve.

The analysis above provides a natural transition to the open economy. With faster consumption growth and higher domestic saving and investment, it is worth investigating the implications for such a host country’s net asset position, hence the next section.
4 The small-open economy case

This section allows for trade in the physical composite good and specializes the analysis to a small-open economy with unlimited access to world markets.

With trading opportunities it is assumed that the typical household still ranks alternative streams of consumption according to (20). Assuming that domestic and foreign physical composites are perfect substitutes, and letting consumers preferences be identical internationally, exports and imports prices are determined in world markets (small open economy). Thus, total expenditures on the physical composite can be expressed as

\[ Y_t = C_t + I_t + EX_t - Q_t IM_t, \quad (31) \]

where \( EX_t \) denotes exports, \( Q_t IM_t \) imports in foreign real prices, and \( Q_t = 1 \) (law of one price).

4.1 The current account dynamics

In a model like ours, the picture of general equilibrium effects can be quite blurry in an open economy. To clarify this picture, we restrict our attention to long-term effects by emphasizing the current account’s behavior in the steady state.

Definition 3 (Steady state equilibrium) A steady state equilibrium is a general equilibrium which in addition satisfies \( K_t^* = K_{t-1}^* = K^* \), and \( H_t^* = H_{t-1}^* = H^* \) for all \( t \), where \( K^* \) and \( H^* \) denote the steady-state values of physical and human capital.

Holding the nominal effective exchange rate constant, the current account balance can be expressed as following:

\[ CA_t = \Delta F_{t+1} = EX_t - IM_t + r_t F_t, \quad (32) \]

where \( r_t \) is the real interest rate, and \( F_t \) the net asset position. We note that the real interest rate is also exogenously determined (small open economy), so that \( r_t = \bar{r} \).
Combining (31) and (32) yields the following expression for the current account:

\[ CA_t = \Delta F_{t+1} = Y_t - C_t - I_t + \bar{r} F_t, \]

thus, the steady state net foreign asset holdings can be expressed as:

\[ F^* = \frac{-1}{\bar{r}} (Y^* - C^* - I^*), \]  

(33)

hence the result:

**Proposition 5** Let condition (28) hold. If in addition \( B < \bar{B} \), then in the very long run (steady state), inward outsourcing improves the host country’s net asset position.

**Proof.** While proving Proposition 1 we showed (in the appendix section) that 
\( Y_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] AK_t H_t^{1-\gamma} \). On the other hand, combining (23) and (27) using 
\( K_t/H_t = (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta \) yields 
\( H_t/\psi_t = (1 - \varepsilon)^\gamma [(1 - \gamma) \theta \beta]^{-\gamma} B^{-1} \). Thus, using (30), (29) and \( K_t^* = K_{t+1}^* = I^* = K^* \), equation (33) can be re-written as follows:

\[ \frac{F^*}{\psi^*} = \frac{\theta \beta}{(1 - \varepsilon)^2 \bar{r} B} \left[ (1 - \gamma) A - B + \left( \frac{1 - \varepsilon}{\theta \beta} \right)^\gamma (1 - \gamma)^{1-\gamma} \right] - \frac{A - B}{\bar{r} B}. \]

Next, differentiating the above with respect to \( \varepsilon \) yields:

\[ \frac{d}{d\varepsilon} \left( \frac{F^*/\psi^*}{\bar{r} B} \right) = \frac{\theta \beta}{(1 - \varepsilon)^2 \bar{r} B} \left[ (1 - \gamma) A - B + (1 - \gamma)^{2-\gamma} \left( \frac{1 - \varepsilon}{\theta \beta} \right)^\gamma \right]. \]

Therefore, for \( d(F^*/\psi^*)/d\varepsilon > 0 \), one needs \( B < \bar{B} \), where

\[ \bar{B} = (1 - \gamma) A + (1 - \gamma)^{2-\gamma} \left( \frac{1 - \varepsilon}{\theta \beta} \right)^\gamma. \]

This ends the proof. ■

The condition that \( B < \bar{B} \) states that the host country’s TFP should not be too high for this result to unfold. Since developing countries are generally characterized by low levels of TFP, we hypothesize that overall, our theory applies more to such countries. Thus Proposition 5 suggests that developing countries that initially benefit from inward outsourcing and FDI are also likely to adopt the very same investment behavior later on in their development process.\(^9\) This analysis aligns well with some recent development experiences, especially those of emerging countries, including China, Brasil, South Korea, etc.

\(^9\)In fact, Bosworth and Collins (1999), and Kumar (2007) document a positive effect of FDI on the host country’s current account.
5 Concluding remarks

This paper develops an intertemporal general equilibrium framework to highlight the effects of inward outsourcing on factors accumulation and as such, on economic growth. Provided foreign entrepreneurs can undertake production activities in an economy, positive demand shocks to intermediate goods cause both prices and (even more so) returns on factor inputs to rise, which in turn provide individuals with the necessary incentives for accumulation. We find that while outsourcing-induced growth may initially be detrimental to welfare, such strategy spurs consumption growth thereafter. Also of interest is our result that in the steady state of the economy, a host country’s net asset position will be higher the larger the initial outsourcing inflows.

Overall, the results suggest that empirical investigations to connect outsourcing and economic growth might be more conclusive provided they attempt to link private decisions to accumulate factor inputs to inward outsourcing. In fact, one can hardly argue against the premise that the keen interest of millions of young Indians or Chinese for vocational training and other specialized curricula including engineering has something to do with the massive inflows of outsourcing their countries have been experiencing over the recent decades.
Appendix: Proof of lemmas and propositions.

Deriving the decision rule of monopolist firms in the intermediate sector.

When confronted with the wave of inward outsourcing, firm \( j \) in the intermediate good sector picks \( P_{jt} \) and \( \tilde{Y}_{jt} \) to

\[
\max \Pi_j \equiv P_{jt} \left( \tilde{Y}_{jt} \right) \tilde{Y}_{jt} - R_t K_{jt} - W_t H_{jt},
\]

subject to

\[
\begin{align*}
H_{jt} &= A^{-1/(1-\gamma)} K_{jt}^{-\gamma/(1-\gamma)} \left( \tilde{Y}_{jt} \right)^{1/(1-\gamma)}, \\
K_{jt} &= A^{-1/(\gamma)} H_{jt}^{-(1-\gamma)/\gamma} \left( \tilde{Y}_{jt} \right)^{1/\gamma}.
\end{align*}
\]

where \( \tilde{Y}_{jt} \) is the modified demand for type\(-j\) physical intermediate. Substituting (34) and (35) back into the objective function and taking the derivative with respect to \( \tilde{Y}_{jt} \) yield:

\[
\left[ 1 + \frac{1}{P_{jt} d\tilde{Y}_{jt}/Y_{jt} dP_{jt}} \right] P_{jt} = \frac{1}{\gamma} A^{-1/\gamma} H_{jt}^{-(1-\gamma)/\gamma} \left( \tilde{Y}_{jt} \right)^{1/\gamma} \left( \tilde{Y}_{jt} \right)^{-1} R_{jt},
\]

That is, using \((P_{jt} dY_{jt}/Y_{jt} dP_{jt}) = -1/(1 - \theta)\) and arranging terms,

\[
\gamma \theta P_{jt} \tilde{Y}_{jt} = R_{jt} K_{jt}.
\]

or, equivalently

\[
(1 - \varepsilon)^{-1} \theta \gamma P_{jt} Y_{jt} = R_{jt} K_{jt},
\]

since

\[
\tilde{Y}_{jt} = (1 - \varepsilon)^{-1} Y_{jt} = (1 - \varepsilon)^{-1} (P_{jt})^{-1/(1-\theta)} Y.
\]

Similarly it can be shown that

\[
(1 - \varepsilon)^{-1} (1 - \gamma) \theta P_{jt} Y_{jt} = W_{jt} H_{jt}.
\]

Following the same steps as above for firm \( i \) thus yields:

\[
(1 - \varepsilon)^{-1} \theta \gamma \rho_{it} X_{it} = R_{it} K_{it},
\]

\[
(1 - \varepsilon)^{-1} (1 - \gamma) \theta \rho_{it} X_{it} = W_{it} H_{it}.
\]
Proof of lemma 1:

Using (14) and (16), it follows that:

\[
R_{jt} = \gamma \theta (1 - \varepsilon) \frac{P_{jt}Y_{jt}}{K_{jt}} = R_{yt} = \gamma \theta (1 - \varepsilon) \frac{Y_{t}}{K_{yt}} \\
R_{it} = \gamma \theta (1 - \varepsilon) \frac{\rho_{it}X_{it}}{K_{it}} = R_{xt} = \gamma \theta (1 - \varepsilon) \frac{\rho_{t}X_{t}}{K_{xt}}
\]

In equilibrium, \( R_{yt} = R_{xt} = R_{t} \). Thus

\[
R_{t} = \gamma \theta \frac{Y_{t}}{K_{yt}} = \gamma \theta \frac{\rho_{t}X_{t}}{K_{xt}} = \gamma \theta \frac{(Y_{t} + \rho_{t}X_{t})}{K_{t}}. \tag{36}
\]

Likewise, using (15) and (17) we have that

\[
W_{t} = \frac{(1 - \gamma) \theta Y_{t}}{(1 - \varepsilon) H_{yt}} = \frac{(1 - \gamma) \theta \rho_{t}X_{t}}{(1 - \varepsilon) H_{xt}} = \frac{(1 - \gamma) \theta (Y_{t} + \rho_{t}X_{t})}{H_{t}}. \tag{37}
\]

The result then follows by way of differentiation with respect to \( \theta \) and \( \varepsilon \).

This ends the proof.

Proving the theorem:

Combining (25) with (18) using \( K_{t+1} = I_{t} \) yields

\[
\frac{I_{t}}{C_{t}} = (1 - \varepsilon)^{-1} \gamma \theta \beta E_{t} \left[ \frac{Y_{t+1} + \rho_{t+1}X_{t+1}}{C_{t+1}} \right] \tag{38}
\]

Similarly, combining (24) with (19) using \( H_{t+1} = X_{t} \) also yields

\[
\frac{\rho_{t}X_{t}}{C_{t}} = (1 - \varepsilon)^{-1} (1 - \gamma) \theta \beta E_{t} \left[ \frac{Y_{t+1} + \rho_{t+1}X_{t+1}}{C_{t+1}} \right].
\]

Summing the two equations above yields

\[
\frac{I_{t} + \rho_{t}X_{t}}{C_{t}} = (1 - \varepsilon)^{-1} \theta \beta E_{t} \left[ \frac{Y_{t+1} + \rho_{t+1}X_{t+1}}{C_{t+1}} \right].
\]

Using \( Y_{t} = C_{t} + I_{t} \), this amounts to

\[
\frac{I_{t} + \rho_{t}X_{t}}{C_{t}} = (1 - \varepsilon)^{-1} \theta \beta E_{t} \left[ 1 + \frac{I_{t+1}}{C_{t+1}} + \frac{\rho_{t+1}X_{t+1}}{C_{t+1}} \right]
= (1 - \varepsilon)^{-1} \theta \beta + (1 - \varepsilon)^{-2} \theta \beta E_{t} \left[ 1 + \frac{I_{t+2}}{C_{t+2}} + \frac{\rho_{t+2}X_{t+2}}{C_{t+2}} \right]
\]
Thus, applying the law of iterated expectations while letting $\theta \beta < (1 - \varepsilon)$ gives

$$I_t + \rho_t X_t = \frac{\theta \beta}{1 - \varepsilon - \theta \beta},$$  \hfill (39)

which upon combination with (38) and $Y_{t+1} = C_{t+1} + I_{t+1}$ yields

$$I_t + \rho_t X_t = \frac{\gamma \theta \beta}{1 - \varepsilon - (1 - \gamma) \theta \beta}.$$  \hfill (40)

Thus, from $Y_t = C_t + I_t$, it follows that

$$C_t = \frac{(1 - \varepsilon - \theta \beta)}{1 - \varepsilon - (1 - \gamma) \theta \beta} Y_t,$$  \hfill (41)

$$I_t \equiv K_{t+1} = \frac{\gamma \theta \beta}{1 - \varepsilon - (1 - \gamma) \theta \beta} Y_t.$$  \hfill (42)

Next, combining (39), (40) and (41) gives

$$\frac{\rho_t X_t}{Y_t} = \frac{(1 - \gamma) \theta \beta}{1 - \varepsilon - (1 - \gamma) \theta \beta}.$$  \hfill (43)

Thus, from (36) it follows that

$$\frac{K_{xt}}{K_t} = \frac{\rho_t X_t}{(Y_t + \rho_t X_t)} = (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta.$$  \hfill (44)

Similarly, (37) implies that

$$\frac{H_{xt}}{H_t} = \frac{\rho_t X_t}{(Y_t + \rho_t X_t)}.$$  \hfill (45)

Therefore,

$$\frac{K_{xt}}{K_t} = \frac{H_{xt}}{H_t} = (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta,$$

or, equivalently,

$$\frac{K_{xt}}{H_{xt}} = \frac{K_t}{H_t} = (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta.$$  \hfill (44)

This ends the proof.

**Proof of proposition 1:** The proof proceeds in two steps.

Since $R_t = \gamma \theta (1 - \varepsilon)^{-1} (Y_t + \rho_t X_t) / K_t$ - see equation (18) - using (43) and arranging terms yield

$$R_t = \gamma \theta (1 - \varepsilon)^{-1} \frac{(1 + \rho_t X_t / Y_t)}{K_t / Y_t} = \frac{\gamma \theta}{1 - \varepsilon - (1 - \gamma) \theta \beta} \frac{Y_t}{K_t}.$$  \hfill (45)
Similarly, as $W_t = (1 - \gamma)(1 - \varepsilon)^{-1} \theta (Y_t + \rho_t X_t) / H_t$ - see equation (19) - using (43) and arranging terms yield

$$W_t = \frac{(1 - \gamma) \theta Y_t}{1 - \varepsilon - (1 - \gamma) \theta \beta H_t}. \quad (46)$$

**Step one: Computing** $Y_t/K_t$ and $Y_t/H_t$.

Market clearing for factor inputs imposes that

$$\frac{K_{yt}}{K_t} = \frac{H_{yt}}{H_t} = 1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta, \quad (47)$$

which states that the $y$-industry uses a fraction $1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta$ of resources available economy-wide. Therefore the total supply of the $y$-good can be derived from:

$$Y_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] AK_t H_t^{1-\gamma}, \quad (48)$$

or, equivalently,

$$\frac{Y_t}{K_t} = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] \left(\frac{H_t}{K_t}\right)^{1-\gamma} A. \quad (49)$$

The above equation can be rewritten using (44) as follows

$$\frac{Y_t}{K_t} = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] \left[\frac{(1 - \varepsilon)}{(1 - \gamma) \theta \beta}\right]^{1-\gamma} A. \quad (49')$$

(Note that under (28) $(1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta < 1$).

Likewise (48) carries the implication that

$$\frac{Y_t}{H_t} = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] \left(\frac{K_t}{H_t}\right)^{\gamma} A.$$

Once again, using (44) yields

$$\frac{Y_t}{H_t} = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] [(1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta]^\gamma A \quad (50)$$

**Step two:** We now turn to proving our claims.

**Claim 1:** The return to physical capital is higher, the larger outsourcing inflows.

We proof this by combining (45) and (49).

$$R_t = (1 - \varepsilon)^{-\gamma} [(1 - \gamma) \beta]^{-(1-\gamma)} \gamma \theta^\gamma A.$$
Differenting the above with respect to $\varepsilon$ yields the result.

**Claim two:** The price of human capital good and the wage rate are higher, the larger outsourcing inflows.

Since the price of human capital goods is given by $\rho_t = (1 - \gamma) \gamma^{-1} E_t (K_{t+1}/H_{t+1})$ - see (27) - we use (44) to get that

$$\rho_t = (1 - \varepsilon)^{-1} (1 - \gamma)^2 \gamma^{-1} \theta \beta.$$

On the order hand, combining (46) and (50) yields

$$W_t = (1 - \varepsilon)^{-(1+\gamma)} (1 - \gamma)^{1+\gamma} \theta^{1+\gamma} \beta^\gamma A.$$

The result follows by differentiating the two equations above with respect to $\varepsilon$.

This ends the proof.

**Proof of proposition 2:**

Using logs we first rewrite (48) as follows

$$y_t = a + \gamma k_t + (1 - \gamma) h_t + \log \left[ 1 - (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta \right]. \quad (51)$$

Similarly, (44) implies that the total supply for the $x$-good can be derived from:

$$X_t = (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta Z K_t \gamma H_t^{1-\gamma}, \quad (52)$$

i.e., in logs terms

$$x_t = z + \gamma k_t + (1 - \gamma) h_t + \log \left[ (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta \right]. \quad (53)$$

Next, combining (42) and (48) gives the level of accumulated physical capital in the next period (in log terms) as:

$$k_{t+1} = a + \log (\gamma \theta \beta) - \log (1 - \varepsilon) + \gamma k_t + (1 - \gamma) h_t, \quad (54)$$

or, equivalently,

$$k_{t+1} - k_t = a + \log \left( \frac{\gamma \theta \beta}{1 - \varepsilon} \right) - (1 - \gamma) (k_t - h_t). \quad (55)$$
Similarly, using (52) and $H_{t+1} = X_t$ we find the level of accumulated human capital in the next period (in log terms) as:

$$h_{t+1} = z + \gamma k_t + (1 - \gamma) h_t + \log \left[ (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta \right],$$

(56)

that is

$$h_{t+1} - h_t = z + \gamma (k_t - h_t) + \log \left[ \frac{(1 - \gamma) \theta \beta}{(1 - \varepsilon)} \right].$$

(57)

On the other hand, (54) and (56) can be combined to get that

$$\gamma k_{t+1} + (1 - \gamma) h_{t+1} = \gamma \log (\gamma \theta \beta) - \gamma \log (1 - \varepsilon) + \gamma k_t + \gamma a + (1 - \gamma) z$$

$$+ (1 - \gamma) h_t + (1 - \gamma) \log \left[ (1 - \gamma) (1 - \varepsilon)^{-1} \theta \beta \right].$$

(58)

In clear, using (56),

$$k_{t+1} - h_{t+1} = a - z + \log (\gamma \theta \beta) - \log [(1 - \gamma) \theta \beta].$$

(59)

Substituting (59) back into (55) and (57) yields

$$g_k = g_h = \gamma a + (1 - \gamma) z + \log \left[ (1 - \varepsilon)^{-1} \theta \beta \right] + \gamma \log \gamma + (1 - \gamma) \log (1 - \gamma).$$

where $g_h$ and $g_k$ stand for the growth rate of human and physical capital. The result then follows by way of differentiation with respect to $\varepsilon$.

This ends the proof.

**Proof of proposition 3:** In generalizing proposition 2 we consider the following production functions:

$$Y_{jt} = AK_j^\gamma H_j^{1-\gamma}, \ 0 < \gamma < 1,$$

(60)

$$X_{it} = ZK_i^\alpha H_i^{1-\alpha}, \ 0 < \alpha < 1.$$  

(61)

We first prove the claim that inward outsourcing raises the capital intensity of physical goods.
From the first order conditions for capital, it can be shown that

\[
R_{jt} = \frac{\theta}{(1 - \varepsilon)} \gamma P_{jt} Y_{jt} = R_{yt} = \frac{\theta}{(1 - \varepsilon)} \gamma Y_t,
\]

\[
R_{it} = \frac{\theta}{(1 - \varepsilon)} \alpha \rho_{it} X_{it} = R_{xt} = \frac{\theta}{(1 - \varepsilon)} \alpha \rho_t X_t.
\]

In equilibrium, \( R_{yt} = R_{xt} = R_t \). Thus

\[
R_t = \frac{\theta}{(1 - \varepsilon)} \gamma Y_t = \frac{\theta}{(1 - \varepsilon)} \alpha \rho_t X_t = \frac{\theta}{(1 - \varepsilon)} (\gamma Y_t + \alpha \rho_t X_t).
\] (62)

Similarly, from the first order conditions for human capital, it can be shown that

\[
W_t = \frac{\theta}{(1 - \varepsilon)} (1 - \gamma) Y_t = \frac{\theta}{(1 - \varepsilon)} (1 - \alpha) \rho_t X_t,
\]

\[
= \frac{\theta}{(1 - \varepsilon)} [(1 - \gamma) Y_t + (1 - \alpha) \rho_t X_t].
\] (63)

Combining (62) and (25) using \( K_{t+1} = I_t \) yields:

\[
\frac{I_t}{C_t} = \frac{\theta}{(1 - \varepsilon)} \beta E_t \left[ \frac{\gamma Y_{t+1} + \alpha \rho_{t+1} X_{t+1}}{C_{t+1}} \right].
\] (64)

Likewise, combining (63) and (24) using \( H_{t+1} = X_t \) yields

\[
\frac{\rho_t X_t}{C_t} = \frac{\theta}{(1 - \varepsilon)} \beta E_t \left[ \frac{(1 - \gamma) Y_{t+1} + (1 - \alpha) \rho_{t+1} X_{t+1}}{C_{t+1}} \right].
\] (65)

Following the same steps as before yields

\[
\frac{I_t + \rho_t X_t}{C_t} = \frac{\theta \beta}{1 - \varepsilon - \theta \beta},
\]

that is,

\[
\frac{\rho_t X_t}{C_t} = \frac{\theta \beta}{1 - \varepsilon - \theta \beta} - \frac{I_t}{C_t}.
\] (66)

Substituting (66) back into (64),

\[
\frac{I_t}{C_t} = \frac{\theta \beta}{(1 - \varepsilon)} \left( \gamma + \frac{\alpha \theta \beta}{1 - \varepsilon - \theta \beta} \right) + \frac{(\gamma - \alpha) \theta \beta}{(1 - \varepsilon)} E_t \left[ \frac{I_{t+1}}{C_{t+1}} \right]
\]

\[
= \frac{\theta \beta}{(1 - \varepsilon)} \left( \gamma + \frac{\alpha \theta \beta}{1 - \varepsilon - \theta \beta} \right) + \frac{(\gamma - \alpha) \theta \beta}{(1 - \varepsilon)} \frac{\theta \beta}{(1 - \varepsilon)} \left( \gamma + \frac{\alpha \theta \beta}{1 - \varepsilon - \theta \beta} \right)
\]

\[
+ \left[ \frac{(\gamma - \alpha) \theta \beta}{(1 - \varepsilon)} \right]^2 E_t \left( \frac{I_{t+2}}{C_{t+2}} \right).
\]

26
Under condition 1 \((\gamma - \alpha) \theta \beta < (1 - \varepsilon)\) and it can be shown that

\[
\frac{I_t}{C_t} = \frac{[(1 - \varepsilon) \gamma - (\gamma - \alpha) \theta \beta \theta \beta]}{(1 - \varepsilon - \theta \beta) [(1 - \varepsilon) - (\gamma - \alpha) \theta \beta]}.
\]

(67)

Thus, using \(Y_t = C_t + I_t\), minor algebraic manipulations yield

\[
\frac{\rho_t X_t}{Y_t} = \frac{\theta \beta (1 - \gamma)}{(1 - \varepsilon - \theta \beta + \alpha \theta \beta)}.
\]

Equation (62) and market clearing for physical capital then imply that

\[
\frac{K_{yt}}{K_t} = \frac{\gamma (1 - \varepsilon - \theta \beta + \alpha \theta \beta)}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta},
\]

(68)

so that

\[
\frac{K_{xt}}{K_t} = \frac{(1 - \gamma) \alpha \theta \beta}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta}.
\]

(69)

Similarly, it can be shown that

\[
\frac{H_{yt}}{H_t} = 1 - (1 - \alpha) (1 - \varepsilon)^{-1} \theta \beta,
\]

(70)

and

\[
\frac{H_{xt}}{H_t} = (1 - \alpha) (1 - \varepsilon)^{-1} \theta \beta.
\]

(71)

Thus, the capital intensity in the sector of physical goods relative to that of the economy is given by:

\[
\frac{K_{yt}/K_t}{H_{yt}/H_t} \equiv \frac{K_{yt}/K_t}{K_{yt}/H_t} \equiv \kappa_y = \frac{(1 - \varepsilon) \gamma}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta}.
\]

Differentiating the above with respect to \(\varepsilon\) then yields

\[
\frac{d\kappa_y}{d\varepsilon} = \frac{(\gamma - \alpha) \gamma \theta \beta}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta}.
\]

We now turn to showing that \(g_k = g_h\).

Given (69), (68), (70) and (70) we now have that

\[
Y_t = A \left[ \frac{\gamma (1 - \varepsilon - \theta \beta + \alpha \theta \beta)}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta} K_t \right]^{\gamma}\left[1 - (1 - \alpha) (1 - \varepsilon)^{-1} \theta \beta H_t\right]^{1-\gamma},
\]

\[
X_t = Z \left[ \frac{(1 - \gamma) \alpha \theta \beta}{\gamma (1 - \varepsilon) - (\gamma - \alpha) \theta \beta} K_t \right]^{\alpha}\left[1 - (1 - \alpha) (1 - \varepsilon)^{-1} \theta \beta H_t\right]^{1-\alpha}.
\]
Likewise, combining (67) and $Y_t = C_t + I_t$ now yields

$$C_t = \frac{(1 - \varepsilon - \theta \beta) [(1 - \varepsilon) - (\gamma - \alpha) \theta \beta]}{(1 - \varepsilon) [1 - (1 - \alpha) \theta \beta]} Y_t,$$

$$I_t \equiv K_{t+1} = \frac{[(1 - \varepsilon) \gamma - (\gamma - \alpha) \theta \beta] \theta \beta}{(1 - \varepsilon) [1 - (1 - \alpha) \theta \beta]} Y_t.$$ 

Following the same steps as before, the counterparts to (55), (57) and (58) respectively are:

$$k_{t+1} - k_t = a - (1 - \gamma) (k_t - h_t) + \log \theta \beta - \log (1 - \varepsilon) + \gamma \log \gamma$$
$$- \log [1 - (1 - \alpha) \theta \beta] - (1 - \gamma) \log (1 - \varepsilon)$$
$$+ (1 - \gamma) \log [(1 - \varepsilon) \gamma - (\gamma - \alpha) \theta \beta] + \log [1 - \varepsilon - (1 - \alpha) \theta \beta], \quad (72)$$

$$h_{t+1} - h_t = z + \alpha (k_t - h_t) - (1 - \alpha) \log (1 - \varepsilon)$$
$$+ \alpha \log (1 - \gamma) a \theta \beta$$
$$- \alpha \log [(1 - \varepsilon) \gamma - (\gamma - \alpha) \theta \beta] + (1 - \alpha) \log [(1 - \alpha) \theta \beta], \quad (73)$$

$$\alpha k_{t+1} + (1 - \gamma) h_{t+1} = a \alpha + (1 - \gamma) z + \alpha k_t + (1 - \gamma) h_t + \alpha \log \theta \beta$$
$$+ \alpha \gamma \log \gamma - \alpha \log [1 - (1 - \alpha) \theta \beta] + (1 - \gamma) \alpha \log (1 - \gamma) a \theta \beta$$
$$+ \alpha \log [1 - \varepsilon - (1 - \alpha) \theta \beta] - \alpha \log (1 - \varepsilon) - (1 - \alpha) \gamma \log (1 - \varepsilon)$$
$$+ (1 - \gamma) (1 - \alpha) \log [(1 - \alpha) \theta \beta] - (1 - \gamma) (1 - \alpha) \log (1 - \varepsilon). \quad (74)$$

We note that (74) shows that $\alpha (k_{t+1} - k_t) + (1 - \gamma) (h_{t+1} - h_t)$ is constant, so that $g_k = g_h$. This in turn implies that the right-hand sides in (72) and (73) are equal and

$$(1 + \alpha - \gamma) (k_t - h_t) = a - z + \gamma \log \gamma - (1 - \alpha) \log (1 - \alpha)$$
$$- \log [1 - (1 - \alpha) \theta \beta] - (1 + \alpha - \gamma) \log (1 - \varepsilon)$$
$$+ \log [1 - \varepsilon - (1 - \alpha) \theta \beta] - \alpha \log (1 - \gamma) \alpha$$
$$+ (1 + \alpha - \gamma) \log [(1 - \varepsilon) \gamma - (\gamma - \alpha) \theta \beta].$$
Substituting the above back into (72) for \((k_t - h_t)\) and arranging terms finally yields:

\[
g_k \equiv k_{t+1} - k_t = \log (1 - \varepsilon)^{-1} \theta \beta + \frac{\alpha}{1 + \alpha - \gamma} \log \left[ \frac{1 - \varepsilon - (1 - \alpha) \theta \beta}{1 - (1 - \alpha) \theta \beta} \right] + \frac{\alpha a + (1 - \gamma) z}{1 + \alpha - \gamma} \\
+ \frac{(1 - \gamma)(1 - \alpha)}{1 + \alpha - \gamma} \log (1 - \alpha) + \frac{(1 - \gamma)}{1 + \alpha - \gamma} \log (1 - \gamma) \alpha + \frac{\alpha \gamma}{1 + \alpha - \gamma} \log \gamma.
\]

This ends the proof.

**Proof of proposition 4:**

**Claim 1:** Inward outsourcing is initially welfare-reducing.

To prove this, we combine (22) with (39) using \(K_{t+1} = I_t\) and then arranging terms:

\[
\frac{C_t}{\psi_t} = 1 - \frac{\theta \beta}{1 - \varepsilon},
\]

Upon differentiation with respect to \(\varepsilon\) this yields

\[
\frac{d}{d\varepsilon} \left( \frac{C_t}{\psi_t} \right) = \frac{-\theta \beta}{(1 - \varepsilon)^2} < 0.
\]

This ends the proof.
References


