Monopolistic Competition, International Trade and Firm Heterogeneity
- a Life Cycle Perspective -

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Abstract
This paper presents a dynamic international trade model based on monopolistic competition, where observed intra-industry differences at a given point in time reflect different stages of the firm’s life cycle. New product varieties of still higher quality enter the market every period rendering old varieties obsolescent in a process of creative destruction. For given technology (variety) production costs decrease after an infant period due to learning. It is shown that several patterns of exports may arise depending primarily on the size of fixed trade costs. At a given point in time firms arise depending due to different age, although all firms are symmetric in a life cycle perspective. The paper thus offers an alternative view on firm heterogeneity compared with other recent papers, where productivity differences appear as an outcome of a stochastic process.

Keywords: Product innovations, learning, creative destruction, firm heterogeneity, export performance.

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1. Introduction

The distinctive feature of the ‘new-new’ trade theory is different productivities for firms within the same industry, and due to such differences firms differ with respect to size, profitability and export orientation. This offshoot of international trade theory is developed in the seminal paper by Melitz (2003), but essential contributions are also found in several other recent papers (e.g. Schmitt and Yu, 2001; Montagna, 2001; Helpman et al., 2004). The theoretical framework in these papers is monopolistic competition based on a ‘love of variety’ specification of preferences as suggested by Dixit and Stiglitz (1977). Dixit-Stiglitz preferences have previously been used for explaining intra-industry foreign trade by Krugman (1979b, 1980). However, in Krugman’s analyses firms were homogenous, so all firms are exporters and earn zero profits due to free entry and exit. By introducing differences of productivities, the recent literature in the Melitz tradition thus appears to be a generalized version of the Krugman model.

In the Melitz (2003) model the differences of productivities follow from the outcome of a lottery process, which all producers have to go through. Before a prospective entrant starts to produce his productivity is uncertain. The productivity is first revealed for the producer after he has paid sunk costs in form of fixed entrance costs. All producers are identical, but only prior to entry as their productivity parameter is drawn from a common distribution. Afterwards, the productivities differ due to the actual outcomes of the individual producer’s participation in the productivity lottery. The productivity is thus exogenous given and time invariant, when the outcome of the lottery has been revealed. This formulation of productivity heterogeneity captures what most perceive as twin facts, namely that producers have different abilities to manage a company, but these abilities are to some extent unknown before actual management is exercised. Management or entrepreneurship is in other words elusive, which only reveals itself through practicing.

However, analyses in the Melitz tradition disregard essential dynamic technological processes related both to product innovations, where new products of higher quality drive out older products, and process innovations, where existing products of given qualities are produced at lower costs. Firm heterogeneity observed at a given point in time may therefore reflect different stages of individual firms in their life cycle and not necessarily outcomes of a lottery, where some producers are lucky and others less so.

The life cycle view of firm’s internationalization has been stressed by Vernon (1966) in his product life cycle theory, and later formalized by Krugman (1979a). Both relate the issue to a North-South perspective, where production is shifted from North to South as techniques of production becomes more standardized. An evolutionary perspective is also stressed in the Uppsala internationalization model (Johanson and Valne, 1977, 1990; Johanson and Wiedersheim-Paul, 1975), where firms also follow different stages in their internationalization, and where decisions to export are made incrementally due to market uncertainty. Finally, the business literature has in recent years presented examples of new types of internationalization, where some firms – the so called ‘born globals’ - orient themselves to the world market from the beginning of their life cycle, while others are
only active on the domestic market (see e.g. Jolly et al., 1992; Lindqvist, 1991; Oviatt et al.; Rennie, 1993). Such mixed patterns of internationalization has been confirmed empirically, both in the business literature through case studies, but also recently by e.g. Bernard and Jensen (2004), who present evidence from the US showing that the export behavior of an individual firm vary from period to period. Their results show that on average 10% exit, while 14% of the firms enter the export market per year. Such differences in export behavior may, as shown in our paper, be related to the life cycle of the firm.

The aim of this paper is to present a simple model, where differences of productivity, profitability and export behavior of firms at a given point in time reflect different layers of innovations and experiences due to different ages of the firms. The model describes equilibrium on a market with monopolistic competition. The preferences are of the ‘love of variety’ type, but generalized to include both horizontal and vertical product differentiation (varieties of the same and different qualities respectively). The flavor of the model follows from two crucial assumptions, both related to the dynamics of technology.

First, it is assumed that every period offers a number of inventions or designs of product varieties of higher quality compared with designs from previous periods. Inventions are turned to actual innovations materialized through entry of new producers. This reduces operating profits of producing old varieties and ultimately, when operating profits become insufficient to cover the fixed production costs, production of these varieties ceases due to obsolescence. Better varieties thus steadily drive out varieties of lower quality in a process of ‘creative destruction’, as originally stressed by Schumpeter (1942). In the last decade several growth models include this mechanism, see e.g. Aghion and Howitt (1992), Barro and Sala-i-Martin (1995) and Aghion et al. (2001).

Secondly, we assume that process innovations also take place. Through learning marginal costs of producing a specific variety may be reduced. The learning is assumed to be firm specific, i.e. we neglect learning across varieties. This assumption has been supported among others by Benkard (2000), who investigates organizational forgetting in the aircraft industry and shows that firm’s production experience depreciates over time in a process with incomplete spillovers of production expertise from one generation of aircraft to the next. We also assume that the productivity gain from learning is limited as experiences relate to a specific technology, i.e. production of a specific variety, and hence, after an initial period of learning the potential for further productivity gains are depleted. This idea of a limited potential for productivity gain from learning linked to a specific technology is also found in some contributions to the theory of economic growth, see e.g. Brezis et al. (1993).

To keep the formal analysis simple we analyze the case, where each variety is produced in three periods only, which we term: the infant, the mature, and the aged period. In the infant period the varieties are of the highest quality on the market, but marginal costs are relatively high due to lack of experiences. In the mature period the varieties are not the most advanced varieties on the market, but marginal costs are relatively low due to
learning from production in the infant period. In the aged period the varieties are close to be outdated, but still produced due to low marginal costs. The three period’s analysis allows us to analyze several export patterns of firms found in the empirical literature. Sunk costs constrain the inflow of new varieties and fixed costs ensure a flow of exits of old varieties. In an open economy fixed trade costs may postpone entry, but precipitate exit from the export market, i.e. varieties may only be exported in the mature period.

The model predicts at a given point in time an intra-industry structure broadly similar to the structure described in the Melitz (2003) model. However, all firms are symmetric in a life-cycle perspective, so we need not include a lottery process explaining productivity differences. In more general models, the life-cycle aspect of our model, as well as the mechanism in the Melitz model may both be included, so the two approaches appear to be complementary. While the Melitz model illustrates intra-industry trade in horizontally differentiated varieties, the life-cycle approach allows for intra-industry trade in both horizontally and vertically differentiated varieties. As shown in the model trade liberalization may influence not only the total number of varieties offered for the consumers, but also the composition between new high quality varieties and older varieties of lower quality.

The paper is organized as follows. Section 2 presents the basic model and analyzes market equilibrium in a closed economy. Section 3 introduces a foreign country which allows for intra-industry trade although exporting firms incur variable and fixed trade costs for exporting to the foreign country. The section describes market equilibrium and identifies the group of exporting firms. Section 4 discusses the results derived in Section 3. Section 5 concludes. The Appendix derives operating profits relevant for the firms’ decision on exit from both the domestic and foreign market or from the foreign market exclusively. The results are used to describe the constraints of the parameters, which make the market equilibrium feasible.

2. The basic model for a closed economy

Assumptions
This section develops a dynamic one-sector model for a closed economy characterized by increasing returns to scale and monopolistic competition based on a love of variety specification of preferences. Each period adds a new generation of a large number of varieties to the total supply of varieties on the market. Each of these varieties enters symmetrically in demand, when compared with other varieties of the same generation. Between generations varieties differ in quality, so varieties from the most recent generation are of better quality than varieties from the preceding periods. Varieties of a certain age are thus becoming obsolete and not produced any longer, so only varieties from a limited number of generations are actually produced on the market.

To simplify, we analyze the case, where only three generations of product varieties are produced. Varieties introduced in the present period indexed by 1 are in the infant period; varieties introduced one period ago indexed by 2 are in the mature period, and finally,
varieties introduced two periods ago indexed by 3 are in the aged period. Older varieties are not produced due to obsolescence which materializes when fixed production costs cannot be recovered any longer by potential operating profit. We do not claim specific realism for this three period assumption, which constrains the value of the parameters, but only that it allows for a simple analysis of interesting cases with respect to productivities, firm size and export behavior.

A representative consumer’s preferences in the present period for varieties from the three generations are specified by a quality augmented love of variety utility function:

\[
U = \sum_{i=1}^{n_1} \phi^2 c_{1,i}^\theta + \sum_{i=1}^{n_2} \phi c_{2,i}^\theta + \sum_{i=1}^{n_3} c_{3,i}^\theta; \quad \phi > 1; \quad 0 < \theta < 1
\]

\(U\) stands for utility and \(c_{i,1}, c_{i,2}, \text{and } c_{i,3}\) indicate an individual’s consumption of variety \(i\) from generation 1, 2 and 3, respectively. \(\phi\) is a parameter describing the effect of quality improvements on utility, and \(n_1, n_2, n_3\) the number of varieties of each of the three generations. The consumers have also a latent demand for unproduced varieties with preferences similar to (1).

The consumer’s behavior is defined by:

\[
\max U \text{ s.t. } \sum_{i=1}^{n_1} p_{i,1} c_{1,i} + \sum_{i=1}^{n_2} p_{i,2} c_{2,i} + \sum_{i=1}^{n_3} p_{i,3} c_{3,i} = 1 \quad \text{(2)}
\]

where the common wage rate is normalized to one.; \(p_{i,1}, p_{i,2}, p_{i,3}\) is the present prices for varieties introduced in the present period, and one and two periods ago, respectively.

Each firm produces one variety only and the concept ‘firm’ is therefore used synonymous with variety. All firms belonging to a given generation are symmetric, i.e. have the same cost function. The only factor of production is labour. The costs of producing the newest generation of varieties and that of earlier generations introduced one and two periods ago, respectively, are, measured in labor units:

\[
l_{i,1} = S + f + \beta x_{i,1}; \quad S, f, \beta > 0
\]

\[
l_{i,j} = f + (\frac{\beta}{\mu}) x_{i,j}; \quad j = 2, 3; \quad \mu > 1
\]

\(S\) is sunk costs, which all firms pay prior to entry, \(f\) is the (recurrent) fixed costs and \(x\) output. Marginal production costs in the infant period are \(\beta\), but for the remaining periods at the lower level \(\beta/\mu\). This decrease of marginal costs represents a very simple specification of a learning effect, where the producer becomes fully experienced in producing his variety after one period.\(^1\) Learning is thus related to the specific variety

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\(^1\) Firm specific learning from experience in an international (oligopolistic) trade setting has e.g. been formalized by Dasgupta and Stiglitz (1988).
with a limited potential, which is realized swiftly. Labor is inelastic supplied from a given total labor force at \( L \). Assuming full employment, the resource constraint is given by:

\[
L = \sum_{t=1}^{n} l_{t,1} + \sum_{t=1}^{n} l_{t,2} + \sum_{t=1}^{n} l_{t,3}
\]  

(4)

Output of the individual firm \( x_{ij} \) equals total demand for its variety \( Lc_{ij} \), i.e:

\[
x_{ij} = Lc_{ij}; \quad j = 1, 2, 3
\]  

(5)

The firm maximizes its value \( V_i \) over its life-cycle. We disregard discounting between periods, and the firm’s value is thus given by:

\[
V_i = \pi_{i,1} + \pi_{i,2} + \pi_{i,3} - (S + 3\mu)
\]  

(6)

where operating profit \( \pi_{ij} \) is given by:

\[
\pi_{i,1} = (p_{i,1} - \beta)x_{i,1}
\]

and:

\[
\pi_{i,j} = (p_{i,j} - \beta)\frac{\mu}{\theta}x_{i,j} \quad j = 2, 3
\]  

(7)

The value of the firm is driven to zero due to free entry and exit, and symmetry over the life cycle, i.e.:

\[
V_i = 0
\]  

(8)

**Market equilibrium**

In the following we describe market equilibrium, and specify the constraint of the parameters for obsolescence after producing in three periods only. First, we determine prices, output and operating profits for firms of different ages using the standard procedure for deriving the equilibriums conditions for models of this type (see Krugman, 1979b, 1980). All consumers are identical. The first order condition of the individual consumer’s utility maximization gives the (inverse) market demand functions of product varieties introduced in current and earlier periods:

\[
p_{i,j} = \left( \frac{\theta p^{\theta-1}}{\lambda} \right) c_{i,j}^{\theta-1}; \quad j = 1, 2, 3
\]  

(9)

where \( \lambda \) is the shadow price of the budget constraint, i.e. marginal utility of money.

The large number of producers ensures a constant elasticity of market demand at \(-1/(1-\theta)\). Maximizing operating profits, the firms charge a price equal to a mark-up of marginal cost with the mark-up factor \( 1/\theta \), i.e.:

\[
p_{i,1} = \beta / \theta
\]

\[
p_{i,j} = \left( \frac{\beta}{\mu} \right) / \theta, \quad j = 2, 3
\]  

(10)
Firms of a given age all produce the same output, therefore we hereafter drop the notation $i$, i.e. $\forall i,j, \; x_{i,j} = x_j, \; c_{i,j} = c_j, \; p_{i,j} = p_j$ for $j=1,2,3$.

The output structure or relative output of each firm during its life cycle depends only on the parameters $\mu$ and $\varphi$. From (5), (9) and (10) we have:

$$x_j = \left(\frac{\mu}{\varphi^{j-1}}\right)^{\frac{1}{\vartheta}} x_1, \; j = 2,3$$

This factorizes the development over time of production into two effects:

a) a positive effect on production of a decrease of marginal costs ($\mu$) due to learning by doing, which stimulates production of old-kind varieties in latter periods,

b) a negative effect on production of lower quality of older varieties ($\varphi^{j-i}$), because consumers increasingly turn their demand to the more recent introduced varieties of higher quality.

Output is lower in the aged period compared with the mature period, as no further decrease of marginal costs takes place, i.e. $x_3 < x_2$. The rank of output in the mature period compared with the infant period is ambiguous as $x_2 \gtrless x_1$ for $\varphi \gtrless \mu$. Hence, at a given point in time firm size differs.

Having determined relative output in the various stages of the firms life cycle we still miss to determine the absolute level of output. This follows from the zero value condition (8).

Inserting (7), (8), (10) and (11) into (6) gives:

$$V = (p_1 - \beta)x_1 + (p_2 - \frac{\beta}{\mu})x_2 + (p_3 - \frac{\beta}{\mu})x_3 - (S + 3f)$$

$$= \beta\frac{1-\theta}{\theta}(x_1 + \frac{1}{\mu}x_2 + \frac{1}{\mu}x_3) - (S + 3f)$$

$$= \beta\frac{1-\theta}{\theta}x_1(1 + (\frac{\mu}{\varphi})^{\frac{1}{1-\vartheta}} + (\frac{\mu}{\varphi^2})^{\frac{1}{1-\vartheta}}) - (S + 3f) = 0$$

Solving this equation with respect to $x_1$ gives:

$$x_1 = \frac{\theta}{(1-\theta)} \frac{(S + 3f)}{\beta H}$$

where:

$$H = \left(1 + (\frac{\mu}{\varphi})^{\frac{1}{1-\vartheta}} + (\frac{\mu}{\varphi^2})^{\frac{1}{1-\vartheta}}\right)$$
Using (12) together with (11) gives the full solution for output in each period of the firm’s life cycle. Finally, we calculate operating profit in each periods of the life cycle by inserting (10), (11) and (12) in (7). This gives:

\[ \pi_i = \beta \left( \frac{1-\theta}{\theta} \right) x_i = \frac{(S+3f)}{H_{1-3}} \]

and:

\[ \pi_j = \frac{\beta}{\mu} \left( \frac{1-\theta}{\theta} \right) x_j = \left( \frac{\mu^\theta}{\varphi_{j-1}} \right)^{1-\theta} \pi_1, \ j = 2,3 \]

Operating profit is thus lower in the aged period compared with the mature period, while the rank of profit between the mature period and the infant period is ambiguous, as \( \pi_2 \geq \pi_1 \) for \( \varphi \geq \mu^\theta \).

The total labor force constrains the number of firms, which can produce on the market in the given period. Total demand for labor is given by inserting the equilibrium values of output given by (11) and (12) in the cost function (3). Inserting this result for total demand for labour in the full employment condition (4) gives:

\[
\begin{align*}
   n_1(\beta)x_1 + n_2 \left( \frac{\beta}{\mu} \right) x_2 + n_3 \left( \frac{\beta}{\mu} \right) x_3 + n_1S + (n_1 + n_2 + n_3)f \\
   = \left( n_1\mu^{\frac{\theta}{1-\theta}}\varphi^{1-\theta} + n_2\mu^{\frac{\theta}{1-\theta}}\varphi^{1-\theta} + n_3\mu^{\frac{\theta}{1-\theta}}\varphi^{1-\theta} \right) \beta x_1 + n_1S + (n_1 + n_2 + n_3)f \\
   = L
\end{align*}
\]

The number of firms established in the past, \( n_2 \) and \( n_3 \), are exogenously given in the present period, while \( n_1 \) is endogenously determined by the condition (14). The relation (14) thus describes the dynamic path of the number of firms.

The market is in a dynamic steady state equilibrium when the rate of growth of the number of entries is constant, which leaves the age structure of the population of producing firms constant. In our case of a constant labour force, the only possible steady state equilibrium is a constant number of entries each period, i.e. a rate of growth of zero. The constant number of entries in the three period steady state equilibrium for the closed economy \( \hat{n} \) is determined by solving (15) for \( n_1=n_2=n_3=\hat{n} \), which gives:

\[ \hat{n} = \frac{L(1-\theta)}{(S+3f)} \] (15)

Till now we have just assumed that all product varieties become obsolete after having been produced in just three periods. This constrains the values of the parameters. A life cycle of three periods will only exist if the individual producer neither has an incentive to
produce his variety for a fourth period nor to cease production of the variety after two periods only. This is the case if operating profits can cover fixed costs in the third period, but not for the subsequent fourth period, i.e:

$$\tilde{\pi}_4 < f < \pi_3$$

(16)

where ‘tilde’ indicates the potential value of the variable. Operating profit $\pi_3$ is given by (13) and $\tilde{\pi}_4$ is calculated in Appendix, see (A1). Inserting these results in (16) gives:

$$\frac{S}{f + 3} < 1 < \frac{S}{\phi^{\tilde{\sigma}}}$$

(17)

where

$$M = \left(1 + \phi^{\tilde{\sigma}} + \mu^{\tilde{\sigma}} \phi^{2\tilde{\sigma}}\right)$$

Noticing that $\partial M/\partial \mu < 0$, while $\partial M/\partial \phi > 0$, both the left-hand side and the right-hand side of (17) varies positively with $S$ and $\mu$, but negatively with $f$ and $\phi$. High entry costs make it more likely that the left hand side of (17) is violated, so the producer has an incentive to produce in a fourth period. The reason is that the high entry costs increases the product runs and operating profits for producing a variety of a given age. Low entry costs bring the right-hand side of (17) into focus as small product runs make it more likely that the producer closes production in a third period. High fixed production costs make it more likely that the right hand side of (17) is violated. High fixed costs increases the product runs and operating profits, but the increase in operating profits are here less than the increase of fixed costs, so production may be closed already after a second period. The opposite effect follows from small fixed costs, which shifts focus to the left-hand side of (17). A high rate of growth of quality improvements or a small learning effect of a given variety may precipitate obsolescence, while a low rate of quality improvement or a large learning effect have the opposite effect.

Figure 1 illustrates a case, where the parameters fulfill the condition for producing in just three periods.
3. Open economy model

This section generalizes the model above by supposing two symmetric countries, a home country and a foreign country. The two countries’ resource endowment (labour force), preferences and technologies are similar to the model just analyzed. Trade is possible, but the markets are only partially integrated due to fixed trade costs $m \geq 0$ and variable iceberg trade costs specified by the parameter $g (0 < g < 1)$, where $(1-g)$ represents the share of output, which disappears as trade costs. We assume that trade costs are independent of the firm’s stage in its life cycle, i.e. we disregard that learning may take place in the export activity\(^2\). Similar to the analysis in the previous section we assume that the parameters ensure the establishment of equilibrium with a three period life cycle.

Even in this simplified three periods life cycle several configurations of export behavior are possible, see Figure 2.

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2 In his review over the empirical literature on the relation between exports and productivity Wagner (2007) states that evidence on learning-by-exporting is somewhat mixed.
The love of variety preferences (with a constant elasticity) ensures that the foreign product always will be demanded irrespective the size of variable trade costs. However, a foreign variety is only supplied when the fixed trade costs can be covered, i.e. operating profit for exports should at least be equal to fixed trade costs. If variable and fixed trade costs are very high, exports may be unprofitable in all stages of the firm’s life as indicated by case VII in Figure 2, i.e. both countries operate in this case as closed economies. In the opposite case, where variable and fixed trade costs are very low, firms export in all stages on their life cycles as indicated by the case I in Figure 2. Between those extreme cases the firm export in some, but not all periods as shown by the cases II to VI. However, case VI newer materializes as it is inferior to at least one of the other cases. If exports are loss-making for the firm in its mature period due to insufficient demand, the market will be even more meager in the aged period and hence, the fixed trade costs cannot be covered for that period. The feasible ‘mixed’ cases of exports in some, but not all periods, thus narrow into the four cases II to V.

The procedure for solving the model in the various cases is similar. This section only looks at two cases. First, we analyze case I, where all firms act as ‘born globals’ and export throughout their life cycle, and next we look at case IV, where all firms only export in their mature period. Especially case IV is illustrative for firm heterogeneity due to different export behavior during a firm’s life cycle.

**Exports in all periods**

The consumers in each country are offered domestic and foreign varieties from all three generations of domestic and foreign firms as in the closed economy. The preferences are specified as in the closed economy by the utility function:

\[
U = \sum_{i=1}^{n_1} \phi^2_i c_{i,1}^\vartheta + \sum_{i=1}^{n_1} \phi_i c_{i,2}^\vartheta + \sum_{i=1}^{n_1} \phi_i c_{i,3}^\vartheta + \sum_{i=1}^{n_1} \phi_i c_{i,1}^\varphi + \sum_{i=1}^{n_1} \phi_i c_{i,2}^\varphi + \sum_{i=1}^{n_1} \phi_i c_{i,3}^\varphi > 1; \quad 0 < \vartheta < 1
\]

where * indicates foreign produced varieties. Utility is maximized subject to

\[
\sum_{i=1}^{n_1} p_{i,1}^\vartheta c_{i,1}^\vartheta + \sum_{i=1}^{n_1} p_{i,2}^\vartheta c_{i,2}^\vartheta + \sum_{i=1}^{n_1} p_{i,3}^\vartheta c_{i,3}^\vartheta + \sum_{i=1}^{n_1} p_{i,1}^\varphi c_{i,1}^\varphi + \sum_{i=1}^{n_1} p_{i,2}^\varphi c_{i,2}^\varphi + \sum_{i=1}^{n_1} p_{i,3}^\varphi c_{i,3}^\varphi = 1
\]

The other assumptions in the closed economy model are unchanged, with the exception that the producer in addition to production costs also incurs variable and fixed trade costs.

The first-order condition for utility maximization gives the (inverse) demand functions:

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3Additionally there may of course also be sunk market access costs. In such cases we broaden the interpretation of S to include both sunk production and market access costs.
where \( p_{i,j}^* / g \) is the price of foreign varieties paid by domestic consumers and hence, \( p_{i,j}^* \) is the price the foreign producer obtains. Due to the symmetry \( p_{i,j}^* / g \) also illustrates the price of domestic varieties paid by foreign consumers and \( p_{i,j}^* \) the price the domestic producer obtains for output to export. Using the assumed constant demand elasticity, the producer prices are:

\[
p_{i,1} = p_{i,1}^* = \beta / \theta \\
p_{i,j} = p_{i,j}^* = \left( \frac{\beta}{\mu} \right) / \theta; \; j = 2, 3
\]

Firms from the same generation face the same demand, costs, and prices. Hence, output is identical for all firms of the same age, and subscripts are therefore neglected in the following.

The firms produce both to the domestic market \( x_{d,j} \) and the foreign market \( x_{f,j} \), i.e. total output is given by:

\[
x_j = (x_{d,j} + x_{f,j}); \; j = 1, 2, 3
\]

To meet demand on the domestic market the firm produces \( Lc_j \). To meet demand on the foreign market \( Lc_j^* \) the firm has to produce \( Lc_j^* / g \), as a part of the output ‘melts’ away due to trade costs. Hence

\[
x_{d,j} = Lc_j
\]

and

\[
x_{f,j} = L \frac{c_j^*}{g}; \; j = 1, 2, 3
\]

Using (20) and (21) in (23) to calculate output produced for export relative to output produced for the domestic market gives:

\[
\frac{x_{f,j}}{x_{d,j}} = \frac{(c_j^*/g)}{c_j} = g^{\frac{\theta}{1-\theta}}; \; j = 1, 2, 3
\]

and hence, total output given by (22):
Using (20) and (21) once more gives the following expressions for the structure of total output for the mature and aged period relative to the infant period:

\[ x_j = \left( \frac{\mu}{\varphi^{j-1}} \right)^{1-\theta} x_i; \quad j = 2, 3 \]  

which is identical with the output structure for the closed economy.

The zero value condition allows us to determine the absolute levels of output. Inserting (7), (8), (21) and (26) into (6) (generalized with fixed trade costs) gives:

\[
V_i = (p_1 - \beta)x_1 + (p_2 - \beta / \mu)x_2 + (p_3 - \beta / \mu)x_3 - (S + 3(f + m)) = 
\]

\[
\beta^{1-\theta} \left( \frac{1}{\mu} x_1 + \frac{1}{\mu} x_2 + \frac{1}{\mu} x_3 \right) - (S + 3(f + m)) = 
\]

\[
\beta^{1-\theta} x_1 \left( 1 + \left( \frac{\mu^\theta}{\varphi} \right)^{1-\theta} + \left( \frac{\mu^\theta}{\varphi^2} \right)^{1-\theta} \right) - (S + 3(f + m)) = 0
\]

Solving with respect to \( x_i \) gives:

\[
x_i = \frac{\theta}{(1-\theta)} \frac{S + 3(f + m)}{\beta H^{1-3}}
\]

where \( H^I = \left( 1 + \left( \frac{\mu^\theta}{\varphi} \right)^{1-\theta} + \left( \frac{\mu^\theta}{\varphi^2} \right)^{1-\theta} \right) \)

The full solution for output in each period of the firms’ life cycle follows from (26) and (27). Although output in relative terms is the same as in the closed economy (see (11)) output in absolute terms is larger in case of trade. The reason is that the producers should earn more operating profit to be able to cover fixed trade costs.

Having solved for the absolute value of output in the infant period, the absolute level of operating profit follows straightforwardly from (7), (21), (26) and (27).

\[
\pi_i = \frac{(S + 3(f + m))}{H^I}
\]

\[
\pi_j = \left( \frac{\mu^\theta}{\varphi^{j-1}} \right)^{1-\theta} \left( \frac{S + 3(f + m)}{H^I} \right) = \left( \frac{\mu^\theta}{\varphi^{j-1}} \right)^{1-\theta} \pi_i; \quad j = 2, 3
\]

Finally, the number of firms in the steady state equilibrium is derived by inserting the results for output, (26) and (27), in the full employment condition (14) (expanded with fixed trade costs), and restricting \( n_1 = n_2 = n_3 = \hat{n} \). This gives:
\[
\hat{n} = \frac{L(1-\theta)}{S + 3(f + m)} \quad (29)
\]

Comparing (29) with (15) shows that the number of firms in each country in the open economy is less than in the closed economy, while output of each firm as previously shown is larger. However, it should be noticed that the consumers in the open economy case have access both to domestic and foreign varieties.

The restrictions on the values of the parameters for the existence of the described equilibrium is more complex compared with the case of a closed economy. The reason is that the firm has the option to exit partially by leaving the export marked, while still producing to the domestic market. To analyze this we dissolve total operating profits \( \pi_j \) into its two components, operating profit on the domestic market \( \pi_{d,j} \), and operating profit on the export market \( \pi_{f,j} \), i.e.:

\[
\pi_j = (\pi_{d,j} + \pi_{f,j}); \quad j = 1, 2, 3 \quad (30)
\]

To ensure that all producers produce in a three period life cycle, each producer should have an incentive to produce in the third period, while no producer should have an incentive to produce in a fourth period, neither to both markets nor to the domestic market only. This gives the conditions:

\[
\tilde{\pi}_4 < (f + m) < \pi_3 \quad (31)
\]

and

\[
\tilde{\pi}^d_4 < f \quad (32)
\]

Moreover, for exporting in all three periods operating profit from exports in each period \( \pi_{f,j}^j \) \((j = 1, 2, 3)\) should exceed the fixed trade costs \( m \). Operating profit for exporting in the mature period is always larger than operating profit for exporting in the aged period, so the binding condition for exporting in all three periods reduces to:

\[
\text{Min. } \pi_{d,1}^f, \pi_{d,3}^f > m \quad (33)
\]

Operating profit \( \pi_3 \) is given by (28), and potential operating profit for producing and exporting in a fourth period \( \tilde{\pi}_4 \) is derived in Appendix, see (A2). Using these results in (31) gives:

\[
\left( \frac{S}{f + m} + 3 \right) \frac{1}{\phi^{1-\theta} M^f} < 1 < \left( \frac{S}{f + m} + 3 \right) M^f
\]

where

\[
M^f = 1 + \phi^{1-\theta} + \mu^{1-\theta} \phi^{2-\theta}
\]

where \( M^f \) is equal to \( M \) for the closed economy.
(34) is broadly identical with the condition for a three period life cycle for a closed economy given by (17), apart from the additional fixed trade costs, which in case of left-hand side of (34) is not fulfilled induce the producer to exit after two periods only.

The Appendix also reports potential operating profit $\hat{\pi}_d^4$ for producing to the domestic market exclusively in a fourth period, see (A3). Inserting this expression for $\hat{\pi}_d^4$ in (32) gives:

$$\left(\frac{S}{f + m} + 3\right)\left(1 + \frac{m}{f}\right)\left(\frac{\phi^{1-\theta} M^I}{(1 + g^{1-\theta})}\right) < 1 \quad (35)$$

If the first parenthesis is less than 1, the left hand side condition of (34) is reproduced and no incentive exists for producing both to the domestic and foreign market in a fourth period. If the second parentheses also is less than 1, (35) is fulfilled and no incentive exists for producing in a fourth period for the domestic market exclusively. Small trade costs (low values of $m/f$ and values of $g$ close to 1) may ensure this condition.

Using (A4) and (A5) from Appendix shows that $\hat{\pi}_1^f \geq \hat{\pi}_1^f$ for $\mu^0 \geq \phi^2$, i.e. operating profit on the export market is smallest in the first period, if learning is strong and quality growth small, and vice versa in the opposite case.

Inserting (A4) and (A5) in (33) gives the constraint:

$$\min \left(\frac{S}{m} + \frac{3f}{m} + 3\right), \left(\frac{S}{m} + \frac{3f}{m} + 3\right) > 1 \quad (36)$$

To satisfy (36) variable and fixed trade costs should be relatively low.

To conclude on case I, the firm experiences different productivities and market positions of its products during its life cycle. This impacts its output as well as its export behaviour. The export orientation, i.e. export to output, is the same in all three periods, where the firm produces. The assumed constant parameter for iceberg trade costs is main responsible for this structural constancy. If the variable trade costs were sensitive to learning, i.e. the iceberg parameter increased after the infant period the firms would be more export oriented in late periods of their life compared with their infant period. However, even in case of constant trade costs during all periods of the firm’s life cycle substantial fixed trade costs may cause radical changes in firms export orientation during its life-cycle. In the following we analyze this more carefully.
Exports only in the mature period
For some values of the parameters market equilibrium exist, where the life cycle make up three periods, but the firms are only export active in their mature period (case IV in Figure 2). As in the analysis above we first describe the structure of the market equilibrium and then the preconditions for this specific equilibrium.

The market is provided by domestically produced varieties from all three periods, but only foreign varieties from the mature period and hence, the consumer’s preferences are described by the utility function:

\[
U = \sum_{i=1}^{n} \varphi^2 c_{i,1}^{\theta} + \sum_{i=1}^{n} \varphi c_{i,2}^{\theta} + \sum_{i=1}^{n} \varphi c_{i,2}^{s} + \sum_{i=1}^{n} c_{i,3}^{\theta} \quad \varphi > 1; \quad 0 < \theta < 1
\]  

(37)

Utility is maximized subject to budget constraint:

\[
\sum_{i=1}^{n} p_{i,1} c_{i,1} + \sum_{i=1}^{n} p_{i,2} c_{i,2} + \sum_{i=1}^{n} p_{i,2}^{s} + \sum_{i=1}^{n} p_{i,3} c_{i,3} = 1
\]

(38)

The cost functions are similar to the previous case although trade costs only are relevant in the mature period. Due to symmetry for firms of same age we only indicate age by subscript.

Using (20), (21), (22) and (23), and keeping in mind that the firm is only export active in the mature period, output in the three periods is given by:

\[
x_1 = x_1^d = Lc_1
\]

\[
x_2 = x_2^d + x_2^f = L(c_2 + c_2^{s} / g) = Lc_2(1 + g^{1-\theta}) = \left(\frac{\mu}{\varphi}\right)^{1-\theta} (1 + g^{1-\theta}) x_1
\]

(39)

\[
x_3 = x_3^d = Lc_3 = \left(\frac{\mu}{\varphi}\right)^{1-\theta} x_1
\]

Using the zero value condition (7) for determination of output in the infant period, \(x_I\), i.e. inserting (7), (8), (20) and (39) into (6) (generalized with fixed trade costs in the mature period only) gives:
\[ V = (p_1 - \beta) x_1 + (p_2 - \beta / \mu) x_2 + (p_3 - \beta / \mu) x_3 - (S + 3f + m) \]
\[ = \beta \frac{1 - \theta}{\theta} \left( x_1 + \frac{1}{\mu} x_2 + \frac{1}{\mu} x_3 \right) - (S + 3f + m) \]
\[ = \beta \frac{1 - \theta}{\theta} x_1 \left( 1 + \left( \frac{\mu^\theta}{\varphi^{1-\theta}} \right)^{\frac{1}{1-\theta}} \left( 1 + g^{\frac{\varphi}{1-\theta}} \right) + \left( \frac{\mu^\theta}{\varphi^{2(1-\theta)}} \right)^{\frac{1}{1-\theta}} \right) - (S + 3f + m) = 0 \]

which gives
\[ x_1 = \frac{\theta (S + 3f + m)}{\beta H^{iv}} \]

where \( H^{iv} = \left( 1 + \left( \frac{\mu^\theta}{\varphi^{1-\theta}} \right)^{\frac{1}{1-\theta}} \left( 1 + g^{\frac{\varphi}{1-\theta}} \right) + \left( \frac{\mu^\theta}{\varphi^{2(1-\theta)}} \right)^{\frac{1}{1-\theta}} \) \]

Inserting the results for prices and output given by (21), (39) and (40) into the expressions for operating profits (7) gives the following results:

\[ \pi_1 = \frac{(S + 3f + m)}{H^{iv}} \]
\[ \pi_2 = \left( \frac{\mu^\theta}{\varphi} \right)^{\frac{1}{1-\theta}} \left( 1 + g^{\frac{\varphi}{1-\theta}} \right) \frac{(S + 3f + m)}{H^{iv}} = \left( \frac{\mu^\theta}{\varphi} \right)^{\frac{1}{1-\theta}} \left( 1 + g^{\frac{\varphi}{1-\theta}} \right) \pi_1 \]
\[ \pi_3 = \left( \frac{\mu^\theta}{\varphi^2} \right)^{\frac{1}{1-\theta}} \frac{(S + 3f + m)}{H^{iv}} = \left( \frac{\mu^\theta}{\varphi^2} \right)^{\frac{1}{1-\theta}} \pi_1 \]

Finally, the number of firms in steady state equilibrium follows from inserting output, (39) and (40), in the full employment condition (14) (expanded by fixed trade costs for the mature period only) for \( n_1 = n_2 = n_3 = \hat{n} \). This gives:

\[ \hat{n} = \frac{L(1-\theta)}{(S + 3f + m)} \]

Given that firms only export in the mature period and only incur one set of fixed trade costs, the number of firms are higher relative to the case with 'born globals' (compare (42) with (29)).

The described equilibrium imposes restrictions on the values of the parameters. To ensure production in a third, but not a fourth period, the following conditions should hold:

\[ \bar{\pi}_4 = \bar{\pi}_4^d < f < \pi_3 = \pi_3^d \]
Furthermore, to ensure export in the mature period, but not in the infant and aged period, the conditions are:

\[ \text{Max. } \pi_1^f, \pi_3^f < m \]  
and  
\[ \pi_2^f > m \]  

To specify the constraints of the parameters explicitly, we use (41), and (A6) from Appendix in (43). This gives:

\[
\frac{1}{\phi^{1-\theta} M^{IV}} \left( \frac{S}{f} + 3 + \frac{m}{f} \right) < 1 \leq \frac{1}{\phi^{1-\theta} M^{IV}} \left( \frac{S}{f} + 3 + \frac{m}{f} \right)
\]

where

\[ M^{IV} = 1 + \frac{-\delta}{\phi^{1-\theta}} + \frac{2}{\phi^{1-\theta}} \left( 1 + g^{1-\theta} \right) \]

The constraint on the parameters for producing in a three period life cycle is very similar to the previous case; see (17) and (34). The constraint illustrates specifically that low variable trade costs \( g \) close to 1 reduces operating profit due to lower product runs, and in case that the right-hand side of (46) is not fulfilled the producer will exit after two periods only.

Inserting (A7) and (A8) from the Appendix in (44) gives the constraint for non-export in the first and third period:

\[
\text{Max} \left\{ \frac{\frac{S}{m} + 3 f + 1}{m} g^{\theta}, \frac{\frac{S}{m} + 3 f + 1}{m} g^{\theta}, \frac{\left( \frac{\mu^\theta}{\phi^3} \right)^{1-\theta}}{M^{IV}} \right\} < 1
\]

Comparing \( \tilde{\pi}_1^f \) with \( \tilde{\pi}_3^f \) shows that \( \tilde{\pi}_1^f \leq \tilde{\pi}_3^f \) for \( \mu \geq \phi^2 \), i.e. the binding constraint is non-export in the first period, if learning is low and quality growth large, while the binding constraint is non-export in the third period for the opposite situation. High variable and fixed trade costs lower operating profits for exporting, and high trade costs thus ensures (47).

For analyzing the constraint for exporting in the second period, we insert (A9) from the Appendix into (45). This gives:
This constraint may be fulfilled for relatively low fixed and variable trade costs. Hence comparing (47) and (48), fixed and variable trade costs should therefore neither be very low nor very small to ensure that export activity only takes place in the second period.

4. Export orientation, intra-industry trade and welfare

In an international trade perspective with variable and fixed trade costs the model opens up for different export patterns, all depending on the value of the parameters of the model: The rate of product innovation: the degree of learning; variable and fixed trade costs; and sunk (entry costs). The export pattern may be ‘born globals’, where firms export over their whole life; or it may be a later expansion into export markets like in the Uppsala model, or it may be export in just some intermediate periods of firms life cycles. All these different export patterns have been observed in the business literature through case studies and /or in empirical trade studies.

The presented model has intra-industry trade (IIT), and since it operates with both horizontal and vertical product differentiation, intra-industry trade may involve both IIT in horizontally and vertically differentiated products. In case of ‘born global’s’, we have IIT at all levels of product qualities, so measured IIT, using e.g. the traditional Grubel-Lloyd index (Grubel and Lloyd, 1975), will involve a mix of IIT in horizontally and vertically differentiated goods. If we on the other hand have trade in mature varieties only, the IIT is purely in horizontally differentiated products. Which type of IIT that will arise is thus sensitive to the parameters in the model.

Concerning the welfare implications of market opening we find the supply of product varieties increases like previous models, e.g. Krugman (1979b, 1980, but which generations of foreign varieties consumers get access to depends on the parameters of the model: In a ‘born global’ framework consumers get access to all foreign varieties, but in other situations, they only get access to a selection of foreign varieties and not necessarily the newest. Trade opening reduces the number of domestic varieties and if we only have trade in mature varieties the consumers may face a reduced number of the most advanced varieties although the total number of varieties available for the consumers has increased.

Trade liberalization through a small reduction of the variable and/or fixed export costs impacts the market equilibrium incrementally, if the feasibility conditions for the life cycle and export configuration still are fulfilled. If in contrast, the decrease of the trade costs violates feasibility conditions, the market equilibrium changes radically as the firms change their strategy on export behavior and/or length of their life cycle. When such profound changes of market equilibrium take place, the number of varieties available for consumers jumps and so does the age structure and social welfare.
5. Conclusion

The above presented model shares important characteristics with previous models of the ‘new-new’ trade theory, but differs also in other important respects. Firm heterogeneity is the common feature, but two alternative stories are told in explaining the reasons for such differences. In previous models of the ‘new-new’ trade theory, e.g. the model of Melitz (2003), the individual firm’s productivity reflects a stochastic outcome of a ‘productivity lottery’ revealed, when the firm enters the market and later on, when the firm is under risk of negative productivity shocks, which ultimately may drive the firm out of business. The lucky firms born with high productivity are large; reap substantial profits and self select into the export marked.

In the model developed above firms are characterized by being symmetric in a life cycle perspective. Firms start up with the newest product technology, but relatively high variable production costs, because of lack of experience in producing this specific variety. After an infant period the producer becomes more experienced and hence more cost efficient. However, an ongoing process of entry of competitors providing the market with more modern varieties influences demand for a producer’s variety negatively, and lastly the producer therefore exits due to obsolescence. These mechanisms give the distinctive nature of the model. At a given point in time observed differences between firms are ascribed to the different ages of the firms. Specifically, the export behavior may differ between generations of firms as each generation of firms are screened by the fixed trade costs for their incentive to export.

The Melitz (2003) model illustrates that exposure to trade makes the more productive firms to enter export markets, while the least productivity firms exit the industry; so on average the productivity of the economy will increase. To put it differently, the more fierce competition in an open economy root out weak (low productivity) firms freeing the resources for more cost efficient firms. This illustrates an important source of welfare from trade liberalization. The same effect may also materialize in our model. The weak firms are here the old firms, which for a given age are equally efficient. Opening for trade may violate the feasibility conditions for the closed economy by rendering it unprofitable to produce in the full life cycle of the closed economy, and in such cases, where trade shortens the length of the life cycle, the efficiency of the population of firms and hence, welfare increases.

The concept firm is used synonymous with variety in all models of the new-new trade theory. When age is the important factor for productivity and market position of the variety, this raises serious empirical challenges for testing the theory. Firm data are usually based on firms as legal units, which from time to time transform their activities from one variety to another more modern variety. This renders the age of the firm into irrelevance in analysis of the life cycle of a variety. For empirical testing more detailed data are therefore needed.

The lottery approach and the life cycle approach thus broadly paint a similar picture of structural intra-industry differences. Larger differences exist, when it comes to policy
implications of the two approaches. A social ruler aiming to maximize welfare may manipulate with the entry process through (positive or negative) subsidies of the sunk costs. In models based on the lottery approach this means that more or less lottery tickets are bought, but the outcome, i.e. the group of entrants, is not on average more efficient than the existing population of firms. In contrast, stimulating entry in models based on the life cycle approach unambiguously leads to larger generations of more modern firms with varieties more preferred by the consumers.
References:


Appendix: Actual and potential operating profits relevant for the producer’s exit decisions

1. **Closed economy**
The producer considers producing the variety in a fourth period on a market, where all competitors produce the variety in three periods. The potential profit for producing in a fourth period $\tilde{\pi}_4$ follows from the optimization conditions for the consumer and producer respectively, (9) and (10), i.e.:

$$\tilde{\pi}_4 = \left( \tilde{p}_4 - \frac{\beta}{\mu} \right) \tilde{x}_4$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \tilde{x}_4 x_1$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \tilde{c}_4 x_1$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \left( \frac{\mu}{\varphi^3} \right)^{1-\theta} x_1$$

$$= \frac{(S + 3 f)}{\varphi^{1-\theta} M}$$

where

$$M = \left( 1 + \mu^{1-\theta} \varphi^{1-\theta} + \varphi^{1-\theta} \right)$$

2. **Open economy, case I**
Actual and potential operating profit are derived by using the consumer and producer optimization (20) and (21) respectively. This gives:

$$\tilde{\pi}_4 = \left( \tilde{p}_4 - \frac{\beta}{\mu} \right) \tilde{x}_4$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \tilde{x}_4 x_1$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \tilde{c}_4 x_1$$

$$= \frac{\beta (1-\theta)}{\mu \theta} \left( \frac{\mu}{\varphi^3} \right)^{1-\theta} x_1$$

$$= \frac{(S + 3 (f + m))}{\varphi^{1-\theta} M}$$
\[
\pi_4^d = \left( \tilde{p}_4 - \frac{\beta}{\mu} \right) \bar{x}_4^d \\
= \frac{\beta(1-\theta)}{\mu \theta} \bar{x}_4^d x_i \\
= \frac{\beta(1-\theta)}{\mu \theta} \frac{\bar{c}_4}{\theta} x_i \\
= \frac{\beta(1-\theta)}{\mu \theta} \left( \frac{\mu}{\varphi^3} \right)^{\frac{1}{1-\theta}} \frac{1_\theta}{(1+g^{1-\theta})} x_i \\
= \frac{(S+3(f+m))}{\varphi^{1-\theta} M^t (1+g^{1-\theta})} \\
\pi_i^f = (p_1 - \beta) x_i^f \\
= \frac{\beta(1-\theta)}{\theta} \frac{c_1}{\varphi} x_i \\
= \frac{\beta(1-\theta)}{\theta} \frac{c_1}{\varphi} \frac{1}{g (c_1 + \frac{c_1}{g})} x_i \\
= \frac{\beta(1-\theta)}{\theta} \frac{g^{\varphi}}{(c_1 + \frac{c_1}{g})} x_i \\
= \frac{\beta(1-\theta)}{\theta} \frac{g^{\varphi}}{(1+g^{1-\theta})} x_i \\
= \frac{(S+3(f+m))}{\left( \frac{\mu^\theta}{\varphi^3} \right)^{\frac{1}{1-\theta}} M^t (1+g^{1-\theta})} \\
\text{(A3)}
\]

\[
\text{(A4)}
\]
\[ \pi_4^f = \left( p_4 - \frac{\beta}{\mu} \right) x_4^f \]

\[ = \frac{\beta(1-\theta)}{\mu \theta} x_4^f x_3 x_1 \]

\[ = \frac{\beta(1-\theta)}{\mu \theta} c_3 \frac{g}{c_3 + \frac{c_3}{g}} c_1 x_1 \]

\[ = \frac{\beta(1-\theta)}{\theta} (1 + g^{1-\theta}) \left( \frac{\mu^{\theta}}{\varphi^3} \right)^{\frac{1}{\mu(1-\theta)}} x_1 \]

\[ = \frac{(S + 3(f + m))}{M^f (1 + g^{1-\theta})} \]

where \( M^f = M \)

(A5)

3. **Open economy, case IV**

Actual and potential operating profit are derived by the consumer and producer optimization conditions (20) and (21).

\[ \bar{\pi}_4^d = \left( \bar{p}_4 - \frac{\beta}{\mu} \right) \bar{x}_4^d \]

\[ = \beta(1-\theta) \frac{x_4^d}{\mu \theta} x_1 \]

\[ = \beta(1-\theta) \frac{\bar{c}_4}{\mu \theta} c_1 x_1 \]

\[ = \beta(1-\theta) \left( \frac{\mu}{\varphi^3} \right)^{\frac{1}{\mu}} x_1 \]

\[ = \frac{(S + 3f + m)}{\varphi^{1-\theta} M^{IV}} \]

where \( M^{IV} = \left( 1 + \mu^{\frac{1-\theta}{\varphi^{1-\theta}}} + \varphi^{\frac{1}{\mu(1-\theta)}} (1 + g^{1-\theta}) \right) \)
\[
\tilde{\pi}_1^f = (p_1 - \beta) \tilde{x}_1^f \\
= \frac{\beta(1-\theta)}{\theta} \tilde{x}_1^f x_1 \\
= \frac{\beta(1-\theta)}{\theta} \left( \frac{c_1}{g \theta} \right) x_1 \\
= \frac{\beta(1-\theta)}{\mu \theta} \left( \frac{\theta}{g^{1-\theta}} \right) x_1 \\
= \frac{(S + 3f + m)}{\left( \frac{\mu^\theta}{\phi^\theta} \right) M^{iv}} \\

\pi_3^f = \left( \frac{p_3 - \beta}{\mu} \right) \tilde{x}_3^f \\
= \frac{\beta(1-\theta)}{\mu \theta} \tilde{x}_3^f \frac{x_3}{x_1} x_1 \\
= \frac{\beta(1-\theta)}{\mu \theta} \left( \frac{c_3}{g \theta} \right) \left( \frac{c_3}{c_1} \right) x_1 \\
= \frac{\beta(1-\theta)}{\theta} \frac{\theta}{g^{1-\theta}} \left( \frac{\mu^\theta}{\phi^\theta} \right)^{\frac{1}{1-\theta}} x_1 \\
= \frac{(S + 3f + m)}{M^{iv}} \frac{\theta}{g^{1-\theta}}
\]
\[ \pi_2^f = \left( p_2 - \frac{\beta}{\mu} \right) \tilde{x}_2^f \]

\[ = \frac{\beta(1-\theta)}{\mu \theta} \frac{x_2}{x_1} \frac{c_2}{c_1} \frac{c_2 + c_1 g}{c_1 + c_2 g} x_1 \]

\[ = \frac{\beta(1-\theta)}{\theta} g^{\frac{\phi}{\theta}} \left( \frac{\mu^\theta}{\phi} \right)^{\frac{1}{1-\theta}} x_1 \]

\[ = \frac{(S + 3f + m)}{\phi^{\frac{1}{1-\theta}} M^{\frac{1}{1-\theta}}} \]

It follows straight forwardly that operating profit by exporting decreases with fixed trade costs, but less than the increase of fixed trade costs. Lowering variable trade costs increases operating profit by exporting, i.e. operating profit by exporting varies positively with \( g \). This follows from (A7) to (A9) isolating the terms with \( g \), i.e.:

\[ g^{\frac{\phi}{\theta}} = \left( \frac{\mu^{\theta}}{\phi} \right)^{\frac{1}{1-\theta}} \left( \mu^{\theta} \phi^{\frac{1}{1-\theta}} + \phi^{\frac{1}{1-\theta}} (1 + g^{\frac{1}{1-\theta}}) + 1 \right) \]

\[ = \frac{1}{g^{\frac{1}{1-\theta}} \mu^{\theta} \phi^{\frac{1}{1-\theta}} + \phi^{\frac{1}{1-\theta}} (1 + g^{\frac{1}{1-\theta}}) + g^{\frac{1}{1-\theta}}} \]

\[ = \frac{1}{\phi^{\frac{1}{1-\theta}} + g^{\frac{1}{1-\theta}} (1 + \mu^{\theta} \phi^{\frac{1}{1-\theta}})} \]

Lowering trade costs, i.e. increasing \( g \) towards 1 increases the right hand side of (A10).