Forward looking versus backward looking behavior in inflation dynamics: a new test

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Abstract

This paper contributes to the debate on inflation dynamics by proposing a novel method to test the relative importance of forward looking and backward-looking behavior. By using Galí and Gertler’s (1999) New Keynesian hybrid model, I derive a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation. Three key advantages of this relationship for estimation purposes are: i) it does not include measures of real marginal costs or output gap, which are very sensitive to different economic assumptions; ii) it does not require assumptions about how expectations are formed and iii) it directly identifies the structural parameters that describe the nature of the price setting. I estimate the proposed equation with Austrian and Spanish data. I find that inflation is far from being exclusively a forward looking phenomenon in these countries.

Keywords: Inflation, Hybrid Phillips curve, Price Rigidities
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1 Introduction

Studying inflation dynamics is crucial for monetary policy analysis; in particular, exploring how plausible it is that inflation is mainly determined by forward-looking behavior. This is especially important for understanding the different sources of inflation persistence, the costs of disinflation processes and the optimal monetary policy. The New Keynesian Phillips Curve (NKPC), which describes the aggregate supply block of the NK model, predicts that inflation is determined exclusively by forward-looking behavior of firms. However, several studies have found evidence of backward-looking behavior. The evidence about its quantitative importance is mixed. Galí and Gertler (1999), Galí et al. (2001) and Galí et al. (2005) find a predominant role for forward-looking behavior. In contrast, Fuhrer and Moore (1995) and Rudd and Whelan (2005) find the backward-looking component to be more important.

In this study, I propose a novel methodology to evaluate the quantitative importance of backward-looking behavior in the form of rule of thumb price setting. By using Galí and Gertler’s (1999) hybrid model, I derive a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation. There are five important features of this relation that make it more attractive than the hybrid NKPC in order to identify rule of thumb behavior. First, the parameters that appear in the equation I derive are only those related with the nature of price-setting: the one that measures the degree of price stickiness and the one that measures the fraction of backward-looking firms. Both of them are identified directly from estimates of that equation. The latter means that the estimates of these parameters are not affected by how real rigidities are modeled and calibrated, as it is the case when they are identified by estimating the hybrid NKPC. Second, the variables that appear in the relationship I propose are predetermined in period $t$. This implies that I do not need an assumption on how expectations about the future are formed. Instead, the estimation of the hybrid NKPC requires to take a position on this issue, given that expected inflation appears in that relation. Third, estimating the equation I propose

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1 Credible disinflations are relatively costless when inflation is determined by the standard NKPC, but are quite costly when backward looking behavior in price setting is quantitatively important. See Ball (1994) and Roberts (1998) for a discussion of this topic.


is not subject to the criticism of Rudd and Whelan (2005), who claim that fitting the hybrid NKPC may be biased in favor of finding a significant role for forward-looking behavior. Fourth, the definition of the variables I use do not change for alternative assumptions on the form of the production function and the way how real rigidities are introduced. Instead, the construction of measures of real marginal costs in the hybrid NKPC is very sensitive to the previous assumptions. Fourth, the dynamic structural relationship that I propose is still very tractable for estimation when departing from the zero inflation steady state assumption made by Galí and Gertler. In contrast, the hybrid NKPC under positive trend inflation contains an infinite sum of expectations about inflation and output.

I estimate the derived structural relationship with Austrian and Spanish monthly data covering the periods 1996-2005 and 1993-2001 respectively. I use the Generalized Method of Moments (GMM) and two different estimators: the continuously updated GMM estimator (CUE) and the usual two step GMM estimator (2S-GMM). Several interesting results stand out. First, the structural relationship proposed in this paper fits the data well. Second, the backward-looking price setting is statistically significant and quantitatively important. Third, the degree of price stickiness implied by the estimates is consistent with the average price duration estimated using only disaggregated data. Fourth, the point estimates imply that, in the hybrid NKPC, the weight on inflation lagged one quarter is at least 45 percent. Fifth, the parameter estimates are not significantly affected when estimating the proposed relationship under positive inflation steady state. Sixth, the parameter estimates are very similar across the two different GMM estimators.

The paper is divided into 6 sections. Section 2 presents the model and the basic assumptions. In Section 3, I derive analytically the dynamic relationship between the cross sectional variance of individual price changes and aggregate inflation. In Section 4, I expose the methodology and the econometric specification used in order to estimate the degree of backward-lookingness. The estimates and related comments are also presented in this

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4Galí and Lopez Salido (2001) show that the definition of real marginal cost can change with the assumptions made on the production function and on the degree of openness of the economy. Additionally, Thomas (2008) finds that the measure of real marginal cost differs from the standard one when real rigidities are introduced considering matching frictions in the labor market.

5Ascari (2004) makes this point for the standard NKPC. It is straightforward to extend his analysis to the hybrid NKPC derived from Gali and Gertler (1999) model.

6Notice that for plausible values of the degree of price stickiness, the maximum weight on inflation lagged one quarter is 52 percent in Galí and Gertler’s hybrid NKPC.
section. In Section 5 I present a detailed comparison of the methodology that I propose in this paper and the one developed by Galí and Gertler (1999), who estimate the hybrid NKPC. In Section 6, I derive the structural relationship proposed in this paper when Galí and Gertler’s model is loglinearized around a positive inflation steady state. Estimates and related comments for this case are also presented in this section. Conclusions are given in Section 7.

2 The New Keynesian Hybrid Model

In this section I briefly describe the hybrid model developed by Galí and Gertler (1999). This model is used in the next section in order to derive the dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation.

2.1 Households

The household purchases differentiated goods and combines them into composite goods using a Dixit-Stiglitz aggregator:

\[ C_t = \left( \int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} \, di \right)^{\epsilon/(\epsilon-1)} \]  

(1)

where \( C_t(i) \) is the differentiated good of type \( i \) and \( \epsilon > 1 \) is the constant elasticity of substitution among goods. The households maximize the index (1) given the total cost of all differentiated goods and their nominal prices \( P_t(i) \). Then, the demand for each good is given by:

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]  

(2)

where \( P_t \) is the aggregate price level and is defined as follows:

\[ P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} \, di \right)^{1/(1-\epsilon)} \]  

(3)
2.2 Firms

In the model, it is assumed a continuum of firms indexed by $i \in [0,1]$. Each firm is a monopolistic competitor and produces a differentiated good $Y_t(i)$ that sells at price $P_t(i)$. Firms set prices as in the sticky price model of Calvo (1983). In this model, during each period, a fraction of firms $(1 - \theta)$ are allowed randomly to change the prices; whereas the other fraction $\theta$ do not change. From those firms resetting prices, only a fraction $(1 - \omega)$ resets price optimally, as in the standard Calvo model. The remaining fraction $\omega$ chooses the (log) price $p_t^h$ according to the simple rule of thumb:

$$p_t^h = p_{t-1}^* + \pi_{t-1}$$

where $p_{t-1}^*$ is the (log) of the average reset price in $t-1$ (across both backward and forward-looking firms) and $\pi_{t-1}$ is inflation in period $t-1$. Galí and Gertler (1999) point out two appealing features of this rule. First, there are no persistent deviations between the rule and the optimal behavior as long as inflation is stationary. Second, the rule implicitly incorporates information about the future, given that $p_{t-1}^*$ is partly determined by forward looking firms.

2.3 Aggregate Price Level Dynamics

After using the law of large numbers and log-linearizing the aggregate price level around a zero inflation steady state, the following expression for the (log) aggregate price level $p_t$ is obtained:

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^*$$

The (log) index for newly set prices is given by the following expression:

$$p_t^* = \omega p_t^h + (1 - \omega)p_t^f$$

where $p_t^f$ is the optimal price chosen by forward looking firms at period $t$. Notice that all firms that reoptimize in period $t$ choose the same value $p_t^f$, given that there are no firm specific state variables.

2.4 The Hybrid NKPC

Although the hybrid NKPC is not necessary to derive the relationship between the cross sectional variance of individual price changes and aggregate inflation, I present it in this section for two reasons. First, it will be useful to discuss the implications of my estimates for $\theta$ and $\omega$ on the dynamics of
inflation. Second, it will be helpful in explaining the main differences between my estimation strategy of $\theta$ and $\omega$ and the one performed by previous studies in which the hybrid NKPC is estimated.\footnote{Gali and Gertler (1999), Galí et al. (2001), Galí and Lopez Salido (2001), and Benigno and Lopez Salido (2006) use the hybrid NKPC derived with rule of thumb behavior to estimate the fraction of backward looking firms.}

Given that the analysis performed in the empirical section is with monthly data, for simplicity I will focus on the case in which $\beta = 1$.\footnote{A quarterly discount factor of 0.99 is equivalent to a monthly discount factor of 0.997, which is very close to 1.} Galí et al. (2001) show that the hybrid NKPC in this case is given by the following expression:

$$\pi_t = \lambda_n \lambda_r \bar{m}\bar{e}_t + \gamma_b \pi_{t-1} + \gamma_f E_t \{\pi_{t+1}\} + \varepsilon_t$$  \hspace{1cm} (7)

with

$$\lambda_n = \frac{(1-\omega)(1-\theta)^2}{\theta + \omega}, \quad \lambda_r = \frac{1-\alpha}{1+\alpha(\epsilon-1)}, \quad \gamma_b = \frac{\omega}{\theta + \omega}, \quad \gamma_f = \frac{\theta}{\theta + \omega}$$

where $\alpha$ measures the curvature of the production function of the firm, which is given by $Y_t(i) = A_t N_t(i)^{1-\alpha}$, $\bar{m}\bar{e}_t$ is average real marginal cost (in percent deviations from its steady state level) and $\varepsilon_t$ is an error term that may arise from either measurement errors or shocks to the desired markup.

Note that the slope coefficient on real marginal cost depends on two different groups of parameters. The first group, given by $\theta$ and $\omega$, are related to the nature of the price-setting. Their impact on real marginal cost is given by $\lambda_n$, which measures the degree of nominal rigidities. The second group, composed by $\alpha$ and $\epsilon$, are associated with real factors of the economy: the structure of the production function and of demand. The effect of these parameters on the real marginal cost is determined by $\lambda_r$, which quantifies the degree of "real rigidities". In this case, these rigidities arise from assuming decreasing returns to scale in labor ($\alpha < 1$).\footnote{There are alternative ways to generate real rigidities. See Woodford (2003), Altig et al. (2004) or Thomas (2008).}

Finally, the coefficients $\gamma_b$ and $\gamma_f$ capture the influence of backward and forward-looking behavior on inflation dynamics. Notice that these coefficients depend only on $\theta$ and $\omega$.\footnote{When the discount factor is lower than one, this parameter also affects $\gamma_b$ and $\gamma_f$.} This implies that the expressions for $\gamma_b$ and $\gamma_f$ are not affected by the assumptions made on the production function and on demand.
3 Relationship Between the Cross Sectional Variance of Individual Price Changes and Aggregate Inflation

In this section, I show that the previous model implies a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation.

Proposition: In the hybrid New Keynesian model, up to a second order approximation around a zero inflation steady state, the cross sectional variance of individual price changes evolves over time according to:

\[
\text{Var}_i \{\pi_t(i)\} = \theta \text{Var}_i \{\pi_{t-1}(i)\} + f(\pi_t, \pi_{t-1}) + (1 - 2\theta)f(\pi_{t-1}, \pi_{t-2})
\] (8)

where \(f(\pi_t, \pi_{t-1})\) is given by:

\[
f(\pi_t, \pi_{t-1}) = \frac{\theta}{1 - \theta} \pi_t^2 + \frac{\omega}{(1 - \theta)(1 - \omega)} (\pi_t - \pi_{t-1})^2
\] (9)

Proof: First, notice that the cross sectional variance of individual prices evolves according to:

\[
\text{Var}_i \{p_t(i)\} = \theta \text{Var}_i \{p_{t-1}(i)\} + f(\pi_t, \pi_{t-1})
\] (10)

Moreover, we know that the cross sectional variance of individual price changes is given by:

\[
\text{Var}_i \{\pi_t(i)\} = \text{Var}_i \{p_t(i)\} - 2\text{Cov}_i \{p_t(i), p_{t-1}(i)\} + \text{Var}_i \{p_{t-1}(i)\}
\] (11)

Using the fact that in the hybrid model, \(\text{Cov}_i \{p_t(i), p_{t-1}(i)\} = \theta \text{Var}_i \{p_{t-1}(i)\}\), the previous expression can be expressed as:

\[
\text{Var}_i \{p_{t-1}(i)\} = \frac{\text{Var}_i \{\pi_t(i)\} - \text{Var}_i \{p_t(i)\}}{1 - 2\theta}
\] (12)

By plugging (12) into (10), the cross sectional variance of individual prices evolves according to:

\[
\text{Var}_i \{p_t(i)\} = \frac{\theta}{1 - \theta} \text{Var}_i \{\pi_t(i)\} + \frac{1 - 2\theta}{1 - \theta} f(\pi_t, \pi_{t-1})
\] (13)

Finally, by using (13) evaluated in periods \(t\) and \(t - 1\); and plugging them into (10), we get (8).

\[11\text{See Steinsson}(2003)\text{ for a formal proof of (10).} \]
4 Empirical Evidence

This part contains two subsections. In the first one, I describe the econometric specification used to estimate equation (8) by applying GMM. In the second one, I present the data and estimates of the model.

4.1 Econometric Specification

In order to perform the GMM technique, an orthogonality condition should be inferred from the model developed in the previous section. In this particular case, the orthogonality condition comes from equation (8) and arises from allowing measurement error in the cross sectional variance of individual price changes. In this case, equation (8) can be written as:

\[ \text{Var}_i^O \{ \pi_t(i) \} = \theta \text{Var}_i^O \{ \pi_{t-1}(i) \} + f(\pi_t, \pi_{t-1}) + (1-2\theta)f(\pi_{t-1}, \pi_{t-2}) + \varepsilon_t - \theta \varepsilon_{t-1} \]

where \( \text{Var}_i^O \{ \pi_t(i) \} \) is the cross sectional variance measured in period \( t \) and \( \varepsilon_t \) is the measurement error in period \( t \). I assume that the measurement error at period \( t \) is not correlated with earlier information. Therefore, the following orthogonality condition can be established:

\[ E_t [\text{Var}_t \{ \pi_t(i) \} - \theta \text{Var}_t \{ \pi_{t-1}(i) \} - f(\pi_t, \pi_{t-1}) - (1-2\theta)f(\pi_{t-1}, \pi_{t-2})] z_{t-2} = 0 \]

where \( f(\pi_t, \pi_{t-1}) \) is given by (9) and \( z_{t-2} \) denotes a vector of variables dated at period \( t-2 \) and earlier.

4.2 Data and Estimates

The data that I use is Austrian and Spanish monthly data running from February 1996 through December 2005 and from February 1993 through December 2001 respectively. In order to measure the evolution over time of the cross sectional variance of individual price changes, a large Austrian and Spanish panel CPI databases have been used. Over the whole sample period, the datasets include product categories that cover around 90 and 70 percent of the expenditures on the Austrian and Spanish CPI basket respectively.\(^{12}\)

Inflation is measured by the percentage change in monthly CPI.

Table 1 presents the GMM estimation of the parameters \( \theta \) and \( \omega \), as well as the average price duration \( (D) \) implied by \( \theta \) and the coefficients \( \gamma_b \)

\(^{12}\)More details on the Austrian and Spanish CPI databases can be found in Baumgartner et.al (2005) and Alvarez and Hernando (2004) respectively.
and $\gamma_f$ that help to measure the relative importance of backward versus forward looking behavior. The last column of the table presents the p-value for the Hansen’s J statistic of overidentifying restrictions. The results are presented for two different GMM estimators: the CUE and the usual 2S-GMM estimator.\(^\text{13}\) Standard errors (with a Newey West correction) for all the estimates are reported in brackets.

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
\textbf{} & \textbf{$\theta$} & \textbf{$\omega$} & \textbf{$D$} & \textbf{$\gamma_b$} & \textbf{$\gamma_f$} & \textbf{J test} \\
\hline
\textbf{AUSTRIA} & & & & & & \\
CUE & 0.922 & 0.946 & 12.887 & 0.506 & 0.494 & 0.572 \\
 & (0.024) & (0.021) & (3.978) & (0.012) & (0.012) & \\
2S – GMM & 0.934 & 0.865 & 15.114 & 0.481 & 0.519 & 0.703 \\
 & (0.011) & (0.050) & (2.592) & (0.017) & (0.017) & \\
\hline
\textbf{SPAIN} & & & & & & \\
CUE & 0.925 & 0.794 & 13.276 & 0.462 & 0.538 & 0.221 \\
 & (0.014) & (0.071) & (2.551) & (0.025) & (0.025) & \\
2S – GMM & 0.938 & 0.759 & 16.121 & 0.447 & 0.553 & 0.185 \\
 & (0.014) & (0.085) & (3.720) & (0.031) & (0.031) & \\
\hline
\end{tabular}
\caption{Estimates of the Structural Parameters}
\end{table}

\textit{Note: Standard errors shown in brackets.}

Several interesting results arise from these estimations, which are robust to the choice of the GMM estimator. First, the point estimates of the fraction of rule of thumb firms are high and statistically different from zero. Second, the point estimates of the average price duration are very close to the ones obtained using the disaggregated data.\(^\text{14}\) Baumgartner et al (2005) find that

\(^{13}\)The instrument set includes aggregate inflation squared and the cross sectional variance of individual price changes from $t - 2$ to $t - 5$.

\(^{14}\)Notice that the average price duration is measured by computing $\frac{1}{1-\theta}$.
the Austrian average price duration for the estimation period is between 10 and 14 months; whereas Alvarez and Hernando (2004) find that the Spanish average price duration for the estimation period is between 11 and 16 months. Third, the estimates of $\theta$ and $\omega$ imply that backward-looking behavior is almost as important as the forward looking one in order to explain inflation dynamics. Fourth, the validity of all the regressions is confirmed by the p-value for the Hansen’s J statistic of overidentifying restrictions with a significance level of 5 percent. Fifth, the CUE and the 2S-GMM estimates of $\theta$ and $\omega$ are very similar. This is evidence that the latter are not plagued by a weak instruments problem.\footnote{See Stock et al (2002).}

5 Comparison with Galí and Gertler (1999)

In this section, I compare my methodology to the one proposed by Galí and Gertler (1999).\footnote{Their methodology has been used by different studies like the ones by Gali et al (2001), Gali and Lopez Salido (2001), Benigno and Lopez Salido (2006), among others.} Basically, these authors propose to estimate the structural parameters $\theta$ and $\omega$ by fitting the hybrid NKPC using the 2S-GMM and the following orthogonality condition:

$$E_t \{ [\pi_t - \lambda_n \lambda_r \hat{m}_{t} - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1}] \Delta t_i \} = 0$$

Condition (16) follows from the fact that the expectational error should be unforecastable with information dated in period $t$ and earlier under rational expectations in inflation.

There are six important differences between the procedure I present and the one developed by Galí and Gertler. First, the way how the orthogonality condition is derived. Notice that Galí and Gertler assume rational expectations in order to infer (16). This is the reason why they can use $\pi_{t+1}$ in their estimation. However, if rational expectations does not hold, then (16) would be incorrect.\footnote{Adam and Padula (2003) provide some evidence that supports that inflation expectations are not rational. They propose to use survey expectations to measure $E_t \pi_{t+1}$ instead of $\pi_{t+1}$ and to formalize the orthogonality condition by allowing a measurement error.} Instead, the condition that is used in this study does not contain expectations about the future. For this reason, I assume that the cross sectional variance of individual price changes contains a measurement error that is uncorrelated with past information. The latter only means that people elaborating statistics are smart enough such that they do not make


\[16\] Their methodology has been used by different studies like the ones by Gali et al (2001), Gali and Lopez Salido (2001), Benigno and Lopez Salido (2006), among others.

\[17\] Adam and Padula (2003) provide some evidence that supports that inflation expectations are not rational. They propose to use survey expectations to measure $E_t \pi_{t+1}$ instead of $\pi_{t+1}$ and to formalize the orthogonality condition by allowing a measurement error.
systematic mistakes. Therefore, my procedure is consistent with any learning scheme that could be used in order to forecast inflation.

The second difference is related with the parameters that appear in the orthogonality conditions and their identification. Condition (16) contains four structural parameters: \( \theta, \omega, \alpha \) and \( \epsilon \). Only two of them can be identified. Galí and Gertler calibrate \( \alpha \) and \( \epsilon \), which means calibrating the degree of real rigidity, in order to identify \( \theta \) and \( \omega \). Therefore, their results are conditional on their identification assumption on the degree of real rigidity. In general, it can be said that their estimation procedure is sensitive to the mechanism that induces real rigidities. In their proposal, the existence of real rigidities arises from the departure of constant returns to scale in labor. However, there are alternative ways to generate real rigidities. Woodford (2003) proposes to consider segmented labor markets. Altig et al. (2004) propose firm specific capital. Thomas (2008) proposes search frictions to induce real rigidities. In all these cases, the slope of the hybrid NKPC is different from the one used by Galí and Gertler.\(^{18}\) Therefore, how real rigidities arise and how the parameters that determine them are calibrated matter for the identification and estimation of \( \theta \) and \( \omega \). On the contrary, my procedure does not require any assumption about the nature or importance of real rigidities. In my view, this is a great advantage, given the uncertainty and absence of consensus on how to model and calibrate real rigidities.

The third difference is related with the power of the estimation procedure to detect backward looking behavior. Rudd and Whelan (2005) criticize the estimation procedure of Galí and Gertler because it is very likely that their estimation is biased in favor of finding a significant role of the forward-looking behavior. The reason for this bias is that it is very plausible that the instrument set contains variables that directly cause inflation but are omitted from the hybrid NKPC specification. If this is the case, it follows that the coefficient next to \( \pi_{t+1} \) is going to capture the effect of the omitted variables. On the other hand, the procedure I propose does not suffer this shortcoming. In fact, a priori there is no reason to believe that my estimation can favor forward or backward-looking price-setting.

The fourth difference is related with the data that appear in the orthogonality conditions. The definition of the variables that I employ is robust to alternative assumptions on the form of the production function and the way how real rigidities are introduced. In contrast, the definition of real marginal cost can change with the previous assumptions. Galí and Lopez Salido (2001) show that the definition of real marginal cost is affected by assumptions on

\(^{18}\)Moreover, in the case of Thomas (2008), the measure of real marginal cost is also different from the one used by Galí and Gertler (1999).
the production function. Additionally, Thomas (2008) finds that the measure of real marginal cost is different from the standard one when matching frictions in the labor market are introduced to generate real rigidities.

The fifth difference is related to the introduction of positive steady state inflation in Galí and Gertler NK hybrid model. It could be shown that when the model is log-linearized around a positive inflation steady state, an infinite sum of expectations about inflation and output would appear in the hybrid NKPC. In contrast, I will show in the next section that the relation I propose in this paper is still very tractable for estimation under positive steady state inflation.

The last difference is related with the potential size of the sample. The relationship I propose can be estimated with monthly data; whereas the hybrid NKPC can be estimated only with quarterly data. This means that for a given period, the procedure I propose could use 3 times the number of observations used with quarterly data.

6 The Case with Positive Inflation Steady State

In this section, I first derive the dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation when Galí and Gertler’s model is log-linearized around a positive inflation steady state inflation $\pi$. Then, I comment the main differences with respect to the zero inflation steady state case. Finally, I present and discuss the econometric estimates for Austria and Spain.

6.1 Relationship Between the Cross Sectional Variance of Individual Price Changes and Aggregate Inflation under Positive Inflation Steady State

Proposition: In the hybrid New Keynesian model, up to a second order approximation around a positive inflation steady state, the cross sectional variance of individual price changes evolves over time according to:

$$Var_i \{\pi_t(i)\} = \theta Var_i \{\pi_{t-1}(i)\} + f(\pi_t, \pi_{t-1}) + (1 - 2\theta)f(\pi_{t-1}, \pi_{t-2})$$

where $f(\pi_t, \pi_{t-1})$ is given by:

$$f(\pi_t, \pi_{t-1}) = -\pi_t^2 + \frac{(1 - \theta)\omega}{(1 - \omega)} \left\{ \frac{\pi_t - \pi_{t-1}}{1 - \theta(1 + \pi)^{\epsilon-1}} \right\}^2 + (1 - \theta) \left\{ \frac{\pi_t - \pi}{1 - \theta(1 + \pi)^{\epsilon-1}} + k \right\}^2$$
and
\[
k = \pi + \frac{1}{\epsilon - 1} \log \left[ \frac{1 - \theta}{1 - \theta(1 + \pi)^{\epsilon-1}} \right]
\]  
(19)

**Proof:** First, notice that the cross sectional variance of individual prices evolves according to:
\[
Var_{i} \{p_t(i)\} = \theta Var_{i} \{p_{t-1}(i)\} + f(\pi_t, \pi_{t-1})
\]  
(20)

Moreover, we know that the cross sectional variance of individual price changes is given by:
\[
Var_{i} \{\pi_t(i)\} = Var_{i} \{p_t(i)\} - 2Cov_{i} \{p_t(i), p_{t-1}(i)\} + Var_{i} \{p_{t-1}(i)\} 
\]  
(21)

Using the fact that in the hybrid model, \(Cov_{i} \{p_t(i), p_{t-1}(i)\} = \theta Var_{i} \{p_{t-1}(i)\}\),
the previous expression can be expressed as:
\[
Var_{i} \{p_{t-1}(i)\} = \frac{Var_{i} \{\pi_t(i)\} - Var_{i} \{p_t(i)\}}{1 - 2\theta}
\]  
(22)

By plugging (22) into (20), the cross sectional variance of individual prices evolves according to:
\[
Var_{i} \{p_t(i)\} = \frac{\theta}{1 - \theta} \left[ Var_{i} \{\pi_t(i)\} - \frac{1 - 2\theta}{1 - \theta} f(\pi_t, \pi_{t-1}) \right]
\]  
(23)

Finally, by using (23) evaluated in periods \(t\) and \(t-1\); and plugging them into (20), we get (17).

6.2 Some Comments about the Previous Relationship

The dynamic structural relationship derived when \(\pi > 0\) is slightly more complicated than the one obtained with \(\pi = 0\). Considering positive inflation steady state introduces two more parameters in the estimation process: \(\pi\) and \(\epsilon\). Unfortunately, all the parameters cannot be estimated simultaneously as it was the case with zero inflation. This means that in order to estimate the parameters \(\theta\) and \(\omega\), it is required to fix \(\pi\) and \(\epsilon\). Consequently, one of the advantages of estimating the relationship I propose disappear with positive inflation steady state. However, notice that all the other advantages that I discuss in the previous section still hold when we consider \(\pi > 0\).

What are the consequences on econometric estimates of considering positive steady state inflation? Qualitatively, it is clear that the estimates of \(\theta\) should be lower with positive steady state inflation. To see this, it is useful to notice that a term \(\theta/(1 + \pi)^{\epsilon-1}\) instead of only \(\theta\) appears now. Therefore,
in the previous section \( \theta \) was also capturing the combined effect of \( \pi \) and \( \epsilon \). The size of the bias in \( \theta \) would depend on how big \( \pi \) and \( \epsilon \) are. The bigger they are, the bigger the bias it is. In the next subsection, it will be shown that the bias is quantitatively very small, given that the monthly inflation steady state is very small. About the bias in \( \omega \), it is very difficult to predict the direction of the bias.

Finally, it is important to mention that the impact of \( \epsilon \) on \( \theta \) and \( \omega \) estimates is always conditional on the steady state inflation. The lower is the steady state inflation, the lower is the influence of \( \epsilon \). Therefore, for very low levels of inflation, it is very likely that the calibrated values of \( \epsilon \) do not affect significantly \( \theta \) and \( \omega \) estimates. Instead, when estimating the hybrid NKPC, the impact of \( \epsilon \) on parameter estimates is conditional on how real rigidities are introduced. Consequently, even if \( \theta \) and \( \omega \) cannot be directly estimated when \( \pi > 0 \), the degree of uncertainty about the influence of the calibrated values of \( \epsilon \) should be lower with the methodology I propose, given that the uncertainty about \( \pi \) is much lower than the one about those parameters that qualify real rigidities in the economy.

6.3 Estimates

In this subsection, I reestimate \( \theta \) and \( \omega \) considering \( \pi > 0 \). The orthogonality condition that I use is the same as the one used in Section 4 but \( f(\pi_t, \pi_{t-1}) \) is now given by (18) and (19). The steady state inflation values for Austria and Spain are set equal to the sample average of monthly inflation (0.001294 and 0.002676 respectively). For both countries, I set \( \epsilon = 11 \), which is the value used by Galí et al (2001). Table 2 presents the results.

The conclusions obtained in Section 4 hold when estimation is performed considering \( \pi > 0 \). The estimates of \( \theta \) and \( \omega \) slightly decrease. As it was anticipated in the previous subsection, the decreases were higher in the case of Spanish estimates, given that the steady state inflation is higher. Quantitatively, the most important changes are in the estimated average price duration. However, all these estimates are still within the range for the average price duration found with disaggregated data.
Table 2
Estimates of the Structural Parameters when $\pi > 0$

<table>
<thead>
<tr>
<th></th>
<th>$\theta$</th>
<th>$\omega$</th>
<th>$D$</th>
<th>$\gamma_b$</th>
<th>$\gamma_f$</th>
<th>$J$ test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AUSTRIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUE</td>
<td>0.909</td>
<td>0.935</td>
<td>11.033</td>
<td>0.507</td>
<td>0.493</td>
<td>0.359</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.033)</td>
<td>(3.513)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>2S – GMM</td>
<td>0.928</td>
<td>0.833</td>
<td>13.930</td>
<td>0.473</td>
<td>0.527</td>
<td>0.537</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.081)</td>
<td>(2.602)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td></td>
</tr>
<tr>
<td><strong>SPAIN</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUE</td>
<td>0.913</td>
<td>0.730</td>
<td>11.432</td>
<td>0.445</td>
<td>0.555</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.098)</td>
<td>(1.726)</td>
<td>(0.036)</td>
<td>(0.036)</td>
<td></td>
</tr>
<tr>
<td>2S – GMM</td>
<td>0.919</td>
<td>0.691</td>
<td>12.367</td>
<td>0.429</td>
<td>0.571</td>
<td>0.201</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.105)</td>
<td>(1.833)</td>
<td>(0.040)</td>
<td>(0.040)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors shown in brackets.

7 Concluding Remarks

The identification of backward-looking behavior to set prices is an important issue in the design of monetary policy because it helps to explain inflation persistence and the costs of disinflation processes. Moreover, it is useful in the design of optimal monetary policy. In this paper, I evaluate the plausibility of the existence of rule of thumb firms to account for backward-lookingness. Previous studies have explored this issue by estimating the hybrid NKPC developed by Galí and Gertler (1999). Instead, by using their model, I derive a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation. I show that the identification of rule of thumb behavior by using this relation does not require assumptions on rationality of expectations, real rigidities and production functions. Moreover, it seems that fitting this relation has more power than estimating
the hybrid NKPC in order to detect backward looking behavior.

By using Spanish and Austrian data, I estimate the structural relationship that I derive in this study. I find that the fraction of rule of thumb firms is statistically significant and quantitatively important. Moreover, I find estimates of the degree of price rigidity that are consistent with estimations based on disaggregated data. From the estimates I present in this paper, it is concluded that backward-looking behavior is almost as important as the forward-looking one in describing Austrian and Spanish inflation dynamics.

Finally, it is worth mentioning that there are three interesting extensions of this study. The first one would incorporate the structural relationship between the cross sectional variance of individual price changes and aggregate inflation in a second order approximation of a DSGE model that uses the hybrid NKPC with rule of thumb firms.\textsuperscript{19} This would allow to identify parameters that affect the degree of real rigidities. The second extension consists in evaluating different rules of thumb. In this sense, it would be interesting to evaluate the one proposed by Nunes (2005). He proposes that backward-looking firms use survey expectations instead of past inflation in order to set prices. I plan to explore these extensions in future research. The last extension would consist in deriving the structural relationship I propose in the case of time varying trend inflation.

\textsuperscript{19}See An and Schorfheide (2005) and Fernández Villaverde and Rubio Ramírez (2005) for estimation of second order approximations of DSGE models.
References


