Abstract

Despite its theoretical dominance, the empirical case in favor of the permanent income hypothesis is at best a weak one. Contrary to one of its basic implications, a growing body of evidence suggests that rich households save a higher proportion of their permanent income than poor households. Following Duesenberry (1949), we propose an overlapping generations economy where households care about relative consumption, the difference between their consumption and the consumption of their reference group. As a result, an individual’s consumption is driven by the comparison of his lifetime income and the lifetime income of his reference group; a permanent income version of the Duesenberry’s (1949) relative income hypothesis. Across households the savings rate increases with income while aggregate savings are independent of the income distribution. Positional concerns lead agents to over-consume, over-work, and under-save, relative to the welfare maximizing levels that a planner would choose. We propose a simple tax schedule that induces the competitive economy to achieve allocative efficiency.

JEL Classification: D62, E21, H21

Key words: relative consumption; relative income hypothesis; permanent income hypothesis.
1 Introduction

James Duesenberry, in his seminal work, Income, Saving and the Theory of Consumer Behavior (1949), introduces the relative income hypothesis in an attempt to rationalize the well established differences between cross-sectional and time-series properties of consumption data. On the one hand, a wealth of studies based on 1935-36 and 1941-42 cross-sectional budget surveys present a saving ratio that increased with income. On the other hand, the data on aggregate savings and income from 1869 to 1929 collected by Kuznets (1942) presents a trend-less saving ratio. Duesenberry (1949) proposes an individual consumption function that depends on the current income of other people. As a result "for any given relative income distribution, the percentage of income saved by a family will tend to be a unique, invariant, and increasing function of its percentile position in the income distribution. The percentage saved will be independent of the absolute level of income. It follows that the aggregate saving ratio will be independent of the absolute level of income" (Duesenberry, 1949, pg. 3).

Despite its empirical success, the relative income hypothesis was quickly replaced by the well-known permanent income hypothesis (Modigliani and Brumberg (1954), Friedman (1957)) as the economists’ workhorse to understand consumption behavior. According to this view the cross-sectional correlation between saving and income is driven by transitory deviations from permanent income, while in the aggregate, most transitory components cancel out, leading to the close relation between consumption and income observed in time series data.

This paper presents a fully specified model of intertemporal choice that formalizes Duesenberry’s intuitions. We consider an overlapping generations economy where households differ in the initial bequest they inherit from their parents. Households derive utility from leisure, bequest and the difference between their consumption and the consumption of others, i.e. relative consumption. In this context, the resulting consumption of an agent is driven by the comparison of his lifetime income and the lifetime income of his reference group; a permanent income version of the relative income hypothesis. As in Duesenberry (1949), individual saving rates increase with relative income while aggregate savings are independent of the income distribution. Positional concerns lead agents to consume and work above the welfare maximizing levels that a benevolent central planner would choose. We propose a simple tax schedule that induces the competitive economy to achieve the efficient allocation. Along the lines anticipated by Frank (2007) it consists on a progressive tax on consumption.

Despite its overwhelming theoretical dominance, the empirical case in favor of the permanent income hypothesis is at best a weak one. Much of the early empirical work (Brady and Friedman (1947) and Mayer (1966)) presents strong evidence against the proportion-
ality of savings rates. Recent empirical work has only confirmed these findings. Browning and Lusardi (1996) conclude that the observed positive relationship between income and saving is difficult to rationalize in terms of consumption smoothing. Dynan, et al. (2004) use panel data to instrument permanent income by education, lagged and future earnings, and measures of consumption. Their careful analysis finds a strong positive relationship between saving rates and lifetime income. The literature on inter-generational saving, bequests and inter-vivos transfers, finds similar results. In recent work, Altonji and Villanueva (2007) estimate that, at the mean of permanent earnings, parents pass on about 2.5 cents of every extra dollar of lifetime resources to their children through bequests. Furthermore, their estimate increases with income, demonstrating that wealthier households bequeath a larger proportion of their income than poor households do. If we are to believe this recent body of evidence we need to depart from the standard version of the permanent income hypothesis\(^1\). Our model does so in an intuitive way, abandoning the independent preference assumption that underlies Friedman’s analysis. The resulting behavior, a mixture of permanent and relative income components, preserves the basic implications of the permanent income hypothesis while being consistent with the empirical evidence on the cross-section of saving out of permanent income.

The assumption that preferences are independent across households, although standard in the economic literature, is not particularly appealing. Indeed, social scientists have long stressed the relevance of status seeking as being an important characteristic of human behavior (see Schoeck (1966) and Rawls (1971)). The idea that the overall level of satisfaction derived from a given level of consumption depends, not only on the consumption level itself, but also on how it compares to the consumption of other members of society, is not new in economics. Though origins of this proposition can be traced as far back as Smith (1759) and Veblen (1899), it was not until the work of Duesenberry (1949) and Pollak (1976) that an effort was made to provide this idea with some micro-theoretic foundations. On the

\(^1\)Several authors have explored departures from the standard permanent income hypothesis to account for the cross-sectional variation in saving rates, with different degrees of success. Zeldes (1989) introduces liquidity constraints in an intertemporal optimization model. He finds that the inability to borrow against future labor income affects the consumption of a significant portion of the population. Ventura and Hugget (2000) analyze the impact on saving rates of the US social security system. Samwick (1998) considers a model where the subjective discount rate is correlated with income. Gentry and Hubbard (2000) assume that entrepreneurs enjoy better access to investment opportunities. As a result, if substitution effects dominate income effects, they will save more. Dynan, et al. (2002, 2004) explore the effects of the introduction of bequest motives and large medical expenses associated with health shocks. The introduction of these expenses implies that low-income households should save more, rather than less, than high-income households. Finally, Carroll (2000) considers the accumulation of wealth as an end in itself, the "capitalist spirit" model. He argues that the implications for saving of his model are virtually indistinguishable from those obtained in a model of interpersonal comparisons.
empirical side, Clark and Oswald (1996), using a sample of 5,000 British workers, find that workers’ reported satisfaction levels are inversely related to their comparison wage rates, supporting the hypothesis of positional externalities. Neumark and Postlewaite (1998) propose a model of relative income to rationalize the striking rise in the employment of married women in the U.S. during the past century. Using a sample of married sisters, they find that married women are 16 to 25 percent more likely to work outside the home if their sisters’ husbands earn more than their own husbands. Bowles and Park (2005), using data from ten OECD economies, find a strong positive correlation between average working hours and the share of consumption of the richest members of society. They interpret this result as indicative of strong emulation motives. Ravina (2007) estimates an Euler equation derived under interdependent preferences. Her results are consistent with preference specifications that place around one third of the weight on relative consumption. Finally, Frank (1985, 2000, 2007) provides a wealth of anecdotal evidence on the effects of positional externalities on individual behavior. On the theoretical side, there is a large literature that explores the effects of preference interdependence for asset pricing (Abel (1990), Gali (1994)), for short-run macroeconomic stabilization policy (Ljungqvist and Uhlig (2000)), for the interaction between saving and growth (Carroll, et al. (1997, 2000)), for capital accumulation (Alvarez-Cuadrado, et al. (2004), Alonso-Carrera, et al. (2005), Liu and Turnovsky (2005)), and for labor supply choices (Fisher and Hof (2000), Alvarez-Cuadrado (2007)). Finally, a growing body of experimental literature highlights the importance of relative rather than absolute payoffs for economic choices (see for instance Alpizar, et al. (2005)).

Our paper contributes to this literature by exploring the interaction between consumption externalities and income inequality. In line with previous results, we find that relative consumption concerns lead to inefficiently low levels of leisure, over-working, and excessive levels of consumption, over-consumption. But in contrast to earlier studies, we also find that consumption externalities are associated with an inefficiently low saving rate as opposed to the over-accumulation results obtained in models with an infinitely lived representative agent (Fisher and Hof (2000), Liu and Turnovsky (2005)). Intuitively, in a representative agent economy with an infinite planning horizon households want to keep up with the Joneses today and on every future day. Being forward-looking they anticipate that reducing current saving relative to their neighbors will lead to an undesirably low level of future consumption. This mechanism underlies the coupling of the over-consumption result (keeping up with the Joneses today) with an over-accumulation result (keeping up with the Joneses in the future) found in previous studies. Our framework, by considering a non-positional saving motive,
enables us to show the opposite effect of consumption externalities on the saving rate.

Finally, our work is closely related to the recent literature on self-reported well-being. Early work by Easterlin (1974, 1995) and Oswald (1997) found differences between the cross-section and time-series properties of happiness data that are quite similar to those reported on savings data more than fifty years before. Self-reported well-being data shows that within a country at a given point in time those with higher incomes are, on average, happier. However, average happiness in developed countries has remained relatively constant over time despite sharp increases in per capita GDP. Clark et al. (2008) highlight the importance of interpersonal comparisons to account for the "Easterlin paradox". Recent work has tried to estimate the direct impact of interpersonal comparisons on self-reported well-being. Luttmer (2005) matches individual-level panel data on well-being from the U.S. National Survey of Families and Households to census data on local average earnings. After controlling for income and other own characteristics, he finds that local average earnings have a significantly negative effect on self-reported happiness. Ferrer-i-Carbonell (2005), using data from a large German panel, concludes that the income of the reference group is about as important as the own income for individual happiness. Dynan and Ravina (2007) find similar results for US households. Their estimates suggest that people’s happiness depends positively on how well they are doing relative to the average in their geographic area, even after controlling for own income levels. Our work formalizes these insights and explores the implications of relative consumption concerns on saving and leisure decisions.

The paper is organized as follows. Section 2 sets out the basic model. Section 3 compares the decentralized and centrally planned solutions under an homogeneous reference group. This section presents the basic implications of a life-cycle version of the relative income hypothesis. Section 4 evaluates the steady state properties of our model. The conclusions are summarized in Section 5, while the Appendices extend our previous analysis allowing for heterogeneous reference groups and provide some technical details.

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2Stevenson and Wolfers (2008) extend Easterlin’s (1974, 1995) country coverage to reassess his paradox. Their results suggest a positive link between GDP and average levels of subjective well-being across countries. These authors conclude that the role for relative income comparisons as drivers of happiness is minimal. We disagree with this interpretation of the evidence since cross-country comparisons of self-reported well-being are problematic. We believe that a definite rebuttal of Easterlin’s paradox requires a careful evaluation of time-series data from individual countries. In this respect, the evidence presented by these authors is mixed.
2 The Model

Consider a small open economy that faces a given world interest rate, $r$. Time is discrete and infinite with $t = 0, 1, 2, \ldots \infty$.

2.1 Production

Every period our economy produces a composite good that may be consumed or invested. Output, $Y$, is produced combining physical capital, $K$, and labor, $1 - L$. The production function, $F(K, 1 - L)$, is homogeneous of degree one and satisfies the usual Inada conditions. Since markets are competitive factors are paid their marginal products and therefore,

$$w_t = f \left( \frac{K_t}{1 - L_t} \right) - \left( \frac{K_t}{1 - L_t} \right) f' \left( \frac{K_t}{1 - L_t} \right)$$

(1)

$$r_t = f' \left( \frac{K_t}{1 - L_t} \right) - \delta$$

(2)

where $f$ denotes the production function in intensive form and capital is assumed to depreciate at the exponential rate $\delta$.

Under the assumption that our economy is open, small, and faces a constant world rate of interest, $r$, the domestic capital-labor ratio is pinned down by (2) with $r_t = r$. The degree of capital intensity, in turn, pins down the domestic wage rate at $w$. Any changes in labor supply are accommodated by capital flows so that the domestic wage and the interest rate remain constant at $w$ and $r$ respectively. We denote the gross return to capital by $R = 1 + r$.

2.2 Households

Individuals live for two periods: "youth" and "old-age". At the end of his youth each individual gives birth to a single offspring and therefore at any point in time there are two generations alive. Each generation is composed of $n$ individuals, indexed by $i = 1, \ldots, n$. Our agents are altruistic toward their children, deriving a "warm-glow" from the bequests they leave to their descendents at the end of their lives (Adreoni (1989)). Within a generation, individuals differ only in their initial levels of wealth bequeathed by their parents. The distribution of wealth in period $t$ is represented by a cumulative distribution function $G_t(b)$. The initial distribution $G_0(b)$ is given. Let’s focus on the $i$-th individual born in period $t$. In the first period of his life he is endowed with one unit of time that he allocates between leisure, $l_t^i$, and work. His labor income, $w(1 - l_t^i)$, together with his inherited wealth, $b_t^i$, is
divided between current consumption, $c_t^i$, and saving, $s_t^i$. His first period budget constraint is given by

$$w (1 - b_t^i) + b_t^i = c_t^i + s_t^i$$

(3)

In the second period of his life, the individual is retired. His only source of income comes from the return on the savings he made when young, $Rs_t^i$. He allocates this income between old-age consumption, $d_{t+1}^i$, and bequest, $b_{t+1}^i$. His old-age budget constraint is

$$Rs_t^i = d_{t+1}^i + b_{t+1}^i$$

(4)

The preferences of an individual born in period $t$ are given by the following life-cycle utility function,

$$U_t (c_t^i, l_t^i, d_{t+1}^i, b_{t+1}^i) = u (c_t^i) + v (l_t^i) + \beta \left[u (d_{t+1}^i) + \phi (b_{t+1}^i)\right]$$

(5)

where $0 < \beta < 1$ is the subjective discount factor. The three subutility functions, $u(\cdot), v(\cdot)$ and $\phi(\cdot)$, are assumed to be increasing, concave and to satisfy the standard Inada conditions.

Our key behavioral assumption is that the satisfaction derived from consumption does not depend on the absolute level of consumption itself but rather in how it compares to the consumption of some reference group. Following Ljungqvist and Uhlig (2000) we adopt an additive specification for relative consumption; $\hat{c}_t^i = c_t^i - \gamma \bar{g}_t^i$ and $\hat{d}_{t+1}^i = d_{t+1}^i - \gamma \bar{o}_{t+1}^i$, where $\bar{g}_t^i$ and $\bar{o}_{t+1}^i$ are the average consumption levels of the reference group of the $i$-th individual when young and old respectively and $0 < \gamma < 1$ is a measure of the relativity concerns$^3$.

3 Homogeneous Reference Group

Following most of the literature on consumption externalities (Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005)) we assume, in this section, that any individuals’ reference group is composed by all the members of his own generation$^4$. Under this assumption

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$^3$We place restrictions on the initial endowments, $G_0 (b)$, so that everyone’s relative consumption is positive.

$^4$See Abel (2005) for an overlapping generation model where the reference group is composed by a weighted average of young and old households. Frank (1985) points out that "the sociological literature on reference group theory stresses that an individual’s personal reference group tends to consist disproportionately of others who are similar in terms of age, education, and various other background variables" (Frank, 1985, pg. 111). Our specification of the reference group, in line with Frank’s (1985) arguments, limits interpersonal comparisons to agents belonging to the same generation. See Appendix I for an extension of our results to an environment with heterogeneous reference groups.
all young households share the same reference group and therefore the reference level of consumption is given by,

$$\tilde{g}_t^i = \tilde{c}_t = \frac{1}{n} \sum_{j=1}^{n} c^i_j$$ (6)

and similarly all old households share the same reference group, with the reference level of consumption being,

$$\tilde{o}^i_{t+1} = \tilde{d}_{t+1} = \frac{1}{n} \sum_{j=1}^{n} d^i_{t+1}$$ (7)

### 3.1 Competitive Solution

The $i$-th individual of the generation born in period $t$ takes as given his inherited wealth, factor prices, and the choices of the other members of his generation, and chooses the amount of time devoted to work, $(1 - l^i_t)$, his level of saving, $s^i_t$, and the second-period consumption, $d^i_{t+1}$, to maximize,

$$u \left( w \left( 1 - l^i_t \right) + b^i_t - s^i_t - \gamma \tilde{c}_t \right) + v \left( l^i_t \right) + \beta \left[ u \left( d^i_{t+1} - \gamma \tilde{d}_{t+1} \right) + \phi \left( R s^i_t - d^i_{t+1} \right) \right]$$

The solution to this problem is characterized by the following optimality conditions,

$$u' \left( c^i_t - \gamma \tilde{c}_t \right) = \beta R \phi' \left( R s^i_t - d^i_{t+1} \right)$$ (8)

$$u' \left( c^i_t - \gamma \tilde{c}_t \right) w = v' \left( l^i_t \right)$$ (9)

$$u' \left( d^i_{t+1} - \gamma \tilde{d}_{t+1} \right) = \phi' \left( b^i_{t+1} \right)$$ (10)

The interpretation of these conditions is standard. Nonetheless, note the effects of interpersonal comparisons. An increase in the consumption of the reference group increases the marginal utility of consumption leading to a reduction in savings and leisure. As we will see, equations (8)-(10) together with the budget constraints, (3) and (4), implicitly define the optimal choices of leisure, saving and bequests as functions of the relative income of the individual.

In order to keep the analysis tractable it is convenient to assume the following logarithmic specification for (5)
where $\mu > 0$ and $\alpha > 0$ denote the importance of leisure and bequest motives respectively.

Given the level of saving, $s_t$, an old individual at time $t + 1$ chooses $d_{t+1}^i$ and $b_{t+1}^i$ to maximize

$$V \equiv \ln (d_{t+1}^i - \gamma ar{d}_{t+1}) + \alpha \ln (b_{t+1}^i)$$

subject to (4). The solution to this problem is

$$d_{t+1}^i = \frac{1}{1 + \alpha} (Rs_t^i + \alpha \gamma \bar{d}_{t+1})$$

(12)

and therefore

$$V(Rs_t^i) = (1 + \alpha) \ln [(Rs_t^i - \gamma \bar{d}_{t+1})] + \ln \left[ \frac{1}{1 + \alpha} \right] + \alpha \ln \left[ \frac{\alpha}{1 + \alpha} \right]$$

The young individual at time $t$ then chooses $c_t^i$, $l_t^i$ and $s_t^i$ to maximize

$$\ln (c_t^i - \gamma \bar{c}_t) + \mu \ln (l_t^i) + \beta V(Rs_t^i)$$

subject to (3). The necessary conditions for problem (14) yield

$$wl_t^i = \mu (c_t^i - \gamma \bar{c}_t)$$

(15)

and

$$s_t^i = \beta (1 + \alpha) (c_t^i - \gamma \bar{c}_t) + \frac{\gamma \bar{d}_{t+1}}{R}$$

(16)

Combining (15), (16) and (3) implies

$$c_t^i(1 + \mu + (1 + \alpha) \beta) = w + b_t^i - \frac{\gamma \bar{d}_{t+1}}{R} + (\mu + (1 + \alpha) \beta) \gamma \bar{c}_t$$

(17)

We begin by characterizing the optimal behavior of the average household, i.e. the household inheriting the average bequest, $\bar{b}_t$. Equation (17) becomes,

$$\bar{c}_t [1 + (\mu + (1 + \alpha) \beta) (1 - \gamma)] = w + \bar{b}_t - \frac{\gamma \bar{d}_{t+1}}{R}$$

(18)
Combining (12) and (13) we reach,

\[ \bar{d}_{t+1} = \frac{1}{1 + (1 - \gamma) \alpha} R \bar{s}_t \]  

(19)

\[ \bar{b}_{t+1} = \frac{(1 - \gamma) \alpha}{1 + (1 - \gamma) \alpha} R \bar{s}_t \]  

(20)

Substituting (19) into (16) we get

\[ \bar{c}_t = \frac{1}{\beta [1 + (1 - \gamma) \alpha]} \bar{s}_t \]  

(21)

Substituting (19) and (21) into (18) we obtain the level of saving of the average individual born in period \( t \),

\[ \bar{s}_t = \beta [1 + (1 - \gamma) \alpha] \chi (w + \bar{b}_t) \equiv \beta [1 + (1 - \gamma) \alpha] \chi \bar{y}_t \]  

(22)

where \( \chi \equiv \frac{1}{1 + (\mu + \alpha \beta)(1 - \gamma) + \beta} \) and \( \bar{y}_t \equiv w + \bar{b}_t \) is the potential lifetime income of the average agent of the generation born at \( t \), i.e. the lifetime income of the agent that inherits the average bequest if all of his time endowment is devoted to work. Saving is just a constant fraction, \( 0 < \beta [1 + (1 - \gamma) \alpha] \chi < 1 \), of this measure of lifetime income.

Combining (15), (18), (19), (20), and (22) we obtain the remaining choices for the average household,

\[ \bar{c}_t = \chi \bar{y}_t \]  

(23)

\[ \bar{T}_t = \frac{\mu(1 - \gamma) \chi \bar{y}_t}{w} \]  

(24)

\[ \bar{d}_{t+1} = R \beta \chi \bar{y}_t \]  

(25)

\[ \bar{b}_{t+1} = (1 - \gamma) \alpha R \beta \chi \bar{y}_t \]  

(26)

we assume \( \frac{\bar{b}_t}{w} < \frac{1 + \beta + \alpha \beta (1 - \gamma)}{\mu (1 - \gamma)} \) so that the constraint \( \bar{T}_t \leq 1 \) is satisfied with strict inequality.

We can use the results for the average household to characterize the behavior of the \( i \)-th individual of the same generation. Substitute (23) and (25) into (17) to reach first-period consumption for the \( i \)-th individual:
\[ c_i^t = \frac{1}{1 + \mu + (1 + \alpha)\beta} \{ \tilde{y}_t^i + (\beta\alpha + \mu)\gamma \tilde{y}_t^i \} \]  

(27)

Combining (27) with (16) and (25) we find that the level of saving is given by,

\[ s_i^t = \frac{(1 + \alpha)\beta}{1 + \mu + (1 + \alpha)\beta} \{ \tilde{y}_t^i - \left( \frac{\alpha - \mu}{1 + \alpha} \right)\gamma \tilde{y}_t \} \]  

(28)

where \( \tilde{y}_t^i \equiv w + b_t^i \) is the potential lifetime income, defined as before, of the \( i \)-th household of the generation born at \( t \). Equation (28) shows that individual saving is a linear function of individual and average income\(^5\). This linearity property ensures that the income distribution plays no role for aggregate saving and the aggregate evolution of our economy.

It is straightforward to solve for the remaining optimal choices as functions of individual and average potential lifetime income,

\[ d_{i+1}^t = \frac{R\beta}{1 + \mu + (1 + \alpha)\beta} \{ \tilde{y}_t^i + (\beta\alpha + \mu)\gamma \tilde{y}_t^i \} \]  

(29)

\[ l_t^i = \frac{\mu}{w (1 + \mu + (1 + \alpha)\beta)} \{ \tilde{y}_t^i - (1 + \beta)\gamma \tilde{y}_t \} \]  

(30)

\[ b_{i+1}^t = \frac{\alpha R\beta}{1 + \mu + (1 + \alpha)\beta} \{ \tilde{y}_t^i - (1 + \beta)\gamma \tilde{y}_t \} \]  

(31)

Consumption of the \( i \)-th household, (27) and (29), is made out of two components. The first increases in the household’s potential lifetime income. The second reflects the influence of interpersonal comparisons and increases in the potential lifetime income of his reference group. As a result leisure and bequests depend on relative, rather than absolute, income. When individual satisfaction depends on consumption comparisons across households, as a growing body of empirical evidence suggests, the relevant variable driving the saving and labor supply choices is the comparison between individual \( i \)'s potential lifetime income and the potential lifetime income of his reference group. The agents populating our economy are not only "disposed, as a rule and on the average, to be forward-looking animals" as those in Modigliani and Brumberg (1954, pg. 430) or Friedman (1957), but are also outward-looking animals as their choices are partially driven by the choices of other members of the

\(^5\)Notice that when \( \mu = \alpha \), \( \gamma \) has no impact on the level of saving although as we will see below \( \gamma \) still impacts the saving rate out of conventionally defined income. In general, an increase in \( \gamma \) will redistribute lifetime resources toward the period in which the basket places relatively more weight on positional goods. If \( \mu \) is smaller than \( \alpha \), the first period basket is relatively more intensive in the positional good than the second period one, and as a result an increase in \( \gamma \) will shift resources toward first period expenditure, resulting in a smaller saving.
community they live in. We can think of these results as an extension of Duesenberry’s (1949) relative income hypothesis to an intertemporal framework.\footnote{It is worth noticing that the combination of quasi-homothetic preferences and perfect capital markets implies that (28)-(31) are affine functions of the level of potential lifetime income. This property of the model ensures that the distribution of wealth does not affect the aggregate evolution of the economy.}

\section*{3.2 The Life-cycle Version of the Relative Income Hypothesis}

Duesenberry (1949), building on work by Brady and Friedman (1947), proposes the relative income hypothesis to rationalize the well established differences between cross-sectional and time series properties of consumption. On the one hand, a wealth of budget studies show a saving ratio that increases with income. On the other hand, Kuznets’ (1942) time series data presents a trend-less saving ratio. Duesenberry (1949) postulates an individual consumption function that depends on the current income of other people. Under this hypothesis, the cross-sectional positive correlation between saving ratios and income levels results from relative consumption concerns (the emulation effect). The long run constancy of the aggregate saving rate arises because the effects of relativity concerns cancel out in the aggregation. The relative income hypothesis was quickly replaced by Friedman’s permanent income hypothesis (Modigliani and Brumberg (1954), Friedman (1957)) as the dominant paradigm for understanding consumption behavior.\footnote{One should point out that Friedman’s view is more nuanced than the profession seems to believe “… and finally, the evidence that we have cited seems to fit it (the Permanent Income Hypothesis) somewhat better… however, this evidence is by no means sufficient to justify a firm rejection of the relative income hypothesis” (Friedman, 1957, p. 169).} According to this view, consumption is driven by permanent income; as a result, saving is proportional to lifetime resources. This view explains the positive correlation between saving and income in cross-sectional data by transitory deviations from permanent income. In the aggregate, most transitory components cancel out, leading to the close relation between consumption and income observed in time series data. Despite the theoretical dominance of the permanent income hypothesis, recent empirical work finds important deviations from its basic predictions. Browning and Lusardi (1996) conclude that the observed positive relationship between income and saving is difficult to rationalize in terms of consumption smoothing. Dynan, et al. (2004) find a strong positive relationship between saving rates and lifetime income. Altonji and Villanueva (2007) report that the propensity to bequeath increases with lifetime income. Our model where agents care, not only about permanent income, but also about relative income provides a straightforward explanation for this evidence in terms of interpersonal comparisons.

In a representative agent economy, Liu and Turnovsky (2005) and Alvarez-Cuadrado
(2007) show that positional concerns lead households to choose levels of consumption and working hours above the welfare-maximizing levels. Our framework, in which consumption externalities interact with income inequality, allows for a more systematic exploration of the differential impact of relative consumption across the income distribution.

Since consumption and bequests are normal goods, their levels increase with wealth (income), although according to the standard version of the permanent income hypothesis their rates should be a constant fraction of lifetime resources. In order to illustrate our permanent income version of the relative income hypothesis it is convenient to define (actual) lifetime income as,

\[ y_i^t = w (1 - l_i^t) + b_i^t = \frac{1 + (1 + \alpha) \beta}{1 + \mu + (1 + \alpha) \beta} (w + b_i^t) + \frac{\mu (1 + \beta) \gamma \chi}{1 + \mu + (1 + \alpha) \beta} (w + \tilde{b}_i) \quad (32) \]

The saving and bequest rates out of (actual) lifetime income are the ratio of (28) and (31) to (32) respectively. Differentiating these ratios with respect to wealth, measured by the initial bequest, we reach the following comparative static results,

\[ \frac{\partial s_i^t}{\partial y_i^t} = \frac{\beta \alpha \gamma \chi \tilde{y}_i^t}{(1 + \mu + (1 + \alpha) \beta) (y_i^t)^2} > 0 \quad (33) \]

\[ \frac{\partial b_i^{t+1}}{\partial y_i^t} = \alpha R (1 + \beta) \frac{\partial s_i^t}{\partial b_i^t} = \frac{\alpha R \beta (1 + \beta) \gamma \tilde{y}_i^t}{(1 + \mu + (1 + \alpha) \beta) (y_i^t)^2} > 0 \quad (34) \]

Finally, differentiating (30) with respect to wealth we reach

\[ \frac{\partial l_i^t}{\partial b_i^t} = \frac{\mu}{w (1 + \mu + (1 + \alpha) \beta)} > 0 \quad (35) \]

In the absence of interpersonal comparisons, \( \gamma = 0 \), the saving rate is proportional to lifetime income and, as in the permanent income hypothesis, independent of the lifetime resources. Once we allow for consumption externalities, \( \gamma > 0 \), the saving rate increases with lifetime income as most empirical evidence suggests. Poor households save a smaller proportion of their income and transfer a lower fraction of their wealth in the form of bequests than richer households do\(^8\). This happens despite the fact that poor households work longer.

\(^8\)Garcia-Penalosa and Turnovsky (2007) explore an economy populated by infinitely lived heterogeneous households in the presence of comparative consumption. As opposed to our framework, in the infinitely lived economy poor households save more than rich ones and the presence of consumption externalities reduces inequality in a growing economy. Their results seem at odds with the empirical evidence cited in this section.
hours than their richer neighbors. These results are consistent with abundant anecdotal evidence on the living conditions of low income households. For instance Newman and Chen (2007) illustrate how "working poor" families in America hold multiple jobs per person while having difficulty to make ends meet.

Finally, note that Duesenberry (1949) dealt with one additional empirical regularity of consumption data: consumption is more stable than income at high frequencies. He explained the rigidity of consumption by appealing to habit formation. In our intertemporal set up with forward-looking agents, this rigidity is a natural result of consumption smoothing, as in the permanent income hypothesis.

3.3 Efficient Solution

In a competitive equilibrium, individual households ignore the effects that their consumption choices have on the utility of other members of their generation. As a consequence, agents’ consumption, leisure, and bequest may diverge from the socially optimal levels chosen by a benevolent central planner. Let us consider a central planner that acknowledges that individual consumption choices create distortions through their effects on average consumption. The planner chooses consumption, labor effort and bequest for each individual within a given generation (taking their initial wealth distribution as given) to maximize the social welfare function,

\[ SW = \frac{1}{n} \sum_{i=1}^{n} \left[ \ln(c_i) + \mu \ln(l_i) + \beta (\ln(d_i) + \gamma \bar{d}) + \alpha \ln(b_{i+1}) \right] \] (36)

subject to the individual’s budget constraints (3), (4), (6), and (7). Solving this program for \( i = 1, 2, 3, \ldots, n \), we get the following optimality conditions,

\[ \frac{1}{c^i - \gamma \bar{c}^p} - \frac{\alpha \beta R}{d^i + R(w + b_{i+1} - w l^i - c^i)} = \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{c^j - \gamma \bar{c}^p} \] (37)

\[ \frac{1}{d^i - \gamma \bar{d}^p} - \frac{\alpha}{d^i + R(w + b_{i+1} - w l^i - c^i)} = \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{d^j - \gamma \bar{d}^p} \] (38)

9 In a version of our model without consumption externalities, poor households will also enjoy less leisure. The presence of consumption externalities only exacerbates this result. Bowles and Parker (2005) estimate that almost 60% of the difference in average working hours between Sweden and the US could be explained in terms of interpersonal comparisons and income inequality.

10 It is worth noticing that in the absence of capital accumulation dynamic inefficiency is not an issue. As a result our planner abstracts from intergenerational efficiency and redistribution. For this reason we drop the time subscripts when unambiguous.
\[
\frac{\mu}{\beta^p} - \frac{\alpha \beta Rw}{d^i_p + R(w + b^i_p - w^{i_p} - c^i_p)} = 0 \quad (39)
\]

where the superscript \( p \) denotes the planner’s choices\(^\text{11} \). Notice that the private marginal utilities of consumption when young are old are given by,

\[
MUC^{i,m}_i \equiv \frac{1}{c^i - \gamma \bar{c}} \quad (40)
\]

\[
MUD^{i,m}_i \equiv \frac{1}{d^i - \gamma \bar{d}} \quad (41)
\]

where the superscript \( m \) refers to the market (i.e. laissez-faire) scenario.

Since the planner realizes that each individual contributes to the externality by a fraction \( \gamma \) of their consumption, the difference between the competitive solution and the planned one lies in the valuation of the utility of consumption. The planner’s counterpart of (40) and (41), the social marginal utilities of consumption for the \( i \)-th household in both periods of his life are given by,

\[
MUC^{i,p}_i \equiv \frac{1}{c^i - \gamma \bar{c}} - \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{c^{j,p} - \gamma \bar{c}^p} \quad (42)
\]

\[
MUD^{i,p}_i \equiv \frac{1}{d^i - \gamma \bar{d}} - \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{d^{j,p} - \gamma \bar{d}^p} \quad (43)
\]

Comparing (40) with (42) and (41) with (43) we see that the social marginal utilities of consumption are composed of two terms. The first term is just the private marginal utility of consumption. The second term captures the negative impact that an additional unit of consumption of the \( i \)-th agent has on his own welfare and on the welfare of other members of his generation through its impact on average consumption. Since this negative impact is independent of the level of consumption of the household, the second terms; \( A \equiv \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{c^{j,p} - \gamma \bar{c}^p} \) and \( D \equiv \frac{\gamma}{n} \sum_{j=1}^{n} \frac{1}{d^{j,p} - \gamma \bar{d}^p} \), are identical for all households of a given generation.

\(^{11}\)We restrict to interior solutions, i.e. the first order conditions implicitly impose restrictions to guarantee that the social marginal utility of consumption is always positive. These restrictions play a similar role to the ones placed in representative agent versions of our model to guarantee that the marginal utility of consumption, after taking into account external effects, is positive. See for instance, Liu and Turnovsky (2005) assumption 1 (i).
As a result, the distortion introduced by relative consumption takes the form of an overvaluation, by a factor \( \frac{1}{1 - A(c^{i,p} - \gamma \bar{c}^p)} = \frac{1}{1 - D(d^{i,p} - \gamma \bar{d}^p)} \), of the marginal utility consumption in each period of lifetime\(^{12}\).

The overvaluation of consumption distorts the marginal rate of substitution between first-period consumption and leisure, which we call the static distortion. This overvaluation also distorts the marginal rate of substitution between first-period consumption and bequests. We refer to this second distortion as the dynamic distortion, since it affects the willingness to shift resources into the future. Combining the marginal utility of leisure, (40), and (42) we obtain a relationship between the marginal rates of substitution between first-period consumption and leisure of the decentralized and centrally planned economies for the \( i \)-th household,

\[
MRS_{c,i}^{s,m} = \frac{l^i}{\mu (c^i - \gamma \bar{c})} > \frac{l^i (1 - A(c^i - \gamma \bar{c}))}{\mu (c^i - \gamma \bar{c})} \equiv MRS_{c,i}^{s,p} \tag{44}
\]

Similarly, combining the marginal utility of bequests, (40), and (42) we reach the following relation for the marginal rates of substitution between first-period consumption and bequest of the two solutions,

\[
MRS_{c,b}^{s,\bar{m}} = \frac{b_{i+1}^i}{\alpha (c^i - \gamma \bar{c})} > \frac{b_{i+1}^i (1 - A(c^i - \gamma \bar{c}))}{\alpha (c^i - \gamma \bar{c})} \equiv MRS_{c,b}^{s,p} \tag{45}
\]

As a result of interpersonal comparisons, households overvalue consumption. Therefore, their willingness to substitute from leisure towards consumption is too high and their willingness to substitute from consumption towards bequests is too low relative to the socially desirable levels. Both distortions lead to a competitive solution characterized by over-consumption, over-working and under-saving. This last result contrasts with the standard one obtained under an infinitely lived representative agent, where consumption externalities induce over-accumulation of capital, i.e. over-saving (Fisher and Hof (2000), Liu and Turnovsky (2005), Alvarez-Cuadrado (2007)). Intuitively with perfect foresight and an infinite planning horizon, households want to keep up with the Joneses today and in every future date. In contrast, our framework, by reducing the planning horizon and introducing a non-positional saving motive, reverses the effects of consumption externalities on the saving rate.

Finally, it is important to notice the differential impact of the distortion across the income distribution. Since the overvaluation factor, \( \frac{1}{1 - A(c^i - \gamma \bar{c})} \), increases with consumption\(^{12}\) See Appendix II for a formal proof.
(income), the relative size of the adjustment made by the planner on the private marginal utility of consumption is larger for high income households. This just reflects the fact that wealthy individuals, with their high levels of consumption, contribute in a disproportionate way to average consumption, creating substantial welfare losses for their neighbors.

### 3.4 Optimal Tax Policy

A competitive economy, where agents are concerned with relative consumption, is characterized by over-consumption, under-saving, and over-working. Under these circumstances the government can restore efficiency by means of distortionary taxation. Combining (3) and (4) we obtain the following lifetime budget constraint for the \( i \)-th household,

\[
w(1 - l^i) (1 - \tau^i_l) + b^i + T^i = (1 + \tau^i_c) \left( c^i + \frac{d^i}{R} \right) + (1 + \tau^i_b) \frac{b^i_{t+1}}{R}
\]  

(46)

where \( \tau^i_l, \tau^i_c, \) and \( \tau^i_b \) denote taxes (subsidies if negative) on labor, consumption, and bequests respectively. Tax revenues are returned to families in the form of lump sum transfers, \( T^i \).

Under the proposed tax structure, we find the relevant marginal rates of substitution for the competitive solution and we equate them to the efficient ones, given by (44) and (45), reaching,

\[
\frac{l^i (1 - \tau^i_l)}{\mu (c^i - \gamma \bar{c}) (1 + \tau^i_c)} = \frac{l^i (1 - A (c^i - \gamma \bar{c}))}{\mu (c^i - \gamma \bar{c})} 
\]  

(47)

\[
\frac{b^i_{t+1} (1 + \tau^i_b)}{\alpha (c^i - \gamma \bar{c}) (1 + \tau^i_c)} = \frac{b^i_{t+1} (1 - A (c^i - \gamma \bar{c}))}{\alpha (c^i - \gamma \bar{c})}
\]  

(48)

From these equations, we can determine at least two alternative optimal tax packages. In the first package, \( \tau^i_c = \frac{A (c^i - \gamma \bar{c})}{1 - A (c^i - \gamma \bar{c})} \) and \( \tau^i_l = \tau^i_b = 0 \). In the second one, \( \tau^i_c = 0 \), \( \tau^i_l = A (c^i - \gamma \bar{c}) \) and \( \tau^i_b = -A (c^i - \gamma \bar{c}) \). Since concerns for relative consumption lead to over-consumption, over-working and under-saving it is not surprising that the optimal fiscal policy penalizes the first two activities while subsidizing the last. The first package consists of a progressive tax on consumption\(^{13}\). Since high income households contribute to a disproportionate share of average consumption, their consumption is taxed at higher rates than the one of low income households. Frank (2007) proposes a similar tax structure and illustrates its practical implementation using only income and saving data.

\[^{13}\text{We define a progressive tax as one such that its effective rate increases with income. An alternative definition of a progressive tax is one which its effective rate increases as the tax base increases. It is worth noticing that we can use (23) and (27) to express the tax rates as functions of parameters and variables that are exogenous from the standpoint of the individual household.}\]
The second package consists of a progressive tax on labor income combined with a subsidy on inter-generational saving, i.e. bequests. Wealthy households face higher labor income tax rates but their saving are also subsidized at a higher rate.

4 Steady State Distribution of Wealth

We now turn to explore the implications of our model economy for the steady state distribution of wealth. Solving (26) with the initial condition, \( b_0 \), the time path of the average bequest is given by,

\[
\bar{b}_t = (\bar{b}_0 - \bar{b}_\infty) [z(1 - \gamma (1 + \beta) \chi)]^t + \bar{b}_\infty
\]

that converges to the unique steady state given by,

\[
\bar{b}_\infty = \frac{z(1 - \gamma (1 + \beta) \chi)}{1 - z(1 - \gamma (1 + \beta) \chi)} w
\]

provided that \( 0 < z \equiv \frac{\alpha R \beta}{1 + \mu + (1 + \alpha) \beta} < 1 \).

Combining (31) and (49) we reach the following difference equation that governs the evolution of wealth (bequests) for the \( i \)-th dynasty,

\[
b_{i+1} = z(1 - \gamma (1 + \beta) \chi) w + z b_i - z \gamma (1 + \beta) \chi \left\{ (\bar{b}_0 - \bar{b}_\infty) [z(1 - \gamma (1 + \beta) \chi)]^i + \bar{b}_\infty \right\}
\]

Given the initial condition, \( b_0 \), this equation has the following solution,

\[
b_i = (\bar{b}_0 - \bar{b}_\infty) z^i - (\bar{b}_\infty - \bar{b}_0) [z(1 - \gamma (1 + \beta) \chi)]^i + \bar{b}_\infty
\]

which implies that the steady state wealth distribution collapses to a single point, where every individual in our economy eventually inherits the average bequest, \( \bar{b}_\infty \). Finally, it is worth noticing the effect of the degree of interpersonal comparisons, measured by \( \gamma \), on the steady state level of wealth, \( \bar{b}_\infty \). Substituting \( \chi \equiv \frac{1}{1 + (\mu + \alpha \beta)(1 - \gamma) + \beta} \) in (50) it is easy to see that as \( \gamma \) approaches one the steady state level of wealth tends to zero.

The convergence of the wealth distribution to a single point, although surprising at first sight, is simply a restatement of the main result in Stiglitz (1969). The intuition is

\[\text{Footnotes:} 14\] Notice that this restriction is the standard stability condition requiring that the rate at which agents discount the future is large relative to the exogenously given interest rate.

\[\text{Footnotes:} 15\] Notice that, given our assumption of interior solution for all households, there is a tradeoff between \( \gamma \) and the admissible spread of our initial wealth distribution, \( G_0(b) \). In the extreme case where \( \gamma = 1 \) the initial distribution of wealth has to be egalitarian for an interior solution.
best understood in the case of exogenous labor. In that case the stability condition on
the evolution of bequests implies that saving out of labor income, which is equal across all
individuals, is larger than the reference level of consumption. Under these circumstances
the rate of growth of bequests is a decreasing function of wealth and therefore the wealth
distribution eventually collapses. To see this, notice that under exogenous labor supply,
\( \mu = 0 \), the present value of the \( i \)-th individual lifetime consumption is given by,

\[
C^i_t = c^i_t + \frac{d^i_{t+1}}{R} = \tilde{c}w + \tilde{c}b^i_t + \tilde{c}_t
\]

(52)

where \( \tilde{c} \equiv \frac{(1 + \beta)}{1 + \beta + \alpha\beta (1 - \gamma)} = (1 + \beta) \chi \) is the average (and marginal) propensity to
consume out of labor income, \( \tilde{c} \equiv \frac{1 + \beta}{1 + (1 + \alpha) \beta} \) is the average propensity to consume out of
inheritances and \( \tilde{c}_t \equiv \frac{(1 + \beta) \beta \alpha \gamma \chi}{1 + (1 + \alpha) \beta} b_t \) is a time-varying autonomous level of consumption.

We can combine (3), (4), and (52) to reach the lifetime budget constraint for the \( i \)-th individual,

\[
\frac{b^{i+1}_t}{b^i_t} = w + b^i_t - C^i_t
\]

(53)

Combining (52) and (53) we reach the following law of motion for bequests,

\[
\Delta b^i_{t+1} = R \left[ (1 - \tilde{c}) w - \tilde{c}_t + \left( 1 - \tilde{c} - \frac{1}{R} \right) b^i_t \right]
\]

(54)

In order to explore the evolution of the distribution of wealth we can divide (54) by \( b^i_t \) to
obtain its proportional rate of change

\[
\frac{\Delta b^i_{t+1}}{b^i_t} = R \left[ \frac{(1 - \tilde{c}) w - \tilde{c}_t}{b^i_t} + \left( 1 - \tilde{c} - \frac{1}{R} \right) \right]
\]

(55)

It becomes clear that the wealth distribution will eventually converge as long as \( (1 - \tilde{c}) w - \tilde{c}_t > 0 \). Intuitively, since \( 1 - \tilde{c} - \frac{1}{R} \) in (54) is proportional to wealth it has no effect on the
evolution of the wealth distribution. On the other hand if saving out of labor income, which
is the same for rich and poor households, is greater than the reference level of consumption,
which is again the same across households, then the bequest of a poor dynasty grows faster
than the bequest of a rich dynasty, since \( (1 - \tilde{c}) w - \tilde{c}_t \) represents a higher fraction of the
bequest of a poor household than of the bequest of a rich household.

Now we can find under what conditions the "aggregate" bequest, \( b_t \), reaches a stable
steady state. Summing (54) across households we reach,
\[ \Delta b_{t+1} = R \left[ (1 - \hat{c}) w - \bar{c}_t + \left( 1 - \hat{c} - \frac{1}{R} \right) b_t \right] \]  

(56)

which implies that the average bequest eventually achieves a steady state iff \( \left( 1 - \hat{c} - \frac{1}{R} \right) < 0 \). This stability condition together with the steady state condition, \( \Delta b_{t+1} = 0 \), implies that \( (1 - \hat{c}) w - \bar{c}_t > 0 \), so that the wealth distribution eventually collapses to a single point. This result is a consequence of some of the simplifying assumptions underlying our model, specifically the homogeneity of the labor force. In line with Stiglitz’s (1969) results, we believe that the introduction of an heterogeneous labor force would lead to a non-degenerate steady state wealth distribution. We leave this exercise for future research.

5 Conclusions

Despite the theoretical dominance of the permanent income hypothesis, there is a growing body of empirical evidence that finds important departures from its basic predictions. Specifically, recent work by Dynan, et al. (2004) and Altonji and Villanueva (2007) provide strong evidence of a saving rate that increases with permanent income, violating the proportionality hypothesis. Our approach departs from the standard version of the permanent income hypothesis in an intuitively appealing way: in line with recent evidence on self-reported well-being, we abandon the independent preference assumption that underlies Friedman’s analysis. We consider an overlapping-generations economy with heterogenous wealth levels. Individuals derive utility from leisure, relative consumption, and bequest. In this context, the resulting consumption of a household is driven by the comparison of his lifetime income and the lifetime income of his reference group, a permanent income version of the relative income hypothesis. As in Duesenberry (1949), individual saving rates increase with relative income while aggregate savings are independent of the income distribution. Positional concerns lead agents to consume and work above the welfare maximizing levels chosen by a benevolent central planner. We propose a simple tax schedule that induces the competitive economy to achieve the efficient allocation.

In sum, one can think of our specification as replacing Keynes’ "fundamental psychological law" with the principle that men are disposed, as a rule and on average, to be not only "forward-looking" but also "outward-looking" animals\(^{16}\).

\(^{16}\)This refers to Keynes’ (1936, p. 96) well known observation about the "fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income."
APPENDIX I

In this Appendix we explore the more general case where reference groups differ across individuals. Our results are qualitatively similar to the ones obtained under a single reference group. Finding the counterparts of (12), (13), (15), (16), and (17) for the \( i \)-th household born in period \( t \), where \( \bar{g}_t^i \) and \( \bar{o}_{t+1}^i \) are the levels of consumption of his reference group when young and old respectively, we obtain

\[
s_t^i = \beta (1 + \alpha) \left[ c_t^i - \gamma \bar{g}_t^i \right] + \frac{\gamma \bar{o}_{t+1}^i}{R} \quad (57)
\]

\[
c_t^i (1 + \mu + (1 + \alpha) \beta) = \bar{g}_t^i + (\mu + (1 + \alpha) \beta) \gamma \bar{g}_t^i - \frac{\gamma \bar{o}_{t+1}^i}{R} \quad (58)
\]

\[
l_t^i = \frac{\mu}{w} \left[ c_t^i - \gamma \bar{g}_t^i \right] \quad (59)
\]

\[
d_{t+1}^i = \frac{1}{1 + \alpha} \left[ Rs_t^i + \alpha \gamma \bar{o}_{t+1}^i \right] \quad (60)
\]

\[
b_{t+1}^i = \frac{\alpha}{1 + \alpha} \left[ Rs_t^i - \gamma \bar{o}_{t+1}^i \right] \quad (61)
\]

For the sake of illustration, suppose there are only two homogeneous income groups, say \( H \) (rich) and \( L \) (poor), and the population is evenly distributed between these two groups. Veblen (1899), Duesenberry (1949), and Frank (2007) eloquently argue that the behavior of successful individuals or groups set the standard for the rest of the community. Ferrer-i-Carbonell (2005) provides convincing microeconometric evidence on the importance of upward comparisons as a determinant of subjective well-being. In line with this evidence, we assume that the reference group of the rich households is made up only of rich households while the reference group of poor households is composed of a weighted average of poor and rich households, with \( \rho \) being the weight placed on poor households. As a result, reference consumption levels for the two groups are given respectively by

\[
\bar{g}_t^H = c_t^H \quad \text{and} \quad \bar{o}_{t+1}^H = d_{t+1}^H \quad (62)
\]

\[
\bar{g}_t^L = \rho c_t^L + (1 - \rho) c_t^H \quad \text{and} \quad \bar{o}_{t+1}^L = \rho d_{t+1}^L + (1 - \rho) d_{t+1}^H \quad (63)
\]

We proceed sequentially. First we solve (57)-(61) together with (62), noting that \( c_t^H = c_t^H \) and \( d_t^H = d_{t+1}^H \), to obtain the optimal choices of the rich households,
\[ c^H_t = \chi \tilde{y}^H_t \]  
(64)

\[ s^H_t = \beta (1 + (1 - \gamma) \alpha) \chi \tilde{y}^H_t \]  
(65)

\[ l^H_t = \frac{\mu (1 - \gamma) \chi \tilde{y}^H_t}{w} \]  
(66)

\[ d^H_{t+1} = R \beta \chi \tilde{y}^H_t \]  
(67)

\[ b^H_{t+1} = (1 - \gamma) \alpha R \beta \chi \tilde{y}^H_t \]  
(68)

Then we solve (57)-(61) together with (63), (64), and (67), noting that \( c^L_t = \tilde{c}^L_t \) and \( d^L_t = \tilde{d}^L_{t+1} \), to obtain the optimal choices of the poor households,

\[ c^L_t = \psi \left[ \tilde{y}^L_t + \gamma (\mu + \alpha \beta) (1 - \rho) \chi \tilde{y}^H_t \right] \]  
(69)

\[ s^L_t = \psi \left[ \beta (1 + (1 - \gamma \rho) \alpha) \tilde{y}^L_t + (\mu - \alpha) \beta \gamma (1 - \rho) \chi \tilde{y}^H_t \right] \]  
(70)

\[ l^L_t = \frac{\mu \psi \left[ (1 - \gamma \rho) \tilde{y}^L_t - (1 + \beta) \gamma (1 - \rho) \chi \tilde{y}^H_t \right]}{w} \]  
(71)

\[ d^L_{t+1} = \psi \left[ R \beta \tilde{y}^L_t + \gamma (1 - \rho) R \beta (\mu + \alpha \beta) \chi \tilde{y}^H_t \right] \]  
(72)

\[ b^L_{t+1} = \psi \left[ R \beta \alpha (1 - \gamma) \tilde{y}^L_t - (1 - \rho) R \beta \gamma \alpha (\beta + 1) \chi \tilde{y}^H_t \right] \]  
(73)

where \( \psi \equiv \frac{1}{1 + (\mu + \alpha \beta) (1 - \gamma \rho) + \beta} \). As before, we calculate actual lifetime income as follows,

\[ y^H_t = w \left( 1 - l^H_t \right) + b^H_t = [1 + \beta (1 + (1 - \gamma) \alpha)] \chi \tilde{y}^H_t \]  
(74)

\[ y^L_t = w \left( 1 - l^L_t \right) + b^L_t = \psi \left[ (1 + \beta (1 + (1 - \rho \gamma) \alpha)) \tilde{y}^L_t + \mu \gamma (1 - \rho) \chi \tilde{y}^H_t \right] \]  
(75)

and we obtain the following relation between the saving rates, out of actual lifetime income, of poor and rich households,
\[
\frac{s^H_t}{y^H_t} = \frac{\beta (1 + (1 - \gamma) \alpha)}{1 + \beta (1 + (1 - \gamma) \alpha)} > \frac{s^L_t}{y^L_t} = \frac{\beta (1 + (1 - \gamma_p) \alpha) \bar{y}^L_t + (\mu - \alpha) \beta \gamma (1 - \rho) \chi \bar{y}^H_t}{(1 + \beta (1 + (1 - \rho \gamma) \alpha)) \bar{y}^L_t + \mu (1 + \beta) \gamma (1 - \rho) \chi \bar{y}^H_t}
\]

(76)

since \(s^H_t y^L_t > s^L_t y^H_t\). Therefore, as in the presence of an homogeneous reference group, poor households save and bequeath smaller fractions of their lifetime income than rich households do. Finally, we can show that the distribution of wealth that converges to a single point given by,

\[
b^H_t = b^L_t = \frac{z(1 - \gamma (1 + \beta) \chi)}{1 - z(1 - \gamma (1 + \beta) \chi)}w
\]

(77)

**APPENDIX II**

Our goal is to show that the overvaluation factor for the \(i\)-th individual remains constant through his lifetime, i.e.

\[
\frac{1}{1 - A (c^{i,p} - \gamma c^p)} = \frac{1}{1 - D (d^{i,p} - \gamma d^p)}
\]

It is easy to see that, under laissez-faire,

\[
\frac{d^{i,m} - \gamma d^m}{c^{i,m} - \gamma c^m} = \beta R
\]

We now show that under the social planner a similar relationship holds:

\[
\frac{d^{i,p} - \gamma \bar{d}^p}{c^{i,p} - \gamma \bar{c}^p} = \beta R
\]

(78)

Let

\[
v^i \equiv \frac{1}{c^{i,p} - \gamma \bar{c}^p}
\]

\[
z^i \equiv \frac{1}{d^{i,p} - \gamma d^p}
\]

Then equations (37) and (38) give

\[
\frac{v^i - \frac{1}{n} \sum_{j=1}^{n} v^j}{z^i - \frac{1}{n} \sum_{j=1}^{n} z^j} = \beta R
\]

Hence

\[
v^i - \frac{\gamma}{n} \sum_{j=1}^{n} v^j = \beta R \left[ z^i - \frac{\gamma}{n} \sum_{j=1}^{n} z^j \right]
\]

(79)

Summing (79) over all \(i\), we get

\[
(1 - \gamma) \sum_{i=1}^{n} v^i = \beta R (1 - \gamma) \sum_{j=1}^{n} z^j
\]

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Hence
\[ \sum_{j=1}^{n} v^j = \beta R \sum_{j=1}^{n} z^j \quad (80) \]

Replacing (80) in (79) we get
\[ v^i = \beta R z^i \]

Thus (78) holds and therefore from (80)
\[ \beta R = \frac{A}{D} \]

Hence (78) gives
\[ A (e^i - \gamma \bar{c}) = D (d^i - \gamma \bar{d}) \]
References


