Quest for Hegemony Among Countries and Global Warming∗

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Abstract

This paper suggests the possibility of a relationship between global warming and the quest for hegemony among countries. We build a game model which starts from the assumption that, in a world with nuclear weapons and uncertainty, a country behaves in a way such as to improve its probability to reach the hegemonic position by its economic strength. The results highlights the crucial role of both country's endowment which is not linked to greenhouses gas emissions and the prize obtained by a country if it reaches the hegemonic position. The results predicts the possibility of an increase in global warming as the numbers of countries in quest for hegemony increases. More over the model shows the possibility that an inequality reduction could increase global warming.

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1 Introduction

1.1 At the heart of the debate over global warming

Concerns about global warming have led to a considerable body of studies. But an important question which has not yet been explored has to do with the relationship between global warming and the quest for the hegemonic role in the international state system. The nuclear revolution has made unlikely the use of a new world war by a country as a route for acquiring the hegemonic role in the world scene, as it was the case in the last centuries. Now, economic development has become the most important source of power\(^1\) in determining the primacy or the subordination of states. A book by Throup (1993) discussed the economic battle for supremacy in the 21st century. See also Huntington (1993),\(^2\) Waltz (1993),\(^2\) Agnew and Corbridge (1995), Leiva (2007), Frieden and Lake (2000), Knorr (1973), Palan et al. (1996), Brawley (2005), Crawford (1977)

But economic and ecological systems are totally interlocked because the most important part of greenhouse gases released into the atmosphere comes from the combustion of fossil fuels) which is the driving force of the economic system (see Stern (2007), Azomahou et al. (2006).

In its turn, increasing economic strength of a country reinforces its political influence in the international system. So, because economic development impacts both the global environmental system and the political ranking in the international system, the fight against global warming may be seen as a dilemma. This dilemma could be seen as a conflict between the goal of the quest for hegemony, in which the main strategy of the players is to reach the economic dominance, and the goal of the reducing the level of greenhouse gases emitted. This dilemma has been recognized by John Marburger III, the White House science adviser in the adminis-

\(^1\)In July 1971, the US president Richard Nixon repeated his opinion to a group of news-media executives in Kansas City that economic power will be the key to other kinds of power and will determine the future of the world.

\(^2\)Waltz (1993) argues that "Now, without a considerable economic capability no state can hope to sustain a world role."
1.2 Increasing number of players in the hegemony game

The quest for hegemony is the quest for the highest political status among countries. Status is defined as a ranking based on traits, assets and actions. The importance of status seeking for explaining behavior has been the subject of various studies in economic literature\(^3\). The country which has the most important political influence in the international state system, and then establishes the rules in the international system is called the hegemon (Rapkin (1990)). History showed that the quest for becoming the hegemonic role is an ever-present of international politics among countries, see Kennedy (1988), Nye (1990), Kennedy (1987), Black (2007), Mosher (2002), Modelski (1987), Ikenberry (2008b). Economic outcomes are a source of hegemonic power in the international system because they helped to gain legitimacy and military power over other states by the achievement of a degree of control over other states, by promotion of the capabilities of friendly and allied countries, weakening the capabilities of opponents states (embargo on the export goods), for coercion, for influencing behavior of the weaker actor. (Knorr (1973), Waltz (1993)\(^4\)).

The number of countries in the set of hegemonic pretenders is increasing. After World War II,


\(^4\)One of the most prominent scholars of international relations has written that "For a country to choose not to become a great power is a structural anomaly. For that reason, the choice is a difficult one to sustain. Sooner or later, usually sooner, the international status of countries has risen in step with their material resources. Countries with great-power economies have become great powers, whether or not reluctantly" (Waltz (1993), page 66)
the United States, with half world’s GNP, the United States ascended to a role of the hegemon in the international state system. But, since the year 1970s, the United States is facing China, India, Brazil, Russia which have made their presence more decisive in the international affairs. See Ikenberry (2008a), Schaffer and Mitra (2005), Elliott (2007), Shenkar (2005). These countries are transforming the hegemony game into a multi-player game (see Shenkar (2005), page 162). Goldman Sachs has published a study by Wilson and Purushothaman (2003) which places China as the world’s largest economy by 2041. It forecasts that only the United States and Japan will still be among the six largest economies by 2050. Another comments from governments show that the emergence of China and India is a source of concern in developed countries

1.3 Consequences on global warming

Economic development is largely based on the fossil fuel energy Chombot (1998), Azomahou et al. (2006), and on several occasions, the authorities of several countries declared that they will not sacrifice their economic development for fighting against global warming. On June 2008, The International Energy Agency (IEA) warned that if countries continue with policy in place to date, CO2 emissions would rise by 130% and oil demand would jump by 70% by the middle of this century. In May 2007, a study by Raupach et al. (2007) found that the annual growth in global CO2 emissions caused by human activity has jumped from an average 1.1% for 1990-1999 to more than 3% for 2000-2004. Emissions increased between 2000 and 2005, at

5The government of Canada has published a book titled Canada's International Policy Statement: A Role of Pride and Influence in the World. In the overview, the government of Canada said that: we recognize that emerging giants, such as China, India and Brazil, are already making their presence felt. Their growing influence - particularly in the economic realm - carries significant implications for Canada. ... If Canada stands idle while the world changes, we can expect our voice in international affairs to diminish.
A similar analysis was made by the National Intelligence Council (2004) which writes that The "arriviste" powers - China, India, and perhaps others such as Brazil and Indonesia - have the potential to render obsolete the old categories of East and West, North and South, aligned and nonaligned, developed and developing. Traditional geographic groupings will increasingly lose salience in international relations. ... Competition for allegiances will be more open, less fixed than in the past
Can the quest for hegemony observed during the last decades, and based on economic competition, have an impact on global warming? This paper gives a theoretical support to answering this question. We will develop a differential game model with countries as players. Our modeling is inspired by the framework developed by Lee and Wilde (1980) for R&D race (another interesting paper in the same way is Reinganum (1982)). Our paper differs from them in several dimensions. Firstly we are not interested by the same question or research field. Secondly, in our model cost function takes into account global warming aspects, as the damage from the stock pollutants and the accumulation dynamic of greenhouses gases in the environment. Thirdly we obtain technical results which are different from the ones derived in their model, fourthly, some parameters are interpreted by referencing to the geopolitical aspect of the question. It’s is organized as follows. Section 2 presents the model and focuses on the steady state of this game. Final section offers concluding remarks.

2 The Model

2.1 Model specification

Consider $N$ countries which are competing to reach the high political position in the international state system, also called the hegemonic position. Each country produces a single commodity for which output is denoted $Q_i(t)$. The probability to reach the characteristics which are required to reach the hegemonic position increases with its output $Q_i(t)$ (economic strength). Production gives rise to the emission $e_i(t) = F(Q_i(t))$. Each country suffers from the global level of pollution. For simplicity, it’s assumed that $e_i(t) = Q_i(t)$, one unit of output gives rise to one unit of pollution.

To visualize the conditions required to be the winner of the game, we can use Greek foot race analogies. The first condition to win the game is to getting to the finish line. The second
condition is to be the first among all players to cross the finish line. The prize of the winner is assumed greater than the prize of the losers. The hegemony game is as an uncertain race which uses economic outcomes. These elements of the hegemonic game are detailed below.

2.1.1 Hegemon’s status prize (winner) and prize of others players (losers)

The advantages to reach the hegemon’s position are prestige and the possibility of setting up on a world scale a whole of standards (political, cultural, economic) in organizing the world. We borrow from the paper of Moldovanu et al. (2007) the notion of "pure status" prize which is related to the intuition that a contestant is happier when he has others countries below him. Hegemon state enjoys "structural power" which Nye (1990) called the "soft power". This structural power permits the hegemon to occupy a central position within the system and to play a leading role in it. Hence the hegemon enjoys a "pure status" prize $A$ of being at date $t$ in the top status category. For simplicity, $A$ is assumed to be constant over time. Denote by $B$ the value of the "pure status" prize derived by any country other than the hegemon. $B$ is also assumed constant. The assumptions of constant $A$ and $B$ can be relaxed by, for example, multiplying each of the two constants by $\exp(mt)$ where $m > 0$ ($m < 0$) is the rate of growth (decline) of the respective "pure status" prize. Assume $A > B$.

2.1.2 Getting to the finish line:

The finish line is constituted, for example, by an economic high level and some others factors that are not directly modeled as good diplomacy, political and social stability, health security. Characteristics levels that constitute the finish line is assumed to be known by all countries. The time it will take for country $i$ to cross the finish line characteristics is a random variable $\tau_i$. Uncertainty about the finishing time $\tau_i$ is determined by what some factors that Cioffi-Revilla (1998) named causal "hazard forces". The causal hazard forces consist of economic, social,
technological or strategic drives that act on countries. Some factors may originate from states (human capital, physical conditions as droughts or others naturals disasters, etc). Others may originate from decisional acts (production). The hazard rate concept provides a useful solution to measure the impact of causal "hazard forces". By definition, the hazard rate \( H_i(t) \) is the ratio of the probability that the value of \( \tau_i \) will be \( t \) (i.e that country \( i \) reach finishing line at \( \tau_i = t \)), relative to the probability that \( \tau_i \) will have greater than \( t \) (i.e that country \( i \) reach finishing line after \( t \)).

In this model, the hazard rate function

\[
H_i(t) = \frac{P_i(t < \tau_i \leq t + dt)}{P_i(t > \tau_i)}
\]

is interpreted as the propensity for country \( i \) to reach the finishing line at time \( t \). The Hazard rate is widely used in many domains and is known by others names as: dissolution rate in comparative politics, intensity function in stochastic processes, instantaneous failure rate in systems reliability, instant mortality rate in demography.

Assume that \( H_i(t) \), which can be interpreted has the propensity for country \( i \) to reach the finishing at time \( t \) is proportional to production or emission. In this game there is a common belief that economy strength is a necessity in the quest for hegemony among countries.

For simplicity take the hazard rate \( H_i(t) = e_i(t) \), then cumulative density function \( F_i(t) \) is given by

\[
F_i(t) = 1 - \exp \left[ - \int_0^t H(u)du \right] = 1 - \exp \left[ - \int_0^t e_i(u)du \right]
\]

(2.1)

It means that the cumulative density function increases with the country \( i \)'s cumulative emissions on the interval \([0, t]\) given by the term \( \int_0^t e_i(u)du \).

Assume that the finish times \( \tau_1, \tau_2, ..., \tau_N \) are independents random variables (In the quest for hegemon status, countries act independently and in pure self-interest).
2.1.3 Crossing the finish line first

The hegemon is the first country to cross the finish line. It’s assumed that each country believes that all the countries are in quest for hegemon’s position in international system. Countries aim to cross the finish line in first position. The first country to cross the finish line becomes the hegemonic role in the international system for-ever. Which country would cross the finish line first in quest? The time instant at which one of the countries become the hegemon is a random variable and is given by $\tau = \min_{i=1,\ldots,N} \tau_i$

It’s the stopping time of this game. It depends stochastically of the countries strategies profile ($e_1(t),\ldots,e_N(t)$). The uncertainty about $\tau$ is consistent with the fact that history of nations has shown that the instant of change in the hegemon’s position among countries is uncertain. Given a profile ($e_1(t),\ldots,e_N(t)$) of emissions levels and a common discount rate $r$, a country $i$ evaluate the instantaneous probability to be the hegemon or not at every time $t$. The instantaneous probability for a country $i$ to win the hegemon’s position on infinitesimal interval $[t,t+dt]$ is computed as:

$$P(t \leq \tau_i \leq t + dt)/\tau_i > t) \times P(\tau_2 > t) \times \cdots \times P(\tau_N > t) = \dot{F}_i(t)dt \times \prod_{j \neq i}^{N} [1 - F_j(t)]$$

The instantaneous probability for a country $i$ to loose is the probability that another of the countries $\{1,...i-1,i+1,...N\}$ win the hegemon’s position between on infinitesimal $[t,t+dt]$. It’s given by the following expression

$$\sum_{j=1,j \neq i}^{N} \dot{F}_j(t)dt \left( \prod_{k=1,k \neq j}^{N} [1 - F_k(t)] \right)$$

Note that the sum of the probability that country $i$ win or loose is not equal to one. It’s means that another possibility is that no country win the race between $[t,t+dt]$
2.1.4 Dynamics of the pollution stock (global warming)

Each Country suffers from the global level of pollution stock. When emissions accumulate, the damage from emissions today is not just the damage today, but the damage in future time periods when today’s emissions are still resident in environment. There is a wide array of cases in what pollution is aggregated and accumulates over time Hence assume that at any given time $t$ emission add to the stock of pollution, denoted by $S(t)$ according to the process

$$\dot{S}(t) = e_1(t) + \ldots + e_N(t) - kS(t) \quad (2.2)$$

where $k > 0$ is the coefficient of natural purification, and $S(0)$ is given. This assumption for dynamic for stocks pollutants is standard in natural resource and environmental economics (see Kolstad (2000), Dockner and Nishimura (1999)). Further assume that pollution damage depends only on $S(t)$. The dependence of this damage on the total pollution stock $S(t)$ gives to the problem another international and externality dimension. Following the literature, (see Dockner and Long (1993)) the damage function is assumed nonlinear increasing convex and for simplicity quadratic.

$$D_i(S(t)) = \frac{b}{2}S(t)^2 \quad (2.3)$$

where $b$ is a positive constant.

2.1.5 Endowment of a country as its Non-Greenhouses Gas emitting resources

In what follows, we define $\pi_i$ as a permanent payoff determined by country factor endowment which is not linked to greenhouses gas emissions. It’s an exogenous parameter that captures the idea of disparity between countries. Countries are assumed to differ solely by the payment of their factor endowments (Non-Greenhouses Gas emitting resources) at each date.
Let the game be played over an infinite horizon. In this model, once a country reach the hegemon’s position, the game ends.

3 A general formulation of the hegemony game

The solution of the dynamic decision problem facing by a country i in quest for hegemony is given by the value function \( V_i(F_1(t), ..., F_N(t), S(t)) \) defined as

\[
V_i(F_1(t), ..., F_N(t), S(t)) = \max_{e_i(t)} \int_0^\infty \exp(-rt) \left\{ A \dot{F}_i(t) \prod_{j \neq i} [1 - F_j(t)] + B \sum_{j \neq i} \left( \dot{F}_j(t) \prod_{k \neq j} [1 - F_k(t)] \right) \right. \\
\left. + (-D_i(S(t)) + \pi_i) \prod_{j=1}^N [1 - F_j(t)] \right\} dt \\
\text{Subject to:} \\
\dot{S}(t) = e_1(t) + ... + e_N(t) - kS(t), \\
H_i(t) = e_i(t) \\
H_{-i}(t) = (e_1(t), ..., e_{i-1}(t), e_{i+1}(t), ..., e_N(t))
\]

(3.1)

where \( k > 0 \) is the coefficient of natural purification, and \( S(0) \) is given, \( H_i(t), \ i=1,...,N \) is the hazard rate of country \( i \).

Recall that there is a well known relationship between hazard rate function and cumulative density function.

It’s a deterministic N-player differential game with controls variables \( e_1(t), ..., e_N(t) \) and state variables \( F_1(t), ..., F_N(t), S(t) \).

This objective functional of country \( i \) consists of three terms. The first reflects net benefits if this country \( i \) succeeds in the quest for hegemon’s position. The second term is the net benefits if country \( i \) looses the quest for hegemon position. The third term represents the pollution damage and endowments at time \( t \) when countries are the hegemon’s quest. All
three components are weighted by their corresponding probabilities.

### 3.1 Dynamic optimization

To analyze the game, let’s introduce the state transformation

\[- \log(1 - F_i(t)) = \int_0^t e_i(u) du = Z_i(t)\]

Differentiating with respect to time gives

\[\dot{Z}_i(t) = e_i(t)\]

Using the expression

\[1 - F_i(t) = \exp \left[ - \int_0^t e(u) du \right]\]

\[= \exp \left[ - Z_i(t) \right]\]

The value function of the country \(i\) is given by

\[V_i(Z_1(t), ..., Z_N(t), S(t)) = \max_{e_i(t)} \int_0^\infty \exp(-rt) \exp \left( - \sum_{j=1}^N Z_i(t) \right) \left[ Ae_i(t) + B \sum_{j \neq i} e_j(t) - \frac{b}{2} S(t)^2 + \pi_i \right] \, dt\]

Subject to:

\[\dot{S}(t) = e_1(t) + \ldots + e_N(t) - kS(t), \]

\[\dot{Z}_i(t) = e_i(t), \]

\[\dot{Z}_{-i}(t) = (e_1(t), ..., e_{i-1}(t), e_{i+1}(t), ..., e_N(t))\]

\[(3.4)\]

It’s suppose that countries know the full state vector \((Z_1(t), ..., Z_N(t))\). But only a function of it, namely the one-dimensional state variable \(N(t) = e^{-\left(\sum_{j=1}^N Z_i(t)\right)t}\) has an effect on payoff or is payoff relevant.

Differentiating \(N(t)\) with respect to time gives a single state equation.

\[\dot{N}(t) = -N(t) \left( \sum_{j=1}^N e_i(t) \right)\]
Then, the value function of the country \( i \) is given by

\[
V_i(N(t), S(t)) = \max_{e_i(t)} \int_0^\infty \exp(-rt)N(t) \left[ Ae_i(t) + B \sum_{j \neq i} e_j(t) - \frac{b}{2} S(t)^2 + \pi_i \right] dt
\]

Subject to:

\[
\dot{S}(t) = e_1(t) + \ldots + e_N(t) - kS(t)
\]

\[
\dot{N}(t) = -N(t) \left( \sum_{j=1}^N e_i(t) \right)
\]

The present value of hamiltonian for country \( i \) is given by

\[
\Phi_i(t) = N(t) \left[ Ae_i(t) + B \sum_{j \neq i} e_j(t) - \frac{b}{2} S(t)^2 + \pi_i \right] + \mu_i(t) \left[ e_1(t) + \ldots + e_N(t) - kS(t) \right]
\]

\[- \lambda_i(t) \left[ N(t) \left( \sum_{j=1}^N e_i(t) \right) \right]
\]

(3.6)

\( \mu_i(t) \) is the contribution which an additional unit of pollution stock state variable \( S(t) \) would make to the change in the value function at the beginning of the period \( t \). It’s also refereed as the shadow price of a unit pollution stock. \( \lambda_i(t) \) is the contribution which an additional unit of variable \( N(t) \) would make to the change in the value function at the beginning of the period \( t \). It’s also refereed as the shadow price of a unit of \( N(t) \). \( e_i(t) \) is required to be nonnegative.

The optimal response for a country \( i \) to the rivals emissions satisfies the following necessary
conditions (obtained by applying the maximum principle to the hamiltonian).

\[
\frac{\partial \Phi}{\partial e_i} = [A - \lambda_i(t)] N(t) + \mu_i(t) \leq 0
\]

\[
\frac{\partial \Phi}{\partial e_i} e_i(t) = 0, \ e_i(t) \geq 0
\]

\[
\dot{\lambda}_i = -\frac{\partial \Phi}{\partial N}
\]

\[
\dot{\mu}_i = -\frac{\partial \Phi}{\partial S}
\]

\[
\dot{S} = \frac{\partial \Phi}{\partial \mu_i} = e_1(t) + \ldots + e_N(t) - kS(t)
\]

\[
\dot{N} = \frac{\partial \Phi}{\partial \lambda_i} = -N(t) \left( \sum_{j=1}^{N} e_i(t) \right)
\]

where \( i = 1, \ldots, N \)

It follows that, if we Assume an interior solution (or that equilibriums emissions are strictly positive), the optimal best response function \( e_i(t) = e_i \left( \sum_{j\neq i} e_j(t) \right) \) for a country \( i \) to the rivals emissions satisfies the following conditions.

\[
[ A - \lambda_i(t)] N(t) + \mu_i(t) = 0 \quad (3.7)
\]

\[
A e_i(t) + B \sum_{j \neq i} e_j(t) - \frac{b}{2} S(t)^2 + \pi_i \] + \lambda_i(t) \left( \sum_{j \neq i} e_j(t) + e_i(t) \right) = \dot{\lambda}_i(t) \quad (3.8)
\]

\[
b N(t) S(t) + k \mu_i(t) = \dot{\mu}_i(t) \quad (3.9)
\]

\[
\dot{S} = \left( \sum_{j \neq i} e_j(t) + e_i(t) \right) - kS(t) \quad (3.10)
\]

\[
\dot{N} = -N(t) \left( \sum_{j \neq i} e_j(t) + e_i(t) \right) \quad (3.11)
\]

where \( i = 1, \ldots, N \)

So, given the total emission of the rivals, the best response emission path might depends on the familiar economic relation between marginal cost and marginal revenue.

In what follows, we will focus on the steady state outcome of this game.
4 Steady state hegemony game

Definition 1 A steady state equilibrium refers to an equilibrium in which pollution stock \( S(t) \) and hazard rate \( H(t) \) are constant over time.

Imposing that hazard rate is constant (\( H_i(t) = e_i \)) implies that

\[
F_i(t) = 1 - \exp(-e_i t)
\]

which is the exponential cumulative distribution function of the exponential. It’s interesting to note that some examples of constant hazard rate are present in world politics in what it’s called the "anarchy condition" in the international system. It states that anarchic force in the international system changes only in the run. For example after year 1648, the war hazard rate has been approximatively constant, see Cioffi-Revilla (1998).

Imposing that pollution stock is constant over time implies (\( \dot{S}(t) = 0 \)), and then \( S = \frac{\theta_i + \theta}{k} \) where \( \theta = \sum_{j \neq i} e_j \) and \( D_i(S) = \frac{b}{2}(\frac{\theta_i + \theta}{k})^2 \).

Taking \( b = 1 \) and \( k = 1 \) gives

\[
D_i(S) = \frac{1}{2}(e_i + \theta)^2
\]

Clearly, we are concerned here with a damage function which depends on stocks. The accumulation of pollution stock produces a significant increment on the environmental damage.

The expected present value of pure status payoff of country \( i \) becomes

\[
\int_0^\infty e^{-(\sum_{j=1}^N e_j + r)t} \left[ Ae_i + B \sum_{j \neq i} e_j - \frac{1}{2}(\sum_{j=1}^N e_j)^2 + \pi_i \right] dt
\]

However, although we suppose that countries know the full state vector \( (e_1 t, ..., e_N t) \), only a function of it, namely the one-dimensional state variable \( \sum_{j=1}^N e_j t \) is payoff-relevant. The
criterion of payoff-relevance says that players condition their actions only upon variables that influence their payoffs. This suggests that countries would not condition their strategies on $(e_1t, ..., e_Nt)$, but rather on $\sum_{j=1}^{N} e_jt$.

Denote by $\Omega(e_i/\theta)$, the country $i$'s payoff, given $\theta = \sum_{j \neq i} e_j$, the total emission of greenhouse gas emitted by other countries.

\[
\Omega(e_i/\theta) = \int_0^{\infty} \exp(e_i + \theta + r)t \left[ Ae_i + B\theta - \frac{1}{2}(e_i + \theta)^2 + \pi_i \right] dt \quad (4.2)
\]

\[
= \frac{Ace_i + B\theta - \frac{1}{2}(e_i + \theta)^2 + \pi_i}{e_i + \theta + r} \quad (4.3)
\]

Country $i$’s best response function gives, for each possible emissions levels of other countries, the expected payoff-maximizing emission of country $i$. Country $i$’s payoff-maximizing emission when others countries total emission is $\theta$ is the output $e_i(\theta) \geq 0$ that maximizes country $i$’s expected payoff.

\[
e_i(\theta) = \arg \max_{e_i} \Omega(e_i/\theta) \text{ subject to } e_i \geq 0
\]

The first derivative with respect to $e_i$ (treating $\theta$ as constant) is

\[
\frac{d\Omega(e_i/\theta)}{de_i} = -\frac{1}{2}(e_i + \theta)^2 - r(e_i + \theta) + 2(A - B)\theta + 2(Ar - \pi_i)
\]

The second derivative with respect to $e_i$ (treating $\theta$ as constant) gives

\[
\frac{d^2\Omega(e_i/\theta)}{de_i^2} = -\frac{(e_i + \theta + r)\Delta(\theta)}{(e_i + \theta + r)^4}
\]

where

\[
\Delta(\theta) = 2(A - B)\theta + r^2 + 2(Ar - \pi_i)
\]

Since $e_i \geq 0$, the concavity of the payoff function will depend uniquely of sign of the term $\Delta(\theta)$ which is the discriminant of the first order condition for an interior solution.
If the payoff function is concave, the first-order necessary conditions is given by \( \frac{d\Omega(e_i/\theta)}{de_i} \leq 0 \) with \( e_i \geq 0 \):

\[-\frac{1}{2}(e_i + \theta)^2 - r(e_i + \theta) + (A - B)\theta + Ar - \pi \leq 0 \text{ with } e_i \geq 0\]

To interpret this condition, rearrange terms and write:

\[D'(e_i + \theta) \leq A - \Omega(e_i/\theta) \quad (4.4)\]

This equation means that in equilibrium, emissions level of greenhouse gases is obtained when the marginal damage is equal the net benefit.

For an interior solution, the first order equation can be written as

\[e_i + \theta - A + \frac{Ae_i + \theta B - \frac{1}{2}(e_i + \theta)^2 + \pi_i}{e_i + \theta + r} = 0 \quad (4.5)\]

**Remark 1** The concavity of the payoff function depends on the sign of the discriminant

\[\Delta(\theta) = 2(A - B)\theta + r^2 + 2(Ar - \pi_i)\]

For interpretation, we will distinguish two cases: The case where the permanent endowment of the country \( i \) is such that \( Ar - \pi_i \geq 0 \) (or \( \frac{\pi_i}{r} \leq A \)) and the case here the permanent endowment of the country \( i \) is such that \( Ar - \pi_i \leq 0 \) (or \( \frac{\pi_i}{r} \geq A \)). Hence heterogeneity is introduced differentiating countries by types, each type identified by the position of the discount permanent endowment relative to the hegemonic prize \( A \), in other words, \( \frac{\pi_i}{r} \leq A \) or \( \frac{\pi_i}{r} \geq A \).

## 5 Best response functions

In what follows, we will call players with permanent endowment such that \( \frac{\pi_1}{r} < A \), "countries of type 1" and players which permanent endowment such that \( \frac{\pi_2}{r} \geq A \), "countries of type 2".
Proposition 5.1 Consider a country of type 1 or in others words with \( \frac{\pi_1}{r} < A \).

Then, the best response function of this country is given by

\[
e_1(\theta) = \begin{cases} 
-r - \theta + \sqrt{2\theta(A - B)} + r^2 + 2Ar - 2\pi_1 & \text{if } \theta \leq \tilde{\theta}_1 \\
0 & \text{if } \theta \geq \tilde{\theta}_1
\end{cases}
\]

where

\[
\tilde{\theta} = \frac{-2(r - (A - B)) + \sqrt{4(r - (A - B))^2 + 8(Ar - \pi_1)}}{2}
\]

Proof. See Appendix

![Figure 1: Best response function \( e_1(\theta) \) of country with type 1](image)

This proposition says that a country of type 1 participates at the game because the payoff is important at its point of view. But it’s not optimal for this country to play a positive emission when rival’s emissions are large.

Proposition 5.2 Consider a country of type 2 or in others words with \( \frac{\pi_2}{r} \geq A \).

If the following conditions are satisfied:

\[
-\frac{r^2 - 2(Ar - \pi_2)}{2(A - B)} \geq 0, \quad \frac{A-B}{r} > 1, \quad r \left(1 - \frac{A-B}{r}\right)^2 + 2(A - \frac{\pi_2}{r}) > 0.
\]
Then the best response function of this country is given by

\[
e_2(\theta) = \begin{cases} 
0 & \text{if } \theta \in [0, \tilde{\theta}_2] \\
-r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi^2} & \text{if } \theta \in [\tilde{\theta}_2, \tilde{\tilde{\theta}}_2] \\
0 & \text{if } \theta \in [\tilde{\tilde{\theta}}_2, +\infty] 
\end{cases}
\]

where

\[
\tilde{\theta}_2 = \frac{-2(r - (A - B)) - \sqrt{(2(r - (A - B)))^2 + 8(Ar - \pi^2)}}{2} \\
\tilde{\tilde{\theta}}_2 = \frac{-2(r - (A - B)) + \sqrt{(2(r - (A - B)))^2 + 8(Ar - \pi^2)}}{2}
\]

Proof. See Appendix

![Figure 2: the non zero best response function $e_2(\theta)$ of a country with type 2](image)

**Proposition 5.3** Consider a country of type 2, in others words with $\frac{\pi^2}{r} \geq A$.

If one of the following conditions are satisfied
1. \( \frac{-r^2 - 2(A - \pi_2)}{2(A - B)} < 0 \), (then \( \Omega(e_2/\theta) \) is convex and decreasing function),

2. \( \frac{-r^2 - 2(A - \pi_2)}{2(A - B)} \geq 0 \), and \( r \left(1 - \frac{A - B}{r}\right)^2 + 2(A - \frac{\pi_2}{r}) \leq 0 \) (then \( e_2(\theta) \) is always negative)

3. \( \frac{-r^2 - 2(A - \pi_2)}{2(A - B)} \geq 0 \), \( \frac{A - B}{r} < 1 \) (then if the exist, zeros of \( e_2(\theta) \) are negatives and then the function is negative for \( \theta > 0 \).

Then, the best response function of this country is given by

\[ e_2(\theta) = 0 \text{ for every } \theta \]

Note that when the third condition implies the second condition. So the real number of eventual conditions are reduced to two. The first and the second one.

The intuition in the second case is that, country of type 2 considers that the gap between the hegemon’s prize and the losers is not enough big and he prefers to play zero.

The intuition in the first case is that country of type 2 is rich but he is not enough rich, so he prefers to play 0.

Figure 3: the zero Best response function \( e_2(\theta) \) of a country with type 2
Remark 2 The table below resumes the best responses of a country of type 2.

When

\[ \frac{-r^2 - 2(Ar - \pi_2)}{2(A - B)} \geq 0 \]

We have seen that

<table>
<thead>
<tr>
<th></th>
<th>(1 &gt; \frac{A-B}{r})</th>
<th>(1 &lt; \frac{A-B}{r})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r (1 - \frac{A-B}{r})^2 + 2(A - \frac{\pi_2}{r}) \leq 0)</td>
<td>BRF=0</td>
<td>BRF=0</td>
</tr>
<tr>
<td>(r (1 - \frac{A-B}{r})^2 + 2(A - \frac{\pi_2}{r}) &gt; 0)</td>
<td>BRF=0</td>
<td>BRF \neq 0</td>
</tr>
</tbody>
</table>
The following figure summarizes graphically the best response function (BRF) of country of type 1 and country of type 2 in all the cases.

![Graph of Best Response Functions]

**Remark 3** In the region between where \( A \leq \frac{\pi_1}{r} \), country of type 1 always plays a non trivial best response function (identically different from zero). This country behaves like a small hungry dog.

In the region between where \( A \leq \frac{\pi_2}{r} \leq A + \frac{r}{2} \), country of type 2 is "just rich and is very happy" with endowment near the hegemon prize, so he plays the trivial best response function
(zero). He is satisfied on his position. This country behaves as a **puppy dog**.

In the region between where \( A + \frac{r}{2} < \frac{\pi}{2} \) and \( 2r \left( 1 - \frac{A-B}{r} \right)^2 + 8(A - \frac{\pi}{r}) \leq 0 \), country of type 2 considers that he is "very rich" because the gap prize is not sufficiently big for him, so he plays zero. This country behaves like a **fat cat**.

In the region between where \( A + \frac{r}{2} < \frac{\pi}{2} \) and \( 2r \left( 1 - \frac{A-B}{r} \right)^2 + 8(A - \frac{\pi}{r}) > 0 \), country of type 2 considers that he is "intermediate rich and the gap between the hegemon and the prize of losers is sufficiently big, so he participates actively in the game and play a non trivial response function. He behaves. If the country of type 2 is an "unsatisfied intermediate rich", because the gap prize \( (\frac{A-B}{r}) \) is sufficiently high for him, so he plays a non trivial response function. He behaves as a **big hungry dog**.

The following proposition is relative to the curvature of the best response function in general

**Proposition 5.4**

- The curvature of the best response emissions functions \( e_1(\theta) \) of country \( i \) of type 1 is non-monotonic and concave function.

- The curvature of the best response emissions functions \( e_2(\theta) \) of country \( i \) of type 2 could be non-monotonic and concave function or constant depending of the level of endowment.

**Proof.** See Appendix

**Remark 4**

- The best-response functions for this game slope upward initially, then downward. They are neither strategic complements nor substitutes.

Beyond the critical point \( \tilde{\theta} = \frac{1}{2} \left[ A - B - \frac{r^2 + 2Ar - 2\pi}{(A-B)} \right] \), further increases in emissions of others countries force country \( i \) to increase its emissions.
• The endowment should not be neglected. It plays a crucial role on the behavior a country in this game and will be important determinants of equilibrium greenhouses gases.

Proposition 5.5 Consider a country of type 1, then given \( \theta \), the best response function of this country, \( e_1(\theta) \) will

1. rise if pure social hegemon prize \( A \) rises,

2. rise if the gap prize \( (A - B) \) rises

3. fall if the level of its endowment \( \pi_1 \) rises

Proof. See Appendix

An increase in the gap prize means that if a country reach the hegemonic position, it will enjoy a payoff very large compared to the payoff enjoyed by a loser. So, it gives more incentive to a country to win, so to emit more more given the total emissions of rivals. An increase in endowment factor means that the country becomes more rich. So, it diminish it incentive to participate at the game, due to the fact that the prize of the game has become less attractive to it.

6 Symmetric equilibrium and global warming in a world with \( N \) countries of type 1 in quest for hegemony

Proposition 6.1 Consider \( N \) identical countries of type 1 in questing for hegemony. Denoted by \( e^*_1(N) \) the symmetric equilibrium emission of the cooperative game for hegemony’s quest, then

\[
\bullet \quad e^*_1(N) = \frac{-[(N-1)(A-B)-Nr]+\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi)}}{N^2}
\]
• \( \lim_{N \to \infty} e_1^*(N) = 0 \)

• \( \lim_{N \to \infty} Ne_1^*(N) = -(A - B) + r + \sqrt{(A - B)^2 + r^2 + 2(Ar - \pi)} > 0 \)

This proposition says that when the number of countries of type 1 is very high, every country emits 0 but the global warming (sum of the emissions) is strictly positive. This result says that no one exercises "market power" but in global there is an externality. It's similar to the case of perfect competition in the presence of externalities.

**Proof.** Recall that \( e(\theta) = -r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi} \).

In a symmetric case, \( \theta = (N - 1)e \). Then it follows that

\[
e(N) = \frac{-(N - 1)(A - B) - Nr + \sqrt{[(N - 1)(A - B) - Nr]^2 + 2N^2(Ar - \pi)}}{N^2}
\]

It's easy to show that

\[
\lim_{N \to \infty} Ne(N) = -(A - B) + r + \sqrt{(A - B)^2 + r^2 + 2(Ar - \pi)} > 0
\]

**Proposition 6.2** Consider \( N \) symmetric countries of type 1. Then the equilibrium level of greenhouse gas \( e_1(N) \) declines decreases as the number of countries in the quest for top status increases

**Proof.** See Appendix
Remark 5 Reaction functions are non-monotonic in general, but negatively sloped in the Nash equilibrium. Hence, we can conclude that emissions functions are strategic substitutes in the non-cooperative equilibrium.

Figure 5: Path of Nash Symmetric equilibrium when the number of countries of type 1 increases

Figure 6: Equilibrium Emission decreases when the number of country of type 1 increases
Proposition 6.3  Consider $N$ symmetric countries of type 1, then global warming $Ne^*_1(N)$ increases as the number of countries increases but remains above a positive number as $N$ grows.

Proof. Using the software Mathematica, it can be shown that the first derivative of the function $Ne^*_1(N)$ hasn’t a zero, independently of the parameters. So the function $Ne^*_1(N)$ is monotonic and increasing.

![Figure 7: Global warming decreases when the number of country of type 1 increases](image)

The intuition of this result is that political fragmentation of sovereign countries of country of type 1 (poor countries) can be global warming enhancing. The explanation is that Political fragmentation increase the number of players in the hegemony game and results in intense competition. Think of the case of the separation of Kosovo from Serbia, Separation of Ossetia from Georgia, etc.

Proposition 6.4 There is a trade-off between the reduction of the expected finishing time of
the hegemony game and the reduction of the global warming, given by the following relation

\[ Ne(N) = \frac{1}{E\left(\min(\tau_1, \ldots, \tau_N)\right)} \]

**Proof.** It follows from the computation of the mean of a random variable which has an exponential distribution.

This proposition states that global warming will is inversely proportional to the expected time of the arrival of a new hegemon. In other words, more players are impatient to reach the hegemonic position, more the global warming increases. May be this relation tell us that there is a trade-off between the Kyoto protocol implementation and the desire of countries to reach the hegemonic position rapidly. Think that some countries will certainly not allow their hegemonic aspirations to be hampered.

## 7 Social planner emissions in a world with \( N \) countries of type 1 in quest for hegemony

**Proposition 7.1** Consider \( N \) symmetric countries of type 1. Denote the optimal social emission by \( e^S_1(N) \), and the symmetric equilibrium emission by \( e^*_1(N) \), then

\[ e^*_1(N) > e^S_1(N) \]

Each country in non cooperative game emits more greenhouse gas than that is socially optimal.

**Proof.** Suppose that \( e^* < e^S \).

Since \( e^* \) is the best response from the sum of the opponents nash strategies \( \theta^* = (N - 1)e^* \), we have that the expected payoff satisfies

\[ \Omega\left(e^*/(N - 1)e^*\right) \geq \Omega\left(e^S/(N - 1)e^*\right) \]

Remark that \( \Omega\left(e^S/(N - 1)e^*\right) \geq \Omega\left(e^S/(N - 1)e^S\right) \) because if the opponents players raise their emissions to \( (N - 1)e^S \), the payoff of a country will diminish.
But $e^S$, the social optimal emission, is solution of following program: \[ \max_e \Omega(e/(N-1)e) \]

These relation implies the following inequality: \[ \Omega(e^*/(N-1)e^*) \geq \max_e \Omega(e/(N-1)e) \]

It’s a contradiction.

8 Effect of differences in endowment or differences in the valuation of winner prize equilibrium in a world with two countries of type 1

**Proposition 8.1** Consider a non symmetric world embodied with two countries of type 1.

The first Country with parameters $A_1$, $B_1$, and $\pi_1$.

The second Country with parameters $\tilde{A}_1$, $\tilde{B}_1$, and $\tilde{\pi}_1$.

- If $\pi_1 < \tilde{\pi}_1$, with $A_1 = \tilde{A}_1$, $B_1 = \tilde{B}_1$, then $e_1^* > e_2^*$
- If $B_1 < \tilde{B}_1$, with $\pi_1 = \tilde{\pi}_1$ and $A_1 = \tilde{A}_1$, then $e_1^* > e_2^*$
- If $A_1 > \tilde{A}_1$, with $\pi_1 = \tilde{\pi}_1$ and $B_1 = \tilde{B}_1$, then $e_1^* > e_2^*$

**Proof.** (it can be shown graphically)

9 Emissions and Global warming in a world with $N$ countries of type 2 in quest for hegemony

In this paragraph, we suppose that the following conditions are satisfied:

\[ \frac{-r^2 - 2(Ar - \pi_2)}{2(A-B)} \geq 0, \quad \frac{A-B}{r} > 1, \quad 4r \left(1 - \frac{A-B}{r}\right)^2 + 8(A - \frac{\pi_2}{r}) > 0 \]

**Proposition 9.1** Consider $N$ symmetric countries of type 2 in questing of hegemony when conditions above prevailed, then the game could have three symmetric equilibriums $e_a^*(N)$, $e_b^*(N)$, $e_c^*(N)$.
Note that $e^*_b(N) < e^*_c(N)$

1. The equilibrium emission $e^*_a(N)$ is constant and equal zero.

2. The equilibrium emission $e^*_c(N)$ is a decreasing function of $N$

- $e^*_2a(N) = 0$ and $\lim_{N \to \infty} Ne^*_a(N) = 0$
- $\lim_{N \to \infty} e^*_2b(N) = 0$
  and $\lim_{N \to \infty} Ne^*_b(N) = 0$
- $\lim_{N \to \infty} e^*_2c(N) = 0$
  and $\lim_{N \to \infty} Ne^*_c(N) = -(A - B) + r + \sqrt{(A - B)^2 + r^2 + 2(Ar - \pi_2)} > 0$

**Proof.** See Appendix

Figure 8: Path of Nash Symmetric equilibrium When $N$ increases
\textit{Proof.} It’s easy to get the intuition graphically. We will prove it mathematically.
The two figures below present emission and global warming as a function of countries in quest for hegemony in the second equilibrium.

Figure 9: Symmetric Equilibrium when $N$ increases, (second equilibrium)

Figure 10: Global warming increases when $N$ increases, (second equilibrium)
The two figures below present emission and global warming as a function of countries in quest for hegemony in the third equilibrium.

Figure 11: Symmetric Equilibrium when $N$ increases, (third equilibrium)

Figure 12: Global warming increases when $N$ increases (third equilibrium)
9.1 Effect of heterogeneity variation on emission

Definition 2 We define a Π-mean preserving spread as the set of all possible pairs \((N_1, N_2)\) real positive numbers such that

\[ N_1\pi_1 + N_2\pi_2 = \Pi \]

where \(\pi_1, \pi_2, \Pi\) are fixed.

The intuition is that a Π-mean-preserving spread maintains the total amount of endowment but could make it more unequally distributed.

In what follows, we consider a heterogeneous world of \(N_1\) countries of type 1 and \(N_2\) countries of type 2, in the sense of a Π-mean preserving spread. We want to analyze how heterogeneity with mean preserving spread affects global warming.

Define \(\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2)\) by the following expression

\[
\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2) = \left\{ 2(N_1 + N_2)(r + \frac{N_2(\pi_1 - \pi_2)}{A - B}) - 2(N_1 + N_2 - 1)(A - B) \right\}^2
- 4(N_1 + N_2)^2 \left[ \left( r + \frac{N_2(\pi_1 - \pi_2)}{A - B} \right)^2 - 2N_2(\pi_1 - \pi_2) - r^2 - 2(Ar - \pi_1) \right]
\]

(9.1)

Proposition 9.2 Suppose that the following conditions are satisfied:

\[
\frac{-r^2 - 2(Ar - \pi_2)}{2(A - B)} \geq 0, \quad \frac{A - B}{r} > 1, \quad r \left( 1 - \frac{A - B}{r} \right)^2 + 2(A - \frac{\pi_2}{r}) > 0
\]

Consider a world composed \(N_1\) countries of type 1 and \(N_2\) countries of type 2, in the sense of a Π-mean preserving spread.

Assume that \(\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2) \geq 0\).

Then the game could have three equilibria computed as follows
\[
\begin{align*}
\left\{ e^*_1(N_1, N_2) &= \max \left\{ -\frac{2(N_1+N_2)(r+N_2(\pi_1-\pi_2))}{A-B} - 2(N_1+N_2-1)(A-B) + \sqrt{\Delta(N_1,N_2,r,A,B,\pi_1,\pi_2)} }{2(N_1+N_2)^2}, 0 \right\} \\
\left\{ e^*_2(N_1, N_2) &= \max \left\{ \frac{\pi_1-\pi_2}{A-B} + e^*_1, 0 \right\} \right. \\
\end{align*}
\]

or

\[
\begin{align*}
\left\{ e^*_1(N_1, N_2) &= \max \left\{ -\frac{2(N_1+N_2)(r+N_2(\pi_1-\pi_2))}{A-B} - 2(N_1+N_2-1)(A-B) - \sqrt{\Delta(N_1,N_2,r,A,B,\pi_1,\pi_2)} }{2(N_1+N_2)^2}, 0 \right\} \\
\left\{ e^*_2(N_1, N_2) &= \max \left\{ \frac{\pi_1-\pi_2}{A-B} + e^*_1, 0 \right\} \right. \\
\end{align*}
\]

or

\[
\begin{align*}
\left\{ e^*_1(N_1, N_2) &= \frac{2(N_1(1-r_1-N_1)+\sqrt{2(N_1(1-r_1-N_1))^2+8N_1(Ar-\pi_1)}}{2N_1^2} \\
\left\{ e^*_2(N_1, N_2) &= 0 \right. \\
\end{align*}
\]

9.2 Effect of heterogeneity variation on global warming

Global in the hegemon game

Proposition 9.3 Consider a world composed \( N_1 \) countries of type 1 and \( N_2 \) countries of type 2 in a sense of a \( \Pi \)-mean preserving spread, in other words, such that

\[ N_1\pi_1 + N_2\pi_2 = \Pi \]

where \( \pi_1, \pi_2, \Pi \) are fixed.

Global warming is,

For the first equilibrium, given by
\[ N_1 e_1(N_1, N_2)^* + N_2 e_2^*(N_1, N_2) \]
\[
= N_1 \max \left\{ \frac{-2(N_1 + N_2)(r + \frac{N_2(\pi_1 - \pi_2)}{A - B}) - 2(N_1 + N_2 - 1)(A - B)}{2(N_1 + N_2)^2} + \sqrt{\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2)} \right\}, 0 \]
\[
+ N_2 \max \left\{ \frac{\pi_1 - \pi_2}{A - B} + e_1^*, 0 \right\}
\]

**For the second equilibrium, given by**

\[ N_1 e_1(N_1, N_2)^* + N_2 e_2^*(N_1, N_2) \]
\[
= N_1 \max \left\{ \frac{-2(N_1 + N_2)(r + \frac{N_2(\pi_1 - \pi_2)}{A - B}) - 2(N_1 + N_2 - 1)(A - B)}{2(N_1 + N_2)^2} - \sqrt{\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2)} \right\}, 0 \]
\[
+ N_2 \max \left\{ \frac{\pi_1 - \pi_2}{A - B} + e_1^*, 0 \right\}
\]

**For the third equilibrium, given by**

\[ N_1 e_1^*(N_1, N_2) + N_2 e_2^*(N_1, N_2) = \frac{2(N_1(1 - r_1 - N_1) + \sqrt{2(N_1(1 - r_1 - N_1))^2 + 8N_1(4r_1 - \pi_1)}}{2N_1} \]

### 9.3 Could a reduction of inequality increases global warming?

It’s easy to prove that

\[ e_2^*(N_1, N_2) < e_1^*(N_1, N_2) \]

So, a country of type 1 emits more than a country of type 2.

One implication is that if the proportion of countries of type 1 in quest for hegemony \( \frac{N_1}{N_1 + N_2} \) is greater than the proportion of countries of type 2 in quest for hegemony, countries of type
1 are the first responsible of climate warming.

Another implication is that

\[ e_2^*(N_1, N_2)(N_1 + N_2) < N_1 e_1^*(N_1, N_2) + N_2 e_2^*(N_1, N_2) < e_1^*(N_1, N_2)(N_1 + N_2) \]

This result lets think that if the number of countries of type 1 increases and the number of countries having the type 2 decreases, then global warming could increase. So there may be a possible tension between the world’s desire to reduce inequality and to help weak, developing, countries become stronger, and the desire to control global warming.

9.4 A graphic representation of the first equilibrium when the number of countries of type 1 in quest for hegemony increases.

![Graph showing a mean preserving spread](image)

Figure 13: A mean preserving spread
Figure 14: Equilibrium emission of type 1 when $N_1$ increases (first equilibrium)

![Graph]

Figure 15: Equilibrium emission of country of type 2 when $N_1$ increases (first equilibrium)

![Graph]

Global warming increases when the number of countries of type 1 in the quest or hegemony increases
Figure 16: Global warming when when $N_1$ increases (first equilibrium)

Figure 17: A mean preserving spread
9.5 A graphic representation of the first equilibrium when the number of countries of type 2 in quest for hegemony increases.

Figure 18: Equilibrium emission of type 1 when the number of countries of type 2 in quest for hegemony increases (first equilibrium)

Figure 19: Equilibrium emission of type 2 when the number of countries of type 2 in quest for hegemony increases (first equilibrium)
Global warming increases when the number of countries of type 2 in the quest for hegemony increases.

Figure 20: An increase in global warming when the number of countries of type 2 in quest for hegemony increases (first equilibrium)
9.6 A graphic representation of the second equilibrium when the number of countries of type 1 in quest for hegemony increases

![Graph 1](image1)

Figure 21: Equilibrium of type 1 when $N_1$ increases (second equilibrium)

![Graph 2](image2)

Figure 22: Equilibrium emission of country of type 2 when $N_1$ increases (second equilibrium)
Figure 23: Global warming $N_1$ increases (second equilibrium)
10 Concluding Remarks

In this paper, the relationship between quest of hegemony and global warming is illustrated through a dynamic model with uncertainty. The paper shows that a common belief that economy strength is a necessity to become an increasingly important geopolitical player could leads to an increases in global warming when the number of countries in quest for hegemony increases. The paper shows the crucial role of the endowment which are not linked to greenhouses gases on the behavior of countries. We show that the comparison of the hegemony prize with the permanent endowment which are not linked to greenhouses gazes play a central role in the behavior of a country and consequently on the issues of impact of hegemony quest on global warming. In particular, best responses functions obtained in this game are inverted-shape in certain cases and identically zero in others cases.

Our results open another interpretation of the Environmental Kuznet Curve which says that at low incomes, pollution initially rises with growth reach a maximum and then decrease at higher levels of income. The results of this paper, may be, suggests to take into account a status index among countries, in others words taking account not only to the level of income but also the position of the income relatively to the hegemony prize. The paper gives some reasons which could explain theoretically why some rich countries as USA or China are dragging their feet in the fight against global warming. The results obtained traduce the importance to take into account of the quest for hegemony in the economic analysis of climate change and international negotiations for a multilateral agreement for greenhouse gas abatements. The paper also gives the intuition of the energy competition between countries. Energy competition is among the many issues included in the debate over how the United States should deal with a rising China. The impact of the quest for hegemony on the global warming could be enormous, and so it will be needed to develop strategies and response to meet this aspect during international discussions on the fight of climate change. This analysis is shared by Scales (2007) who thinks
that the use of economic outcomes for the quest of power in modern societies must be replace by new ways of thinking, more respectful of the environment. In this paper, we have focuss on the steady state game. In another paper we could explore a repeated version of this game. In this paper, the quest for hegemony stops as soon as one of the contestants wins. It would be interesting in another research to analyze this game when the quest for hegemony has many steps with possibility for a country to loose his position at each step of the step of the game. The present model can also be rewrite by taking account disparities in damages among developed and developing countries, disparities in ranking among countries at the beginning of the game, and by introducing an emission constraint, a carbon tax in the model, or by taking into account reputation aspects in the game.

A Appendix

A.1 Proof of the proposition 6.1

The term $\Delta(\theta)$ is equal zero if $\theta = -\frac{r^2 + 2(Ar - 2\pi_1)}{2(A - B)}$. Since $Ar - \pi_1 \geq 0$, the term $\Delta(\theta)$ is positive. It follows that $\frac{d^2 \Omega(e_1/\theta)}{de_1^2}$ is negative and then the payoff function is concave. The unique positive solution is given by

$$e_1(\theta) = \max(0, -r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_1})$$ (A.1)

The zeros of the equation $r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_1} = 0$ gives the interval bounds of the best response function

A.2 Proof of the proposition 6.2

The equation

$$-r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_2} = 0$$
can be rewrite as

$$\theta^2 + 2(r - (A - B)\theta - 2(Ar - \pi_2) = 0$$

It’s a second degree equation. The last assumption implies that the discriminant of the equation which is $4 \left( r - \frac{A - B}{r} \right)^2 + 8(Ar - \pi_2)$ is strictly positive. The sum of solutions gives $-2(r - (A - B))$ and the multiplication of solutions gives $-2(Ar - \pi_2)$ and is strictly positive. Since we assume that $\frac{A - B}{r} > 1$, the sum of solution of the equation is also strictly positive. It follows that solutions denoted by $\tilde{\theta}_k, k = 1, 2$ are all positive.

The unique solution of the equation $\Delta(\theta) = 0$ is $\theta^* = \frac{-r^2 - 2(Ar - \pi_2)}{2(A - B)}$

Note that $\sqrt{\Delta(\theta^*)} = 0 < \sqrt{\Delta(\tilde{\theta}_k)} = r + \tilde{\theta}_k$ and then $\theta^* < \tilde{\theta}_k, \ k=1,2$

Since

$$e_2(\theta) = \max(0, -r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_2}) \quad (A.2)$$

Analyzing methodically, the conditions on the positivity of the best response function, the result is obtained.

A.3 Proof of the proposition 6.3

1. The term $\Delta(\theta)$ is equal zero if $\theta = -\frac{r^2 + 2(Ar - 2\pi_2)}{2(A - B)}$. Since $\frac{r^2 - 2(Ar - \pi_2)}{2(A - B)} < 0$, the term $\Delta(\theta)$ is equal zero for a negative number. It follows that the discriminant of the first order condition is positive because $\theta$ is always positive, and then the payoff is a convex function

$$\frac{d^2\Omega(e_i/\theta)}{de_i^2} \geq 0$$

and decreasing function

$$\frac{d\Omega(e_i/\theta)}{de_i} = -\frac{1}{2}(e_i + \theta)^2 - r(e_i + \theta) + (A - B)\theta + Ar - \pi \quad \frac{(e_i + \theta + r)^2}{(e_i + \theta + r)^2} \leq 0$$

Then the payoff function is maximized at 0.

2. In this case, the function

$$-r - \theta + \sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_2}$$

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don’t have a zero and then is always negative

3. In this case, zeros (when there exist) of the function

\[-r - \theta + \sqrt{2\theta(A-B) + r^2 + 2Ar - 2\pi_2}\]

are negative and then this function is negative for \( \theta \geq 0 \)

A.4 Proof of the proposition 6.4

The first derivative of best response function of a country of type 2 when there is an interval where it’s not zero is given by

\[e'_i(\theta) = -1 - \frac{A - B}{\sqrt{2\theta(A-B) + r^2 + 2Ar - 2\pi_i}}\]

on this interval

Searching the zeros of the equation:

\[e'_i(\theta) = 0\]

gives the solution

\[\hat{\theta} = \frac{1}{2} \left[ A - B - \frac{r^2 + 2Ar - 2\pi_i}{A - B} \right]\]

The second derivative gives:

\[e''_i(\theta) < 0\]

Then it’s a concave function.

The best response function has a global maximum point at \( \hat{\theta} \).

A.5 Proof of the proposition 6.5

\[
\frac{\partial e_1}{\partial A} = \frac{r + \theta}{\sqrt{2\theta(A-B) + r^2 + 2Ar - 2\pi_1}} > 0
\]
\[
\frac{\partial e_1}{\partial (A - B)} = \frac{\theta}{\sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_1}} > 0
\]

\[
\frac{\partial e_1}{\partial \pi_i} = \frac{-1}{\sqrt{2\theta(A - B) + r^2 + 2Ar - 2\pi_1}} < 0
\]

### A.6 Proof of the proposition 7.2

Let

\[
G = [(N - 1)(A - B) - Nr]
\]

and

\[
H = \left[(N - 1)(A - B) - Nr \right]^2 + 2N^2 \left(Ar - \pi_1 \right) > 0
\]

Using these expressions \((G\text{ and } H)\), it follows that the equilibrium emission could be written as

\[
e_1(N) = \frac{2(Ar - \pi_1)}{G + \sqrt{H}}
\]

By differentiating with respect to \(N\), it follows that

\[
\frac{d e_1(N)}{dN} = - \left\{ \frac{(A - B - r) + \frac{1}{2} \left[2(A - B - r)\left[(N - 1)(A - B) - Nr \right] + 4N(Ar - \pi_1)\right]}{(G + \sqrt{H})^2} \right\}
\]

\[
= - \frac{(A - B - r)\sqrt{H} + (A - B - r)G + 2N(Ar - \pi_1)}{\sqrt{H}(G + \sqrt{H})^2}
\]

\[
= - \frac{N(A - B - r)(\sqrt{H} + G) + H - G^2}{N\sqrt{H}(G + \sqrt{H})^2}
\]

\[
= - \frac{(A - B) + \sqrt{H}}{N\sqrt{H}(G + \sqrt{H})^2} < 0
\]

(A.3)

It follows that

\[
\frac{d e_1(N)}{dN} = - \frac{(A - B) + \sqrt{H}}{N\sqrt{H}(G + \sqrt{H})^2} < 0
\]

(A.4)
A.7 Proof of the proposition 10.1

The first country has the following best response function:

\[ e_1 = -r_1 - e_2 + \sqrt{2e_2(A - B) + r_1^2 + 2(Ar_1 - \pi)} \]

The second country has the following best response function:

\[ e_2 = -\tilde{r}_1 - e_1 + \sqrt{2e_1(A - B) + \tilde{r}_1^2 + 2(A\tilde{r}_1 - \pi)} \]

It follows that

\[(e_2 + e_1 + r_1)^2 = \sqrt{2e_2(A - B) + r_1^2 + 2(Ar_1 - \pi)}\]
\[(e_2 + e_1 + \tilde{r}_1)^2 = \sqrt{2e_1(A - B) + \tilde{r}_1^2 + 2(A\tilde{r}_1 - \pi)}\]

By subtraction, we obtain

\[(r_1 - \tilde{r}_1)(2(e_1 + e_2) + r_1 + \tilde{r}_1) = 2(e_2 - e_1) + r_1^2 - \tilde{r}_1^2 + 2A(r_1 - \tilde{r}_1)\]

Which gives the following linear relation between \(e_1\) and \(e_2\):

\[(2(e_1 + e_2) + r_1 + \tilde{r}_1) = \frac{2(e_2 - e_1)}{r_1 - \tilde{r}_1} + r_1 + \tilde{r}_1 + 2A\]

and then it’s easy to compute solutions

A.8 Proof of the proposition 14.1

Country of type 1 has the following best response function

\[
\begin{cases}
  e_1 = -[(N_1 - 1)e_1 + N_2e_2] - r + \sqrt{2\{(N_1 - 1)e_1 + N_2e_2\}(A - B) + r^2 + 2(Ar - \pi_1)} & \text{if } (N_1 - 1)e_1 + N_2e_2 \leq \tilde{\theta}_1 \\
  0 & \text{if elsewhere}
\end{cases}
\]

Country of type 2 has the following best response function
\[
\begin{cases}
0 & \text{if } \theta \in [0, \tilde{\theta}_2] \\
\ddot{e}_2 = -[N_1e_1 + (N_2 - 1)e_2] - r + \sqrt{2(N_1e_1 + (N_2 - 1)e_2)(A - B) + r^2 + 2(Ar - \pi_2)} & \text{if } \tilde{\theta}_2 \leq [(N_1 - 1)e_1 + N_2e_2](A - B) \leq \tilde{\theta}_1 \\
0 & \text{if } \theta \in [\tilde{\theta}_2, +\infty]
\end{cases}
\]

It follows that if the followings conditions are satisfied:

\[\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2) \geq 0, \text{ and } (N_1 - 1)e_1 + N_2e_2 \leq \tilde{\theta}_1, \text{ and } \tilde{\theta}_2 \leq N_1e_1 + (N_2 - 1)e_2 \leq \tilde{\theta}_2\]

The equilibrium are solutions of the followings system:

\[
\begin{align*}
N_1e_1 + N_2e_2 + r &= \sqrt{2((N_1 - 1)e_1 + N_2e_2)(A - B) + r^2 + 2(Ar - \pi_1)} \\
N_1e_1 + N_2e_2 + r &= \sqrt{2(N_1e_1 + (N_2 - 1)e_2)(A - B) + r^2 + 2(Ar - \pi_2)}
\end{align*}
\]

which implies that

\[e_2 = e_1 + \frac{\pi_1 - \pi_2}{A - B}\]

Substituting that expression into one of the equations of the system, one obtain

\[\left[N_1e_1 + N_2e_2 + r\right]^2 = 2((N_1 - 1)e_1 + N_2e_2)(A - B) + r^2 + 2(Ar - \pi_1)\]

which could rewrite as a second degree equation.

\[
(N_1 + N_2) e^2_1 + \left[2(N_1 + N_2) \left( r + \frac{N_2(\pi_1 - \pi_2)}{A - B} \right) - 2(N_1 + N_2 - 1)(A - B) \right] e_1 - 4 \left[ \left( r + \frac{N_2(\pi_1 - \pi_2)}{A - B} \right)^2 - 2N_2(\pi_1 - \pi_2) - r^2 - 2(Ar - \pi_1) \right] = 0
\]

(A.5)

The discriminant of this equation is given by \(\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2)\) and is positive by assumptions of the proposition

Then Equilibrium emission of the country of type 1 is

\[e^*_1 = \frac{-2 \left[ (N_1 + N_2) \left( r + \frac{N_2(\pi_1 - \pi_2)}{A - B} - 2(N_1 + N_2 - 1)(A - B) \right) \right] + \sqrt{\Delta(N_1, N_2, r, A, B, \pi_1, \pi_2)}}{2(N_1 + N_2)^2}\]
and equilibrium emission of type 2 is computed as $e_2^* = \frac{\pi_1 - \pi_2}{A - B} + e_1^*$

The last equilibria is computed by imposing that $e_2 = 0$, then solution is given by the equation

$$N_1 e_1^2 + [2N_1 r - 2(N_1 - 1)] e_1 - 2( Ar - \pi_1 ) = 0$$

This equation gives the solution

$$e_1^* = \frac{2(N_1(1 - r_1 - N_1) + \sqrt{[2(N_1(1 - r_1 - N_1)]^2 + 8N_1(Ar - \pi_1)}}{2N_1^2}$$

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