On the profitability and welfare effects of downstream mergers *

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Abstract

We consider an upstream firm selling an input to several downstream firms through non-discriminatory two-part tariff contracts. Downstream firms can alternatively buy the input from a less efficient source of supply. We show that downstream mergers lead to lower wholesale prices. They translate into lower final prices only when the alternative supply is insufficiently efficient. Downstream mergers are very profitable in this setting and monopolization is the equilibrium outcome of a merger game even for unconcentrated markets.

Key words: downstream mergers, wholesale price, two-part tariff contracts

JEL codes: L11, L13 and L14.

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1 Introduction

In the last few years, we have observed the rise of very large downstream firms in previously more fragmented industries such as retailing, farming, natural resource extraction and health. The effect of downstream mergers on consumer surplus and social welfare is far from being clear for industrial economists. The reason is that they affect two different dimensions. On the one hand, they have an effect on the supply side of the market, by allowing downstream firms to get better deals from suppliers. On the other hand, they reduce competition downstream. The key point to assess the welfare effect of mergers is whether the lower wholesale prices obtained by downstream firms are passed on to consumers.

In order to address this question, we consider a model with an upstream firm selling an input to several downstream firms competing à la Cournot in a homogeneous good final market. They can alternatively obtain the input from a less efficient source of supply. The upstream firm offers non-discriminatory two-part tariff contracts to downstream firms. We get that the optimal wholesale price is increasing in the number of downstream firms. The reason is that the upstream firm uses the wholesale price to control the level of competition downstream and the fixed fee to extract the surplus. This result suggests that downstream mergers, by reducing the number of downstream competitors, could lead to a reduction in the final price paid by consumers. We find that if the alternative supply is inefficient enough (or, in other words, if there is strong market power in the upstream sector), downstream mergers countervail the market power of the dominant supplier leading to a reduction in the final price. As a result, in this context, downstream mergers are pro-competitive.

\footnote{For example, several reports of the European Comission and the OECD show that the grocery retail market of many states of the UE is now dominated by a small number of large retailers. Although market concentration in retailing is less extreme in the US, concern about buyer power is also increasing there.}
Our paper is related to von Ungern-Sternberg (1996) and Dobson and Waterson (1997). The key difference with our paper is that they consider linear supply contracts. They obtain that downstream mergers lead to lower final prices only when there exists enough competition in the downstream market. Caprice (2005,2006) study a similar model but assuming secret supply contracts. In Caprice (2005), it is shown that the upstream firm may be better-off the higher the level of competition downstream. The reason is that competition reduces the outside option of firms. Caprice (2006) shows that banning price discrimination may be welfare improving in the presence of an alternative supply.

After the analysis of the welfare effects of downstream mergers, we turn our attention to their profitability. Again, the degree of competition upstream plays a key role. In particular, we show that when the alternative supply is inefficient, downstream mergers are profitable, because they countervail the dominant position of the upstream firm. Indeed, in an endogenous merger formation game, we get that monopolization is the equilibrium outcome even for very unconcentrated industries. This result deeply contrasts with the one obtained by Kamien and Zang (1990) for Cournot markets, where monopolization only occurs for very concentrated markets.

In a vertical structure, profitability of downstream mergers was also obtained in Lommerud et al. (2005, 2006). In the former paper, they consider three downstream firms, each of them contracting in exclusivity with an upstream firm. The supply contracts are linear. In this setting, the merger of two downstream firms is shown to be profitable because it generates competition between their suppliers, leading to a reduction in input prices. In the latter paper, the same idea is applied to study the pattern of mergers in an international context.

The rest of the paper is organized as follows. In the next section, we analyze the optimal supply contracts taken a market structure as given. In Section 3, we solve an endogenous merger game to analyze the profitability of downstream mergers. Finally, Section 4 concludes. All proofs
are relegated to the Appendix.

2 Model with an exogenous market structure

We consider an upstream firm that produces an input at cost $c_u$. A number $n$ of downstream firms transform this input into a final homogenous good on a one-to-one basis, without additional costs. Downstream firms may alternatively obtain the input from a competitive supply at cost $c < a$. Inverse demand for the final good is given by $P = a - Q$, where $Q$ is the total amount produced.

The upstream firm sets observable vertical contracts that establish the terms under which inputs are transferred. After contracts are set, competition downstream is à la Cournot. More specifically, the game is modelled according to the following timing: first, the supplier offers a two-part tariff contract $(F, w)$ to downstream firms, where $F$ specifies a non-negative fixed amount and $w$ a wholesale price. Second, downstream firms decide whether or not to accept the contract. The ones that accept, pay $F$ to the upstream firm. Finally, they compete à la Cournot.

Assume that $k$ firms have accepted a supply contract $(F, w)$. Firms that have not accepted the contract produce in equilibrium:

$$q_N(k, w) = \begin{cases} 
\frac{a-c(k+1)+wk}{n+1} & \text{if } w > \frac{-a+c(k+1)}{k} \\
0 & \text{otherwise}
\end{cases}$$

Observe that, if $w$ is very low, the firms that do not accept the contract are driven out of the market. On the other hand, the firms that accept the contract produce in equilibrium:

$$q(k, w) = \begin{cases} 
\frac{a+c(n-k)-w(n-k+1)}{n+1} & \text{if } w > \frac{-a+c(k+1)}{k} \\
\frac{a-w}{k+1} & \text{otherwise}
\end{cases}$$
Profits of non-accepting and accepting firms are given, respectively, by $\Pi_N(k, w) = (q_N(k, w))^2$ and $\Pi(k, w) = (q(k, w))^2$.

In the second stage, downstream firms accept the contract offered by the upstream firm whenever $F \leq \Pi(k, w) - \Pi_N(k - 1, w)$. Obviously, as the upstream firm maximizes profits, in order for $k$ firms to accept the contract, it will choose $F$ such that $F = \Pi(k, w) - \Pi_N(k - 1, w)$. This implies that the problem of choosing the optimal contract $(F, w)$ is equivalent to that of choosing $(k, w)$. Then, in the first stage, the upstream solves the following problem:

$$\max_{k, w} k (\Pi(k, w) - \Pi_N(k - 1, w) + (w - c_u)q(k, w))$$  \hspace{1cm} (1)

s.t. $1 \leq k \leq n$ and $w \leq c$.

We proceed as follows. First of all, we prove that the upstream firm finds profitable to sell the input to all firms in the downstream sector. Then, we calculate the optimal wholesale price once we replace $k$ by $n$ in expression (1). As far as the first result is concerned, we know that with a fixed fee contract, the input would be sold to only a subset of firms in order to protect industry profits from competition (Kamien and Tauman (1986)). With a two-part tariff contract, however, the upstream firm can always sell to one more firm without affecting the level of competition, by choosing an appropriate wholesale price. Assume that the upstream firm sells to $k$ firms with a wholesale price $w < c$. It is direct to see that he can always sell to all firms with a $w < w_c < c$ such that the final price remains constant. In other words, the upstream firm may always use the wholesale price to control for competition. Next proposition shows that this particular contract increases the profits of the upstream firm.

\footnote{As $\frac{\partial \Pi(k, w) - \Pi_N(k - 1, w)}{\partial k} < 0$, this is the only equilibrium in the acceptance stage.}

\footnote{Observe that selling to all firms with a wholesale price $w$ would decrease the final price and that selling to all firms with a wholesale price equal to $c$ would increase the price.}

\footnote{This argument is used in Sen and Tauman (2007) to prove that with an auction plus royalty contract, the input would be sold to all firms.}
Lemma 1. Let $\pi(k, w)$ represent the upstream profit if the upstream firm sells to $k$ firms and sets wholesale price $w \leq c$. Then $\pi(n, w_1) \geq \pi(k, w)$ where $w_1$ solves $nq(n, w_1) = (n - k)q_N(k, w) + kq(k, w)$.

Proof. We have that
\[
\pi(k, w) = (P - c_u) ((n - k)q_N(k, w) + kq(k, w)) - k (q_N(k - 1, w))^2 - (n - k) (q_N(k, w))^2 - (c - c_u)(n - k)q_N(k, w).
\]

Observe that if the upstream sells to $n$ firms with the wholesale price $w_1$, the first term in the above expression will also appear in $\pi(n, w_1)$. Then the difference in profits is given by:
\[
\pi(n, w_1) - \pi(k, w) = k (q_N(k - 1, w))^2 + (n - k) (q_N(k, w))^2 + (c - c_u)(n - k)q_N(k, w) - n (q_N(n - 1, w_1))^2.
\]

In order to prove the proposition we have to check the previous expression is non-negative in the following three different regions:

1. When $c \geq w > c + \frac{-a + c}{k}$, where $q_N(k, w) > 0$ and $q_N(k - 1, w) > 0$,
2. When $\frac{-a + ck}{k+1} < w \leq c + \frac{-a + c}{k}$, where $q_N(k, w) = 0$ and $q_N(k - 1, w) > 0$ and
3. When $w \leq \frac{-a + ck}{k+1}$, where $q_N(k, w) = 0$ and $q_N(k - 1, w) = 0$.

If $c \geq w > c + \frac{-a + c}{k}$, we have that $w \leq w_1 = \frac{c(n-k)+kw}{n} \leq c$ and
\[
\pi(n, w_1) - \pi(k, w) = k (q_N(k - 1, w))^2 + (n - k) (q_N(k, w))^2 - n (q_N(n - 1, w_1))^2 = (2)
\]
\[
= \frac{(n-k)k(c-w)^2}{n(1+n)} \geq 0.
\]

If $\frac{-a + ck}{k+1} < w \leq c + \frac{-a + c}{k}$, we have that $w < w_1 = \frac{a(n-k)+k(n+1)w}{(k+1)n} < c$ and $q_N(k, w) = 0$.

We have to distinguish two cases:

1. If $\frac{c(1+k)n^2-a(k+n)^2}{k(n^2-1)} < w \leq c + \frac{-a + c}{k}$, we have that $q_N(n - 1, w_1) > 0$. To sign the difference in profits we obtain that
\[
kq_N(k - 1, w) - nq_N(n - 1, w_1) \geq 0.
\]
This implies that
\[ \pi(n, w_1) - \pi(k, w) = k \left( q_N(k - 1, w) \right)^2 - n \left( q_N(n - 1, w_1) \right)^2 > 0. \]

If \(-a + ck < w \leq \frac{c(1+k)n^2 - a(k+n^2)}{k(n^2-1)}\), then \(w_1 \leq \frac{-a + cn}{-1+n}\) and, therefore, \(q_N(n - 1, w_1) = 0\). Then,
\[ \pi(n, w_1) - \pi(k, w) = k \left( q_N(k - 1, w) \right)^2 > 0. \]

If \(w \leq \frac{-a + ck}{1+k}\), we have that \(w_1 = \frac{a(n-k)+k(n+1)w}{(k+1)n} \leq \frac{-a + cn}{-1+n}\) and, therefore, \(q_N(n - 1, w_1) = 0\).

As we have also that \(q_N(k - 1, w) = 0\), then
\[ \pi(n, w_1) - \pi(k, w) = 0. \]

Let us provide an intuition for the above result. The problem of the upstream firm can be seen as that of maximizing total market profits minus the outside option of downstream firms. We show in the above proposition that it is always possible for the upstream firm to sell the input to all firms and increase the wholesale price accordingly, so that the final price remains constant. By doing so, total market profits increase due to an efficiency effect (buying the input from the alternative supplier increases the price it has to pay for the input from \(w_1\) to \(c\)). The effect with respect to the outside option is ambiguous: on the one hand, selling the input to more firms for a given wholesale price decreases the outside option; on the other hand, by increasing the wholesale price, the outside option increases. As we have shown in the proof, the final effect is negative, so that it is in the interest of the upstream firm to sell the input to all firms.

This result is central to the paper and, therefore, it seems interesting to know whether it holds for more general demand functions. In the Appendix, we show that it holds for concave demands satisfying a technical restriction concerning the third derivative of the inverse demand. We show that it also holds for the class of demands \(P = A - X^b\), where \(b \geq 1\).

Next, we derive the optimal two-part tariff contract to sell to \(n\) firms.
Proposition 1 The upstream firm optimally sells the input to all firms with a wholesale price
\[ w^*(n) = \frac{(n-1)(2n+c_u-c)+2c_u}{2(1-n+n^2)} \]
if \( c < \frac{a-c_u+(a+c_u)n^2}{2n^2} \) and \( w^M(n) = \frac{-a+c_u+(a+c_u)n}{2n} \) otherwise.

Proof. See Appendix

This result has been independently obtained in Erutku and Richelle (2007) for the case of a research laboratory licensing a cost-reducing innovation to a n-firm Cournot oligopoly through observable two-part tariff licensing contracts. However, Lemma 1 allows for a simpler and more intuitive proof and we generalize the result for the case of non-linear demands.

The optimal wholesale price arises from the balance of two conflicting incentives. On the one hand, maximizing industry profits requires a high wholesale price; on the other hand, reducing the outside option of downstream firms asks for a low wholesale price. Observe that whenever \( c \geq \frac{a-c_u+(a+c_u)n^2}{2n^2} \), the alternative supply is irrelevant and the upstream firm obtains the monopoly profits. In this case, as \( n \) increases the wholesale price is adjusted upwards in order to implement the monopoly price in the final market. On the other hand, if \( c < \frac{a-c_u+(a+c_u)n^2}{2n^2} \), \( w^*(n) \) is an increasing function of \( n \) and tends to \( c \) as \( n \) tends to infinity.\(^5\) This is a key result to explain the effect of downstream mergers on social welfare. It implies that downstream mergers, by reducing the number of firms in the downstream sector, lead to a reduction in the optimal wholesale price charged by the upstream firm, which is a necessary condition in order for downstream mergers to be welfare improving. The intuition behind the result is as follows: recall that what the upstream firm maximizes is total market profits minus the outside option of downstream firms. Increasing total market profits calls for a high wholesale price. Reducing the outside option requires a low wholesale price. The optimal wholesale price arises from the balance of these two opposing effects. Now, a downstream merger, by reducing the level of competition in the

\(^5\)This holds for any \( n \geq 2 \). Observe that, if \( c < \frac{a+3c_u}{4} \), \( w^*(1) = c_u > w^*(2) \). Notice also that the restriction that the wholesale price can not be higher than \( c \) is never binding in equilibrium.
downstream market, allows the upstream firm to put more weight on the second effect, and then it reduces the wholesale price in order to reduce the outside option of downstream firms.

It is interesting also to emphasize the key role played by parameter $c$. It affects the way in which the upstream firm adjusts the wholesale price as $n$ changes. The higher the value of $c$, the faster the wholesale price changes with $n$. This is very important because only when the wholesale price adjusts very fast to changes in $n$, it may be the case that a reduction in $n$ leads to a reduction in the final price paid by consumers. And this happens for high values of $c$. When the alternative supply is inefficient, there is strong market power in the upstream sector. In this case, a downstream merger countervails this market power, leading to a reduction in the final price. In other words, when there is strong market power upstream, the optimal merger policy calls for an increase in the degree of market concentration downstream to balance the situation. This result supports the view that "symmetry" between the upstream and downstream markets increases social welfare (Inderst and Shaffer, forthcoming).

From the previous proposition, it is direct to compute the equilibrium price, which is given by:

$$P^*(n) = \begin{cases} \frac{2(n-1)n^2+a(2+n(n-1))+cn(n+1)}{2(1+n^2)} & \text{if } c \leq \frac{a+cu}{2} \text{ or } c > \frac{a+cu}{2} \text{ and } n < \sqrt{\frac{a-cu}{2c-a-cu}} \\ \frac{a+cu}{2} & \text{otherwise} \end{cases}$$  \quad (3)

It is interesting to analyze the evolution of price with respect to $n$. First, if $c > \frac{a+cu}{2}$ and $n \geq \sqrt{\frac{a-cu}{2c-a-cu}}$, it is constant. For $c > \frac{a+cu}{2}$ and $n < \sqrt{\frac{a-cu}{2c-a-cu}}$ or $c \leq \frac{a+cu}{2}$, the sign of the derivative of price with respect to $n$ is given by the sign of the following expression$^6$:

$$R(n,c) = -c_u(-1+n)(1+n)^3 + 2cn(-2+3n+n^3) - a(1-2n+6n^2-2n^3+n^4)$$  \quad (4)

$^6$Note that this refers to the case $n \geq 2$. We have that $P^*(1) > P^*(2)$ regardless of $c$.

$^7$The value of $c'$ is given by $c' = \frac{c_u(1-n)(n+1)^3+al(1-2n+6n^2-2n^3+n^4)}{2n^2(n+1)}$. 

It is direct to see that (4) is positive when $c > c'$, where$^7 c' < \frac{a+cu}{2}$. Therefore, downstream
mergers reduce price whenever the level of competition upstream is low. For illustrative purposes, Figure 1 plots the values \((n, c)\) such that \(R(n, c) = 0\) for the particular case \(c_u = 0\).

Observe that, in this model, given that all downstream firms buy the input from the efficient supplier, social welfare is a decreasing function of price. Then, we have that the welfare effect of downstream mergers depends on their effect on the final price we have just studied. This effect depends on \(c\), that parametrizes the level of competition upstream. For high values of \(c\), horizontal mergers downstream countervail the dominant position of the upstream firm, leading to a price reduction. On the other hand, for low values of \(c\), there is little market power to countervail and then downstream mergers have the main effect of reducing competition, leading to a price increase.
We next compute the equilibrium profits of upstream and downstream firms. They are given, respectively, by:

\[
\Pi^U(n) = \begin{cases} 
\frac{n(a-c_u)((a-c_u)(1+n^2)-2(a-2c_u)+4(a-c)(c-c_u)n^3)}{4(1+n^n+n^n+n^n)} & \text{if } c \leq \frac{a+c_u}{2} \text{ or } c > \frac{a+c_u}{2} \text{ and } n < \sqrt{\frac{a-c_u}{2c-a+c_u}} \\
0 & \text{otherwise.}
\end{cases}
\]

\[
\Pi^D(n) = \begin{cases} 
\left(\frac{a-c_u+(a-2c_u+n^2)}{2(n^n+1)}\right)^2 & \text{if } c \leq \frac{a+c_u}{2} \text{ or } c > \frac{a+c_u}{2} \text{ and } n < \sqrt{\frac{a-c_u}{2c-a+c_u}}. \\
0 & \text{otherwise.}
\end{cases}
\]

Concerning the upstream profits we have that the upstream profits are increasing in \( n \) for \( c > c^* \), where\(^8\) \( c^* < c' < \frac{a+c_u}{2} \). Combining this result with the one on welfare, it is easy to see that any merger that increases the upstream profits reduces social welfare. This result is very intuitive because horizontal mergers increase welfare only when they counteract the selling power of the upstream firm.

Concerning joint downstream profits, we have that they are decreasing in \( n \). The fact that downstream mergers increase joint profits does not imply that there will be private incentives to merge, due to their public good nature. This is what we analyze in the next section, designing an endogenous merger formation game. Here, we only want to emphasize how merger profitability depends on \( c \). A merger of \( k+1 \) firms is profitable if

\[
\Pi^D(n-k) - (k+1)\Pi^D(n) \geq 0.
\]

This condition holds if \( c \geq \frac{a-c_u+(a+c_u)n^2}{2n^2} \) because, in this case, \( \Pi^D(n) = 0 \). Otherwise, it is useful to study profitability rewriting (6) in the following way:

\[
\frac{\Pi^D(n-k)}{\Pi^D(n)} \geq (k+1).
\]

It is direct to see that the left hand side of the inequality is increasing in \( c \). This means that mergers become more likely as \( c \) increases, that is, as the market power of the upstream firm increases. Bru and Fauli-Oller (2003) obtain the same result but considering secret supply contracts. Let us now introduce a merger formation game.

\(^8\)The value of \( c^* \) is given by \( c^* = \frac{c_u(1+n^2)+a(n-1)(1+(-4+n)n)}{2n(4+(-1+n)n)}. \)
3 An endogenous merger game

The most widely accepted merger game is the one developed by Kamien and Zang (1990, 1991, 1993). In Kamien and Zang (1990) each firm simultaneously chooses a bid for each competitor and an asking price. A firm is sold to the highest bidder whose bid exceeds the firm’s asking price. They get that, with linear demand and Cournot competition, monopolization does not occur when we have three or more firms. Buying firms is expensive because, by not accepting a bid, a firm free-rides on the reduction in competition induced by the remaining acquisitions.

In this section, we design a merger game inspired in the previous papers in order to endogenize the market structure. For simplicity, we restrict attention to a simple game where there is only one acquiring firm. Observe this assumption makes it more difficult to get monopolization.

In order to be able to explicitly solve the merger game, we have to fix a value for parameter $c$. We want to analyze how profitable downstream mergers are in the presence of endogenous input prices. Then, given that merger profitability increases with $c$, we choose the largest value such that downstream profits are positive whatever the number of firms in the downstream sector, i.e. $c = \frac{a + c_0}{2}$. As we will see below, with the downstream profits inherited from the contract game of the previous section, the only equilibrium of the merger game is monopolization even for very unconcentrated industries.

More specifically, the timing of the game is the following: we assume that there are, initially, $N$ symmetric downstream firms in the industry. One of them, say firm 1, can make simultaneous bids to acquire rival firms.

In the first stage, firm 1 offers bids $b_i$ to buy firm $i$ ($i = 2, \ldots, n$). In the second stage, these firms decide simultaneously whether to accept the bid or not. If firm $i$ accepts the offer, it sells the firm to firm 1 at the price $b_i$. Given the equilibrium market structure that results at the end
of stage two, the contract game of the previous section is played.

We solve by backward induction starting at stage two. Suppose that at the end of stage 2, there are \( n \) independent downstream firms. They would obtain the following profits in the market stage:

\[
\Pi^D(n) = \frac{(a - c_u)^2}{4(1 + n^3)^2}.
\]

This expression is obtained just by plugging \( c = \frac{a + c_u}{2} \) into expression (5). Observe that, in this case, the profits of downstream firms are always strictly positive.

Firms will accept the offers of firm 1 whenever the bid is not lower than their outside option, which of course depends on the acceptance decisions of the other firms. If, for example, \( k - 1 \) firms (other than firm \( j \)) accepted, the outside option of firm \( j \) would be \( \Pi^D(N - k + 1) \). At the first stage, firm 1 has to decide the number of firms to acquire, taking into account that in order to buy \( k \) firms it has to make a bid of \( \Pi^D(N - k + 1) \). Then, the payoff of firm 1 as a function of the number of acquisitions \( k \) is given by:

\[
\Pi^D(N - k) - k\Pi^D(N - k + 1)
\]

The maximizer of the previous expression is \( k = N - 1 \) if \( N \leq 21 \) and \( k = 0 \) otherwise. This result is summarized in the following proposition:

**Proposition 2** If \( N \leq 21 \), monopolization takes place. Otherwise, no merger occurs.

**Proof.** See Appendix

A natural question to address at this point is whether competition authorities should allow for monopolization. In order to answer this question recall that, in this model, social welfare is inversely related to price, which is given by:

\[
P^*(n) = \frac{a(2 - n + n^3) + c_u(n + n^3)}{2(n + 1)^3}.
\]
This expression is obtained just by plugging \( c = \frac{a + c_u}{2} \) into expression (3). Then, one can check that mergers up to duopoly reduce price and that price is maximized in monopoly. Therefore, the optimal merger policy should allow all mergers except the one leading to monopolization. Similar calculations as in Proposition 3 show that, under the optimal merger policy, mergers up to duopoly would take place whenever \( N < 13 \).

4 Conclusions

We have analyzed how a process of market concentration in the downstream sector affects final prices through its effect on the supply contracts offered by an upstream firm. We show that downstream mergers induce the upstream firm to offer lower wholesale prices. This reduction offsets the anticompetitive effect of mergers, only when the upstream firm enjoys a strong dominant position. In this case, downstream mergers countervail the market power of the dominant supplier, leading to an increase in social welfare.

A natural question is then to ask whether monopolization downstream can be the equilibrium outcome of an endogenous merger formation game. Contrarily to what happens when input prices are exogenous, we obtain that monopolization occurs even for very unconcentrated industries, when the upstream supplier has strong market power. Our results call for a lenient merger policy towards downstream mergers.

To conclude, we want to discuss more carefully the role played by \( c \), the price of the alternative supply. This parameter can be interpreted as a measure of the degree of competition upstream. The larger \( c \) the higher the monopolistic power of the dominant upstream firm. Then, it would be interesting to study settings where the parameter \( c \) is endogenously determined. One possible application would be to consider that the alternative supply is an international market for the input and the upstream supplier is a national firm. In this setting, it would be of interest the
analysis of the optimal tariff. The effect of this trade policy on welfare is not straightforward though. On the one hand, a tariff would increase the final price for a given number of firms, which hurts consumers and welfare. On the other hand, the imposition of a tariff would increase the monopolistic power of the upstream firm, which induces more mergers downstream. But in our model, downstream mergers may be welfare enhancing. The final effect of a tariff would depend on the balance of these two effects. This and some other possible applications of our model are left for future research.

5 Appendix

Proof of Proposition 1 with a general demand

Assume we have $n$ firms and market demand is given by $P(X)$, where $P'(X) < 0$ and $P''(X) \leq 0$. Firms have constant marginal costs. Denote by $C$ the sum of marginal costs. Then in an interior equilibrium we have that:

$$nP(X) - C + P'(X)X = 0 \quad (8)$$

The profits of a firm with cost $c$ is given by:

$$\pi(C) = \frac{(P(X(C) - c)^2}{-P'(X(c))}$$

where $X(C)$ is implicitly defined in (8). Then condition (2) in Proposition 1 can be written as

$$k \left[ \pi(C^*) - \pi(\underline{C}) \right] - (n - k) \left[ \pi(\bar{C}) - \pi(\underline{C}) \right] \geq 0 \quad (9)$$

where $C^* = (n - k + 1)c + (k - 1)w$, $C = (n - 1)w_1 + c$ and $\underline{C} = (n - k)c + kw$. We have also that $C^* - \bar{C} = \frac{(n-k)(c-w)}{n}$ and $\bar{C} - \underline{C} = \frac{k(c-w)}{n}$. (9) can be rewritten as:

$$\frac{\pi(C^*) - \pi(\bar{C})}{\pi(\bar{C}) - \pi(\underline{C})} = \frac{\int_{\underline{C}}^{C^*} \pi'(C)dC}{\int_{\underline{C}}^{\bar{C}} \pi'(C)dC} \geq \frac{n - k}{k}$$
A sufficient condition for this to hold is that \( \pi''(C) > 0 \). Then

\[
\frac{\int^C_0 \pi'(C) dC}{\int^C_0 \pi''(C) dC} > \frac{(C^* - C)\pi'(C)}{(C - C)\pi''(C)} = \frac{n - k}{k}
\]

It is tedious but direct to show that \( \pi''(C) > 0 \) if \( -P'(X) \) is log-concave and \( P''(X) \leq 0 \).

We show that it also holds for the class of demands \( P = A - X^b \), where \( b \geq 1 \).

**Proof of Proposition 2**

We have to find the maximizer of this expression:

\[
Max_w \begin{cases} 
  n \left( \left( \frac{a - w}{n+1} \right)^2 - \left( \frac{a - cn + w(n-1)}{n+1} \right)^2 + (w - c_u) \left( \frac{a - w}{n+1} \right) \right) \\
  \text{if} \quad c \geq w \geq -\frac{a + cn}{n-1}. \\
  n \left( \left( \frac{a - w}{n+1} \right)^2 + (w - c_u) \left( \frac{a - w}{n+1} \right) \right) \\
  \text{if} \quad w < -\frac{a + cn}{n-1}
\end{cases}
\]

\[s.t.w \leq c \tag{10}\]

Direct resolution of this problem leads to the result in Proposition 2.

**Proof of Proposition 3**

The objective of firm 1 is given by expression (7):

\[
F(N,k) = \Pi^D(N - k) - k\Pi^D(N - k + 1)
\]

Simple computations show that whenever \( N < 25 \), the result in the text holds. For \( N \geq 25 \), we proceed as follows. We check that for \( m \geq 9 \)

\[
\frac{\Pi^D(m)}{\Pi^D(m + 1)} < 2 \tag{12}
\]

This implies that for \( N - 9 \geq k \geq 2 \), we have that \( \frac{\Pi^D(N - k)}{\Pi^D(N - k + 1)} < 2 \). This implies that \( F(N,k) < 0 \). For \( N - 1 \geq k \geq N - 8 \), simple computations show that \( F(N,k) < 0 \). Finally, \( k = 1 \) yields less profits that \( k = 0 \), because of (12).
6 References


Salant, S.W., S. Switzer and R.J. Reynolds (1983), "Losses from horizontal mergers: The effects of an exogenous change in industry structure on a Cournot-Nash Equilibrium", The
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