On the Amplification Role of Collateral Constraints*

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Abstract

How important are collateral constraints for the propagation and amplification of shocks? Kiyotaki and Moore (1997) argue that collateralized debt may act as a powerful mechanism by which small shocks propagate into the economy. In contrast, Kochelekota (2000) and Cordoba and Ripoll (2004) demonstrate that collateral constraints per se are unable to propagate and amplify exogenous shocks, unless unorthodox assumptions on preferences and production technologies are made.

This paper explores the role of inefficiencies in the debt enforcement procedures for the amplification of productivity and credit market shocks. We argue that the magnitude of amplification crucially depends on the fraction of the asset used as a collateral in the credit market.

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1 Introduction

Standard Real Business Cycle theories succeed in accounting for business cycle observations of aggregate quantities, such as output, investment and consumption, by relying mainly on large and persistent aggregate productivity shocks. Kiyotaki and Moore (1997) and Kiyotaki (1998) show that if debt is fully secured by collateral, even small and temporary productivity shocks can have large and persistent effects on economic activity. Kiyotaki and Moore’s theoretical work has been very influential and an increasing number of papers have documented the contribution of collateral constraints to business cycle fluctuations. Collateralized debt is becoming a popular feature of business cycle models.\(^1\) A common assumption in this strand of the business cycle literature is that debt enforcement procedures are costly and lenders limit the agents’ ability to borrow to a fraction of the value of their collateral.

Kocherlakota (2000) and Cordoba and Ripoll (2004) demonstrated that collateral constraints \textit{per se} are unable to propagate and amplify exogenous shocks. In particular, Cordoba and Ripoll (2004) document that the endogenous amplification generated by Kyiotaki and Moore (1998) is driven by unorthodox assumptions on agents’ preferences (lenders’ linear utility) and technology (borrowers’ linear technology in the collateral asset). As a result, in a modified version of the model in which all agents face the same concave preferences and production technologies no amplification is found.

Papers on the amplification role of collateral constraints have neglected the role of inefficiencies in the liquidation of the collateralized assets and thus the required level of downpayment. As documented by Djankov, Hart, McLiesh and Shleifer (2006) debt enforcement procedures around the world are significantly inefficient. An average of 48 percent of an insolvent firm’s value is lost in debt enforcement worldwide. Costly debt enforcement procedures provide a rational to higher downpayment requirements. Thus, limiting the amount lent to a fraction of the value of the asset is a reasonable assumption.\(^2\)

\(^1\) For example, on the international transmission of business cycles, see Iacoviello and Minetti (2007); on the role of the housing and collateralized debt in the transmission and amplification of shocks, see Iacoviello (2005) and Iacoviello and Neri (2008); on the macroeconomic implications of mortgage market deregulation, see Campbell and Hercowitz (2005); on the role of nominal debt in sudden stops, see Mendoza (2006) and Mendoza and Smith (2007); and on overborrowing, see Uribe (2007).

\(^2\) Among others Calza et al (2007) report that the typical LTV ratios in the mortgage...
This paper aims to reconcile these two strands of the business cycle literature by exploring the role of downpayment requirements in the transmission of productivity and credit market shocks using a stochastic general equilibrium model with borrowing limits à la Kiyotaki and Moore (1997). We modify the original model as in Cordoba and Ripoll (2004) and assume that all agents face concave production and utility functions and are generally identical, except for the subjective discount factor. We confirm that under these standard assumptions if agents can borrow the full value of the collateral, the endogenous amplification generated by the model is negligible. However, we argue that the magnitude of the endogenous amplification delivered by collateral constraints crucially depends on the fraction of the asset used as a collateral in the credit market.

In the benchmark model, featuring inelastic capital supply, the sensitivity of output to productivity shocks depends on the redistribution of capital between borrowers and lenders and varies in a non-linear way with respect to the degree of downpayment requirement. The intuition is as follows. Since all producers face the same technology, the borrowers, limited in their capital holding by the existence of credit constraints, experience higher marginal productivity of capital. The higher the downpayment requirement, and thus the degree of friction in the credit market, the lower their capital holding. High downpayment requirements lead to a large difference between borrowers’ and lenders’ productivity and sizable gains from a better allocation of resources. However, the redistribution of capital to the borrowers is limited by the high downpayment requirement itself that restrict the access to external funds. In contrast, low downpayment requirements allow for a larger redistribution of capital among agents but are associated to smaller differences in the marginal productivity. Thus, the actual gains from the redistribution are low. We show that the model features negligible amplification in only two parameterization: autarky and fully efficient debt enforcement procedures. At an intermediate level of down-payment requirements, collateral constraints significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

Kocherlakota (2000) and Cordoba and Ripoll (2004) report that models with market vary significantly among OECD countries ranging between 50% in Italy and 110% in the Netherlands.
collateral constraints require an uncommonly high capital share in production in order to generate endogenous amplification of shocks. We show that this result holds only when the agents can borrow the full amount of their collateral or when downpayment requirements are sufficiently low. Standard value for the capital share in production are sufficient to amplify the effect of shocks in economies with higher downpayment requirements.

Corboda and Ripoll (2004) also find that changes in the different assumptions of the benchmark model do not affect the sensitivity of output to productivity shocks. Both the introduction of labor and of capital accumulation reduce the sensitivity of output to productivity shocks. In contrast, we show that both extensions to the benchmark model may generate larger amplification of shocks once we allow for different degrees of downpayment requirement. In particular, when capital is reproducible, movements in the relative price of capital enters the measurement of aggregate output and directly affect the transmission of shocks to output. Since higher down-payment requirements are related to larger differences in productivity between borrowers and lenders, more capital is needed to fill the productivity gap and the sensitivity of the relative price of capital to productivity shocks is larger. We show that the contribution of the relative-price effect to the amplification of shocks is particular sizable in economies with higher downpayment requirements.

This paper also explores the effects of temporary deviations from the established downpayment requirements. Our findings show that for a large range of the degree of credit market inefficiency, collateral constraints significantly amplify the effects of credit market shocks on output even under standard assumptions on preferences and technology.

The paper is organized as follows. Section 2 presents the benchmark model with capital in fixed aggregate supply and studies the amplification of productivity shocks. Section 3 extend the model as to introduce endogenous labor supply. Section 4 studies amplification in the model with reproducible capital. Section 5 analyses the amplification of shocks to the credit market. Section 6 discusses the results and section 7 draws some conclusions.
2 Benchmark Model

We adopt a two-agents close economy model à la Kiyotaki and Moore (1997) modified as in Corboda and Ripoll (2004) to introduce standard assumptions on preferences and technology. The economy populated by two types of agents who trade two kinds of goods: a durable asset and a non-durable commodity. The durable asset \((k)\) does not depreciate and has a fixed supply normalized to one. The commodity good \((c)\) is produced with the durable asset and cannot be stored. At time \(t\) there are two competitive markets in the economy: the asset market, in which one unit of the durable asset can be exchanged for \(q_t\) units of the consumption good, and the credit market. In order to impose the existence of flows of credit in this economy ex-ante heterogeneity is assumed on the subjective discount factor: \(\beta_2 < \beta_1 < 1\). The economy is populated by a continuum of heterogeneous agents of unit mass: \(n_1\) Patient Entrepreneurs (denoted by 1) and \(n_2\) Impatient Entrepreneurs (denoted by 2). This assumption ensures that in equilibrium patient households lend and impatient households borrow.

Agents of type \(i\), \(i = 1, 2\), maximize their expected lifetime utility as given by

\[
\max_{\{c_{it}, k_{it}, b_{it}\}} \lim_{T \to \infty} \sum_{t=0}^{T} (\beta_t)^t U (c_{it})
\]

s.t. a budget constraint

\[
c_{it} + q_t (k_{it} - k_{it-1}) = y_{it} + \frac{b_{it}}{R_t} - b_{it-1}
\]

and a borrowing constraint

\[
b_{it} \leq \gamma E_t \left[q_{t+1} k_{it}\right],
\]

where \(k_{it}\) is a durable asset, \(c_{it}\), a consumption good, and \(b_{it}\), the debt level.

Both agents produce the commodity good using the same technology:

\[
y_{it} = Z_t k_{it-1}^{\alpha}
\]

where \(Z_t\) represents a temporary aggregate productivity shock. Agents have access to the same concave production technology: \(\alpha_1 = \alpha_2 < 1\). Technology is specific to each producer and only the household that started the production
process has the skills necessary to complete the process. Nevertheless, agents cannot precommit to produce. This means that if household \( i \) decides not to put his effort into production between \( t \) and \( t + 1 \), there would be no output at \( t + 1 \), but only the asset \( k_{it} \). Agent are free to walk away from the production process and from debt contracts between \( t \) and \( t + 1 \). This results in a default problem that makes creditors willing to protect themselves by collateralizing the borrower’s asset. Creditors know that in the case where the borrower chooses not to produce and neglects his debt obligations, they can still get his asset. However, we assume that the lenders can repossess the borrower’s assets only after paying a proportional transaction cost, \( [(1 - \gamma)E_t q_{t+1} k_{it}] \). Thus, agents cannot borrow more than a certain amount such that the next period’s repayment obligation cannot exceed the expected value of next period assets, \( b_{it} \leq \gamma E_t [q_{t+1} k_{it}] \). The lower \( \gamma \), the more costly, and thus inefficient, the debt enforcement procedure. The fraction \( \gamma \), referred to as the loan-to-value ratio, should not exceed one and is treated as exogenous to the model.

Agents’ optimal choices of bonds and capital are characterized by:

\[
\frac{U_{c_i,t}}{R_t} \geq \beta_i E_t U_{c_i,t+1}, \tag{2}
\]

and

\[
q_t - \beta_i E_t \frac{U_{c_i,t+1}}{U_{c_i,t}} q_{t+1} \geq \beta_i E_t \frac{U_{c_i,t+1}}{U_{c_i,t}} (F_{k_i,t+1}), \tag{3}
\]

where \( F_{k_i,t} = \alpha Z_t k_{it}^{\alpha-1} \) is the marginal product of capital. The first equation relates the marginal benefit of borrowing to its marginal cost, while the second shows that the opportunity cost of holding one unit of capital, \( [q_t - \beta_i E_t \frac{U_{c_i,t+1}}{U_{c_i,t}} q_{t+1} (1 - \delta)] \), is greater than or equal to the expected discounted marginal product of capital.

For constrained agents, the marginal benefit of borrowing is always bigger than its marginal cost. If \( \mu_{i,t} \geq 0 \) is the multiplier associated with the borrowing constraint, then the Euler equation becomes:

\[
\frac{U_{c_i,t}}{R_t} - \mu_{i,t} = \beta_i E_t U_{c_i,t+1}. \tag{3.a}
\]

Moreover, borrowers internalize the effects of their capital stock on their financial constraints. Thus, the marginal benefit of holding one unit of capital is given not only by its marginal product but also by the marginal benefit of being allowed to borrow more:

\[
q_t - \beta_i E_t \frac{U_{c_i,t+1}}{U_{c_i,t}} q_{t+1} (1 - \delta) = \beta_i E_t \frac{U_{c_i,t+1}}{U_{c_i,t}} (F_{k_i,t+1}) + \gamma E_t q_{t+1} \frac{\mu_{i,t}}{U_{c_i,t}} \tag{10.a}
\]
Collateral constraints alter the future revenue from an additional unit of capital for the borrowers. Holding an extra unit of capital relaxes the credit constraint and thus increases their shadow price of capital. Thus, this additional return encourages borrowers to accumulate capital even though they discount the revenues more heavily than lenders. As long as the marginal product of capital differs from its market price, borrowers have an incentive to change capital stock.

This framework implies impatient agents to be credit constrained in the deterministic steady state. Following previous literature, we analyze the properties of the model in a neighborhood of the steady state. Thus, if the economy fluctuates around the deterministic steady state, the borrowing constraint holds with equality for impatient agents

\[ b_{2,t} = \gamma E_t \left[ q_{t+1} k_{2t} \right] \]

and

\[ k_{2t} = \frac{W_{2,t} - c_{2,t}}{q_t - \gamma E_t \frac{q_{t+1}}{R_t}} \]

where \( W_{2,t} = y_{2,t} + q_t k_{2,t} - b_{2,t-1} \) is the impatient agent’s wealth at the beginning of time \( t \), and \( d_t = \left[ q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right] \) represents the difference between the price of capital and the amount he can borrow against a unit of capital, i.e. the downpayment required to buy a unit of capital. The higher \( \gamma \) the lower the downpayment requirements.

In contrast, patient households are creditors in the neighborhood of the steady state. The creditors’ capital decision is determined at the point where the opportunity cost of holding capital equals its marginal product:

\[ q_t - \beta_1 E_t \frac{U_{c_1,t+1}}{U_{c_1,t}} q_{t+1} = \beta_1 E_t \frac{U_{c_1,t+1}}{U_{c_1,t}} (F_{k_1,t+1}) . \] (3.b)

\^Consider the Euler equation of the impatient household:

\[ \frac{u_{c_2,t}}{R_t} - \mu_{2,t} = \beta_2 E_t u_{c_2,t+1} . \]

In the steady state it implies:

\[ \mu_2 = \left( \frac{1}{R} - \beta_2 \right) u_{c_2} . \]

Since the steady state interest rate is determined by the discount factor of the patient agent:

\[ \mu_2 = \left( \frac{1}{R} - \beta_2 \right) u_{c_2} = (\beta_1 - \beta_2) u_{c_2} ; \]

As long as \( \beta_2 < \beta_1 < 1 \), the lagrange multiplier associated with the borrowing constraint for the impatient household is strictly positive in the deterministic steady state.
The total stock of capital $k_t$ is given by:

$$k_t = n_1 k_{1t} + n_2 k_{2t}. \quad (4)$$

The following conditions also hold

$$y_t = n_1 y_{1t} + n_2 y_{2t} = n_1 c_{1t} + n_2 c_{2t} \quad (5)$$

$$n_1 b_{1t} = -n_2 b_{2t} \quad (6)$$

### 2.1 Benchmark parameter values.

We set the parameters’ value on a quarterly base. Patient households’ discount factor equals 0.99 such that the average annual rate of return is about 4 percent. The discount factor for impatient agents, $\beta_2$, equals 0.95 This value is in line with previous estimates by Lawrence (1991), Samwick (1998) and Warner and Pleeter (2001). See also Hendricks (2007). The baseline choice for the fraction of the population that is borrowing constrained, $n_2$, is set to 50 percent. This value is also in the range of estimates in the literature. Campbell and Mankiw (1989) estimate around 40 percent of the population to be rule-of-thumb consumers. According to Iacoviello (2005), in the U.S. about 55 percent of the population is credit constrained. The coefficient of relative risk aversion, $\sigma$, equals 2.2, which is in the range suggested by several previous authors. As a benchmark value for the share of capital in production we set $\alpha=0.4$. In the section 2.5 we investigate the sensitivity of our results to different parameter values.

### 2.2 Credit Market and the Deterministic Steady State

In what follows, we analyze how the deterministic steady state of the model is affected by the equity requirements as proxied by $\gamma$. Figure 1.a shows the marginal productivity of capital as a function of $\gamma$. Using the equations representing the households’ optimal choice of capital evaluated at the steady state it is possible to show that as long as $\gamma < \frac{1}{\beta_1}$,

$$\frac{F_{k_2}}{F_{k_1}} = \frac{\beta_1 [1 - \beta_2 - \gamma(\beta_1 - \beta_2)]}{(1 - \beta_1) \beta_2} > 1,$$

(7)

where $F_{k_i} = \alpha \left( \frac{K_i}{n_i} \right)^{\alpha - 1}$. Thus, the steady state allocation of capital depends on the subjective discount factors, the population weights for the two groups of
agents, and \( \gamma \).

\[
K_2 = \frac{1}{1 + \frac{n_1}{n_2} \left[ \frac{\beta_2(1-\beta_1)}{\beta_1(1-\beta_2-\gamma(\beta_1-\beta_2))} \right]^{\frac{1}{\pi_f}}}
\]

Compared to the frictionless case, the allocation under credit constraints reduces the level of capital held by borrowers and implies a difference in the marginal productivity of capital for the two groups of producers. The higher \( \gamma \) the lower the difference between borrowers’ and lenders’ marginal productivity and the larger the borrowers’ share of total production. Since total output is maximized when the marginal productivity of the two groups is identical, collateral requirements distort total production below the efficient level. Thus, a more efficient credit market improves the allocation of capital between the two groups of agents and reduces the efficiency loss in terms of output. See figures 1.b-1.c.

A better allocation of capital between the two groups also leads to a higher relative price of capital.\(^4\) See figure 1.d

### 2.3 Model Dynamics

We now consider the response of the model economy to a productivity shock. We assume that the economy is at the steady state level at time zero and then is hit by an unexpected increase in aggregate productivity of 1 percent. For an illustrative purpose we assume a loan-to-value ratio of 80%.\(^5\) The results are reported in figure 2. An aggregate shock rises production and thus the earnings of both groups of agents. As the shock hits the economy, borrowers, initially limited in their capital holdings by borrowing constraints, increase their demand for productive assets. This allows the agents to more easily smooth the effect of the shock. In order for the capital market to clear, lenders have to decrease their demand for capital. The user cost of holding capital increases. Movements in the relative price of capital, altering the value of the collateral asset, affect the

\(^4\)The relative price of capital depends on the marginal productivity of capital and increases with \( \gamma \). The steady state value of the price of the asset, \( q = \frac{\beta_1}{1-\rho_1(1-\gamma)^\frac{1}{\pi_f}} F_{k_1} = \frac{\beta_2}{1-\rho_2(1-\gamma)^\frac{1}{\pi_f}} F_{k_2} \), is always less than unity for any value of \( \gamma < 1 \). The model is not equivalent to the standard one-sector real business cycle model with a one-to-one trasformation between consumption and capital.

\(^5\)Djankov, Hart, McLiesh and Shleifer (2006) find an average of 48 percent of the firm’s value is lost in debt enforcement worldwide, 24 percent among OECD countries and 14 percent in the US.
ability to borrow. Thus, borrowers’ expenditure decisions are affected not only by the direct impact of the shock but also by the larger availability of credit resulting from a rise in the value of their collateral. Due to the higher marginal productivity of capital experienced by the borrowers, the positive effect of an increase in aggregate productivity on total production is propagated over time.

2.4 Credit Market and Amplification

Kiyotaki and Moore’s (1997) theoretical work shows that the amplification generated by the model with collateral constraints may be very large. Cordoba and Ripoll (2004) document that the amplification generated by the model is driven by two unorthodox assumptions: the linearity of the production technology in the collateral asset for the borrowers and the linear utility function for the lenders. According to their results, when agents face concave preferences and technology no amplification is endogenously generated by the model. In this paper we show that even when borrowers and lenders face the same concave production technology and utility, collateral constraints generate significant amplification of productivity shocks for intermediate levels of down-payment requirements.

Since the first impact of the shock is equal to the shock itself, we look at the second-period effect of the shock. See figure 3.a (left panel). The top panel report the results for an iid shock and the bottom panel for a shock with persistence equal to 0.9.

Given that all producers face the same technology, borrowers limited in their capital holding by the existence of credit constraints, experience higher marginal productivity of capital. A redistribution of capital in favor of the borrowers generates significant variations in terms of total production. In the absence of a credit market \((\gamma = 0)\), capital is allocated in a very inefficient way, so the gains from a better allocation of resources allowed by positive productivity shocks are potentially very big. On the other hand, the redistribution of capital induced by the shock itself is limited since impatient agents cannot finance their capital expenditure through the credit market. Thus, the amplification of the shocks on total production is negligible. Lower down-payment requirements allowing for an easier access to external funds generate larger redistribution of capital and enhance the endogenous amplification generated by the model. However,
as $\gamma$ approaches unity the difference in marginal productivity of capital between lenders and borrowers shrinks (see figures 1.a, 1.c). When the allocation of capital between borrowers and lenders is such that the productivity gap is small, the gains from the redistribution of capital are minimal and the amplification generated by the model is in fact negligible.

Strictly speaking, the second-period elasticity of total output with respect to technology shocks can be written as:

$$
\epsilon_{yz} = \epsilon_{ykz} = \frac{F_{k2} - F_{k1}}{F_{k2}} \alpha \frac{y_2}{y} \epsilon_{k2z}.
$$

The first term is the productivity gap between constrained and unconstrained agents (see figure 1.a), $\alpha$ represents the share of capital in production, while $y_2$ is the production share of constrained agents, and $\epsilon_{k2z}$ is the elasticity of borrowers’ capital with respect to the shock – i.e. the redistribution of capital to impatient agents). The elasticity of total output to productivity shocks depends on the production share of constrained agents and the productivity gap. The fraction of total output produced by constrained agents increases with $\gamma$, since more efficient enforcement procedures induce a better allocation of capital in the economy (production share effect). However, for the same reason, the productivity gap decreases with $\gamma$. These two opposite forces contribute to non-linear shape of the second-period impact of the shock on total output.

It is easy to prove that assuming linearity in capital for the borrowers’ production function and linearity for the lenders’ utility function generate larger amplification of shocks for any given $\gamma$. A linear production function for borrowers doesn’t decrease the marginal productivity of capital with respect to $\gamma$ and generates a larger productivity gap for any given $\gamma$. Linearity of the lenders’ utility function instead doesn’t allow for variations in the real interest rate. If after a positive productivity shock lenders are willing to provide additional funds without any rise in the real interest rate borrowers increase in capital expenditure and production is more sizeable. Thus, a constant interest rate implies a higher elasticity of borrowers’ capital to productivity shocks.

Figure 3.a plots the second-period variation in output (left panel) and the cumulative response of output over a 15-quarter period (right panel). Following an i.i.d. shock, the deviation of output from the steady state is below 0.3% even at the peak of amplification. However, the effect of the shock is quite persistent. In fact, over a 15-quarter period the cumulative deviation of output
from the steady state is between 1 and 4 percent. The response of output is largely more pronounced in case of a shock with persistence equal to 0.9 with a peak of 10.5 percent cumulative response.

2.5 Sensitivity

Results presented above show that for values of $\gamma$ below unity the model with collateral constraints can generate amplification and persistence of productivity shocks of non-negligible magnitude. Kocherlakota (2000) and Corboba and Ripoll (2004) argue that models with collateral constraints require an uncommonly high capital share in production in order to generate large amplification. We investigate the sensitivity of the results to the share of capital in production and the discount factor of borrowers. We show that the relation between $\gamma$ and the intensity of the output reaction to productivity shocks is clearly non-linear with respect to both $\alpha$ and $\beta_2$.

Regarding the discount factor of borrowers, we compare the results for three different values of $\beta_2$: 0.91, 0.95 and 0.97. These values are in the ballpark of previous estimates. For the share of capital in the production process, $\alpha$, we follow Angeletos and Calvet (2006) and assume two different values of this parameter: $\alpha=0.4$, which corresponds to the standard definition of capital, and $\alpha=0.7$, which reflects a broader definition and includes both physical and intangible capital.

We document that in the benchmark model, output amplification is not a strictly increasing function of the capital share. In fact, a lower $\alpha$ does not necessarily imply lower amplification of shocks. In the model presented here, this result holds only for sufficiently low down-payment requirements. The same result holds for the degree of heterogeneity in the model as measured by the relative difference in discount factors. A lower $\beta_2$ generate larger amplification only under low downpayment requirements. The region of amplification shrinks with a higher capital share in production. See figures 3.b and 3.c.

3 Introducing Labour Supply

According to Cordoba and Ripoll (2004), if aggregate labor is not fixed but rather optimally supplied, the amplification role of collateral constraints is dramatically reduced. To explore the robustness of the results presented above, we
now consider the case where household work is also an input of production. We assume that each household works in its own firm and gets utility from leisure.

**Model 1.** Following Greenwood *et al.* (1988), we adopt the following utility function

\[ U(c_{it}, L_{it}) = \frac{1}{1-\sigma} \left( c_{it} - \frac{L_{it}}{\eta} \right)^{1-\sigma} \]  

(9)

and production function

\[ y_{it} = Z_t k_{it-1}^\alpha L_{it}^{1-\alpha}. \]  

(10)

Figure 4 shows the response to a 1% increase in productivity. Endogenous amplification of the shocks is already present in the first period.\(^6\) However, it is possible to show that the first period amplification is independent of \(\gamma\). Given the household’s labor supply,

\[ \chi L_{it}^{\eta-1} = (1 - \alpha)Z_t k_{it}^\alpha L_{it}^{-\alpha}, \]  

(11)

individual’s production can be written in terms of the capital input:

\[ y_{it} = Z_t^{\frac{\eta}{\alpha+\eta-1}} k_{it-1}^{\alpha + \frac{(\alpha - \alpha \eta)}{\alpha+\eta-1}} \frac{1 - \alpha}{\chi}. \]

When productivity decreases by 1 percent, output decreases by \(\frac{\eta}{\alpha+\eta-1} = 1.47\) percent. The second impact still varies with the degree of credit friction. The elasticity of total output with respect to technology shocks can be written as in equation (7), but now multiplied by \(\frac{\eta}{\alpha+\eta-1}\):

\[ \epsilon_{yz} = \frac{\eta}{\alpha + \eta - 1} \epsilon_{ykz} \epsilon_{kzz}. \]  

(12)

Figure 4.a shows that the output response delivered by the model with variable labor supply can be much stronger and persistent than the response generated by the benchmark model.

**Model 2.** In order to take into account the implications for amplification of the wealth effects on labor supply the following utility function is assumed\(^7\):

\[ U(c_{it}, L_{it}) = \frac{1}{1-\sigma} \left( c_{it}^{(1-\varphi)} (1 - L_{it})^\varphi \right)^{1-\sigma}. \]  

(13)

\(^6\)We calibrate the labor supply elasticity to 0.5 (\(\eta=2\)). The weight on leisure is chosen so that hours worked in the initial steady state is around 1/3 of total time depending on \(\gamma\) (\(\chi=1\)).

\(^7\)We calibrate \(\varphi = 0.6\), so that hours worked in the initial steady state are around 1/3 of total time.
As in Cordoba and Ripoll (2004), when $\gamma = 1$, introducing labor supply according to a utility function of this type is detrimental for the amplification of shocks. However, the result only holds for a very limited range of the downpayment requirement parameter. Figure 4.b shows that for any value of $\gamma < 0.98$, the magnitude of the second-period amplification is bigger than that produced by the benchmark version of the model.\(^8\)

### 4 Reproducible Capital

We follow Cordoba and Ripoll (2004) and consider the case in which the durable asset, $k$, is reproducible and depreciates at the rate of $\delta$. Agents of type $i$, $i = 1, 2$, maximize their expected lifetime utility s.t. the following budget constraint

$$c_{it} + q_t(k_{it} - (1 - \delta) k_{it-1}) = F_{it} + \frac{b_{it}}{K_t} - b_{it-1}$$

Real production is given by

$$F_{it} = y_{it} + q_t h_{it}$$

where $y_{it}$ represents the technology for producing consumption goods and $h_{it}$ is the production for capital goods

$$y_{it} = Z_t \left( k_{it-1}^c \right)^{\alpha_y} \quad h_{it} = Z_t \left( k_{it-1}^h \right)^{\alpha_h} \quad (14)$$

with $k_{it-1}^j - j = c, h$ – being the stock of capital used as an input of production in the two sectors. We allow for reproducible capital and assume that each agent is able to produce both consumption and investment goods.\(^9\) For simplicity, we assume that both types of production are identical.\(^10\)

It is possible to express the amount of capital allocated to each type of production as a fraction of the total capital owned by each agent, as follows:

$$k_{it-1}^c = \theta_t k_{it-1} \quad (15)$$

where $\theta_t(q) = \frac{q_{it-1}}{1 + q_{it-1}}$. Thus, the allocation of existing capital between the two productions depends on the current relative price of capital. The total

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\(^8\)According to Djankov et al. (2006), the degree of efficiency in the debt enforcement procedures around the world does not reach such a high level.

\(^9\)In this way we avoid creating a rental market for capital, and make the model directly comparable to those of Kiyotaki and Moore (1997).

\(^10\)The assumption of decreasing returns in the production of investment goods is equivalent to assume convex adjustment costs for investments.
production of each individual can be expressed as

$$F_{it} = k_{it}^{\alpha}Z_{it}[\theta_{it}^{\alpha} + q_{it}(1 - \theta_{it})^{\alpha}] .$$  \hspace{1cm} (16)

Each agent capital stock evolves according to

$$k_{it} = (1 - \delta)k_{it-1} + h_{it}.$$  \hspace{1cm} (17)

The total stock of capital $k_t$ is given by:

$$k_t = n_1k_{1t} + n_2k_{2t}.$$  \hspace{1cm} (18)

### 4.1 Impulse Responses.

Figure 5 shows the response of total aggregate output to the productivity shock.\footnote{The productivity parameter, $\alpha$, is set to 0.4, in all sector. The capital depreciation rate equals 0.03.} After a one percent increase in aggregate productivity, total output increases by approximately 1.2 percent in the first period and further in the second. In response to a neutral technology shock, the model generates co-movement between consumption and capital good production. However, the production of the capital good shows evidence of significant amplification, while the production of the consumption good reacts much less markedly.

After an exogenous increase in productivity, borrowers increase their demand for capital. As in the benchmark model, the redistribution of capital in favor of borrowers propagates the positive effects of the shock to future production.\footnote{The allocation of capital in this version of the model is given by}

Further more, the increase in the relative price of capital, $q_t$, generated by the redistribution of capital between borrowers and lenders, strongly contribute to the first-period amplification of the shock. The increase in the relative price of capital implies a more profitable use of the productive input in the capital good sector and thus a more efficient allocation of capital as factor in this production. Capital is reallocated towards the production of investment goods in coincidence of the two major peaks of amplification. With asset prices increasing and the production of investment good reacting strongly, the response of aggregate real output is greatly amplified.

\footnote{The allocation of capital in this version of the model is given by}

$$\frac{K_1}{K_2} = \left[ \frac{\beta_1(1 - \beta_2(1 - \delta) - \gamma(\beta_1 - \beta_2))}{\beta_2} \right]^{\frac{1}{1-\gamma}} > 1.$$  

See figure 1.e.
4.2 Amplification

Figure 6.a compares the output’s reaction to the productivity shock for different values of $\gamma$. The first-, second-period and cumulative response of output is displayed. In the first-period output amplification falls as $\gamma$ rises. More specifically, a higher $\gamma$ magnifies the reaction of consumption goods production while weakening the response of investment goods production. The difference between the reactions of the two sectors is explained by the dynamics of the price of capital which drives the capital reallocation between sectors. Given that an economy with a lower degree of credit rationing has a smaller productivity gap between lenders and borrowers, less capital is redistributed to the borrowers. Thus, borrowers’ demand for capital rises by a smaller margin, which dampens the increase in the relative price of capital. It thus becomes less profitable to reallocate capital to the production of investment goods. As shown in Figure 6.b (top panel), reducing credit market frictions lowers the sensitivity of asset prices to productivity shocks and consistently reduces the magnitude of capital reallocation between sectors. Since in the model with reproducible capital variations in relative price enter the measurement of total output, the decline in the sensitivity of the relative price of capital directly affects the sensitivity of total output to productivity shocks. In the second period, both the allocations of capital between sectors of production and the redistribution of capital between groups of producers contribute to amplification. Thus, as in the model with capital in fixed supply, both the second-period response and the cumulative impact of the shock on total output displays a non-linear shape.

5 Credit Market Shocks

5.1 Impulse Responses

Now we consider the response of the economy to temporary deviations from the established downpayment requirements. Figures 7.a-7.b show the impact of a positive shock in the benchmark version of the model with capital in fixed supply and in the model with reproducible capital, respectively. A sudden increase in the availability of external funds increases the ability of borrowers to finance
their capital expenditure. Aggregate variables move in the same direction as in the case of a positive productivity shock. Only exception is the behavior of the relative price of capital. The asset price equation derived from the model is given by

\[ q_t = E_t \sum_{j=0}^{\infty} \beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}} F_{k1,t+j}, \]  

where \( \beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}} \) is the stochastic discount factor or \textit{pricing kernel} and \( F_{k1,t+j} \) is the marginal productivity of capital. Agents demand for capital is such that the marginal productivity of capital, discounted by \( \beta_1^j \frac{u_{c1,t+1}}{u_{c1,t}} \), is equal to the relative price of capital. Thus, movements in the real interest rate and the productivity of capital determine the behavior of the relative price of capital. A reduction in the required downpayment means that lenders have to increase their supply of available funds. For this to happen, the real interest rate needs to increase. Since the capital stock can only be used in production with a one-period lag, under credit market shocks, movements in the real interest rate determine the first-period behavior of asset prices. Thus, the price of capital first declines and then increases.

In the model with aggregate capital in fixed supply the decrease in the relative price of capital has an indirect effect on output via the collateral constraints. See figure 7.a. In the model with reproducible capital since variations in relative prices enter the measure of total output, the decline in the relative price of capital leads to a first-period decline in total output. See figure 7.b.

5.2 Amplification

In what follows, we consider how the sensitivity of output to a temporary change in the availability of credit depends on \( \gamma \). Figure 8.a shows the second-period impact and the cumulative response of output over a 15-quarter period for the benchmark model. The sensitivity of output to credit market shocks and productivity shocks displays a similar relationship with the degree of efficiency in the debt enforcement procedures. However, contrary to the productivity shock, an iid shock to the availability of credit generates stronger second-period responses of output than a persistent shock. The reason is as follows. A one-period increased availability of credit makes borrowers increase their borrowing ability as much as possible. Thus, the less persistent the credit market shock,
the lower the substitution effect and the larger the reaction of the real interest rate. This leads to larger initial movements in capital and lending and thus, larger variations in production. Still the resulting cumulative effect is larger for more persistent shocks.

Figures 8.b-8.c display the reaction of total output and the relative price of capital in the model with reproducible capital. A higher \( \gamma \) implies a larger first-period reduction in the relative price of capital and thus in total output. However, despite the initial decrease in total output, the cumulative response is increasing with respect to \( \gamma \).

We find substantial differences in the amplification if we measure output including variations in the relative price of capital or not. If only quantities affect movements in total output the first impact is negligible and the second impact is only driven by the redistribution of capital between agents. So as in the benchmark model the cumulative response of output to the credit market shock displays an inverted U shape. However, this result does not hold for iid shocks. Figures 8.d shows the response of total production measured at steady state prices.

6 Elastic vs Inelastic Capital supply

The degree of credit market inefficiency affects the reaction of output to productivity shocks differently in the two models. See figure 9.a for the cumulative response of output. When aggregate capital is fixed in supply and only one good is produced, the only source of amplification after the shock is the redistribution of capital between borrowers and lenders. Thus, the model doesn’t feature amplification in the first period. In contrast, in the model with reproducible capital, movements in relative prices significantly affect the amplification both in the first and subsequent periods. In accordance with Cordoba and Ripoll (2004) when \( \gamma=1 \) and standard parameter values are used to calibrate the technology and utility functions, in both versions of the model the amplification generated by the model is negligible. However, the results are not robust to different assumptions on the degree of inefficiencies. In both models larger amplification of productivity shocks is generated at intermediate levels of credit market inefficiency.

In the benchmark model the sensitivity of output to both productivity and
credit market shocks displays an inverted U relationship with the degree of efficiency in the debt enforcement procedures. In contrast in the model with reproducible capital, the degree of credit market inefficiency affects the elasticity of total output to technology shock and credit market shocks in a different way. The lower the degree of credit frictions the higher the impact of credit shocks on output due to the relative price effect. The total effect is much larger in the two sector model. See figure 9.b.

7 Conclusion

The aim of this paper is to quantify the amplification generated by collateral constraints in relation to the degree of frictions in the credit market. To this end, we analyze a business cycle version of the Kiyotaki and Moore (1997) model. In contrast to previous papers that call into question the quantitative relevance of collateralized debt as a transmission mechanism, we document that the existence of costly debt enforcement plays an important role in the endogenous amplification generated by the model. We also show how different assumptions on the reproducibility of the asset used as a collateral affect the transmission and amplification of both productivity and credit market shocks.
8 References

Figure 1.a shows how the steady state productivity gap in total production between the two groups of agents varies with respect to $\gamma$.

Figure 1.b
Figure 1.c shows how the steady state value of total output changes with respect to the degree of credit market development $\gamma$ in the model with capital in fix supply.

Figure 1.d shows how the steady state values of the model's variables change with respect to the degree of credit market development $\gamma$ in the model with capital in fix supply.
Figure 1.e shows how the steady state values of the model's variables change with respect to the degree of credit market development $\gamma$ in the model with reproducible capital.
Figure 2. Responses of the model economy to an unexpected 1% increase in aggregate productivity; $\rho=0.9; \gamma=0.8$. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.
Figure 3.a Sensitivity of Output for any given $\gamma$ in the second period (left panel) and cumulative over a 15 quarters period (right panel). Benchmark Model.
Figure 3.b Sensitivity of Output for any given $\gamma$ in the second period (top panel) and cumulative over a 15 quarters period (bottom panel) for $\alpha=.4$ (dashed line) and $\alpha=0.7$ (dotted line). Iid productivity shock, benchmark Model.

Figure 3.c Sensitivity of Output for any given $\gamma$ in the second period (top panel) and cumulative over a 15 quarters period (bottom panel) $\beta=.91$ (dotted line), $\beta=.95$ (dashed line) and $\beta=.97$(solid line). Iid productivity shock, benchmark Model.
Figure 4.a Sensitivity of Output for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel). Model with Labor supply (1).
Figure 4.b Sensitivity of Output for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel). Model with Labor supply (2)
Figure 5.a Responses of the model economy to an unexpected 1% increase in aggregate productivity; $\rho=0.9$; $\gamma=0.8$. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.
Figure 5. Responses of the model economy to an unexpected 1% increase in aggregate productivity; $\rho=0.9; \gamma=0.8$. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.
Figure 6.a Sensitivity of Output (top) and the Relative Price of Capital (bottom) for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel). Model with reproducible capital.
Figure 6.b Sensitivity of Output (top) and the Relative Price of Capital (bottom) for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel).
Figure 6.c Second-period sensitivity of Production by sector and reallocation of capital for any given $\gamma$. 
Figure 6.d Sensitivity of Output to an iid credit market shock (top) and a shock an AR(1) shock with persistence equal to 0.9 (bottom) for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel).
Figure 7.a Responses of the model economy to an unexpected 1% increase in the required down payment; $p=0.9$; $\gamma=0.8$. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.
Figure 7.b Responses of the model economy to an unexpected 1% increase in the required down payment; $\rho=0.9$; $\gamma=0.8$. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.
Figure 8.a Sensitivity of Output for any given $\gamma$ in the second period (left panel) and cumulative over a 15 quarters period (right panel).
Figure 8. b Sensitivity of Output (top) and the Relative Price of Capital (bottom) for any given γ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel).
Figure 8.c Sensitivity of Output (top) and the Relative Price of Capital (bottom) for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel).
Figure 8.d Sensitivity of Output to an iid credit market shock (top) and a shock an AR(1) shock with persistence equal to 0.9 (bottom) for any given $\gamma$ in the first period (left panel), second period (middle panel) and cumulative over a 15 quarters period (right panel).
Figure 9.a Sensitivity of Output for any given $\gamma$ cumulative over a 15 quarters period in the model with capital in fix supply (left panel) and reproducible capital (right panel).
Figure 9.b Sensitivity of Output for any given $\gamma$ cumulative over a 15 quarters period in the model with capital in fix supply (left panel) and reproducible capital (right panel).
Figure 9.c Sensitivity of Output for any given γ cumulative over a 15 quarters period in the model with capital in fix supply (left panel) and reproducible capital (right panel).