Abstract

Significant attention has been paid to two related issues in climate change economics and policy: uncertainty in global damages and the potential for technological progress to lower future abatement costs. Our paper extends this discussion by looking at the role that uncertainty over the rate of technological progress may play in the abatement decision, and how this may interact with uncertain climate damages. Our results show that, as a result of the non-linear nature of learning-by-doing, stochastic learning rates lead to lower equilibrium emissions control at any given point in the state space, but that expected emissions over time will be significantly lowered relative to a situation with deterministic learning. We find that uncertain future abatement costs complement the Weitzman (2009) result on thick tails. If you’re going to buy significant greenhouse insurance, you want access to abatement cost lottery tickets.

Key words: Keywords.
JEL classification: Q30; Q42; Q54

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Preliminary version.
1 Introduction

Significant attention has been paid to two related issues in climate change economics and policy: uncertainty with respect to potential global damages and the potential for technological progress to lower future abatement costs. Our paper extends this discussion by looking at the role that uncertainty over the rate of technological progress may play in the abatement decision, and how this may interact with uncertain climate damages. Our results show that, as a result of the non-linear nature of learning-by-doing, stochastic learning rates lead to lower equilibrium emissions control at any given point in the state space, but that expected emissions over time will be significantly lowered relative to a situation with deterministic learning. We find that uncertain future abatement costs may complement the Weitzman (2009) result on thick tails - simply put, if you’re going to buy significant greenhouse insurance, you want access to abatement cost lottery tickets.

A key issue in global climate modeling is establishing the appropriate range for climate sensitivity to increased greenhouse gases (GHG), usually expressed as the expected long-run temperature change due to a stabilized doubling of atmospheric GHGs. Given a probability density for these effects, and a relationship between climate change and economic impacts, economic models can assess the impact of changes in emissions today on the distribution of future damages. The Intergovernmental Panel on Climate Change [2] Fourth Assessment Report catalogs (see Table 9.3) 13 studies which provide estimates of the climate sensitivity to CO$_2$ doubling. The results of their survey are shown below in Figure 1, which clearly shows that there is almost no probability of zero temperature change due to increased carbon
Climate-economy models have traditionally assumed that damages may be represented by a reduction in total factor productivity, with the reduction being quadratic, but generally deterministic, in the magnitude of temperature change. Several economic models have solved for optimal policy where the planner considers the distribution of potential future climate changes, not simply a deterministic path. Regardless of the specific form of the damage function, it is generally true that with more uncertainty with respect to the mechanisms of climate change, a risk-averse planner will choose a more aggressive abatement strategy.\(^1\)

\(^1\) For an important review of the role of the choice of the form of the planner’s utility function on the response to uncertainty, see Gollier et al. [1].
Adding uncertainty over the structural parameters of climate change adds significantly to the computational complexity of the solution. Our paper draws on previous work by Kolstad [5], Kolstad [6], Kelly and Kolstad [3], Kelly et al. [4], and Leach [7] which each solve optimal climate policy models with uncertainty over climate change parameters. As in Leach [7], we assume that the planner is uncertain with respect to both the sensitivity of climate to atmospheric carbon and with respect to the persistence of global temperature shocks. The planner then accounts for this uncertainty in forming expectations about the future impacts of emissions abatement and investment choices today.

This debate over the formulation of optimal climate policy under uncertainty has been altered recently with the examination of the so-called thick tails of potential climate damage by Weitzman [13] and subsequent research. This work argues that, even with low probability of potential catastrophic damages far into the future, abatement should be much more aggressive when structural uncertainty with respect to climate system parameters is present. In fact, the structural uncertainty may prevent the bounding of future climate impacts, in which case the proper response in terms of abatement effort today is also unbounded (or at least bounded only by the complete abatement of both the stock and the flow of GHGs). The model we propose does not explicitly include catastrophic damages, but is indirectly inspired by this work. There is, clearly, uncertainty with respect to future climate change impacts, and these uncertainties should factor in to our abatement decisions - something Weitzman refers to as an “inconvenient truth” - and should likely lead us to pursue much more aggressive abatement strategies as a form of greenhouse insurance. However, it seems important to ask whether we are certain with respect to the costs of abatement. In other
words, if abatement costs are known, we should buy more greenhouse insurance, but if they are uncertain, should we still buy more insurance even if that insurance is held in the form of lottery tickets?

The role of uncertainty in potential abatement costs has received less attention than uncertain climate damages in the literature, so the answer to this question is not clear. We know that, if the planner considers induced technological progress through learning-by-doing in deciding on emissions abatement today, abatement should increase, all else equal. These results have been shown in optimal climate policy models by Nordhaus [10] and Popp [11, 12]. However, to the best of our knowledge, our work constitutes the first optimal climate policy model with stochastic technological progress in emissions abatement costs.

Abatement activities, whether direct abatement or the development of alternative energy technologies, are generally expected to exhibit learning-by-doing. The accepted formulation for technological progress in this case is an exponential learning curve, along which a doubling of cumulative experience or installed capacity leads to a proportional reduction in unit costs. Cost reductions have been consistently observed in the deployment of new technology, as reviewed by McDonald and Schrattenholzer [8], which finds learning rates between 1 and 35% depending on the nature of the technology. The distribution shown in Figure 2 shows the significant variation in \textit{ex post} learning rates found in their analysis.

Wind energy provides a specific example in which cost decreases have not been monotonic or predictable. In Figure 3, we see that the installed cost of wind power has increased in recent years, as manufacturing pressures driven by government subsidies pushed equilibrium
In discussing the impact of learning-by-doing, McDonald and Schrattenholzer [8] state that learning, “introduces ... both non-linearities and positive feedbacks (the more a technology is used, the greater the incentive for using it more). This drastically increases model complexity and problematic non-convexities, both of which result in large computational requirements.”
Both of these statements are unquestionably true, and play a significant role in driving the results that we find in our paper. Because learning is non-linear, expected future progress is higher than the progress at expected future learning rates. In periods where learning is high, technology improves rapidly, leading to a positive feedback of more deployment and thus more learning. This positive feedback effect drives our results, which show that stochastic learning leads to higher cumulative emissions reductions, and lower future climate change impacts.  

In the remainder of the paper, we introduce, calibrate and solve an integrated assessment model, and report on model results under various uncertainty formulations.

2 The Integrated Assessment Model

In order to assess whether increased uncertainty over abatement costs will drive more or less abatement, and how this uncertainty interacts with structural uncertainty over climate change, we build an integrated assessment model. The model economy is based on the Nordhaus Dice-2009 model. A planner determines optimal investment in capital and chooses emissions abatement to maximize the welfare of society over time. Welfare is the net present value of the expected utility of per capita consumption, and there is assumed to be constant relative risk aversion at the agent-level. Production generates GHG emissions which

\footnote{For the parameter values we use, the impact of introducing stochastic learning-by-doing is significantly larger than the impact of allowing for uncertain future climate change, although these results are not robust to all potential parameter values.}

\footnote{The Dice-2010 model has been augmented to include a sea-level rise module, but adapting this characterization will increase the state-space of the model, so we have decided to adopt the less complex version.}
contribute to climate change which in turn causes a loss of economic productivity over time. Emissions abatement is an experience good, and so costs per unit of abatement decline with cumulative abatement, but this decline may be stochastic and is determined by a Markov process. We examine scenarios in which the rate of technological progress in emissions abatement and/or the damages from climate change are uncertain.

2.1 Production and investment

A single good is used for both consumption, $C$, and capital investment, $I$. Production uses Cobb-Douglas technology with inputs of capital, $K$, and labour, $L$ as follows:

$$Y(G, K, t) = \Omega(G, t)K^\alpha L(t)^{1-\alpha}. \quad (1)$$

Total factor productivity, $\Omega$, depends on two factors. First, it evolves exogenously with a time trend discussed below in (12). Second, total factor productivity declines with global temperature increases due to climate change, $G$, as specified below in (11). Labour supply, $L(t)$, is also determined exogenously as a function of time $t$.

The capital stock, $K$, evolves with investment, $I$, chosen by the social planner, and depreciation rate, $\delta_k \in (0, 1]$, according to:

$$K_{t+1} = (1 - \delta_k)K_t + I_t. \quad (2)$$

Aggregate consumption is equal to production, net of investment in physical capital and the
cost of emissions abatement, \( \mu \), given by:

\[
C = (1 - \Lambda_t \mu_t^{b_2})Y(G, K, t) - I. 
\]  

(3)

Our specific characterization of emissions abatement costs is discussed below in (6).

### 2.2 Emissions and Abatement

Emissions are a function of production, time, and endogenous abatement. In each period, the planner chooses the level of costly emissions control, \( \mu_t \), and emissions are then given by:

\[
E_t = (1 - \mu_t)\phi(t)Y(G, K, t). 
\]  

(4)

In (4), \( \phi(t) \) is the exogenous rate of emissions intensity of production, the law of motion for which is defined in (12).

We add uncertainty and learning-by-doing to the cost of emissions control. The DICE-2009 model characterizes the cost of abatement as quadratic in the relative emissions reductions, such that:

\[
C(\mu) = 1 - b_1 \mu^{b_2} 
\]  

(5)

We adjust this by adding a parameter, \( \Lambda \), which determines the cost of abatement over time as follows:

\[
C(\mu_t, \Lambda_t) = (1 - \Lambda_t b_1 \mu_t^{b_2}), 
\]  

(6)

so, for equivalent parameter values of \( b_1 \) and \( b_2 \), the cost functions will be equivalent where \( \Lambda = 1 \). We allow \( \Lambda \) to evolve through a stochastic learning-by-doing process, whereby more
cumulative abatement reduces future abatement costs, but at a rate which is unknown to
the planner in advance of making the abatement decision. Abatement costs also decrease
over time, albeit exogenously, in DICE-2009.  

Our model differs from that of Nordhaus in that decreases in abatement cost will be driven
by abatement intensity through a stochastic learning-by-doing process. Modeling abatement
costs in this manner is consistent with the evidence presented in the introduction for emis-
sions abatement through deployed wind power as well as through realized abatement costs
under the US Acid Rain program. We assume that the stochastic evolution of the progress
ratio follows a Markov process which allows for persistence in the current progress rate state.
A draw from a $\beta(2, 2)$ distribution, re-scaled to have support on the interval $[0, 5]$, provides
a new potential progress rate in each period.

The Markov process is described as follows. Let $\xi$ and $\xi'$ represent the rates of progress in
current and future states, and let $p$ represent the probability of remaining in the current
progress ratio state. In each period, the transition is as follows:

$$
\xi' = \begin{cases} 
\xi & \text{if } \varsigma < p \\
v & \text{if } \varsigma \geq p.
\end{cases}
\quad v \sim \beta(2, 2)/2, \; \varsigma \sim U(0,1).
$$

To model a stochastic learning curve, we make an assumption about the way experience and
cost interact. We assume that, in each period, a progress ratio is drawn from the Markov
process, and we treat the current abatement cost index as a Markovian state variable which

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4 In DICE-2009, Nordhaus specifies that the abatement cost is equal to $b_1\mu b_2$, where the initial
value of $b_1 = 0.051$ declines at a declining rate over time. The initial rate of decline, for the first 10
years, is 1.9% per year, and by the end of the simulation period, in the year 2595, $b_1 = 0.001279,$
or 97.5% below its initial value.
captures the history of accumulated experience. As such, if abatement cost is at a particular level, \( \hat{\Lambda} \), for a given level of abatement in the following period, the \( \hat{\Lambda}' \) will be the same for any given progress ratio regardless of the learning history which brought the economy to \( \hat{\Lambda} \) in the first place. This requires a two-step learning-by-doing process as follows:

1. Based on the current cost and newly drawn progress ratio, calculate *implied experience* as \( (\Lambda/C_0)^{(-1/\xi)} \). This is the amount of experience which it would have taken, at the current progress rate, to reach this cost level.

2. Calculate the new cost index, based on current abatement as \( \Lambda' = ((\Lambda/C_0)^{(-1/\xi)} + \mu Y)^{-\xi} \).

We will consider three versions of this transition. The first, which we term the *abatement cost certainty* process, sets \( p = 1 \) so that progress rates are held constant at the starting value in each period. The second, our *uncertain abatement costs* scenario, sets \( p = 0.9 \), so that learning states are highly persistent, and the variance of potential cost decreases over time is increased significantly. Finally, for sensitivity analysis, we introduce a *random* process with \( p = 0 \) so that each period sees a new draw from the re-scaled \( \beta(2, 2) \) distribution. The expected learning paths for \( \Lambda \) over time for each of the processes, holding abatement effort constant, are shown in Figures 4 and 5.

With a learning-by-doing process, the gains to abatement today are, in part, determined by the expected future cost reductions generated as a result. If future progress is uncertain, then the reduction in future abatement cost as a result of current abatement is also uncertain. Uncertain but endogenous future abatement costs are the natural analog to endogenous but uncertain future climate changes - both depend on the long-term accumulation of a stock, but the value of that stock is unknown at the time at which it is accumulated.
Fig. 4. The expected transition, along with high and low 95% confidence intervals, for the random ($p = 0$) process, assuming that $\mu_t$ is 1GtC in each period.

Fig. 5. The expected transition, along with high and low 95% confidence intervals, for the uncertain abatement cost ($p = 0.9$) process, assuming that $\mu_t$ is 1GtC in each period.

2.3 Climate Change

As with most economic models in the tradition of Nordhaus [9] DICE, we model total factor productivity, $\Omega_t$, as endogenously determined by emissions-induced changes in global temperature, which we denote by $G$. We use a single-reservoir atmospheric carbon model for simplicity, since the three-reservoir model used in DICE-2009 would increase our state space and thus our computational constraints significantly, without appreciably affecting our results of interest. Atmospheric carbon, $m_t$ evolves according to:

$$m_{t+1} = E_t + \delta_m m_t,$$

where $\delta_m$ defines the rate of natural decay of atmospheric carbon content.

Climate change occurs when increased atmospheric carbon generates increased radiative forcing which, in turn, increases global surface temperature, $G$. Climate change is regulated by inertia modeled via ocean temperature, $O$. Climate change occurs through the following
two-equation system:

\[ G_t = \lambda_1 G_{t-1} + \eta \ln \frac{m_{A,t}}{m_{A}^*} + \lambda_2 O_{t-1} + \epsilon, \quad \epsilon \sim NID(0, \sigma^2_\epsilon) \]  
(9)

\[ O_t = \lambda_3 O_{t-1} + (1 - \lambda_3) G_{t-1}. \]  
(10)

The radiative forcing parameter, \( \eta \), determines the climate’s sensitivity to CO\(_2\) accumulation, discussed above in the literature review, expressed as the long-run temperature change from a doubling a CO\(_2\), \( G_{2\times CO_2} = \frac{\eta}{1 - \lambda_1 - \lambda_2} \). Random variable \( \epsilon \) introduces stochasticity into the evolution of global temperature - this assumption is necessary for internal consistency if we assume that the planner can observe temperature changes and atmospheric carbon, but is uncertain about the underlying climate dynamics.

The social planner is assumed to only know the distributions of \( \eta \) and \( \lambda_1 \), but not their actual values. The planner’s estimates of \( \lambda_1 \) and the climate sensitivity to CO\(_2\) doubling are denoted \( \hat{\lambda}_1 \) and \( \hat{G}_{2\times CO_2} \) respectively, and are assumed to be distributed according to normal distributions with parameters chosen to match distributions estimated in Leach [7]. As was shown in Leach [7], the time to learn the true values of even such a simple climate system through observation will be on the order of thousands of years, so we assume without significant loss of generality that uncertainty remains constant over time.\(^5\)

Changes in global temperature in turn affect total factor productivity via:

\[ \Omega_t = \frac{\Omega_t}{(1 + b_1 G_t + b_2 G_t^2)}. \]  
(11)

\(^5\) We recognize that active research may narrow the variance of the distributions of key parameters over time, but adding either active or passive learning within the context of this model would add significantly to the complexity without, in our opinion, altering the results.
In (11), \( \Omega_t \) represents exogenous change in total factor productivity. Damage parameters, \( b \), and the climate sensitivity, \( G_{2xCO_2} \), determine the level of future benefit from investment in emissions abatement today. The return on abatement will be uncertain since \( \eta \) is unknown to the planner. The transition parameters \( \delta_m \) in (8) and \( \lambda_1 \) to \( \lambda_3 \) in (9-10) determine the time lag between actions to reduce or increase emissions and improved total factor productivity.

### 2.4 Exogenous trends

There are three exogenous trends which we treat as deterministic functions of calendar time, \( t \), which we use as a state variable in solving the model. We apply the following generic law of motion:

\[
J(t) = J_0 \exp \left( \frac{\gamma_J}{\delta_J} (1 - e^{-\delta_J t}) \right),
\]

where \( J \in \{ \Omega, L, \phi \} \), and initial conditions are defined by \( J_0 \), growth rates are defined by \( \gamma_J \), and growth rates convergence to zero over time at rate \( \delta_J \). We cap the transition of calendar time \( t \) at \( t = 450 \), by which time all transitions have stabilized.

### 2.5 Dynamic Optimization

Assume that a social planner maximizes expected welfare through choices of aggregate investment and emissions abatement in each period. Welfare is defined as the expected, discounted stream of utility, where utility has constant relative risk aversion form. The state of the economy by \( S = \{ K, m, G, O, \Lambda, \xi, T \} \). The solution to the planner’s recursive problem
with learning-by-doing is characterized using Bellman’s equation as:

\[ V(S) = \max_{I, \mu} U(C, T) + \beta E[V(S')] \] subject to: \[ (13) \]

\[ U(C_t, T) = \frac{1}{1 - \sigma} \left( \frac{C_t}{L(T)} \right)^{1 - \sigma} + 1 \] \[ (14) \]

\[ C_t = (1 - \Lambda_t b_1 \mu_t) Y(G, K, T) - I_t. \] \[ (15) \]

\[ Y(G, K, T) = \Omega(G, T) K^\alpha L(T)^{1-\alpha}. \] \[ (16) \]

\[ K_{t+1} = (1 - \delta_k) K_t + I_t \] \[ (17) \]

\[ m_{t+1} = (1 - \mu_t) \phi(T) Y(G, K, T) + (1 - \delta_m) m_{t-1} \] \[ (18) \]

\[ G_{t+1} = \lambda_1 G_t + \eta \ln \frac{m_t}{m_b} + \lambda_2 O_t + \epsilon, \lambda_1 \sim N(\hat{\lambda}_1, \sigma_{\lambda_1}^2), \]
\[ \eta = \frac{G_{2xCO_2}}{1 - \lambda_1 - \lambda_2}, \ G_{2xCO_2} \sim N(\hat{G}_{2xCO_2}, \sigma_{G_{2xCO_2}}^2), \ \epsilon \sim N(0, \sigma_{\epsilon}^2) \] \[ (19) \]

\[ O_{t+1} = \lambda_3 O_t + (1 - \lambda_3) G_t \] \[ (20) \]

\[ \Lambda_{t+1} = C_0 * ((\Lambda_t / C_0)^{(-1/\xi)} + a_t)^{-\xi}, a_t = \mu_t \phi(T) Y(G, K, T) \] \[ (21) \]

\[ \xi_{t+1} = \begin{cases} \xi_t & \text{if } \varsigma < p \\ v & \text{if } \varsigma \geq p \end{cases}, \ v \sim \beta(2, 2)/2, \varsigma \sim U(0, 1). \] \[ (22) \]

\[ T' = T + 1. \] \[ (23) \]

### 3 Computation

The recursive problem described in equations (13-23) is solved using the solution technique described in Leach [7], which characterizes the solution using an iterative algorithm combined with the use of an 18-node artificial neural network approximation of the value function over a finite set of grid points.

We establish a grid, using a low-discrepancy sequence, of 3000 grid points over values of the
seven state variables with ranges chosen such that the transition is a contraction mapping over the grid (i.e. for all state variables on the grid, the state in the following period will also lie on the grid). The ranges used on the state-space grid are shown in Table 2. We then solve for each iteration of the value function conditional on \( \tilde{V} \), a neural network approximation to \( V \) calculated over the grid points. In particular, let each iteration follow:

\[
V_i(S) = \max_{I, \mu} U(C, T) + \beta E[\tilde{V}_{i-1}],
\]

subject to the definitions and state transition equations specified in (13-23). Convergence of the solution to the Bellman equation is determined by the difference between realized values \( V_i(S) \) and previous iteration values of \( V_{i-1}(S) \).

### 3.1 Parameterization

The intent of the present study is to provide qualitatively-informative comparative dynamics, but the model remains sufficiently simplistic that the output should not be relied upon for predictive accuracy. The parameter values used in the solutions and simulations of the model are shown in Table 1 in the Appendix. We discuss key assumptions made to parameterize the model below.

The base parameterization of the model is derived from Nordhaus [9], with some modifications and additions based on other papers in the literature. We also make some assumptions which are required to assure convergence of our model to a steady state, at least asymptotically, which allows for solution via value function iteration.

The first area in which our parameterization diverges from DICE is with respect to the
level of uncertainty over climate transitions. Here, we draw in part on Leach [7] and also on work by the Intergovernmental Panel on Climate Change [2]. We suppose that the planner is uncertain both about the autoregressive component of temperature change and the degree to which atmospheric greenhouse gases affect temperature over time. We begin with the prior for the auto-regressive parameter, \( \lambda_1 \), which we parameterize according to the estimation using Hadley Center data from Leach [7], to have a mean estimate of 0.9123, with a standard error of .02633. We censor the numerical integral over this transition such that the expectations are bounded from above by 1, to maintain the stability of the model.\(^6\) We then impose a normal distribution for the degree of temperature change from changes in atmospheric CO\(_2\) to reflect the information in Figure 1. We approximate this with a mean of 3°C and a standard error of 0.75°C for the benchmark model. The distributions for \( \lambda_1 \) and \( G_{2\times CO_2} \) determine the distribution of the imputed value for \( \eta \).

Our second departure from the Nordhaus [9] model is in the characterization of emissions abatement cost. We adopt the same functional form, but scale the cost down with cumulative experience. The key parameters for interpreting our results will be the starting value for the cost, relative to that of Nordhaus, and the degree to which we assume learning to have already taken place to reach that cost (i.e. what do we assume was the starting value for abatement cost in the past when society had acquired no relevant experience.) The rate of growth during the time-period of our simulations will be slower as we assume that more accumulated experience already exists, since experience acquired during the course of our

\(^6\) This does not rule out positive-feedback effects in climate change, since these would be driven by increased exogenous forcing at higher temperatures. We don’t include these effects here, but doing so would be possible within this framework.
simulations will represent a smaller proportion of total experience.

We benchmark our abatement cost decline rate such that the results in the certainty case are comparable to those stipulated by the exogenous decline in Nordhaus [9]. In DICE-2009, abatement costs are halved from their starting values in 40 years, in response to 6GtC of cumulative abatement, and halved again by 2095 after 36GtC of cumulative abatement. Under our certainty case, abatement costs are half of the starting value after 32 years, and are 25% of the starting value after 105 years.

The final changes we make with respect to the Nordhaus [9] model are in the specification of the exogenous trends for technological change and the emissions:output ratio. Our calibrated values are chosen to match the first 300 years of the Nordhaus model calibration, after which our values for the emissions:output ratio and for technological change converge more quickly. This allows us to limit the scope of the time state variable to 450 years, while solving for an infinite horizon model, without significant loss of generality. Our exogenous trend for labour supply matches the Nordhaus population, which converges after 100 years.

Initial values for the state variables used in simulations are drawn from the 2010 time period for the Nordhaus DICE-2009 model, and are shown in Table 2.

In the results which follow, we make use of 4 scenarios with different assumptions with respect to uncertainty, the parameter values for which are detailed in Table 3. The four scenarios are as follows. First, we examine a benchmark certainty case in which the planner knows both the learning-by-doing process and the structural parameters of climate change.
Second, the *uncertainty* scenario proposes simultaneous uncertainty over both climate change and technological progress in abatement costs. The third and fourth scenarios, *Climate Uncertainty* and *Abatement Cost Uncertainty* test uncertainty in one dimension and assume, respectively, that learning-by-doing and climate change structural parameters are known with certainty.

Fig. 6. Choices of emissions control rate (left panel) and capital investment (right panel) over time under the four basic scenarios.

### 4 Results

The first question we want to ask is how the introduction of abatement cost uncertainty affects optimal behaviour. We can address this in two ways: first, by looking at simulation results over time, and second by looking at optimal strategies.
First, as shown in Figure 6, the different uncertainty structures alter emissions abatement decisions in material ways over time, but do not appreciably change investment decisions. Specifically, compared to a scenario with perfect certainty, adding uncertainty with respect to climate change affects abatement over time in a counter-intuitive manner - emissions abatement is lower (by a small amount) when the planner is uncertain about the severity of climate change and the evolution of global temperature. Adding uncertainty with respect to technological progress in abatement has the opposite effect, as it leads to a much higher average level of abatement activity over time.

Fig. 7. Optimal emissions control rate, as a function of state variables for surface temperature, $G$, (left panel) and the abatement cost index, $\Lambda$, (right panel) for each of the four basic scenarios. All other state variable values held constant at starting values given in Table 2.

To understand the effects on emissions control rates, it’s instructive to look at the strategies
underlying the simulations. In Figure 7, we project the optimal emissions control rate against surface temperature \( G \) and abatement cost \( \Lambda \) states, holding all other state variables constant at the simulation starting values given in Table 2. In this Figure, the basic comparative statics are intuitive. In the left-hand panel, we see that the more advanced is climate change, the more you want to control emissions, all else equal. In the right hand panel, we see that the level of emissions control increases as the cost of emissions control decreases, again with all else equal.\(^7\) The statics with respect to uncertainty also make sense here. All else equal, in the left hand panel, the most abatement takes place under the most uncertain scenario, and in both scenarios where future climate change is unknown, the abatement choice is significantly higher. The relationship between learning rate uncertainty and abatement is not consistent - a little less abatement takes place when moving from full uncertainty to \textit{Climate Uncertainty} scenario, and the opposite effect holds between \textit{Certainty} and \textit{Abatement Cost Uncertainty} scenarios. The introduction of uncertainty over learning-by-doing potential leads to more abatement if climate change processes are also uncertain, and less abatement if they are known with certainty, all else equal.

We see the same effect demonstrated in the right-hand panel, albeit with different relative magnitudes. Across all values of the abatement cost index, we see more abatement when the climate change process is uncertain than when it is certain, and less abatement, all else equal, when the learning-by-doing effect is stochastic than when it is deterministic. The effect is more pronounced, as would be expected, at higher relative abatement costs. This takes

\(^7\) The value on the right-hand panel does not limit to 1 because we include a fixed component in the abatement cost function, such that it asymptotically approaches 10\% of the Nordhaus costs.
place for two reasons - first, when abatement costs are high, the equilibrium costs of climate change are higher as well, so you’d spend more on climate insurance via extra abatement, all else equal, when your costs are high. Further, at higher cost levels, the learning-by-doing impact of a ton of emissions abatement is larger in expectation, so the uncertainty over future abatement costs as a function of today’s costs and today’s abatement will be higher when current costs are higher.

How does a strategy which stipulates more abatement effort under certainty about learning by doing lead to higher long-run abatement, on average, when uncertainty is present, and less when there is only uncertainty about the climate system? The answer lies in the relevant roles of uncertainty in each dimension. First, consider the role played by an uncertain learning-by-doing rate. In a given period, if the planner is certain about the future learning rate, then the derivative of $\frac{\partial V_t}{\partial \mu_t}$ is known and the planner includes this benefit of abatement today in his decision-making. If the planner is uncertain, than the planner will apply the same first-order condition, but with an expectation over the value the same derivative of future value with respect to current abatement. A risk averse planner will choose less abatement at any given state as shown above. However, the non-linear nature of learning plays a role here - since the learning process itself is stochastic, the realized learning rate may be higher or lower than the expected learning rate. When learning rates are high, the realized value of $\frac{\partial \Lambda_{t+1}}{\partial \mu_t}$ is substantially higher than when learning rates are low - as such, while the expected progress ratio is equal to the certainty case, the expected learning is not, holding abatement choice $\mu_t$ constant. So, when the learning draw is high, and learning takes place more quickly than expected, there is a larger movement along the strategy curve shown in the right hand
panel of Figure 7, leading to higher abatement in the periods after a high-learning-rate draw. 

This is why, in the left-hand panel of Figure 6, the first periods do show higher expected abatement in the cases with certainty in learning-by-doing, but the non-linear learning effect quickly trumps and the abatement grows more quickly in the stochastic progress ratio case. In short, the expected future progress is greater than the progress at expected future learning rates - a Jensen’s inequality result due to the non-linear learning process.

The message in the previous discussion is not that uncertainty is inherently good with respect to learning-by-doing - this is an artefact of our particular assumptions on the nature of uncertainty. The result would not hold in the same way if the probability distribution were skewed such that expected future progress were made equivalent to progress at expected future learning rates despite the non-linear nature of learning. However, it does illustrate the degree to which comparative dynamic results can differ in important ways from the interpretation of static results when non-linear processes are considered -this is true both for non-linear effects on the damage side and on the abatement costs side. It affirms that, from a climate change mitigation point of view, there is value in the potential for winning lottery tickets.

Given that the nature of uncertainty with respect to future progress will impact the abatement decisions, we seek to develop more precise understanding of the nature of these effects, by examining our additional scenarios for uncertainty in which the Markov process is adjusted such that the probability of remaining in the current progress state in any given period is set to 0, and so the next period’s progress rate becomes a random draw from the re-scaled
In Figure 8, we show the impact of considering these two additional scenarios. The results, again, are somewhat surprising. In the situation where the variance with respect to next-period’s learning rate is highest, the expected abatement is also highest. Figure 9, which shows the expected progress ratio as well as 95% confidence intervals based on 100 simulations of the stochastic model, provides some context for these results. In the right-hand panel, the Markov process used for the basic simulations in the paper is such that the starting value is persistent in 90% of cases, so the confidence intervals are initially tighter than in the case with the random draw. The first-order effect is that the planner has better information about future progress rates the higher is \( p \), but the second-order effect is that there is less
chance in the early periods of very fast progress when it matters most. Again, since progress is non-linear, we will expect to see faster learning in cases where there is more probability of higher learning rates, even where that is compensated for by a matching higher probability of slower learning rates.

Fig. 9. Progress ratio means and confidence intervals based on values observed over 100 simulations from identical starting values for two scenarios for uncertainty (Markov with $p=0.9$ and $p=0$) as well as the certainty case.

We see this intuition validated in the simulations. In Figure 10 we show the simulation means and confidence intervals for the abatement cost index, with the random process with $p = 0$ in the left panel and the base Markov process with $p = .9$ in the right panel. The random process has tighter confidence intervals, since the mean progress ratio over each simulation will be close to 0.25, the expectation of the random draw. The Markov process with lower
transition probability implies that the economy could remain in a high- or low-progress-rate state for long periods of time - the probability of remaining in the same progress rate state for 50 years is over 50%. This manifests in the much wider divergence in potential cost states over time.

The broad conclusion of this section is that, while at any given point in time or within the state space, a risk-averse planner will choose less abatement where the future value realized through learning-by-doing is uncertain, if the future progress ratio is uncertain but the uncertainty is symmetric around the mean, the expected future progress and expected future abatement will be higher with uncertainty than without.

![Diagram](image)

Fig. 10. Simulation means and confidence intervals for the abatement cost index, \( \Lambda \), based on values observed over 100 simulations from identical starting values for two scenarios for uncertainty (Markov with \( p=0.9 \) and \( p=0 \)) as well as the certainty case. The simulation means correspond to the series shown in the left panel of Figure 8.
The final question is whether these conclusions are relevant for climate change policy - does the added abatement resulting from uncertainty over learning-by-doing returns translate to significantly lower emissions and less climate change? The answer is yes, but with a couple of caveats. For the assumptions of our model, the increases in realized abatement and changes in uncertainty over future emissions and abatement were significant. These changes, however, do not translate into large changes in climate change expectations over the time horizon considered.

Figure 11 shows the impact of the addition of uncertainty over abatement costs alone on emissions over time. When $p = 0$, although the expected learning rate is identical to that
used in the certainty case, emissions are lower by up to 5-6% in expectation, and emissions levels over 10% lower fall within the 95% confidence interval for the simulations. These are significant, as they correspond to up to a 25% increase in expected emissions abatement as compared to the optimal policy with certainty.

Figure 12 shows the climate change impacts of these changes in abatement, and they are small. In both cases, surface temperatures are lower in the uncertainty cases than in the certainty case, but not significantly so. Cumulative emissions are 5% lower over the simulation path, but this is not sufficient to translate to significant mitigation of temperature changes in-and-of itself.
5 Conclusion

The motivation for this paper was to consider whether uncertainty in the future costs of abatement would have an analogous impact to the impact of structural uncertainty with respect to climate change. In Weitzman [13], the fact that climate change damages may be unbounded leads to a conclusion that we should engage in aggressive (or even complete) abatement in the near term. Of course, there is also significant uncertainty with respect to what the costs of near or complete carbon emissions abatement would be - we do not really know what a zero-net-emissions world would look like or what the costs of achieving this would actually be. Given this context, we ask whether one should still buy greenhouse insurance, as suggested by Weitzman, even though the cost of that insurance is a lottery.

We find that, in fact, the lottery complements the Weitzman result. For the intuition, we need to look at the relative bounds and how they interact. With climate change, damages are, at least theoretically, infinite - they are unbounded. Abatement costs, however, are bounded from above by today’s costs and may decrease. Given the potential for rapid decrease, the introduction of stochasticity will, in expectation, drive more abatement than a situation with deterministic future abatement costs. In other words, if you’re going to buy significant greenhouse insurance, you want access to lottery tickets.

The analysis presented in this paper omits several key factors, in particular with respect to climate change uncertainty. We do not include the potential for exacerbated damages due to sea level rise, as is now the case with newer versions of the DICE model, nor do
we account for extreme events, thresholds, or positive feedbacks. All of these will lead to larger deviations in abatement between cases with climate process certainty and those where structural uncertainty about the process remains. However, we have not emphasized the size of responses to uncertainty, and have concentrated on the comparative dynamics. We would expect the basic results to be robust to the inclusion of more stochasticity in climate system or more uncertainty over the underlying parameters driving it.

References


Table 1  
Calibrated Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
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<tbody>
<tr>
<td><strong>Inter-temporal Utility Function</strong></td>
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<tr>
<td>$\sigma$</td>
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<td>$\beta$</td>
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<td>$\alpha$</td>
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<td>$\delta_k$</td>
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<td>Initial growth rate of emissions: output ratio</td>
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<td>Rate of decline of $\gamma_\phi$</td>
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<td>$\omega$</td>
<td>Transfer between ocean and surface temperature</td>
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<td>Climate sensitivity to CO$_2$ doubling ($^\circ$C)</td>
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<td>$\sigma_{G_{2C}}$</td>
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<td>$\sigma_\epsilon$</td>
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<td><strong>Emissions Control Technology</strong></td>
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<td>$b_1$</td>
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<td>$b_2$</td>
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<td>$C_0$</td>
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<tr>
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<td><strong>Productivity Loss from Climate Change</strong></td>
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Table 2
Parameter values and state variable initial conditions used for simulations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Starting Value</th>
<th>Grid Range</th>
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<td><strong>Factors of production</strong></td>
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<td>$K$</td>
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<td>$L$</td>
<td>Population ($US trillions)</td>
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<td>Emissions:output ratio (t/$ million)</td>
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<td>0.01192-0.1445</td>
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<td>$G$</td>
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Table 3
Distribution parameters for different uncertainty scenarios

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<th>Standard Deviation</th>
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<td>$\hat{\lambda}_1$ Planner’s estimate of $\lambda_1$</td>
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<td>0</td>
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<tr>
<td>$G_{2\times CO_2}$ Planner’s Estimate of $G_{2\times CO_2}$</td>
<td>3</td>
<td>0</td>
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<tr>
<td>$\epsilon$ Random component of temperature change</td>
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<td>$p$ Probability of remaining in the same progress state</td>
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<td>$G_{2\times CO_2}$ Planner’s Estimate of $G_{2\times CO_2}$</td>
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<td>0.75</td>
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<td>$\epsilon$ Random component of temperature change</td>
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<tr>
<td>$G_{2\times CO_2}$ Planner’s Estimate of $G_{2\times CO_2}$</td>
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<td>0.75</td>
</tr>
<tr>
<td>$\epsilon$ Random component of temperature change</td>
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<td>0.11</td>
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<tr>
<td>$p$ Probability of remaining in the same progress state</td>
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<td><strong>Abatement Cost Uncertainty</strong></td>
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<td>0</td>
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<tr>
<td>$\epsilon$ Random component of temperature change</td>
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<tr>
<td>$p$ Probability of remaining in the same progress state</td>
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<td><strong>Random Abatement Cost</strong></td>
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<td>$G_{2\times CO_2}$ Planner’s Estimate of $G_{2\times CO_2}$</td>
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<td>0</td>
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<tr>
<td>$\epsilon$ Random component of temperature change</td>
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<tr>
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