Non-Renewable Resource Supply: Substitution Effect, Compensation Effect, and All That*

by

Julien Daubanes
CER-ETH, Center of Economic Research at ETHZ, Swiss Federal Institute of Technology Zurich
E-mail address: jdaubanes@ethz.ch

and

Pierre Lasserre
Département des sciences économiques, Université du Québec à Montréal,
CIRANO and CIREQ
E-mail address: lasserre.pierre@uqam.ca

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Address. Please address all correspondence to Pierre Lasserre, Département des sciences économiques, Université du Québec à Montréal, C.P. 8888, Succursale Centre-Ville, Montréal, Québec, Canada H3C 3P8. E-mail: lasserre.pierre@uqam.ca

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The non-renewable-resource-extraction literature has extensively studied the effects of several policy instruments on the market equilibrium extraction quantities. In this note, we examine the reaction of an individual resource supplier to given prices. In a synthetic resource supply theory, we compute instantaneous supply functions and decompose the effects of price changes on supplied quantities. In a pure Hotelling model, we show that price changes entail an intertemporal substitution which is reminiscent of a green paradox. When the reserves stock is endogenous, this effect is completed by a stock effect of opposite direction. This gives rise to a resource supply decomposition equation, which is the counterpart of Slutsky equation in demand theory. We show that the substitution effect always dominates so that a price decrease at some date always causes supply to increase at all other dates. Moreover, there is no phenomenon of the kind of the Giffen paradox. When this supply picture is introduced into a partial equilibrium framework, we find that the above results explain the effects of demand-reducing policies.

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1. **Introduction**

The so-called "green paradox" phenomenon refers to the fact that future, anticipated policies aiming at reducing the demand for an extracted exhaustible resource increase the present rate of extraction of that resource. Hans-Werner Sinn (2008) coined the expression, implying "that good intentions do not always breed good deeds." (page 380).

More generally, the credible threat of a "gradual greening of economic policies" (Sinn, 2008, page 360) causes suppliers to extract their stock more rapidly. This phenomenon has received particular attention in the context of climate change economics and policies. Fossil fuels are responsible for the bulk of greenhouse gas emissions, a pollution that has been labelled "the ultimate commons problem of the twenty-first century" (Stavins,
Environmental policies should attempt to slow fossil fuel exploitation. In this context, a policy entailing a green paradox would instead accelerate current and near-future consumption. Of major practical concern is the possibly undesired effect of a lag in green policies’ implementation\(^1\) or of the expectation that green policies will be introduced gradually to be more and more stringent over time.

The phenomenon is not counter-intuitive to any resource economist. Since Hotelling’s (1931) seminal contribution on exhaustible resources, such commodities have been thought of as a fixed, finite initial endowment that is to be allocated over time. A profit-maximizing extraction path should equalize the present-value marginal spot profits of extracting at all dates. In this context, it is well-known that there exist neutral tax paths. Such paths reduce present-value marginal profits in the same proportion at all dates, leaving the intertemporal arbitrage of resource producers unaffected (Dasgupta et al., 1980). Tax paths that are rising faster over time deteriorate the marginal profitability of extracting at distant dates relatively to early dates, thus leading producers to extract less of the stock in the future and to shift production to earlier dates. From Long’s (1975) study, such policies can be interpreted as credible threats of future expropriation.

A "green paradox" arises when a policy is believed to distort relative marginal profitabilities in a way that penalize future extraction. Sinn argued that such a distortion not only arises from tax policies but may also result from subsidies to substitutes for the resource or from policy induced improvements in current efficiency. The "green paradox" can be described as a pure substitution between resource units extracted at different dates. In this respect, Sinn’s (2008) metaphor of a closed "pneumatic system of various pipes connecting various pistons" (page 378) is illuminating: "If only one piston is pressed down, the others go up." In other words, when total cumulative supply is perfectly inelastic, demand-reducing policies only shift quantities away to other dates.

In the recent literature, authors have departed from the pneumatic metaphor by arguing that green policies may affect the cumulative quantity of the resource that is to be extracted ultimately, introducing a leak in the pneumatic system: a reduction in total extraction tends to mitigate the phenomenon. In Hoel (2010), in Gerlagh (2010) and

\(^1\)On that, see Smulders et al. (2010).
in van der Ploeg and Withagen (2010, 2011), the total quantity extracted is determined endogenously: marginal extraction costs are constant at each date in the sense that they are independent of current extraction, but depend on past cumulative extraction. Furthermore, a substitute can be produced at some cost. Extraction stops at the date when cumulative extraction is such that the unit cost of extraction overtakes the unit cost of the substitute. Green policies aim at reducing the cost of the substitute, thus reducing the equilibrium total amount of resource extracted and increasing the part of the stock that is left unexploited. The authors show that the green paradox survives this generalization.

In this note, we formulate a simple, parcimonious, theory of resource supply that can be used to analyze the green paradox or other policy induced changes in extraction as standard supply phenomena in a general framework. We do not focus on specific demand reducing policies nor do we impose any restriction on their time trajectories. In a first part, we develop a theory of resource supply: we assume a path of producer prices to be given and we study the effect of a price change. Later, we extend our results to a partial equilibrium setting where prices are endogenously determined on competitive markets so that we can focus on the effect of demand-reducing policies – we might also say "green policies" although we do not deal explicitly with environmental issues. These policies can consist of any combination of taxes on the extracted resource, subsidies to substitutes for the resource, and efficiency-improving policies. We assess the effect of implementing such policies at some dates on extraction flows at these dates and at other dates. When total cumulative extraction is taken as given, such policies modify the relative marginal profit from extracting in a clear direction and thus entail a pure intertemporal substitution effect shifting extraction from the dates at which the policies are implemented to other dates. This is the essential cause of the green paradox as initially formulated.

We also do away with the Hotelling assumption of a given reserve stock. However, we don’t follow Hoel (2010), Gerlagh (2010) and van der Ploeg and Withagen (2010, 2011) in considering that some part of the resource may be left unexploited. Instead, we assume that the stock of reserves is made available prior to extraction via exploration and
development; developed reserves are completely exhausted during the extraction phase, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). However, exploration and development are sensitive to the net-of-tax rent that accrues to the extractors during the exploitation of the resource. This rent is precisely the component which is affected by policies and policy changes. Consequently, the introduction of policies that affect resource rents has an effect on ultimate extraction through its effect on exploration and development via the extraction rent. This way to endogenize the stock of resource turns out to be particularly adapted to deal with the green paradox as it allows us to separate the pure intertemporal substitution effect described above from what we call the stock effect of green policies. Any reduction in the resource price faced by producers, or any attempt to implement a demand-reducing policy, may cause a reduction in the ultimately exploited reserve stock. The former increases the extracted resource flow at dates when policies are not implemented while the latter tends to reduce it. We show that the substitution effect always dominates the stock effect. Resource supply at any given date is thus increasing in the resource price at this date and decreasing in the resource prices at all other dates.

Although classical supply theory is different from classical demand theory, exhaustible resource supply has something in common with classical demand theory: resource producers allocate a stock of resource to different dates in a way that is comparable to the way consumers allocate their revenue to different expenditures. That is why combining the intertemporal substitution effect and the stock effect of a resource price change gives rise to an equation that is reminiscent of Slutsky’s (1915) decomposition equation. The stock effect in resource supply theory resembles the income effect in classical demand theory. Resource supply theory differs from the regular theory of supply because cross-price effects arise in resource supply even when costs of extraction are independent over time. This happens because reserve development is necessarily subject to decreasing returns to scale as exploration prospects are finite. If such was not the case, non renewable resources would be indefinitely reproducible like conventional commodities, so that demand reducing policies would have no effect on resource rents: under constant returns to
scale the rent reduces to the constant quasi-rent associated with exploration and reserve development.

Although the parcimonious model analyzed up to this stage illustrates perfectly how the effect of a price change occurring over part of the extraction period can be decomposed into a substitution effect and a stock effect, it does not take into account the lack of homogeneity of non-renewable resources. In reality a non-renewable resource such as oil is supplied from many heterogenous deposits with different cost of extraction and cost of exploration and development characteristics. We show how to adapt the model to such variety and show that the results are unchanged.

Revoir le planSection 2 proposes a synthetic theory of resource supply. Section 3 analyzes the effects of a price change at a single date. Section 4 shows the general dominance of the intertemporal substitution effect over the stock effect. Section 5 exploits the analogy with Slutsky’s decomposition equation in classical demand theory. Section 6 extends the result to price changes at a subset of dates. Section 7 extends the result to a partial equilibrium setting where we deal with the effect of a change in the stringency of demand-reducing policies.

2. A SYNTHETIC THEORY OF EXHAUSTIBLE RESOURCE SUPPLY

A quantity \( x_t \geq 0 \) of a non-renewable resource is supplied at each of a countable set of dates \( t = 0, 1, 2, \ldots \). The initial stock \( X \) of the resource is finite and treated as exogenous at this stage, with \( \sum_{t \geq 0} x_t \leq X \). The producer price is denoted by \( p_t \geq 0 \). The stream of prices \( p \equiv (p_t)_{t \geq 0} \) is taken as given by the producers and treated as exogenously given at this stage.\(^2\) Spot extraction profits are denoted \( \pi_t = \pi_t(x_t, p_t) \), where \( \pi_t \) may be time varying, is increasing in both arguments, twice differentiable, and satisfies \( \frac{\partial^2 \pi_t(\cdot)}{\partial x_t^2} < 0 \) and \( \frac{\partial^2 \pi_t(\cdot)}{\partial x_t \partial p_t} > 0 \).\(^3\)

The stock of reserves to be exploited by a mine does not become available without some prior exploration and development efforts. Although exploration and exploitation

\(^2\)In Section 7, we will extend the results to a partial equilibrium setting where prices are endogenously determined on competitive markets.

\(^3\)For instance, profits might write \( \pi_t = p_t x_t - C_t(x_t) \), where time-dependent total extraction cost \( C_t(\cdot) \) is a strictly convex function such that \( C_t(0) \geq 0 \).
often take place simultaneously (e.g. Pindyck, 1978, and Quyen, 1988\textsuperscript{4}), a convenient and meaningful simplification consists in representing them as taking place in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way to model the supply of reserves is particularly adapted to the problem under study because it provides a simple and natural way to isolate the effect of an anticipated price change on the size of the exploited stock. Specifically, the cost $E(X)$ of developing an initial, exploitable stock $X$ at date 0 is twice differentiable, strictly increasing and convex, and satisfies $E(0) = 0$ and $E'(0) = 0$. The property $E'(0) = 0$ that the marginal cost of reserves development is zero at the origin is introduced because it is sufficient to ensure that a positive amount of reserves is developed. It thus rules out uninteresting situations where resource prices do not warrant the production of any reserves.

For simplicity, we assume the rate $r \geq 0$ at which profits are discounted to be independent of time\textsuperscript{5}.

Since the development of reserves is costly, producers’ optimum plans will always bind the exhaustibility constraint. In other words, leaving part of the developed stock ultimately unexploited will never maximize profits. Hence, for a given stream of prices $p$, the problem faced by a producer is

$$\max_{(x_t)_{t \geq 0}, X} \sum_{t \geq 0} \pi_t(x_t, p_t)(1 + r)^{-t} - E(X)$$

subject to

$$\sum_{t \geq 0} x_t = X.$$  \hspace{1cm} (2)

Denoting by $\lambda$ the Lagrange multiplier associated with constraint (2), the necessary first-order conditions for the choice of an optimum extraction path are

$$\frac{\partial \pi_t(x_t, p_t)}{\partial x_t}(1 + r)^{-t} = \lambda, \forall t \geq 0,$$  \hspace{1cm} (3)

and

$$E'(X) = \lambda.$$  \hspace{1cm} (4)

\textsuperscript{4}See Cairns (1990) for a comprehensive survey of related contributions.

\textsuperscript{5}Although our results survive a time-dependent discount rate, such an assumption would make mathematical expressions more complex without providing much insight.
(3) is the Hotelling rule stating that the marginal profit from extraction must be constant over time in present value, equal to $\lambda$, which is therefore the unit present-value of reserves underground, i.e. the unit Hotelling scarcity rent. (4) can be interpreted as defining the supply of reserves as a strictly increasing function of the unit rent:

$$X = X(\lambda) \equiv E^{t-1}(\lambda). \quad (5)$$

(3) implicitly defines the solution $x_t$ as a function which is increasing in the current price $p_t$ and decreasing in the rent $\lambda$:

$$x_t = x_t(p_t, \lambda), \forall t \geq 0. \quad (6)$$

Combining with (2), we obtain that the rent is a function increasing in all prices $p \equiv (p_t)_{t \geq 0}$ and decreasing in the stock $X$:

$$\lambda = \lambda(p, X). \quad (7)$$

Substituting (7) into (6) gives the conditional supply functions

$$\bar{x}_t = \bar{x}_t(p, X) \equiv x_t(p_t, \lambda(p, X)), \forall t \geq 0. \quad (8)$$

Conditional on the initial reserve stock $X$ and given prices $p$, these functions determine how the suppliers allocate extraction from the stock to different dates. $\bar{x}_t$ is increasing in $X$ and decreasing in any $p_{t'}, t' \neq t$. Its partial derivative with respect to $p_t$ has an ambiguous sign.

xxxxNow consider the choice of initial reserves, denoting by $X^*$ the optimum stock of reserves at the producers’ optimum given the price sequence. The value of the unit rent at the producers’ optimum is $\lambda^* = \lambda(p, X^*)$. By (5), the optimum amount of reserves satisfies $X^* = X(\lambda^*) = X(\lambda(p, X^*))$, which implicitly defines $X^*$ as a function of $p$ only:

$$X^* = X^*(p). \quad (9)$$

xxThen the optimum extraction flow $x^* = (x^*_t)_{t \geq 0}$ is given by the supply functions at each date:

$$x^*_t = x^*_t(p) \equiv \bar{x}_t(p, X^*(p)), \forall t \geq 0. \quad (10)$$
3. Effect of a Price Change at a Single Date: Intertemporal Substitution and Stock Effects

Let us now study the effect of a change in the price at one single date, say $T \geq 0$, on the resource supplies at other dates $t \neq T$. From the definition of the supply functions (10), we have

$$\frac{dx_t^*}{dp_T} = \left. \frac{\partial \tilde{x}_t(.)}{\partial p_T} \right|_{X=X_0^*} + \frac{\partial \tilde{x}_t(.)}{\partial X} \frac{\partial X^*(.)}{\partial p_T}, \quad \forall t \neq T. \quad (11)$$

The definition of the conditional supply functions $\tilde{x}_t(.)$ in (8) implies that the first term on the right-hand side is a pure substitution effect, since the initial stock of reserves is kept unchanged; the substitution effect measures the impact of a change in $p_T$ on the way this stock is allocated to extraction at different dates. The second term thus isolates the stock effect, i.e. the effect of $p_T$ on $x_t^*$ via the induced change in $X^*$. (11) is thus a decomposition equation, reminiscent of Slutsky decomposition equation in the classical demand theory.

As (8) makes clear, $x_t, t \neq T$, is only affected by $p_T$ via $\lambda$, i.e. only to the extent that prices affect the rent. Using (6), the substitution effect is

$$\frac{\partial \tilde{x}_t(.)}{\partial p_T} = \frac{\partial x_t(.)}{\partial \lambda} \frac{\partial \lambda(.)}{\partial p_T}, \quad (12)$$

which is negative. The stock effect is clearly positive.

A decrease in price $p_T$ has thus two effects of opposite direction on resource supply dates $t \neq T$. On the one hand, it shifts resource production from date $T$ to other dates, which tends to increase $x_t$. On the other hand, it reduces the total cumulative extraction $X$, which tends to decrease supply at all dates.

As far as the effect of a price change on contemporary extraction, the substitution and the stock effects of a change in $p_T$ on $x_T$ are both positive: a reduction in $p_T$ causes a drop in $x_T^*$. This will be very clear after the next section.

4. The Substitution Effect Dominates the Stock Effect

Let us reconsider the two components of the effect of $p_T$ on $x_t, t \neq T$, as given by (11). By (12), the substitution effect depends on the effect of $p_T$ on the rent $\lambda$; by (8), the
stock effect can be decomposed as \( \frac{\partial x_t}{\partial p_T} \frac{\partial x^*_t}{\partial p_T} = \frac{\partial x_t}{\partial X} \frac{\partial X^*_t}{\partial p_T} \). Hence, both terms in (11) can be factorized as follows:

\[
\frac{dx_t^*}{dp_T} = \frac{\partial x_t(.)}{\partial \lambda} \left( \frac{\partial \lambda(.)}{\partial p_T} \bigg|_{X=X^*} + \frac{\partial \lambda(.)}{\partial X} \frac{\partial X^*(.)}{\partial p_T} \right), \quad \forall t \neq T,
\]

where the term between brackets is the total derivative of \( \lambda \) with respect to \( p_T \); it gives the total effect of a price change on the unit rent, decomposed into a direct price effect at constant initial reserves, and the effect on the rent of the change in initial reserves induced by the price change:

\[
\frac{\partial \lambda(.)}{\partial p_T} \bigg|_{X=X^*} + \frac{\partial \lambda(.)}{\partial X} \frac{\partial X^*(.)}{\partial p_T} = \frac{d\lambda^*}{dp_T}.
\]

It can be shown that resource prices at all dates positively affect the rent, i.e. \( \frac{d\lambda^*}{dp_T} \geq 0 \).

6 Consequently, \( \frac{dx_t^*}{dp_T} = \frac{\partial x_t(.)}{\partial \lambda} \frac{d\lambda^*}{dp_T} \leq 0, \quad \forall t \neq T, \)

implying that the positive stock effect never more than compensates the negative substitution effect. Since the effect of a drop in \( p_T \) is positive on total extraction and negative on supplies at all dates \( t \neq T \), it must be that its effect is always positive on \( x_T \).

Our analysis highlights the crucial role played by the scarcity rent \( \lambda \). In particular, it makes clear that the substitution effect always overtakes the stock effect because this unit rent is bound to decrease when the resource price decreases at any date. Certainly, the stock effect mitigates the decrease in the rent but it does not fully compensate it. This effect of prices on the unit scarcity rent, together with the fact that a higher unit rent tends to reduce reserve supply, drives the result.

It is worth recalling here that we have assumed decreasing returns to the development of reserves – increasing marginal cost of development, i.e. strict convexity of the cost function \( E(X) \). We have argued that this assumption is necessary to take account of the finiteness of extraction prospects. In the following, we will show that this fundamental feature of exhaustible resources is essential to the result.

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6Formally, the definition of \( X_t^* \) yields \( \frac{\partial X_t^*}{\partial p_T} = \frac{X_t^*(.) \frac{\partial p_T}{\partial p_T}}{1 - \frac{\partial X_t^*}{\partial X}} \bigg|_{X_t=nX_t^*} \), implying that the left-hand side of (14) can be factorized as \( \frac{d\lambda^*}{dp_T} = \frac{\partial \lambda(.)}{\partial p_T} \bigg|_{X=nX_t^*} \left( \frac{1}{1 - \frac{\partial X_t^*}{\partial X}} \right) \), which is positive since \( \frac{\partial \lambda(.)}{\partial X_t} \) is negative.
Assume for a while that the development of reserves is subject to constant returns to scale, as would be the case for a regular, producible – non exhaustible – commodity whose total quantity $X$ would need to be produced prior to its allocation to several alternative uses indexed by $t = 0, 1, 2, \ldots$. Let the marginal cost $e$ of developing reserves be constant: $E(X) = eX$. As before, $\lambda$ is the present value of each unit of the commodity under study that the producer decides to develop at the beginning of the program, when the price trajectory becomes known. It must be that $\lambda = e$; it is given by the technology and insensitive to variations in prices $p$. Then, the quantity $x_t$ dedicated to a particular use $t$ only depends on its own price $p_t$. Exhaustibility is no longer relevant. The substitution effect vanishes and the stock $X$ fully absorbs any changes in $x_t$ induced by a change in $p_t$.

Hence, to the extend that the reserve stock can adjust to the price change, constant returns to scale in the development of $X$ makes all cross-price effects on extraction vanish, just like in the classical theory of supply under separable cost. The specificity of resource supply results from decreasing returns in reserves development, implying that cross-price effects occur – via the rent – even though extraction costs are independent from each other. The case of a Hotelling resource is extreme in that the stock of reserves is perfectly inelastic so that no stock effect comes to attenuate the cross-price effects.

5. **Analogy of resource supply theory with demand theory**

Although the decomposition of the effects of a resource price change into a substitution effect and a stock effect is reminiscent of the Slutsky decomposition, resource supply is not isomorphic to demand theory. In the theory of demand, disentangling the substitution and income effects requires to compensate for the level of utility: holding the objective level unchanged, the move from the consumer’s optimum under the slope of the budget constraint induced by the price change represents the substitution effect. In the resource supply theory, to isolate the substitution effect, it is sufficient to hold the stock unchanged: the exhaustibility constraint, completely determined by the level of the stock, is not modified; the price change induces the form of the extraction profit function to change; the move to the new optimum under the initial exhaustibility constraint represents the
substitution effect. So, while isolating the substitution effect in demand theory requires to keep the level of the objective unchanged (same iso-utility curve) and to consider the modified slope of the constraint, in resource supply theory, one should take the level of the constraint unchanged and consider the modified slope of the iso-profits curves; unlike indifference curves, isoprofit curves at different prices cross each other.

The substitution effect and the stock effect of a resource price change are illustrated in Figure 1 for the case of two periods, which corresponds to the two-good representation of demand theory. Assuming prices $p_0$ and $p_1$, point $A = (x_0, x_1)$ in Figure 1 depicts the producer optimum. The developed stock is $X$ is determined so that producers reach the highest possible iso-profit curve\(^7\) $\bar{\pi}$. The optimum allocation $(x_0, x_1)$ is thus at the point of tangency between the iso-profit curve of level $\pi$ and the exhaustibility constraint which trades quantities extracted between Period 1 and Period 2 in such a way that $x_0 + x_1 = X$.

Consider now a decrease in $p_1$ to $p_1' < p_1$. If the stock remains at $X$, since the price change implies all iso-profit curves to become steeper, the new tangency point is along the same exhaustibility constraint and along the iso-profit curve of level $\bar{\pi} < \pi$, at point $\bar{A}$ below $A$, so that $\bar{x}_0 > x_0$ and $\bar{x}_1 < x_1$. The move from $A$ to $\bar{A}$ represents the substitution effect.

However the drop in price leads producers to reduce reserve development to $X'$. Taking this stock effect into account brings the new optimum to $A'$. It is clear that $x_1 > \bar{x}_1 > x'_1$: there is no possibility of a commodity analogous to a Giffin good, whose supply would increase as a result of a drop in its price. Since the total effect of a price change occurring in Period 1 on Period 0’s extraction is ambiguous, the graph does not \textit{a priori} indicate whether $x'_0$ should be greater or lower than $x_0$. However our previous analysis has shown that $x_0 < x'_0 < \bar{x}_0$: there is no such thing as an inferior commodity on the supply side.

To reiterate, when the demand for one good decreases as a result of a drop in the price of another good, that is to say when the substitution effect dominates the income

\(^7\)From our assumption that profits are concave in extraction flows, iso-profits curves are decreasing and convex.
effect, those goods are said to be substitutable. On the contrary when the income effect dominates the substitution effect, the two goods are said to be complements. In the case of non renewable resource supply, quantities extracted at different dates are always substitutes.

6. Simultaneous price changes at a subset of dates

xxJ’ai déplacé le contenu de cette section dans la suivante.

7. Demand related policy changes in partial equilibrium

Policy changes are more complex than the above analysis of supply for two main reasons. First the policy related price changes usually take place over an extended period rather than at a single date; second the policy usually affect prices indirectly, because they affect the demand for the resource. For example the green paradox is often described as the effect on current or near-future resource supply of policies reducing resource demand over some extended future period via various forms of help to alternative energy sources.

Suppose that resource demand decreases at all dates \( T \in \Omega \), where \( \Omega \) is a strict subset of dates. The analysis of Section 4 applies to each of the changes occurring at dates belonging to \( \Omega \). While these changes jointly contribute to increase supply \( x_t \) at all \( t \notin \Omega \), their effect on supply at date \( T \in \Omega \), depends on the magnitude of the price change occurring at that date relative to the changes occurring at other dates \( T' \in \Omega \). However, the analysis of Section 4 indicates that cumulative extraction over \( \Omega \) is reduced. This is because \( X \) decreases while supply at all dates \( t \notin \Omega \) increases.

Suppose further that prices are determined by the equilibrium of supply and demand where supply is defined as above: taking prices \( p \equiv (p_t)_{t \geq 0} \) as given, producers solve (1) subject to (2). Equations (3)–(6) hold as before. Assume that the demand for the resource at date \( t \geq 0 \) only depends on the date, on the resource price at that date \( p_t \) and on the stringency of date-\( t \) demand-reducing policies, synthesized by the index \( \theta_t \). Formally, let demand be given by the function\(^8\) \( D_t(p_t, \theta_t) \), continuously differentiable and decreasing

\(^8\)A less synthetic demand function would be for instance \( D_t(p_t) \equiv f_t((\alpha_t)dt(p_t + \beta_t, p_t^s - \gamma_t)) \), where \( f_t(.) \) is decreasing, \( dt(.) \) is decreasing in its two arguments and where \((\alpha_t)_{t \geq 0}, (\beta_t)_{t \geq 0}, (p_t^s)_{t \geq 0}, (\gamma_t)_{t \geq 0}\) denote respectively the given time paths of an index of demand-reducing technical progress, a tax on
in both arguments. The path of policy stringency $\Theta \equiv (\theta_t)_{t \geq 0}$ is exogenously given. The inverse demand function $P_t(x_t, \theta_t)$ is continuously differentiable and decreasing in its two arguments.

In equilibrium, it must be that $x_t = x_t(P_t(x_t, \theta_t), \lambda)$, where the function $x_t(\cdot)$ is defined by (6). This implicitly defines the equilibrium quantity as a function $\phi_t$ which is decreasing in its two arguments $\theta_t$ and $\lambda$ only:

$$x_t = \phi_t(\theta_t, \lambda), \forall t \geq 0. \quad (16)$$

The rest of the analysis follows the same steps as in Sections 2 and 3, where the exogenous indexes $\theta_t$ will replace the formerly given prices $p_t$. Combining (2) with (16) gives the resource rent as a function that is decreasing in all indexes $\Theta$ and in the stock $X$; to simplify notation we redefine $\lambda$ to be that equilibrium rent function:

$$\lambda = \lambda(\Theta, X). \quad (17)$$

Substituting (17) into (16) yields the equilibrium extraction flows, conditional on the stock of reserves; redefining $\widetilde{x}_t$ to denote that function, we have:

$$\widetilde{x}_t = \widetilde{x}_t(\Theta, X) \equiv \phi_t(\theta_t, \lambda(\Theta, X)), \forall t \geq 0. \quad (18)$$

Equilibrium extraction quantities are increasing in $X$ and in $\theta_{t'}$, $t' \neq t$. The partial derivative of $\widetilde{x}_t(\cdot)$ with respect to $\theta_t$ has an ambiguous sign.

Denote by $X^e$ the equilibrium amount of reserves, which is also the total cumulative extraction. The unit rent in equilibrium is thus $\lambda^e = \lambda(\Theta, X^e)$. By (5), the equilibrium stock of reserves satisfies $X^e = X(\lambda^e) = X(\lambda(\Theta, X^e))$, which implicitly defines $X_0^e$ as a function of $\Theta$ only:

$$X^e = X^e(\Theta). \quad (19)$$

The equilibrium stock of initial reserves is decreasing in all $\theta_t$, $t \geq 0$. Finally, the equilibrium extraction flows $x^e_t$ are determined by

$$x^e_t = x^e_t(\Theta) \equiv \widetilde{x}_t(\Theta, X^e(\Theta)), \forall t \geq 0. \quad (20)$$

the extracted resource, the price of a substitute and a subsidy to this substitute. An increase in $\theta_t$ at any date $t$ can thus represent a technical change, an increase in resource taxation, an increase in any substitute’s subsidy or any combination of such policies that reduce the demand for the resource at given price.
Let us now assess the total effect of an increase in date-$T$ policy stringency $\theta_T$. By (18), the effect of a change in the stringency of a policy at $T$, on the equilibrium extraction quantities $x_t^e$ at all dates $t \neq T$, can be decomposed in (11). Remembering that a more stringent policy reduces resource demand, the total effect of an increase in stringency at $T$ is the combination of a pure intertemporal substitution effect (positive) and a stock effect (negative). As before an increase in $\theta_T$ can be shown to reduce the equilibrium rent $\lambda^c$. From (16), $\theta_T$ affects $x_t$, $t \neq T$, only to the extent that the rent is affected. Since $\phi_t(.)$ is a decreasing function of the rent, it follows that $\theta_T$ always affects $x_t$ positively, for $t \neq T$. Although more stringent demand-reducing policies at some dates result in lower total cumulative extraction (stock effect), they always lead to increased equilibrium extraction flows at the other dates (the substitution effect dominates).

8. The supply of an heterogeneous resource

Non-renewable resources come in a great variety of forms which differ by extraction costs, exploration and development costs, location, etc. Let these various sources be identified by $j$, $j = 1, ..., J$ and let resource supply at date $t$ be

$$S_t = \sum_{j=1}^{J} x_t^j \quad (21)$$

where $x_t^j \geq 0$ is the quantity of resource $j$ supplied at date $t$. Spot extraction profits are $\pi_t^j = \pi_t^j(x_t^j, p_t)$, with the same properties as in the single source case. Since $x_t^j$ may be zero, there is no loss of generality in assuming that the same countable set of dates applies for all sources. Each source is constrained by its own exhaustibility constraint:

$$\sum_{t \geq 0} x_t^j = X^j, \quad j = 1, ..., J; \quad (22)$$

each source is characterized by its own exploration and development cost $E^j(X^j)$ whose qualitative properties are the same as in the case of a single resource, with the following minor difference. The property $E'(0) = 0$ is replaced with $E''(0) \geq 0$, so that a resource whose marginal exploration and development is too high for profitability need not be developed. However the same deposit that is not economic at date zero may be
developed when prices become higher and/or when the technology encompassed in the functions \( \pi^j_t (\cdot) \) allows it.

As before, it is supposed that exploration and development are instantaneous, are undertaken only once for each deposit, and that extraction may take place only after deposit development. All potential producers face the same given price stream. For source \( j \), the producer solves:

\[
\max_{(x^j_t)_{t \geq 0}, X^j} \sum_{t \geq 0} \pi^j_t (x^j_t, p_t) (1 + r)^{-t} - xx \frac{1}{(1 + r)^t} E^j_t (X^j) 
\tag{23}
\]

subject to (22) and to:

\[
x^j_t = 0, \ t < \tau^j,
\]

where \( \tau^j \geq 0 \) is the development date for deposit \( j \). Suppose that \( \tau^j > 0 \). No production occurs before that date, so that \( x_t = 0, \ t < \tau^j \). It would not be rational to develop reserves at \( \tau^j \) if it was not in order to produce immediately thereafter; consequently whether \( \tau^j = 0 \) or \( \tau^j > 0 \), \( x^j_{\tau^j} > 0 \), although nothing rules out that extraction be interrupted later on during periods of low prices. There is one specific resource rent \( \lambda^j \) associated with each deposit. Clearly, the problems to be solved for each source are independent of each other. Thus the sole difference with the one-resource case analyzed earlier is the fact that all resources need not be developed at date zero. Roughly, given a price path, resources whose extraction cost is higher and/or whose cost of exploration and development is high will be developed later.

We are interested in resource supply at dates \( t \geq 0 \); in particular we want to determine how supply \( S_t \) reacts to a change in price at \( T \neq t \). Since each component \( x^j_t \) of \( S_t \) is determined independently of the others, consider deposit \( j \) in particular.

Suppose at this stage that \( \tau^j \) is given. Then the derivations and properties established in Section 2. for problem (1) can be readily adapted to problem (23). If \( \tau^j = 0 \) the solution is identical; if \( \tau^j > 0 \), \( x^j_t = 0, \ t < \tau^j \) and, for \( t \geq \tau^j \), the first-order conditions for the choice of the optimum extraction path are, instead of (3) and (4),

\[
\frac{\partial \pi^j_t (x^j_t, p_t)}{\partial x^j_t} (1 + r)^{-(t-\tau^j)} = \lambda^j, \ \forall \ t \geq \tau^j,
\tag{24}
\]

\[9\]On the order of extraction of resource deposits, see, among others, Herfindahl, Amigues et al., Gaudet and Lasserre...
and

\[ E_t^j (X^j) = (1 + r)^{\tau^j} \lambda^j. \]  

(25)

where the unit Hotelling scarcity rent \( \lambda^j \) is evaluated at date zero, although development occurs at \( \tau^j \) rather than at zero.

The difference between (24) and (3) amounts to a redefinition of the time scale for each deposit. Subject to that adjustment, all properties of the supply functions established in Sections 2., 3., and 4. apply \textit{mutatis mutandis} for all deposits. Still holding \( \tau^j \) constant, consider the effect of a drop in price at date \( T \) on date \( t \) supply, where \( T \geq t \). All deposits developed at or before \( t \) contribute to \( S_t \). Consequently the effect is the sum of the changes in the supply from all deposits such that \( \tau^j \leq t \leq T \). Among them one must separate deposits \( i \) such that \( \tau^i = T \) from deposits \( j \) such that \( \tau^j < T \). The first instance corresponds to the case \( t = T \) in Section 3.: a reduction in \( p_T \) causes a drop in \( x_t^i \). The second instance corresponds to \( t \neq T \) in Section 4.: the substitution effect dominates the stock effect, so a drop in \( p_T \) causes a rise in \( x_t^j \). Consider the sum of all effects: if \( t < T \), a drop in \( p_T \) increases extraction from all active deposits at \( t \) thus increases supply at \( t \). If \( t = T \), a drop in \( p_T \) reduces the contribution from all deposits, thus reduces supply at \( t \).

Now allow optimal development dates to adjust to the price change. If it is positive, the optimal development date \( \tau^{j*} (p) \) of deposit \( j \) must be such that,

\[
\frac{\partial \pi_t^j (x_t^j, p_t)}{\partial x_t^j} (1 + r)^{-\tau^*} = \lambda^j
\]

(26)

\[
\frac{\partial \pi_{t-s}^j (0, p_{t-s})}{\partial x_{t-s}^j} (1 + r)^{-(\tau^* - s)} < \lambda^j \forall \text{ positive integer } s
\]

(27)

where \( \lambda^j \) is optimum: \( \lambda^j = \lambda^{j*} \left( p, X^j \ast (p, \tau^{j*}), \tau^{j*} \right) \equiv \lambda^{j*} (p) \). This means that production from the optimal amount of reserves is spread over time in such a way that present value marginal profits are equalized and that there is no date at which production is zero while a small amount \( \varepsilon \) of production would cover the resource rent cost.\(^{10}\)

Consider a change in the price schedule from \( p \) to \( p' \) where \( p' \) departs from \( p \) because of a drop in \( p_T \), \( T > \tau^{j*} \). We know that \( \lambda^{j*} (p') < \lambda^{j*} (p) \). Then since \( p_t \) is unchanged,

\(^{10}\)A condition similar to (27) must hold after exhaustion. We do not make use of it here.
the concavity of \( \pi^j_i \) in \( x^j \) implies that (26) is still satisfied at \( \tau^j = \tau^{j*}(p) \), for some positive level of \( x^j \):

\[
\frac{\partial \pi^j_i(x^j, p_r)}{\partial x^j}(1 + r)^{-\tau^{j*}(p)} = \lambda^{j*}(p').
\]  

(28)

There are two possibilities: (27) is still satisfied \( \tau^j = \tau^{j*}(p) \), or it is violated. If it is still satisfied, the development date is unchanged. If it is violated then \( \tau^{j*}(p') \neq \tau^{j*}(p) \), \( \tau^{j*}(p') \) cannot be higher than \( \tau^{j*}(p) \) because (28) is satisfied: indeed, for any positive integer \( s \), if \( \tau^{j*}(p') = \tau^{j*}(p) + s \), then (28) can be written

\[
\frac{\partial \pi^j_i(x^j, p_r)}{\partial x^j}(1 + r)^{-\tau^{j*}(p')-s} = \lambda^{j*}(p')
\]

which implies that

\[
\frac{\partial \pi^j_i(0, p_r)}{\partial x^j}(1 + r)^{-\tau^{j*}(p')-s} \geq \lambda^{j*}(p') \]

by the concavity of \( \pi^j_i \) in \( x^j \), violating (27). Hence

\[
\tau^j(c)(p') \leq \tau^j(c)(p).
\]  

(29)

This implies that \( \forall s \in [\tau^j(c)(p'), \tau^j(c)(p)] \) production from deposit is non-negative at the new price schedule \( p' \) while it is null at the old price schedule \( p \).

Since this derivation applies to any deposit, we have shown that the supply from all deposits already developed at \( t \) increases as a result of a drop in price at any date \( T > t \). This increase in supply may involve earlier development of deposits.

9. References


