Modelling Regular and Estimable Inverse Demand Systems: A Distance Function Approach

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Abstract

To be useful for policy simulation in the current climate of rapid structural change, inverse demand systems must remain regular over substantial variations in quantities. The distance function is a convenient vehicle for generating such systems. It also allows convenient imposition of prior ideas about the structure of preferences required for realistic policy work. While the distance function directly yields Hicksian inverse demand functions via the Shephard-Hanoch lemma, they are usually explicit in the unobservable level of utility (u), but lack a closed-form representation in terms of the observable variables. Note however that the unobservability of u need not hinder estimation. A simple one-dimensional numerical inversion allows the estimation of the distance function via the parameters of the implied Marshallian inverse demand functions. This paper develops the formal theory for using distance functions in this context, and reports on initial trials on the operational feasibility of the method.

1. INTRODUCTION

In recent years, there has been increasing interest in investigating systems of inverse demand functions in which normalised prices are functions of quantities demanded.[1] Such development is timely because important regulatory issues in natural resource and agricultural markets can naturally be cast within the inverse demand framework. For example, in the natural resource literature, inverse demand functions are regarded as marginal willingness to pay functions, which can be employed to measure the welfare changes from environmental quality change.

Two approaches to the derivation of inverse demand functions may be identified. The first one is the Rotterdam methodology, which is a direct approximation of the conceptual inverse demand relationships without imposing the rigid structure that is implied by utility maximization.[2] An alternative to the Rotterdam methodology based on a dual representation of preferences is typified by Huang (1983), which is based on a specified functional form of the direct utility function. While both approaches lead to inverse demand functions, which can provide a first-order approximation to an arbitrary inverse demand system, such systems suffer from two practical shortcomings:

1) In both approaches it may be inconvenient to incorporate prior ideas about the structure of preferences, which is always required when working with highly disaggregated inverse demand systems. Note that such information (which must be matched to the aggregation level at which estimation is to proceed) often takes the form of statements about relative substitutability among items within different commodity groups.[3]

2) Such systems remain regular over ranges that often are two narrow for (i) realistic policy simulation in the current climate of rapid structural change; (ii) for historical analysis over long periods; or (iii) for empirical analysis of inverse demands using international comparisons data spanning countries widely separated on the development spectrum.

The purpose of this paper is to propose a new approach to the specification of inverse demand systems, which in principle is free from these limitations but is readily applicable for empirical estimation.

Due to the relative ease with which curvature can be related to the properties of the Antonelli matrix, the distance function is a convenient vehicle for generating inverse demand

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[1] See, for example, Kim (1997) on welfare measurement, Huang (1983), Eales & Unnevehr (1994), Brown et al. (1995), Eales et al. (1997) and Beach & Holt (2001) on specification, and Barten & Bettendorf (1989) on estimation.

[2] See Theil (1976), Huang (1988), Barten & Bettendorf (1989), Brown et al. (1995), and Eales et al. (1997).

[3] See Clements & Smith (1983).

systems incorporating structural features required for most policy analysis applications. Furthermore, since concavity (the curvature property of the distance function) is preserved under addition and nesting of increasing concave functions, a straightforward way of generating wider classes of regular distance functions is readily available. Duality theory suggests that systems of inverse demand functions can be derived from the distance function via simple differentiation, according to the Shephard-Hanoch lemma. Whilst these functions are conditioned on an unobservable variable (utility), in most cases they do not have an explicit closed-form representation as the Marshallian inverse demand functions i.e. in terms of the observable variables such as quantities. As pointed out by McLaren et al. (2000) in the context of the expenditure function, and Cooper et al. (2001) in the context of the consumer's profit function, the unobservability of utility need not hinder estimation. A simple one-dimensional numerical inversion allows us to estimate the parameters of a particular distance function via the parameters of the implied Marshallian inverse demand functions. The formal theory for using a distance function in this context will be developed and illustrated in the next section of this paper.

The remainder of this paper proceeds as follows. Section 2 develops the theoretical foundations formally. These include relevant concepts and results from static duality theory as well as the idea of the numerical inversion estimation method. In Section 3, we explore some options for the specification of the distance function. Descriptions of the data and estimation method are provided in Section 4, followed by the interpretation of empirical estimates in Section 5. Finally, Section 6 recapitulates and concludes.

2. THEORETICAL FOUNDATIONS

Let $\Re$ represent the N-tuples of real numbers, let Ω^N represent the non-negative orthant, and let Ω_+^N represent the strictly positive orthant. Let $\mathbf{x}\in\Omega^N$ represent an N vector of commodities, $\mathbf{p}\in\Omega_+^N$ the corresponding N vector of prices, $c\in\Omega_+^1$ a level of expenditure, and $\mathbf{r} = \mathbf{p}/c \in\Omega_+^N$ an N vector of normalised prices. Inverse demand functions conditioned on the quantity vector $\mathbf{x}$ will be referred to as "Marshallian" functions, whilst those conditioned on the quantity vector $\mathbf{x}$ and utility u will be termed "Hicksian" functions.

Consider individual preferences that can be represented over commodity vectors $\mathbf{x}$ by means of a direct utility function $u=U(\mathbf{x})$ satisfying the following properties (**Conditions RU**):[4]

RU1: $U: \Omega^N \rightarrow \Re$
RU2: U is continuous
RU3: U is non-decreasing in $\mathbf{x}$
RU4: U is quasi-concave in $\mathbf{x}$.

[4] The notation $u=U(\mathbf{x})$ is indicative of that used in the rest of this paper. Upper case letters denote functions, and the corresponding lower case letters denote the scalar values of those functions.

Preferences can also be equivalently defined in terms of a function defined over normalised price vectors **r**, giving a dual representation of preferences by means of an indirect utility function V(**r**). Then, inverse demand functions can be obtained by solving the following problem:

$$U(\mathbf{x}) = \text{Minimise}_{\mathbf{r}} \{V(\mathbf{r}): \mathbf{r}\,\mathbf{x} = 1\}. \tag{1}$$

The solution to this problem, gives the Marshallian (or uncompensated) inverse demands, which satisfy the Hotelling-Wold identity :

$$r_i = R_i^{MI}(\mathbf{x}) = \frac{\partial U(\mathbf{x}) / \partial x_i}{\sum_j \left[\partial U(\mathbf{x}) / \partial x_j\right] x_j}, \tag{2}$$

where the superscript MI is to indicate that (2) represents the Marshallian inverse demand functions. Since normalised prices and quantities of commodities are observable variables, it is natural to begin with a direct utility function satisfying Conditions **RU**, exploit the Hotelling–Wold identity to derive the Marshallian inverse demands, and then statistically estimat the parameters that characterise the inverse demand functions given data on **x** and **r**.

On the other hand, there may be advantages to explore other approaches to the specification of regular inverse demand systems. For instance, any specification involving a large number of commodities may require that prior structure (such as separability) be imposed on the system of inverse demands. As noted by McFadden (1978), such structure can be built by simple operations such as composition of lower dimensional functions, and the preservation of regularity conditions will be simpler under some specifications than others. For example, it is well known that increasing concave functions of concave functions are concave, whereas an analogous result may not be true for quasi-concave functions. In general, it is the distance function specification that is most attractive in this regard. Finally, one might argue that specifications of empirical models should be matched with the final aim of the empirical analysis, rather than by the intermediate step of parameter estimation. In welfare analysis, for example, an empirically calibrated distance function is obviously more useful than the corresponding direct utility function.[5]

The starting point of this paper is the distance function defined implicitly as:

$$u = U[\mathbf{x} / D(\mathbf{x}, u)], \tag{3}$$

which gives the maximum amount (distance) by which the commodity quantities **x** must be inflated or deflated in order to reach the base utility contour defined by u.[6] Clearly u = U(**x**) if and only if

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[5] See Kim (1997).

[6] See Anderson (1980), Cornes (1992).

$$D(\mathbf{x}, u) = 1. \tag{4}$$

Therefore, the distance function may be appropriately thought of as an implicit representation of the direct utility function. Under the assumptions that the direct utility function U(**x**) satisfies Conditions **RU**, the distance function will inherit the regularity conditions **RD**:

RD1: D: $\Omega^{N} \times \Re \rightarrow \Omega^{1}_{+}$
RD2: D is continuous
RD3: D is non-decreasing in **x**
RD4: D is non-increasing in u
RD5: D is concave in **x**
RD6: D is homogeneous of degree one (HD1) in **x**.

Consider now the possibility of using a distance function to specify preferences, but with the use of observable data to estimate the implied Marshallian inverse demand functions. Take as given an arbitrary distance function satisfying Conditions **RD**. Application of the Shephard- Hanoch lemma to the distance function yields the Hicksian (compensated) inverse demand functions:

$$R_i^{HI}(\mathbf{x}, u) = \frac{\partial D(\mathbf{x}, u)}{\partial x_i} = D_{x_i} \tag{5}$$

where the superscript HI is to indicate that (5) represents the Hicksian inverse demand functions. Unlike Marshallian inverse demands, Hicksian inverse demands are not directly estimable because they are defined with the level of unobservable utility u. This makes estimation a bit more complicated, but does not create as many difficulties as one might expect. To motivate what follows, note that if the explicit functional form of the corresponding direct utility were available, then Hicksian inverse demands could be "Marshallianised" by replacing the u by:

$$u = U(\mathbf{x}) \tag{6}$$

to give:

$$R_i^{MI}(\mathbf{x}) = R_i^{HI}[\mathbf{x}, U(\mathbf{x})] = D_{x_i}[\mathbf{x}, U(\mathbf{x})] \tag{7}$$

where U(**x**) in (7) is the analytical inversion of (4). Indeed, this was exactly the procedure followed by Eales & Unneveher (1994) [or Beach & Holt (2001)] in deriving their Inverse Almost Ideal Demand System (or the Normalised Quadratic Inverse Demand System), whereby they first specified the distance function D, then derived the Hicksian inverse demands R_i^{HI}, and lastly inverted D explicitly at the optimum (D = 1) to give the direct utility function U(**x**) that was used to eliminate u. In practice, however, such an explicit inversion of (4) in u is not always available; it depends heavily on the particular function form of D. If it is available, then this procedure is equivalent to beginning with the corresponding direct utility function, and there is no gain in generality in starting with a distance function.

This paper focuses on the class of distance functions for which such explicit inversion is not available, and exploits the fact that the implied Marshallian inverse demand functions derived from any distance function can be expressed implicitly by the following system:

$$R_i^{HI}(\mathbf{x},\ u) = D_{x_i},\ i = 1,\ \ldots\ldots,\ N \tag{8}$$

$$D(\mathbf{x}, u) = 1. \tag{9}$$

Provided that conditions RU3 and RD4 are strengthen to be strictly increasing and decreasing respectively, then it becomes feasible to numerically invert u in (9) to express u as a function of **x**. Note that the dimension of the numerical inversion is not related to the dimension of the commodity vector $\mathbf{x} = (x_1, x_2, \ldots\ldots, x_N)$ so that the order of numerical complexity does not increase with the number of commodities.

Given a specific functional form for a distance function D and a vector of parameters $\boldsymbol{\theta}$, the corresponding inverse demand system can be written as:

$$r_i = D_{x_i}(\mathbf{x},\ u;\ \boldsymbol{\theta}) = R_i^{HI}[\mathbf{x},\ U(\mathbf{x};\ \boldsymbol{\theta});\ \boldsymbol{\theta}] = R_i^{MI}(\mathbf{x};\ \boldsymbol{\theta})$$

where $U(\mathbf{x}; \boldsymbol{\theta})$ is the numerical solution of the identity function:

$$D(\mathbf{x}, u; \boldsymbol{\theta})=1 \tag{10}$$

for u, solved at the given values of **x** and $\boldsymbol{\theta}$. At each iterative step of the maximisation of the likelihood function, there is a given set of parameter values. For these parameter values, (10) can be numerically inverted to recover the value of utility u consistent with the given values of commodities **x**. Then, this value of utility can be used to eliminate the unknown value of u from the Hicksian inverse demand system.

We conclude this section by noting the elasticity functions of inverse demands. Let F_{ij}^{HI} denote the Hicksian quantity elasticities for commodity i with respect to x_j, F_{ij}^{MI} the Marshallian quantity elasticities for commodity i with respect to x_j, and S_i^{MI} the Marshallian scale elasticity of commodity i. To facilitate thinking about preferences in terms of a distance function, the quantity and scale elasticity functions can be written in terms of **x** and u:[7]

$$F_{ij}^{HI} = \frac{\partial \log(R_i^{HI})}{\partial \log(x_j)},$$

$$F_{ij}^{MI} = \frac{\partial \log(R_i^{MI})}{\partial \log(x_j)} = F_{ij}^{HI} - \frac{\partial \log(R_i^{HI})}{\partial u}\frac{w_j}{\partial D / \partial u},$$

$$S_i^{MI} = \sum_j F_{ij}^{MI} = \sum_j F_{ij}^{HI} - \frac{\partial \log(R_i^{HI})}{\partial u}\frac{1}{\partial D / \partial u},$$

[7] See Anderson (1980) for the derivation of the quantity and scale elasticity functions.

where $w_i = r_i x_i$ is the budget share functions of commodity i, and $w_j/(\partial D / \partial u) = -\partial U \partial \log(x_j)$.[8]

Quasi-concavity of the direct utility function implies that F_{ii}^{MI} and F_{ii}^{HI} are non-positive. We further note that interpretation of quantity and scale elasticities can be made in a manner similar to price and expenditure elasticities. For instance, demand for commodity i is said to be flexible (or inflexible) if F_{ij}^{MI} and F_{ij}^{HI} are less than (or greater than) minus one. Likewise, commodities i and j are classified as net (or gross) substitutes and complements according to whether F_{ij}^{HI} (or F_{ij}^{MI}) is negative and positive respectively. Lastly, commodities are termed luxuries (or necessities) if their scale elasticities S_i^{MI} are greater than (or less than) minus one. (See Eales and Unnevehr (1994))

3. DISTANCE FUNCTION SPECIFICATION

Having discussed the theoretical framework, we now explore some options for the specification of the distance function. These are meant to provide a bridge to our empirical analysis rather than to be taken seriously in their own right. A good starting point is the Linear Inverse Demand System (LIDS), which has explicit closed forms for the direct utility and distance functions. As will be clear from the following discussion, the LIDS is parametrically similar to the expenditure function underlying Stone's (1954) Linear Expenditure System (LES). Therefore, most of the desirable theoretical properties attributed to the LES carry over to LIDS. We then move to a mild generalisation of LIDS, namely the Generalised LIDS (GLIDS) form, in which the direct utility function lacks an explicit closed form, demonstrating that the numerical inversion method is feasible for the estimation of this GLIDS system. Finally, we exploit the notion of implicit separable structure to derive a multistage inverse demand system, which allows a straightforward derivation of a consistent parameterisation of inverse demand relations at various budgeting stages.

3.1 The Linear Inverse Demand System (LIDS)

The LIDS is obtained from the following specification of the distance function:

$$D(\mathbf{x}, u) = X1 + X2 \cdot F(u) \tag{11}$$

where F(u) is a non-negative, real, decreasing and continuous function, and Xk (k=1, 2) are functions of quantities of goods satisfying the following regularity conditions (**RX**):

RX1: Xk: $\Omega^N \rightarrow \Omega^1$
RX2: Xk is continuous
RX3: Xk is HD1 in **x**
RX4: Xk is non-decreasing in **x**

[8] This expression follows from differentiating the identity function $D[\mathbf{x}, U(\mathbf{x})] = 1$ with respect to $\log(x_j)$.

RX5: Xk is concave in **x**.

Using some intuition stemming from Stone's LES (1954), we choose the F(u) and Xk (k=1, 2) as follows:

$$F(u) = 1/u,$$

$$X1 = \sum_j \gamma_j x_j \text{, and}$$

$$X2 = \prod_j x_j^{\beta_j} \tag{12}$$

where γ_i and β_i are parameters with $\sum_j \beta_j = \sum_j \gamma_j = 1$. Note that for $0 \le \gamma_i, \beta_i \le 1$, the LIDS is regular over the region $u \ge 0$.

Functions (11) and (12), on application of the Shephard – Hanoch lemma, generate the following system of Hicksian budget share equations:

$$W_i^{HI} = \frac{\partial D(\mathbf{x},\ u)}{\partial \log(x_i)} = \gamma_i x_i + \beta_i\left(\frac{X2}{u}\right). \tag{13}$$

The impacts of quantities and the scale of consumption on commodity prices can be evaluated with the use of the own/cross quantity and scale elasticities, which in the case of the LIDS are given by:

$$F_{ij}^{HI} = \frac{\beta_i}{w_i}\frac{X2}{u}\left(-\delta_{ij} + \beta_j\right)$$

$$F_{ij}^{MI} = \frac{\beta_i}{w_i}\left[-\gamma_j x_j - \delta_{ij}(1 - X1)\right]$$

$$S_i^{MI} = \frac{-\beta_i}{w_i},$$

where δ_{ij} is the Kronecker delta.

Elimination of u from (13) by the analytical inversion of (11) at the optimum [setting (11) equal to one] leads immediately to the Marshallian inverse demand system, given by:

$$W_i^{MI} = \gamma_i x_i + \beta_i(1 - X1).$$

It is also transparent that, given the values of parameters and quantities of goods, the numerical inversion of (11) at the optimum to give u in terms of **x** and $\boldsymbol{\theta}$, and its substitution in (13) would give the same results as analytical inversion.

3.2 The Generalised LIDS

Two appealing features of the LIDS are the economy of parameters (involving only 2N-2 independent parameters), and the ease of imposing and maintaining regularity conditions. It should be noted, however, that this model has two drawbacks for empirical application; namely, the facts that all pairs of goods must be net complements but gross substitutes, and that own quantity elasticities cannot exceed (minus) unity (so that the demand for each good must be inflexible). Thus, care must be taken when interpreting results of estimation based on such a system. A generalisation that does not have these restrictions arises from the extension of (11):

$$D(\mathbf{x},\ u) = \left[X1^{\kappa} + (X2/u)^{\kappa}\right]^{1/\kappa}, \tag{14}$$

where κ is a parameter, and the quantity functions X1 and X2 have the familiar LIDS structure:

$$X1 = \sum_j \gamma_j x_j,\ \sum_j \gamma_j = 1, \text{ and } X2 = \prod_j x_j^{B_j}, \tag{15}$$

but the β_i from (12) is replaced by the utility varying coefficients $B_i = \dfrac{\alpha_i + \beta_i e^u}{1 + e^u}$ with $\sum_j \alpha_j = \sum_j \beta_j = 1$. This model generalises and nests the LIDS which is obtained when κ=1 and $\alpha_i = \beta_i$, $\forall$ i though it is still parsimonious in the number of independent parameters (involving 3N – 2 independent parameters). The sufficient conditions to ensure (14) to be a regular distance function over the region $u \geq 0$ are:

$$0 \leq \alpha_i, \gamma_i, \beta_i \leq 1, \text{ and } \kappa \leq 1.$$

Applying the Shephard–Hanoch lemma to (14), and after some manipulation, we obtain the GLIDS budget share equations:

$$W^{HI} = \left[X1^{\kappa} + (X2/u)^{\kappa}\right]^{1/\kappa} \cdot [Z \cdot E1_i + (1-Z) \cdot E2_i] \tag{16}$$

where $Z = X1^{\kappa}/[X1^{\kappa}+(X2/u)^{\kappa}]$, and $Ek_i = \dfrac{\partial \log(Xk)}{\partial \log(x_i)}$ (k=1 ,2) in which $\Sigma_j Ek_j = 1$. It is evident that from (14) it is impossible to solve explicitly for the value of u in terms of parameters and quantities. In order to convert (16) to a Marshallian system, the unobservable u in (14) has to be replaced by the numerical inversion of (14) at D = 1.

The specifications of own/cross quantity and scale elasticity equations associated with the system (16) are expressed as follows:

$$F_{ij}^{HI} = -\delta_{ij} + w_j + [Z_j \cdot E1_{ij} + Z_j(E1_i - E2_i)]/[Z \cdot E1_i + (1-Z) \cdot E2_i]$$

$$F_{ij}^{MI} = -\delta_{ij} + \frac{1}{w_i}[Z \cdot (E1_{ij} - E2_{ij}) + E2_{ij} + (Z_j + Z_j^*) \cdot (E1_i - E2_i)]$$
$$S_i^{MI} = -1 + \frac{1}{w_i}\left[\left(\sum_j E2_{ij}\right) \cdot (1 - Z) + \left(\sum_j Z_j^*\right) \cdot \left(E1_i - E2_i\right)\right]$$

where

$$Ek_{ij} = \frac{\partial Ek_i}{\partial \log(x_j)}, \ k = 1,\ 2,$$
$$Z_j = \frac{\partial Z}{\partial \log(x_j)} = \kappa \cdot Z \cdot (1 - Z) \cdot (E1_j - E2_j),$$
$$Z_j^* = \frac{\partial Z}{\partial U} \cdot \frac{\partial U}{\partial \log(x_j)} = \frac{\partial Z}{\partial u} \cdot \left(\frac{-w_j}{\partial D / \partial u}\right), \text{ and}$$
$$\frac{\partial D}{\partial u} = (1 - Z) \cdot \left\{\sum_j \left[\left(\frac{\partial E2_j}{\partial u}\right) \cdot \log(x_j) - \frac{1}{u}\right]\right\} \text{ in which } \frac{\partial E2_j}{\partial u} = \frac{e^u(\beta_i - \alpha_i)}{(1 + e^u)^2}.$$

3.3 An Implicitly Separable Inverse Demand Systems (AISIDS)

An alternative method by which a functional form may be generated is by the imposition of separable structure. Following Blackorby et al. (1978), let the set of indices of the quantities $\mathbf{x}$ be:

$$\mathbf{I} = \{1, 2, \ldots\ldots, N\},$$

and order these quantities in M separable groups defined by the mutually exclusive and exhaustive partition $\hat{\mathbf{I}} = \{\mathbf{I}^1, \ldots, \mathbf{I}^m, \ldots, \mathbf{I}^M\}$ $(M \le N)$ of the set $\mathbf{I}$. Then preferences are said to be implicitly separable in the partition $\hat{\mathbf{I}}$ if the distance function has the structure:

$$D(\mathbf{x},\ u) = \overline{D}\left[u,\ D^1(\mathbf{x}^1,\ u), \ldots,\ D^m(\mathbf{x}^m,\ u), \ldots,\ D^M(\mathbf{x}^M,\ u)\right], \tag{17}$$

where $\mathbf{x}^m$ is a vector of quantities in the partition $\hat{\mathbf{I}}$, and the D^m are quasi-distance functions depending only on group quantities $\mathbf{x}^m$ and utility level u.[9] Blackorby et al. (1978, Chapter 5) refer to this type of structure on preferences as felicitous decentralisability: in order to allocate expenditure optimally within group m, the consumer needs to know the share of total expenditure allocated to group m, the quantities ($\mathbf{x}^m$) in group m, and the level of utility u[10].

[9] See Blundell & Robin (2000), p. 60.

[10] McFadden (1978, p.60) shows that the distance function has the structure in (17) if and only if the corresponding cost function is implicitly separable, which is the structure to which Blackorby et. al. refer.

Their argument is based the fact that the budget share equations corresponding to (17) have the multiplicative forms given by:

$$w_{i\in I^m} = w^m \cdot w_i^m = W^m(\mathbf{x},\ u)\cdot W_i^m(\mathbf{x}^m,\ u)$$

where $w^m = \sum_{j\in I^m}\frac{\partial \log(D)}{\partial \log(x_j)} = \frac{\partial \log(\overline{D})}{\partial D^m}\sum_{j\in I^m}\frac{\partial D^m}{\partial \log(x_j)} = \frac{\partial \log(\overline{D})}{\partial \log(D_m)}$ is the expenditure share allocated to goods belonging to group m, and $w_i^m = \frac{\partial \log(D)/\partial \log(x_i)}{\sum_{j\in I^m}\partial \log(D)/\partial \log(x_j)} = \frac{\frac{\partial \log(\overline{D})}{\partial D^m/D^m}\cdot\frac{\partial D^m/D^m}{\partial \log(x_i)}}{\frac{\partial \log(\overline{D})}{\partial \log(D^m)}} = \frac{\partial \log(D^m)}{\partial \log(x_j)}$ is the ith good share of the expenditure allocated to group m.

The use in (17) of functions $\overline{D}$ and D^m that satisfy Conditions **RD** is a sufficient condition to generate a regular distance function,[11] and hence provides an attractive means of construction of regular distance functions from more basic regular generating functions. Using the intuition stemming from the GLIDS and Perroni & Rutherford's (1995) N-stage CES functional form, we obtain an implicitly separable inverse demand system (AISIDS). Specifically, each quasi-distance function D^m is written as:

$$D^m = \left(\sum_{j\in I^m} B_j^m x_j^{\kappa_m}\right)^{1/\kappa_m},$$

whilst $\overline{D}$ is written as:

$$\overline{D} = \left\{\sum_{m=1}^{M}\left[\frac{D^m}{u^{\eta_m}}\right]^{\kappa_0}\right\}^{1/\kappa_0}, \tag{18}$$

where $B_j^m = \frac{\alpha_i^m + \beta_i^m e^u}{1+e^u}$ are the utility varying coefficients in which $\sum_{j\in I^m}\alpha_j^m = \sum_{j\in I^m}\beta_j^m = 1$, and η_m, κ_o, κ_m, α_i^m, and β_i^m are parameters. Provided $u \geq 0$, the sufficient conditions to ensure (18) to be a regular distance function are:

$$\kappa_m \leq 1,\ \kappa_o \leq 1,\ \eta_m \geq 0, \text{ and } 0 \leq \alpha_i^m,\ \beta_i^m \leq 1.$$

[11] That implies $\overline{D}$ is decreasing in u, and concave, HD1 and increasing in D^m $\forall$ m, whereas D^m are decreasing in u, and concave, HD1 and increasing in $\mathbf{x}^m$.

Applying the Shephard-Hanoch lemma, logarithmic differentiation of (18) with respect to quantities produces the AISIDS share equations:

$$W^{HI}_{i\in I^m} = \overline{D}\cdot E^o_m \cdot E^m_i \tag{19}$$

with the corresponding elasticity equations:

$$\begin{aligned}
F^{HI}_{ij,\, i,\, j\in I^m} &= -\delta_{ij} + w_j + \frac{E^o_{mj}}{E^o_m} + \frac{E^m_{ij}}{E^m_i}\\
F^{HI}_{ij,\, i\in I^m,\, j\in I^\ell,\, m\neq\ell} &= w_j + \frac{E^o_{mj}}{E^o_m}\\
F^{MI}_{ij,\, i,\, j\in I^m} &= -\delta_{ij} + \frac{E^o_{mj} - E^o_{mu}\cdot w_j / D_u}{E^o_m} + \frac{E^m_{ij} - E^m_{iu}\cdot w_j / D_u}{E^m_i}\\
F^{MI}_{ij,\, i\in I^m,\, j\in I^\ell,\, m\neq\ell} &= \frac{E^o_{mj} - E^o_{mu}\cdot w_j / D_u}{E^o_m} - \frac{E^m_{iu}\cdot w_j / D_u}{E^m_i}\\
S^{MI}_{ij,\, i,\, j\in I^m} &= -1 - \frac{1}{D_u}\left(\frac{E^o_{mu}}{E^o_m} + \frac{E^m_{iu}}{E^m_i}\right)
\end{aligned}$$

where

$$E^o_m = \frac{(D^m / u^{\eta_m})^{\kappa_0}}{\sum_\ell \left(D^\ell / u^{\eta_\ell}\right)^{\kappa_0}},$$

$$E^m_i = \frac{B^m_i x_i^{\kappa_m}}{\sum_j B^m_j x_j^{\kappa_m}},$$

$$\begin{aligned}
E^o_{mj} = \frac{\partial E^o_m}{\partial \log(x_j)} &= \kappa_o \cdot E^o_m \cdot (1 - E^o_m)\cdot E^m_j \text{ for } j\in I^m\\
&= -\kappa_o \cdot E^o_m \cdot E^o_\ell \cdot E^\ell_j \text{ for } j\in I^\ell \ (m\neq\ell),
\end{aligned}$$

$$E^m_{ij} = \frac{\partial E^m_i}{\partial \log(x_j)},$$

$$E^m_{iu} = \frac{\partial E^m_i}{\partial u} = \frac{(\partial B^m_i / \partial u)\cdot x_i^{\kappa_m}}{\sum_j (\partial B^m_j / \partial u)\cdot x_j^{\kappa_m}} - E^m_i \frac{\sum_j (\partial B^m_j / \partial u)\cdot x_j^{\kappa_m}}{\sum_j B^m_j x_j^{\kappa_m}} \text{ in which } \frac{\partial B^m_i}{\partial u} = \frac{e^u(\beta^m_i - \alpha^m_i)}{(1+e^u)^2},$$

$$E^o_{mu} = \frac{\partial E^o_m}{\partial u} = \kappa_o \cdot \left(V_m - E^o_m \cdot \sum_{m=1}^{M} V_m\right) \text{ in which } V_m = E^o_m \cdot (D^m_u - \eta_m / u),$$

$$D^m_u = \frac{\partial \log(D^m)}{\partial u} = \frac{\sum_j (\partial B^m_j / \partial u)\cdot x_j^{\kappa_m}}{\kappa_m \sum_j B^m_j x_j^{\kappa_m}}, \text{ and}$$

$$D_u = \frac{\partial D}{\partial u} = \frac{\overline{D} \cdot \sum_{\ell=1}^{M} (D^{\ell} / u^{\eta_\ell})^{\kappa_o} \cdot (D_u^{\ell} - \eta_\ell / u)}{\sum_{\ell=1}^{M} (D^{\ell} / u^{\eta_\ell})^{\kappa_o}}.$$

Note that the AISIDS share equations involve only 2N+1 independent parameters. Additionally, if $\kappa_o = 1$, then the distance function that results is additive or strongly separable in Partition $\hat{I}$; i.e.,

$$\overline{D} = \sum_{\ell} \frac{D^{\ell}(\mathbf{x}^{\ell},\ u)}{u^{\eta_\ell}} = \sum_{\ell} \Phi_\ell(\mathbf{x},\ u),$$

which is a generalisation of Rimmer & Powell's (1996) An Implicitly Additive Demand Systems model.

4. DATA, ESTIMATION AND RESULTS

4.1 Brief Remarks on the Data Base

For illustrative purposes, equation systems (13), (16) and (19) are estimated on quarterly time series data for Japanese fish consumption and prices covering the period 1980-1994. The data base are developed by Eales et al. (1997), and are deseasonalised prior to estimation.[12] A six-commodity breakdown of the goods is used:

1. x_1 = High value fish (including tuna, sea bream, flatfish, and yellowtail);
2. x_2 = Medium value fish (including horse mackerel, bonito, flounder, and salmon);
3. x_3 = Low value fish (including sardines, mackerel, saury, and cod);
4. x_4 = Lobster, shrimp, and crab;
5. x_5 = Cuttlefish, squid, and octopus; and
6. x_6 = Shellfish.

We further divide the six commodities into two separable groups (M=2) according to the following classification:

Group 1: x_1: High value fresh fish
x_2: Medium value fresh fish
x_3: Low value fresh fish

[12] See Eales, Durham & Wessells (1997), p. 1157 for a complete description of the data.

Group 2: x_4: Lobster, shrimp and crab
x_5: Shellfish
x_6: Cuttlefish, squid, and octopus.

In this case, the implicitly separable distance function given by (18) takes the form:

$$\overline{D} = \left\{ \left[\frac{D^1(\mathbf{x}^1,\ u)}{u^{\eta_1}} \right]^{\kappa_0} + \left[\frac{D^2(\mathbf{x}^2,\ u)}{u^{\eta_2}} \right]^{\kappa_0} \right\}^{1/\kappa_0},$$

where $D^1 = \left(\sum_{j=1}^{3} B_j^1 x_j^{\kappa_1} \right)^{1/\kappa_1}$ and $D^2 = \left(\sum_{j=4}^{6} B_j^2 x_j^{\kappa_2} \right)^{1/\kappa_2}$. The main reason for restricting our form to the case of two separable groups is to minimise the number of parameters. Admittedly, the number of parameters increases substantially with the number of separable groups, which may increase the complexity of estimation.

4.2 Estimation Technique

Since the GAUSS language is ideally suited for handling the implicit representation of functional relationships, all equation systems may be estimated by using the GAUSS 3.4.22 computer package with the modules NLSYS and CML. For purposes of estimation, an error term e_{it} is appended additively in all budget share systems. The estimation method is non-linear full information likelihood technique, and the last equation in each system, which is the budget share equation for shellfish, is deleted to ensure non-singularity of the error covariance matrix. As usual, the estimation should be independent of which equations are excluded.

Before we proceed to the estimation of the systems, two statistical problems have to be addressed. The first issue deals with the stochastic properties of the error terms. Following Beach & MacKinnon (1979), we introduce the first order autoregressive scheme:

$$\mathbf{e}_t = \rho \mathbf{e}_{t-1} + \boldsymbol{\varepsilon}_t,\ \ t = 2, \ldots\ldots, T$$

where $\mathbf{e}_t$ is the vector of error terms e_{it}, ρ is the autocorrelation coefficient, and $\boldsymbol{\varepsilon}_t$ is the vector of serially uncorrelated error terms characterised by a multivariate normal distribution with zero mean and a constant contemporaneous covariance matrix Ω. By writing the system in a more compact form:

$$\mathbf{w}_t = \mathbf{W}(\mathbf{x_t}, u_t; \boldsymbol{\theta}) + \mathbf{e}_t,\ t=1, \ldots\ldots, T,$$

and by transforming the first order autoregressive scheme, the following system is obtained:

$$\mathbf{w}_t = \rho \mathbf{w}_{t-1} + \mathbf{W}(\mathbf{x}_t, u_t; \boldsymbol{\theta}) - \rho \mathbf{W}(\mathbf{x}_{t-1}, u_{t-1}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t,\ \ t=2, \ldots\ldots, T \tag{20}$$

where **W**(.) is the vector of deterministic components of the budget share equations. Thus, estimation of the equation systems with first order autoregressive error terms can be carried out using the estimation procedure of a singular budget share system based on (20), with one additional parameter ρ to estimate in addition to parameters $\boldsymbol{\theta}$.

The second issue deals with the structure of the contemporaneous covariance matrix Ω. As argued by many econometricians, when the number of commodities in a budget share system becomes large, maximum likelihood estimation procedures may become numerically unstable as the estimated covariance matrix tends to become singular. In response to this concern, we follow the procedures of Selvanathan (1991) and Rimmer and Powell (1996) to impose a simple but sensible structure on the variance covariance matrix of the error terms $\boldsymbol{\varepsilon}_t$. Define Γ as the variance covariance matrix of $\boldsymbol{\varepsilon}_t$ after deleting the sixth budget share equation. Assume that Γ has the following parametric form:

$$\Gamma = \lambda^2 \cdot \Xi$$

where $\Xi = \mathbf{w}^* - \boldsymbol{\varpi}'\boldsymbol{\varpi}$ in which $\mathbf{w}^* = \text{diag}(\overline{w}_1, \overline{w}_2, \overline{w}_3, \overline{w}_4, \overline{w}_5)$, $\boldsymbol{\varpi} = (\overline{w}_1, \overline{w}_2, \overline{w}_3, \overline{w}_4, \overline{w}_5)'$, $\overline{w}_i$ (i=1 to 5) is the sample mean of the ith budget share, and λ is the parameter to be estimated. Clearly this procedure requires the additional estimation of only a single parameter λ. Furthermore, this structure has two interesting implications: i) it allows for larger error variances for commodities that occupy larger shares, and ii) the covariances between goods are proportional to the products of their average budget shares.

4.3 Empirical Results and Their Interpretation

4.3.1 Analysis of the Estimates

Comparative results for the LIDS, GLIDS and AISIDS are presented in Table 1. Estimates of asymptotic t-ratios are reported in parentheses, and L represents the system log-likelihood values. The results for the LIDS are derived under the implicit estimation scheme rather than the standard method in order to provide a basis for comparison with the other two models.

The most important point to highlight from the results in Table 1 is that all parameter estimates (for all models) satisfy the sufficient conditions for global regularity without the need to impose constraints. We further find that the estimated scalar value of u is consistently positive in the sample period. These lead to the conclusion that the LIDS, GLIDS and AISIDS satisfy all regularity conditions (**RD**) for all observations.

The summary statistics suggest that all three inverse demand systems provide a reasonably good fit, given that the data is quarterly and estimation is in share form: the share equation R^2 values range from 52% for low value fish (for LIDS) to 87% for lobster (for AISIDS). Not unexpectedly, the R^2 value for the share equation of low value fish (for all models) is the lowest relative to the other share equations. Probably, this exhibits signs of dynamic misspecification. More likely, this may be caused by the failure to allow for imperfect adjustment to quantity changes as the share of low value fish has a reasonable high amount of variation. The serial correlation properties of the error terms as shown in the Durbin-Watson statistics are no longer severely pathological, although there is still evidence of negative serial correlation. Probably, this indicates the less than perfect appropriateness of the correction for autoregressive errors.

In terms of fit, GLIDS and AISIDS perform better on the basis of comparisons of likelihood values and R^2 values. Furthermore, a chi-squared test of the restrictions ($\alpha_i = \beta_i$ and κ=1) implied by the LIDS specification results in a calculated χ^2 statistic of 26.410, to be compared with a critical value of $\chi^2_{0.01,\,6}$= 18.475. It might be concluded that the restrictions required in moving from GLIDS to LIDS are not supported by the data. On prima facie grounds, our preferred models are GLIDS and AISIDS.

4.3.2 Analysis of the Elasticity Estimates

The quantity and scale elasticity estimates for the LIDS, GLIDS and AISIDS evaluated at the sample means of the variables are reported in Table 2. Columns 1 to 6 report the estimates of the Hicksian own/cross quantity elasticities. These estimates show how much the price of commodity i must change to induce the consumer to absorb marginally more of commodity j while maintaining the same utility level. Overall, these estimates offer no surprise. The own quantity elasticities (f_{ii}^{HI}) for all models are negative but generally greater than minus one, which obey the inverse law of demand, and indicate that all types of fish are inflexible. We also find that the cross quantity elasticities (f_{ij}^{HI}) for all models are less than one in absolute value. For instance, for the GLIDS, a 1% increase in the consumption of high value fish will increase the price of low value fish by only 0.06%. This contradicts the notion that all fish are mutual substitutes. We further find that the cross quantity elasticity estimates are unstable across functional forms. Results confirm that some of the reported elasticities exhibit sign reversals, and there are many substantial differences in magnitude. In particular, f_{34}^{HI}, f_{43}^{HI}, f_{36}^{HI}, f_{63}^{HI}, f_{46}^{HI} and f_{64}^{HI} vary from positive to negative depending on functional forms. Thus, it is not clear what conclusion can be reached about the net complementarity relationships between low value fish & lobster, low value fish & cuttlefish, and lobster & cuttlefish.

Scale elasticities (s_i^{MI}), reported in column 7, measure the potential response of commodity price to a proportionate increase in all commodities. For example, the scale elasticity for high value fish for GLIDS is -1.01, which indicates that a 1% proportionate increase in all commodities will reduce the price of this fish category by about 1.01%. All the estimates of s_i^{MI} for all models are consistently negative whilst cuttlefish has the largest scale effect. For GLIDS and AISIDS (the preferred models), the estimated s_i^{MI} $\forall$ i are close to minus one, suggesting that preferences are homothetic. This essentially supports the findings in Barten & Bettendorf (1989), Eales et al. (1997), Park & Thurman (1999) and Beach & Holt (2001), though they adopt more flexible functional forms.

Not surprisingly, there are some discrepancies in scale elasticity estimates among the three models. Specifically, the scale elasticities for lobster and cuttlefish implied by GLIDS and AISIDS are substantially smaller in absolute terms than those implied by LIDS, whereas just the opposite response is recorded for high value fish and shellfish. The differences in the basic demand responses may indicate that model choice is important for analysing substitution possibilities among commodities.

The Marshallian own/cross quantity elasticities (f_{ij}^{MI}) reported in columns 8 to 13 describe the gross effects of changes in quantities on commodity prices by allowing for substitution and scale effects. As expected, the Marshallian own quantity elasticities (f_{ii}^{MI}) are larger in absolute value than of the corresponding Hicksian elasticities (f_{ii}^{HI}). Note also that the results that most fish pairs appear to be net complements changes to gross substitutes when the definition of the Marshallian elasticities is used. This might appear to be a contradiction but it could be resolved easily when one considers scale effects in the final calculation. In this respect, it is useful to note that $f_{ij}^{MI} = f_{ij}^{HI} + s_i^{MI} w_j$. It is now apparent that only if the net substitution effect (measured by the term f_{ij}^{HI}) dominants the scale effect (measured by the term $s_i^{MI} w_j$) will f_{ij}^{MI} have the same sign as f_{ij}^{HI}.

As with the findings of Barten & Bettendorf (1989), Eales et al. (1997) and Beach & Holt (2001), results show that demand for all types of fish are inflexible in the sense of exhibiting Marshallian own quantity elasticities below unity. For the GLIDS (or AISIDS), high value fish exhibits the highest own quantity elasticity [f_{11}^{MI} = -0.958 (or -0.787)] whilst cuttlefish reveals the lowest own quantity elasticity of –0.807 (or –0.258). With respect to the derived cross quantity elasticities (f_{ij}^{MI}), they are generally low and illustrate weak gross substitutability between all types of fish, which is similar to the results obtained in Belgium by Barten & Bettendorf (1989). Overall, the magnitudes of f_{ij}^{MI} confirm that for all models price for high value fish is relatively less sensitive to low value fish and shellfish than medium value fish. Similarly, for all models, price for cuttlefish is relatively more sensitive for medium value fish than high value and low value fish.

5. CONCLUDING REMARKS

This paper advocates a more general use of the distance function in specifying estimable and regular inverse demand systems. Note that we only focus on the type of distance functions for which it is not necessary to have closed functional forms for the Marshallian inverse demand equations, nor for the direct utility function. The technical aspects on how to estimate the Hicksian inverse demand functions have been discussed in considerable detail. In particular, a method based on the numerical inversion approach developed by McLaren et al. (2000) and Cooper et al. (2001) is adopted to deal with the unobservability of utility.

The implementation of the proposed method relies on some simple functional forms to specify the LIDS, GLIDS and AISIDS. Such specifications seem particularly attractive for the purpose of modelling complete and multi-stage inverse demand systems since they can be easily constrained to be regular over an unbounded region, since the numbers of additional parameters to be estimated are small, and since it allows convenient imposition of prior ideas about the preference structure. The models were illustrated with an application to Japanese fish demand. Results indicate that this method is operationally feasible and that all models satisfy their required regularity conditions for all observations in the sample period. These lead to the conclusions that the distance function approach is a promising tool of empirical analysis of inverse demand systems subject to tight theoretical conditions. This opens up a further avenue for ultimately obtaining systems of inverse demand equations, which are simultaneously more flexible and regular than those currently employed in applied consumer demand analyses.

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Table1: Empirical Results (t-ratios in parentheses)

	Models						
	LIDS		GLIDS			AISIDS	
Parameters							
α_1	–	–	0.046	(2.579)	α_1^1	0.136	(7.697)
α_2	–	–	0.495	(15.423)	α_2^1	0.588	(22.309)
α_3	–	–	0.064	(5.012)	α_3^1	0.276	(11.860)
α_4	–	–	0.180	(5.196)	β_1^1	0.162	(8.653)
α_5	–	–	0.080	(3.551)	β_2^1	0.575	(22.171)
α_6	–	–	0.135	(7.826)	β_3^1	0.263	(11.956)
β_1	0.051	(4.268)	0.120	(4.595)	κ_1	0.461	(9.009)
β_2	0.434	(11.465)	0.425	(14.170)	η_1	0.150	(5.731)
β_3	0.071	(6.186)	0.073	(4.141)	α_4^2	0.465	(16.063)
β_4	0.207	(6.509)	0.164	(4.281)	α_5^2	0.170	(4.797)
β_5	0.085	(3.751)	0.094	(3.821)	α_6^2	0.365	(10.339)
β_6	0.152	(9.080)	0.124	(4.895)	β_4^2	0.465	(16.350)
γ_1	0.025	(2.493)	0.029	(3.627)	β_5^2	0.185	(5.471)
γ_2	0.250	(9.956)	0.316	(10.761)	β_6^2	0.350	(9.885)
γ_3	0.181	(9.683)	0.161	(9.455)	κ_2	0.612	(7.462)
γ_4	0.227	(8.966)	0.200	(8.916)	η_2	0.183	(6.260)
γ_5	0.116	(5.000)	0.107	(5.030)	κ_0	0.795	(13.754)
γ_6	0.201	(8.893)	0.187	(8.139)	ρ	0.995	(179.41)
κ	–	–	0.599	(10.172)			
ρ	0.998	(224.93)	0.998	(233.83)			

Table 1: (Continued)

Summary Statistics	LIDS	GLIDS	AISIDS
L	1115.359	1128.564	1128.860
k[♥]	10	16	15
R^2			
High Value Fish	0.524	0.566	0.567
Medium Value Fish	0.594	0.629	0.630
Low Value Fish	0.523	0.565	0.565
Lobster	0.854	0.866	0.867
Shellfish	0.779	0.798	0.799
Cuttlefish	0.609	0.643	0.644
Durban-Watson Statistics			
High Value Fish	2.424	2.484	2.417
Medium Value Fish	2.917	2.972	2.872
Low Value Fish	2.733	2.631	2.628
Lobster	3.172	3.147	3.228
Shellfish	2.867	2.874	2.794
Cuttlefish	2.749	2.692	2.662

Likelihood Ratio Test of LIDS against GLIDS ($\beta_i = \alpha_i$ & $\kappa = 1$):
Test Statistic = 26.410 and $\chi^2_{0.01,\ 6} = 18.475$

[♥] k denotes the number of free parameters.

Table 2: Elasticity Estimates Compared Across Models[2]

	Hicksian Quantity Elasticities						Scale Elasticities	Marshallian Quantity Elasticities					
Commodity i	f_{i1}^{HI}	f_{i2}^{HI}	f_{i3}^{HI}	f_{i4}^{HI}	f_{i5}^{HI}	f_{i6}^{HI}	s_{i}^{MI}	f_{i1}^{MI}	f_{i2}^{MI}	f_{i3}^{MI}	f_{i4}^{MI}	f_{i5}^{MI}	f_{i6}^{MI}
							LIDS						
High Value Fish	-0.055	0.021	0.006	0.007	0.019	0.003	-0.218	-0.106	-0.055	-0.008	-0.024	-0.011	-0.014
Medium Value Fish	0.256	-0.456	0.032	0.042	0.107	0.020	-1.254	-0.038	-0.890	-0.047	-0.136	-0.061	-0.082
Low Value Fish	0.231	0.106	-0.489	0.038	0.096	0.018	-1.129	-0.034	-0.284	-0.560	-0.123	-0.055	-0.074
Lobster	0.298	0.138	0.037	-0.621	0.124	0.023	-1.461	-0.044	-0.367	-0.055	-0.828	-0.071	-0.096
Shellfish	0.130	0.060	0.016	0.021	-0.238	0.010	-0.637	-0.019	-0.160	-0.024	-0.069	-0.323	-0.042
Cuttlefish	0.381	0.176	0.047	0.062	0.159	-0.825	-1.866	-0.056	-0.469	-0.070	-0.203	-0.091	-0.977
							GLIDS						
High Value Fish	-0.721	0.319	0.060	0.134	0.131	0.077	-1.010	-0.958	-0.030	-0.003	-0.009	-0.004	-0.006
Medium Value Fish	0.211	-0.522	0.047	0.098	0.111	0.055	-0.965	-0.015	-0.855	-0.013	-0.039	-0.018	-0.024
Low Value Fish	0.230	0.254	-0.787	0.115	0.123	0.065	-1.028	-0.011	-0.101	-0.852	-0.031	-0.014	-0.020
Lobster	0.224	0.234	0.051	-0.689	0.119	0.061	-1.013	-0.013	-0.116	-0.012	-0.832	-0.017	-0.023
Shellfish	0.231	0.289	0.057	0.125	-0.773	0.072	-1.012	-0.006	-0.061	-0.006	-0.018	-0.908	-0.012
Cuttlefish	0.230	0.217	0.050	0.104	0.120	-0.720	-1.054	-0.017	-0.147	-0.016	-0.045	-0.021	-0.807
							AISIDS						
High Value Fish	-0.556	0.259	0.043	0.088	0.117	0.050	-0.986	-0.787	-0.082	-0.019	-0.052	-0.015	-0.032
Medium Value Fish	0.185	-0.306	0.015	0.011	0.093	0.003	-0.999	-0.049	-0.651	-0.048	-0.131	-0.040	-0.080
Low Value Fish	0.174	0.086	-0.309	-0.019	0.084	-0.015	-1.005	-0.062	-0.261	-0.372	-0.162	-0.050	-0.098
Lobster	0.133	0.043	-0.003	-0.214	0.062	-0.022	-1.012	-0.104	-0.306	-0.066	-0.357	-0.073	-0.105
Shellfish	0.196	0.232	0.038	0.077	-0.587	0.043	-0.998	-0.038	-0.112	-0.024	-0.064	-0.720	-0.039
Cuttlefish	0.128	0.031	-0.006	-0.038	0.059	-0.174	-1.021	-0.111	-0.322	-0.070	-0.183	-0.077	-0.258

[2] f_{ij}^{HI}, s_{i}^{MI} and f_{ij}^{MI} are the point estimates of the elasticity equations F_{ij}^{HI}, F_{ij}^{MI} and S_{i}^{MI} respectively.