The Translog Utility Function and the Demand for Money in the United States: Is There a Problem?*

Apostolos Serletis† and Asghar Shahmoradi
Department of Economics
University of Calgary
Calgary, Alberta, T2N 1N4

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Abstract

In this paper we use sampling theoretic as well as Bayesian inference to revisit the demand for money in the United States in the context of the basic translog flexible functional form. In doing so, we impose local curvature, using recently suggested procedures, and argue that a breakthrough from the current state of using locally flexible specifications that violate theoretical regularity to the use of such specifications that are more consistent with the theory will be through the use of Bayesian inference. We also demonstrate, very conclusively, that the basic translog does not perform well in terms of describing U.S. money demand in a manner that satisfies the restrictions imposed by microeconomic theory and gives rise to stable elasticity estimates.

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*Serletis gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada.

†Corresponding author. Phone: (403) 220-4092; Fax: (403) 282-5262; E-mail: Serletis@ucalgary.ca; Web: http://econ.ucalgary.ca/serletis.htm.

1 Introduction

This paper focuses on the demand for money in the United States, building on a large body of recent literature, which Barnett (1997) calls the 'high road' literature, that takes a microeconomic- and aggregation-theoretic approach to the demand for money. This literature follows the innovative works by Chetty (1969), Donovan (1978), and Barnett (1980, 1983) and utilizes the demand systems approach to investigating the inter-related problems of monetary aggregation and estimation of monetary asset demand functions — see, for example, Ewis and Fisher (1984, 1985), Serletis and Robb (1986), Serletis (1991), Fisher and Fleissig (1997), and Fleissig and Serletis (2002), among others.

These works are interesting and attractive as they include estimates of the demand for money and of the degree of substitutability between money and near-monies using locally flexible functional forms [see Ewis and Fisher (1984), Serletis and Robb (1986), and Serletis (1991)], effectively globally regular functional forms [see Barnett (1983)], and globally flexible functional forms [see Ewis and Fisher (1985) and Fleissig and Serletis (2002)]. Even though model comparisons haven't been carried out realistically, research has indicated that the simple-sum approach to monetary aggregation cannot be the best that can be achieved, in the face of cyclically fluctuating incomes and interest rates.

However, the usefulness of flexible functional forms depends on whether they satisfy the theoretical regularity conditions of positivity, monotonicity, and curvature, and in the literature there has been a tendency to ignore regularity. In fact, as Barnett (2002, p. 199) put it in his *Journal of Econometrics* Fellow's opinion article, without satisfaction of all three theoretical regularity conditions "the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid." In short, the recent advances in the 'high road' literature to the inter-related problems of monetary aggregation and estimation of money demand functions are an important step in a positive direction, but have not yet produced money demand estimates consistent with optimizing behavior.

In this paper we revisit the demand for money in the United States in the context of one of the most widely used flexible functional form — the basic translog, using recent state-of-the-art advances in microeconometrics. Although this is a locally flexible form, we employ it in this paper for three reasons. Most importantly, it has been used extensively in the monetary literature and sometimes performs well, even in comparison with members of the class of globally flexible models, such as the Fourier and the Asymptotically Ideal models. Secondly, it is well understood and provides a benchmark for the (usually) better globally flexible models. Finally, in this paper we use recent advances in microeconometrics for imposing local curvature conditions and also estimate the model using Bayesian procedures.

In doing so, we pay explicit attention to all three theoretical regularity conditions (positivity, monotonicity, and curvature) using sampling theoretic as well as Bayesian estimation procedures. We argue that unless regularity is attained by luck, flexible functional forms should always be estimated subject to regularity, as suggested by Barnett (2002) and Barnett and Pasupathy (2003), because without satisfaction of regularity the resulting inferences are virtually worthless, since violations of regularity violate the maintained hypothesis and invalidate the duality theory that produces the estimated model. In fact, we impose local curvature, using procedures recently introduced by Ryan and Wales (1998) and Moschini (1999), and show that even when local curvature is imposed the model performs badly at other points within the region of the data, thereby producing inference about the demand for money and near-monies inconsistent with optimizing behavior and duality theory.

We also follow Terrell (1996) and take a Bayesian approach to incorporating the theoretical regularity restrictions into the basic translog functional form. In doing so, we use the Metropolis-Hastings algorithm to generate an initial sample from the posterior probability density function for the parameters for a prior that ignores theoretical regularity. For each parameter vector in the sample, we evaluate the theoretical regularity restrictions in the price domain over which inferences are drawn and use accept-reject sampling to generate a sample of the parameter vectors that is consistent with theoretical regularity. Then we use marginal posterior density functions consistent with theory to draw inferences about income elasticities, own- and cross-price elasticities, as well as the elasticities of substitution. We also evaluate the model in terms of its ability to describe U.S. monetary demand in a manner that satisfies the restrictions imposed by microeconomic theory and gives rise to stable elasticity estimates.

The paper is organized as follows. Section 2 briefly sketches out the neoclassical monetary problem while Section 3 discusses monetary aggregation and measurement matters and uses the Divisia index to aggregate monetary assets. In Sections 4, 5, and 6 we use sampling theoretic procedures to estimate the basic translog functional form and assess the results in terms of their consistency with optimizing behavior. In Section 7 we use Bayesian estimation procedures and explore the economic significance of our results. The final section concludes the paper.

2 The Demand for Monetary Services

We assume in accordance with the Federal Reserve's a priori assignment of assets to monetary aggregates, that financial decisions are weakly separable from consumption decisions, so that the representative money holder faces the following problem

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} = m \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_8)$ is the vector of monetary asset quantities described in Table 1; $\boldsymbol{p} = (p_1, p_2, \cdot \cdot \cdot, p_8)$ is the corresponding vector of monetary asset user costs, defined as in Barnett (1978); and m is the expenditure on the services of monetary assets. Because the flexible functional forms are parameter intensive we rationalize the estimation to a small set of monetary asset demand equations by imposing the following separable structure of preferences

$$f(\boldsymbol{x}) = f\left(f_1\left(x_1, x_2, x_3, x_4\right),\ f_2\left(x_5, x_6\right),\ f_3\left(x_7, x_8\right)\right)$$

where the subaggregator functions f_i $(i = 1, 2, 3)$ provide subaggregate measures of monetary services. Here the subaggregates will be thought of as Divisia quantity indexes that can allow for less than perfect substitutability among the relevant monetary components — see Barnett (1980) for more details.

3 The Data

Because the economic agent involved in this study is the household, it is important to work with data that reflect this composite agent's selection of monetary services. In practice, the assets in the official M2 definition of money are appropriate, but we have excluded the rapidly growing retail money market mutual funds, as does much of the empirical literature, mainly because satisfactory monetary aggregates cannot be obtained using this asset most probably because the household employs this particular asset for its investment properties and not for its monetary services. This judgment in the literature is the result of revealed preference testing that reaches this conclusion. As already noted, the list of assets that are employed is provided in Table 1.

In the table we have separated the group of assets into three collections based on empirical pre-testing. The main reason for employing subaggregates, rather than studying all eight items is that our model is very parameter intensive. The pre-testing, for which there is a large literature [see Barnett, Fisher, and Serletis (1992)] is based on the NONPAR GARP procedure of Varian (1982, 1983). The specific collection used here is very much like that reported in the literature but specifically is described in Anderson, Jones, and Nesmith (1997), as reported in Table 1.

The Federal Reserve Bank of St. Louis, in its Monetary Services Index project, provides index numbers for monetary quantities as well as user costs, for the eight items listed in Table 1 (and many others, up through the L definition of money in the Federal Reserve's lexicon). For our empirical work we require per capita real data and to that end we have divided each measure of monetary services by the U.S. CPI (for all items) and total U.S. population. That data are quarterly from 1970:1 to 2003:2 (a total of 134 observations). The calculation of the user costs, which are the appropriate prices for monetary services, is explained in several online publications of the Federal Reserve Bank of St. Louis or in Barnett, Fisher, and Serletis (1992), Barnett and Serletis (2000), and Serletis (2001).

In order to provide the three subaggregates shown in Table 1, we employ a Divisia quantity index. What this does, up to a third order remainder term, is preserve the microeconomic characteristics of the underlying monetary assets — see Fisher (1989) for an explanation. The collection of assets, then are as follows: Subaggregate A is composed of currency, travelers checks and other checkable deposits including Super NOW accounts issued by commercial banks and thrifts (series 1 to 4 in Table 1). Subaggregate B is composed of savings deposits issued by commercial banks and thrifts (series 5 and 6) and subaggregate C is composed of small-time deposits issued by commercial banks and thrifts (series 7 and 8). Finally, Divisia user cost indexes are calculated by applying Fisher's (1922) weak factor reversal test.

4 The Basic Translog

Our objective is to estimate a system of demand equations derived from an indirect utility function. The most important advantage of using the indirect utility approach is that prices enter as exogenous variables in the estimation process and the demand system is easily derived by applying Roy's identity.

In this paper, the demand system used in estimation is derived by approximating the indirect aggregator function, dual to the direct aggregator function $f(\boldsymbol{x})$, with the basic translog function, which for n subaggregates can be written as

$$\log V(\boldsymbol{p}, m) = \alpha_0 + \sum_{k=1}^{n} \alpha_j \log(\frac{p_k}{m}) + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{ik} \log(\frac{p_k}{m}) \log(\frac{p_j}{m})$$

where the symmetry restriction $\beta_{ij} = \beta_{ji}$, for all $i, j = 1, \cdots, n$, is imposed.

The share equations, derived using the logarithmic form of Roy's identity,

$$s_i = -\frac{\partial \log V(\boldsymbol{p}, m)/\partial \log p_i}{\partial \log V(\boldsymbol{p}, m)/\partial \log m}, \quad i = 1, \ldots, n$$

are

$$s_i = \frac{\alpha_i + \sum_{k=1}^{n} \beta_{ik} \log\left(\frac{p_k}{y}\right)}{\sum_{k=1}^{n} \alpha_k + \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{jk} \log\left(\frac{p_k}{y}\right)}, \quad i = 1, \ldots, n \tag{2}$$

where α_i and β_{ij} (all $i, j = 1, \cdots, n$) are the unknown parameters to be estimated.

5 The Stochastic Specification

In order to estimate share equation systems such as (2), a stochastic version must be specified. Since these systems are in share form and only exogenous variables appear on the right-hand side, it seems reasonable to assume that the observed share in the ith equation, $i = 1, ..., n$, deviates from the true share by an additive disturbance term u_i. Furthermore, we assume $\boldsymbol{u} \sim N(\mathbf{0}, \boldsymbol{\Omega} \otimes I_T)$ where $\mathbf{0}$ is a null matrix and $\boldsymbol{\Omega}$ is the $(n \times n)$ symmetric positive definite error covariance matrix.

With the addition of additive errors, the share equation system (2) can be written in matrix form as

$$\boldsymbol{s}_t = \boldsymbol{g}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \boldsymbol{u}_t \tag{3}$$

where $\boldsymbol{s} = (s_1, \cdots, s_n)'$, $\boldsymbol{g}(\boldsymbol{v}, \boldsymbol{\theta}) = (\boldsymbol{g}_1(\boldsymbol{v}, \boldsymbol{\theta}), \cdots, \boldsymbol{g}_n(\boldsymbol{v}, \boldsymbol{\theta}))'$, $\boldsymbol{\theta} = (\boldsymbol{\alpha}, \boldsymbol{\beta})$ are the parameters to be estimated, and $\boldsymbol{g}_i(\boldsymbol{v}, \boldsymbol{\theta})$ is given by the right-hand side of equation (2). Finally, the normalization $\sum_{k=1}^{n} \alpha_k = 1$, which ensures adding-up, is imposed in estimation.

6 Maximum Likelihood Estimation

The assumption that we have made about $\boldsymbol{u}_t$ in (3) permits correlation among the disturbances at time t but rules out the possibility of autocorrelated disturbances. This assumption and the fact that the shares satisfy an adding up condition (because this is a singular system) imply that the disturbance covariance matrix is also singular. Barten (1969) has shown that full information maximum likelihood estimates of the parameters can be obtained by arbitrarily deleting any equation in such a system. The resulting estimates are invariant with respect to the equation deleted and the parameter estimates of the deleted equation can be recovered from the restrictions imposed.

The likelihood function can be written [see Judge *et al.* (1988, p. 553) for more details] as

$$\begin{aligned} P(\boldsymbol{s} \,|\boldsymbol{\theta}, \boldsymbol{\Omega}) &\propto |\boldsymbol{\Omega}|^{-T/2} \exp\left[-\frac{1}{2}(\boldsymbol{s}-\boldsymbol{g})'\left(\boldsymbol{\Omega}^{-1} \otimes I_T\right)(\boldsymbol{s}-\boldsymbol{g})\right] \\ &\propto |\boldsymbol{\Omega}|^{-T/2} \exp\left(-\frac{1}{2} trc \boldsymbol{\Phi} \boldsymbol{\Omega}^{-1}\right) \end{aligned} \tag{4}$$

where $\propto$ denotes 'proportional to,' trc refers to the trace of a matrix, and $\boldsymbol{\Phi}$ is a 2×2 matrix with the jkth element equal to

$$\Phi_{jk} = [\boldsymbol{s}_k - \boldsymbol{g}_k(\boldsymbol{v}, \boldsymbol{\theta})]' \left[\boldsymbol{s}_j - \boldsymbol{g}_j(\boldsymbol{v}, \boldsymbol{\theta})\right]$$

All estimation is performed in TSP/GiveWin (version 4.5) using the LSQ procedure — our programs are available upon request. The results are reported in the first column of Table 2, with standard errors in parentheses. We also report the number of positivity, monotonicity, and curvature violations. These theoretical regularity conditions are checked as follows:

- Positivity is checked by direct computation of the values of the estimated budget shares, $\widehat{\boldsymbol{s}}_t$. It is satisfied if $\widehat{\boldsymbol{s}}_t \geq 0$, for all t.
- Monotonicity is checked by choosing a normalization on the indirect utility function so as to make $V(\boldsymbol{p}, m)$ decreasing in its arguments and by direct computation of the values of the first gradient vector of the estimated indirect utility function. It is satisfied if $\widehat{V}_{\boldsymbol{p}}(\boldsymbol{p}, m) < 0$ and $\widehat{V}_m(\boldsymbol{p}, m) > 0$.
- Curvature is checked by performing a Cholesky factorization of the Slutsky matrix and checking whether the Cholesky values are nonpositive [since a matrix is negative semidefinite if its Cholesky factors are nonpositive — see Lau (1978, Theorem 3.2)].

Although the model satisfies positivity and monotonicity at all 134 sample observations, it violates curvature at over 70% of the observations, rendering itself useless for inference purposes.

6.1 Imposition of Local Curvature

In a recent article, Ryan and Wales (1998) suggested a relatively simple procedure for imposing at a point (that is, locally) the curvature conditions implied by economic theory. Their procedure applies to those demand systems for which at the point of approximation, $y = p_k = 1$ ($\forall\ k$), the $n \times n$ Slutsky matrix $\mathbf{S}$ can be written as

$$\mathbf{S} = \mathbf{B} + \mathbf{C} \tag{5}$$

where $\mathbf{B}$ is an $n \times n$ symmetric matrix, containing the same number of independent elements as the Slutsky matrix, and $\mathbf{C}$ is an $n \times n$ matrix whose elements are functions of the other parameters of the demand system. Curvature requires the Slutsky matrix to be negative semidefinite. Ryan and Wales (1998) draw on related work by Lau (1978) and Diewert and Wales (1987) and impose curvature by replacing $\mathbf{S}$ in (5) with $-\mathbf{KK}'$, where $\mathbf{K}$ is an $n \times n$ lower triangular matrix, so that $-\mathbf{KK}'$ is by construction a negative semidefinite matrix. Then solving explicitly for $\mathbf{B}$ in terms of $\mathbf{K}$ and $\mathbf{C}$ yields

$$\mathbf{B} = -\mathbf{KK}' - \mathbf{C} \tag{6}$$

meaning that the models can be reparameterized by estimating the parameters in $\mathbf{K}$ and $\mathbf{C}$ instead of the parameters in $\mathbf{B}$ and $\mathbf{C}$. That is, we can replace the elements of $\mathbf{B}$ in the estimating share equations by the elements of $\mathbf{K}$ and the other parameters of the model, thus ensuring that $\mathbf{S}$ is negative semidefinite at the point $y = p_k = 1$ ($\forall$ k), which could be any data point.

Applying the Ryan and Wales (1998) procedure for imposing local curvature to the basic translog, the Slutsky terms can be written as

$$S_{ij} = \beta_{ij} - \alpha_i \delta_{ij} - \alpha_i \sum_{k=1}^{n} \beta_{kj} - \alpha_j \sum_{k=1}^{n} \beta_{ik} + \alpha_i \alpha_j \left(1 + \sum_{k=1}^{n} \sum_{m=1}^{n} \beta_{km} \right)$$

for $i, j = 1, ..., n$. Ryan and Wales (1998) argued that in the case of the basic translog replacing $\mathbf{S}$ by $-\mathbf{KK}'$ is of little help in imposing local curvature because the ijth element of $\mathbf{S}$ contains not just β_{ij} but also the terms $\sum_{k=1}^{n} \beta_{kj}, \sum_{k=1}^{n} \beta_{ik}$, and $\alpha_i \alpha_j \left(1 + \sum_{k=1}^{n} \sum_{m=1}^{n} \beta_{km}\right)$. As they noted, there are $n(n+1)/2$ independent β_{ij} parameters, but only $n(n-1)/2$ independent elements in $\mathbf{S}$, rendering it no longer possible to express the β_{ij} in terms of the elements of $\mathbf{K}$ and of the other parameters of the model.

However, Moschini (1999) suggested a possible reparameterization of the basic translog to overcome the problem noted by Ryan and Wales (1998). In particular, he showed that by letting $\theta_i = \sum_{j=1}^{n} \beta_{ij}$ we can rewrite (2) as

$$s_i = \frac{\alpha_i + \sum_{k=1}^{n-1} \beta_{ik} \log\left(\frac{p_k}{y}\right) + \theta_i \log\left(\frac{p_n}{y}\right)}{1 + \sum_{k=1}^{n} \theta_k \log\left(\frac{p_k}{y}\right)}, \quad i = 1, \ldots, n-1 \tag{7}$$

with s_n given by $s_n = 1 - \sum_{i=1}^{n-1} s_i$. With this parameterization, the Slutsky terms can be expressed in terms of a matrix of dimension $(n-1) \times (n-1)$, with the ijth element written as

$$S_{ij} = \beta_{ij} - \alpha_i \delta_{ij} - \alpha_i \theta_j - \alpha_j \theta_i + \alpha_i \alpha_j \left(1 + \sum_{k=1}^{n} \theta_k \right) \quad i, j = 1, \cdots, n-1 \tag{8}$$

Note that now in equation (8) there are exactly $n(n-1)/2$ S_{ij} terms as there are $n(n-1)/2$ β_{ij} terms.

Next, we estimate (3), with (8) imposed. As noted by Ryan and Wales (1998), the ability of locally flexible models to satisfy curvature at other sample observations other than the point of approximation, depends on the choice of approximation point. Thus, we estimated the model 134 times (a number of times equal to the number of observations) and report results for the best approximation point (best in the sense of satisfying the curvature conditions at the largest number of observations). The results are reported in the second column of Table 2 (with the approximation point being in 1971:4); note that parameters without standard errors have been recovered from the restrictions imposed. Our findings in terms of curvature violations when the local curvature conditions are imposed are disappointing, as the number of curvature violations drops only from 96 to 91. This suggests that inferences about money demand (including those about income and price elasticities as well as the elasticities of substitution) are worthless in trying to understanding real world money demand.

Given these disappointing results with the basic translog, for comparison purposes we also estimated two other well-known locally flexible functional forms — the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) and the normalized quadratic (NQ) of Diewert and Wales (1988). The share equations and the local curvature restrictions associated with each of these demand systems are discussed in detail in Ryan and Wales (1998). Like the basic translog, both the almost ideal demand system and the normalized quadratic always satisfy the positivity and monotonicity restrictions at each data point. However, when local curvature is not imposed, the almost ideal violates curvature at all 134 observations and the normalized quadratic at 118 observations. When local curvature is imposed, the number of curvature violations declines from 134 to 98 with the almost ideal and from 118 to 94 with the normalized quadratic.

This suggests that locally flexible functional forms estimated using sampling theoretic procedures are likely to produce money demand estimates inconsistent with optimizing behavior. It is to be noted, however, that this is a data specific conclusion. For example, Ryan and Wales (1998) use three locally flexible functional forms (the almost ideal, the normalized quadratic, and the linear translog) and annual Canadian per capita data on three broad nondurable commodity groups, food, clothing, and miscellaneous, over the period from 1947-1993 (a total of 47 observations), and show that when local curvature is imposed, curvature conditions are satisfied not only at the point of imposition but at every data point.

In the next section, we follow Terrell (1996) and take a Bayesian approach to the estimation of the basic translog demand system, with the objective of producing money demand estimates consistent with economic theory.

7 Bayesian Estimation

Following Judge *et al.* (1988) the likelihood function of the whole sample $\boldsymbol{s}$ can be written as

$$\begin{aligned} P(\boldsymbol{s}\,|\boldsymbol{\theta},\boldsymbol{\Omega}) &\propto |\boldsymbol{\Omega}|^{-T/2}\exp\left[-\frac{1}{2}(\boldsymbol{s}-\boldsymbol{g})'\left(\boldsymbol{\Omega}^{-1}\otimes I_T\right)(\boldsymbol{s}-\boldsymbol{g})\right] \\ &\propto |\boldsymbol{\Omega}|^{-T/2}\exp\left(-\frac{1}{2}trc\mathbf{A}\boldsymbol{\Omega}^{-1}\right) \end{aligned}$$

where $\mathbf{A}$ is a $(n\times n)$ matrix defined as follows

$$\mathbf{A}=\left[(\boldsymbol{s}_i-\boldsymbol{g}_i)'\left(\boldsymbol{s}_j-\boldsymbol{g}_j\right)\right], \quad i,j=1,\cdots,n$$

Assuming *a priori* independence of $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$, a constant prior probability density function for $\boldsymbol{\theta}$, and the conventional noninformative prior for $\boldsymbol{\Omega}$, $P(\boldsymbol{\Omega})\propto|\boldsymbol{\Omega}|^{-n/2}$, then the joint prior probability density function for all the unknown parameters can be written as

$$P(\boldsymbol{\theta},\boldsymbol{\Omega})\propto|\boldsymbol{\Omega}|^{-n/2}$$

Using Bayes' theorem, the joint posterior probability density function for all the parameters can be written as [the likelihood function of the sample, $P(\boldsymbol{s}\,|\boldsymbol{\theta},\boldsymbol{\Omega})$, times the prior probability density function for the parameters, $P(\boldsymbol{\theta},\boldsymbol{\Omega})$]

$$P(\boldsymbol{\theta},\boldsymbol{\Omega}\,|\boldsymbol{s})\propto|\boldsymbol{\Omega}|^{-(T+n)/2}\exp\left[-\frac{1}{2}trc\left(\mathbf{A}\boldsymbol{\Omega}^{-1}\right)\right] \tag{9}$$

In a Bayesian investigation, equation (9) is the source of all inferences about the unknown parameters. It can be used to obtain the marginal posterior probability density function for the parameters

$$P(\boldsymbol{\theta}\,|\boldsymbol{s})=\int P(\boldsymbol{\theta},\boldsymbol{\Omega}\,|\boldsymbol{s})\,d\boldsymbol{\Omega}\propto|\mathbf{A}|^{-\frac{T}{2}} \tag{10}$$

and calculate their posterior means and corresponding standard deviations.

However, equation (9) is too complicated for analytical integration (to obtain the marginal posterior probability density function for each element of $\boldsymbol{\theta}$). One solution to this problem is the use of simulation techniques, such as Gibbs sampling, introduced by Geman and Geman (1984), and the Metropolis-Hastings algorithm, due to early work by Metropolis *et al.* (1953) and Hastings (1970). Such simulation techniques provide a way of drawing observations from the joint posterior probability density function. These generated observations are then used to construct histograms and calculate sample means and variances to provide consistent estimates of the marginal posterior probability density functions and the posterior means and variances (that is, the Bayesian counterparts of sampling theory point estimates and variances) of the elements in $\boldsymbol{\theta}$ — see, for example, Chib and Greenberg (1995, 1996) for a detailed discussion.

In this paper we use the Metropolis-Hastings algorithm, because Gibbs sampling is suitable for linear seemingly unrelated regression models. The steps involved in the Metropolis-Hastings algorithm are as follows — see Griffiths and Chotikapanich (1997, p. 333) for more details.

1. Select initial values for $\boldsymbol{\theta}$, say $\boldsymbol{\theta}_0$. Perform the remaining steps with τ set equal to 0.

2. Compute a value for $P\left(\boldsymbol{\theta}_\tau \,|\, \boldsymbol{s}\right)$, based on equation (10).

3. Generate $\mathbf{z}$ from $N(\mathbf{0}, \kappa\mathbf{V})$, where $\mathbf{V}$ is an adjusted covariance matrix of the maximum likelihood estimates and κ is chosen in line with convention (so that the acceptance rate for $\boldsymbol{\theta}^*$ is approximately 50%).

4. Compute $\boldsymbol{\theta}^* = \boldsymbol{\theta}_\tau + \mathbf{z}$.

5. Compute a value for $P\left(\boldsymbol{\theta}^* \,|\, \boldsymbol{s}\right)$ and the ratio of the probability density functions
$$r = \frac{P\left(\boldsymbol{\theta}^* \,|\, \boldsymbol{s}\right)}{P\left(\boldsymbol{\theta}_\tau \,|\, \boldsymbol{s}\right)}.$$

6. If $r \geq 1$, set $\boldsymbol{\theta}_{\tau+1} = \boldsymbol{\theta}^*$ and return to step 2; otherwise proceed with step 7.

7. Generate a uniform random variable y from the interval $(0, 1)$. If $y \leq r$, set $\boldsymbol{\theta}_{\tau+1} = \boldsymbol{\theta}^*$; otherwise set $\boldsymbol{\theta}_{\tau+1} = \boldsymbol{\theta}_\tau$ and return to step 2.

Clearly, the Metropolis-Hastings algorithm provides a means for drawing observations consistent with the marginal posterior probability density function for the parameters, $P(\boldsymbol{\theta} \,|\, \boldsymbol{s})$. In particular, the vector $\mathbf{z}$ in step 3 represents a potential change from the last drawing of $\boldsymbol{\theta}$ and the potential new value $\boldsymbol{\theta}^*$ is given by the random walk process in step 4. In step 6 a new observation is accepted if it is more probable than the previous one; if it is less probable, it is accepted in step 7 with probability given by the ratio of the two probability density functions. Thus, as Griffiths and Chotikapanich (1997, p. 334) put it, "the procedure explores the posterior pdf yielding a relatively high proportion of observations in regions of high probability and a relatively low proportion of observations in regions of low probability."

In simulation procedures, like the Metropolis-Hastings algorithm, because observations are drawn artificially using computer software, we can make the estimated marginal posterior probability density functions as accurate as we like, by drawing as many observations as required. However, these generated observations are not independent and as a result the sample means and variances are not as efficient as they would be from uncorrelated observations. One way to produce independent observations is to run one long chain and select observations at a specified interval, say every tenth or twentieth observation. Alternatively, we can run a large number of chains and select the last observation from each chain.

In this paper we use the (unrestricted) ML parameter estimates from the first column of Table 1 as $\boldsymbol{\theta}_0$, we follow steps 1-7 and run one chain with 102,000 draws and select every fiftieth observation after deleting the first 2,000 samples to avoid sensitivity to the choice of $\boldsymbol{\theta}_0$. Posterior probability density functions, as well as posterior means and variances (the Bayesian counterparts of sampling theory point estimates and variances) are then estimated from these observations for each of the parameters of the model — all Bayesian estimation in this paper is performed in MATLAB (version 6.5) and our programs are available upon request. The posterior moments are presented in the third column of Table 2 (under unrestricted), together with information regarding positivity, monotonicity, and curvature violations.

As in the case with maximum likelihood estimation, although positivity and monotonicity are satisfied globally, curvature is violated at about 69% of the observations. As already noted, without satisfaction of all three theoretical regularity conditions (positivity, monotonicity, and curvature), the resulting inferences are virtually worthless, since violations of regularity violate the maintained hypothesis and invalidate the duality theory that produces the estimated model. In what follows, we follow Terrell (1996) and incorporate the theoretical regularity restrictions into the prior distribution.

7.1 Bayesian Estimation and Theoretical Regularity

In a Bayesian framework, to impose the theoretical regularity conditions of positivity, monotonicity, and curvature, we define an indicator function $h(\boldsymbol{\theta})$ which is equal to one if the specified theoretical regularity condition holds and zero otherwise, as follows

$$h(\boldsymbol{\theta}) = \begin{cases} 1 & \text{if the specified theoretical regularity condition holds} \\ 0 & \text{otherwise} \end{cases}$$

Using this indicator function, we define the informative prior for our model parameters as

$$\begin{aligned} P_1(\boldsymbol{\theta}) &= h(\boldsymbol{\theta}) \times P_0(\boldsymbol{\theta}) \\ &\propto h(\boldsymbol{\theta}) \times \text{constant} \end{aligned}$$

thereby assigning zero weight to estimated vectors which lead to violation of the specified theoretical regularity condition(s).

Columns 4, 5, 6, and 7 of Table 1 report posterior means and standard deviations and the violations of the theoretical regularity conditions when positivity, monotonicity, curvature, and all three regularity conditions are incorporated into the prior distribution, respectively. We find that when each of positivity and monotonicity are imposed the frequency of curvature violations increases with the basic translog specification in our application. However, the imposition of curvature does not increase the frequency of monotonicity violations — in this regard, Barnett and Pasupathy (2003) argue that imposition of global curvature may increase the frequency of monotonicity violations.

Although with the use of Bayesian inference we have been able to significantly reduce the frequency of curvature violations, it seems that the basic translog does not perform well in terms of describing the demand for money in the United States in a manner that satisfies the restrictions imposed by microeconomic theory. In particular, the parameter estimates in the last column of Table 2 violate curvature at about 30% of the observations.

7.2 The Substitutability of Money and Near-Monies

With this in mind, in this section we focus on the problem of interpreting the parameter estimates by computing the income elasticities (η_i), the price elasticities (η_{ij}), the Allen elasticities of substitution (σ_{ij}), and the Morishima elasticities of substitution (σ_{ij}^m), with a model that is not completely consistent with economic theory. These elasticities, evaluated at the mean of the data (where curvature is satisfied) are reported in Tables 3 and 4 — see Serletis (2001, Chapter 18) for the elasticities formulas.

The estimated expenditure elasticities in Table 3 reveal a clear-cut pattern. All three assets are 'normal goods' ($\eta_i > 0$), with savings deposits (asset B) being a 'luxury good' ($\eta_i > 1$). The own- and cross-price elasticities in Table 3 also reveal a pattern consistent with demand theory. All own-price elasticities are negative, and all assets are own-price inelastic ($|\eta_{ii}| < 1$), with time deposits being much more so than M1 (asset A) and savings deposits. Moreover, the assets are found to be (gross) complements ($\eta_{ij} < 0$).

The estimated Allen elasticities of substitution show a quite different pattern; all own Allen elasticities of substitution are negative while the assets appear to be net substitutes ($\sigma_{ij} > 0$) except for savings and time deposits that appear to be net complements. However, as Blackorby and Russell (1989) have shown, the Allen elasticity of substitution is quantitatively and qualitatively uninformative and that the Morishima elasticity of substitution is the correct measure of the substitution elasticity. The Morishima elasticity of substitution, σ_{ij}^m, is defined as [see Blackorby and Russell (1989) for more details]

$$\sigma_{ij}^m = s_i(\sigma_{ji}^a - \sigma_{ii}^a)$$

and addresses the impact on the ratio of two goods, x_i/x_j. In particular, it categorizes goods as complements ($\sigma_{ij}^m < 0$) if an increase in the price of j causes x_i/x_j to decrease. If $\sigma_{ij}^m > 0$, goods are Morishima substitutes — see also Serletis (2001) for more details. As documented in Table 4, where the asymmetrical Morishima elasticities are reported, all assets are Morishima substitutes.

Finally, in order to assess whether the basic translog model works well in terms of describing the U.S. money demand in a manner that produces stable elasticity estimates, we present the estimated income elasticities and the Morishima elasticities of substitution in Figure 1 and Figures 2-4, respectively, along with information regarding the 40 data points at which curvature is violated (these are the vertically shaded points on the x axis). Clearly, there is considerable elasticity volatility in the data regions that curvature is not satisfied. In the literature, such elasticity volatility has been attributed to things other than model failure, such as, for example, in the case of the United States, to double-digit inflation after 1979, monetary decontrol, and the disinflation of the early 1980s. Here, we argue that it is the horrible regularity results that lead to the crazy plots of wildly varying elasticities. In fact, the model produces so extremely unstable elasticity estimates that is certainly useless for modelling the demand for money in the United States.

8 Robustness to Dynamic Specifications

We have used a static model, implicitly assuming that the pattern of demand adjusts to a change in exogenous variables instantaneously. No attention has been paid to the dynamic structure of the model used, although many recent studies have focused attention to the development of dynamic generalizations of the traditional static models — see, for example, Serletis (1991). In this section we investigate the robustness of our results to dynamic specifications of the translog model by allowing the possibility of autocorrelation in error terms; autocorrelation in money demand systems is a common result and may be caused by institutional constraints which prevent people from adjusting their asset holdings within one period.

In doing so, we assume a first-order autoregressive process, such that

$$\boldsymbol{u}_t = \mathbf{R}\boldsymbol{u}_{t-1} + \boldsymbol{e}_t$$

where $\mathbf{R} = [R_{ij}]$ is a matrix of unknown parameters and $\boldsymbol{e}_t$ is a non-autocorrelated vector disturbance term with constant covariance matrix. In this case, estimates of the parameters can be obtained by using a result developed by Berndt and Savin (1975). They showed that if one assumes no autocorrelation across equations (i.e., $\mathbf{R}$ is diagonal), the autocorrelation coefficients for each equation must be identical. Consequently, by writing equation (3) for period $t-1$, multiplying by $\mathbf{R}$, and subtracting from (3), we can estimate stochastic budget share equations given by

$$\boldsymbol{s}_t = \mathbf{g}(\boldsymbol{v}_t, \boldsymbol{\theta}) + \mathbf{R}\boldsymbol{s}_{t-1} - \mathbf{R}\mathbf{g}(\boldsymbol{v}_{t-1}, \boldsymbol{\theta}) + \boldsymbol{e}_t \tag{11}$$

We estimated equation (11) and observed that serial correlation correction produces violations of both positivity and monotonicity. Although we haven't examined other locally flexible functional forms, we believe that dynamic specifications of demand systems based on locally flexible functional forms will not produce results consistent with theoretical regularity.

9 Conclusion

In this paper, we have used sampling theoretic as well as Bayesian procedures to revisit the demand for money in the United States in the context of one of the most popular locally flexible functional forms, the basic translog. We have argued that inferences based on such locally flexible functional forms are virtually worthless unless all three theoretical regularity conditions of positivity, monotonicity, and curvature are satisfied, since violations of regularity violate the maintained hypothesis and invalidate the duality theory that produces the estimated model. Moreover, unless theoretical regularity is attained by luck, locally flexible functional forms should always be estimated subject to regularity.

Using recent monetary data for the United States, we have shown that with maximum likelihood estimation procedures the imposition of local curvature, using methods recently suggested by Ryan and Wales (1998) and Moschini (1999), does not assure theoretical regularity, because of curvature violations at other points within the region of the data. We believe that this is a typical result in the literature that uses locally flexible functional forms and alert researchers to the kinds of problems (mentioned earlier) that arise when all three theoretical regularity conditions are not satisfied — see also Barnett (2002) and Barnett and Pasupathy (2003).

The main objective of this paper, however, has been to suggest a possible solution to this problem. The solution lies in incorporating the theoretical regularity restrictions into flexible functional forms using Bayesian inference, as in Terrell (1996). We have used the Metropolis-Hastings algorithm to generate an initial sample from the posterior probability density function for the parameters for a prior that ignores theoretical regularity. As in Terrell (1996), for each parameter vector in the sample, the theoretical regularity restrictions are evaluated in the price domain over which inferences are drawn and accept-reject sampling generates a sample of the parameter vectors that is consistent with theoretical regularity. Then marginal posterior density functions consistent with theory can be used to draw inferences about income elasticities, own- and cross-price elasticities, as well as the elasticities of substitution.

We have shown that relative to sampling theoretic inference, Bayesian inference increases the basic translog's ability to model U.S. monetary demand, suggesting that a breakthrough from the current state of using locally flexible specifications of tastes and technology that violate the theoretical regularity properties to the use of such specifications that are more consistent with the theory will be through the use of Bayesian inference. We have demonstrated, however, very conclusively, that the basic translog model does not perform well in terms of describing the demand for money in the United States in a manner that satisfies the restrictions imposed by microeconomic theory and gives rise to stable elasticity estimates. We are positively challenged to apply the standards used in this paper to find a better functional form that could be used for policy purposes.

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TABLE 1

Monetary Assets/Components

1	Currency + Travelers checks
2	Demand deposits
3	Other checkable deposits at commercial banks including Super Now accounts
4	Other checkable deposits at thrift institutions including Super Now accounts
5	Savings deposits at commercial banks including money market deposit accounts
6	Savings deposits at thrift institutions including money market deposit accounts
7	Small denomination time deposits at commercial banks
8	Small denomination time deposits at thrift institutions

Source: Anderson, Jones, and Nesmith (1997, p. 61).

Table 2

Basic Translog Parameter Estimates Under Maximum likelihood and Bayesian Estimation

	Maximum likelihood estimation		Bayesian (Metropolis-Hastings) estimation				
Parameter	Unrestricted	Curvature	Unrestricted	Positivity	Monotonicity	Curvature	All
α_A	.412	.413	.413	.413	.413	.412	.418
	(.004)	(.004)	(.004)	(.004)	(.004)	(.003)	(.003)
α_B	.290	.290	.291	.291	.291	.288	.292
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)
β_{AA}	.770	.786	.724	.725	.725	.847	.766
	(.062)		(.085)	(.083)	(.083)	(.067)	(.068)
β_{AB}	.117	.135	.062	.066	.066	.241	.128
	(.075)		(.094)	(.091)	(.090)	(.068)	(.069)
β_{AC}	.461	.475	.414	.416	.416	.540	.506
	(.064)		(.074)	(.072)	(.072)	(.061)	(.061)
β_{BB}	.249	.257	.215	.217	.217	.299	.204
	(.050)		(.065)	(.062)	(.062)	(.049)	(.050)
β_{BC}	.228	.238	.188	.191	.191	.306	.245
	(.053)		(.063)	(.060)	(.060)	(.046)	(.047)
β_{CC}	.511	.517	.480	.482	.481	.536	.533
	(.056)		(.056)	(.054)	(.054)	(.049)	(.047)
LogL	499.051	496.286					
Positivity violations	0	0	0	0	0	0	0
Monotonicity violations	0	0	0	0	0	0	0
Curvature violations	96	91	92	103	123	41	40

Notes: The sample size is 134. Numbers in parentheses are standard errors. Parameters without standard errors have been recovered from the restrictions imposed.

Table 3

Income and Price Elasticities

Subaggregate i	Income elasticities	Price elasticities		
	η_i	η_{iA}	η_{iB}	η_{iC}
(A)	.912	−.575	−.262	−.074
(B)	1.917	−.791	−.791	−.808
(C)	.236	−.318	−.306	−.590

Notes: Sample period, quarterly data 1970:1-2003:2 ($T = 134$).

Table 4

Allen and Morishima Elasticities of Substitution

Subaggregate i	Allen elasticities			Morishima elasticities		
	σ_{iA}	σ_{iB}	σ_{iC}	σ_{iA}^m	σ_{iB}^m	σ_{iC}^m
(A)	−.481	.0005	.664		.199	.472
(B)		−.885	−.828	.255		.016
(C)			−1.738	.718	.272	

Notes: Sample period, quarterly data 1970:1-2003:2 ($T = 134$).

Figure 1. Income Elasticities

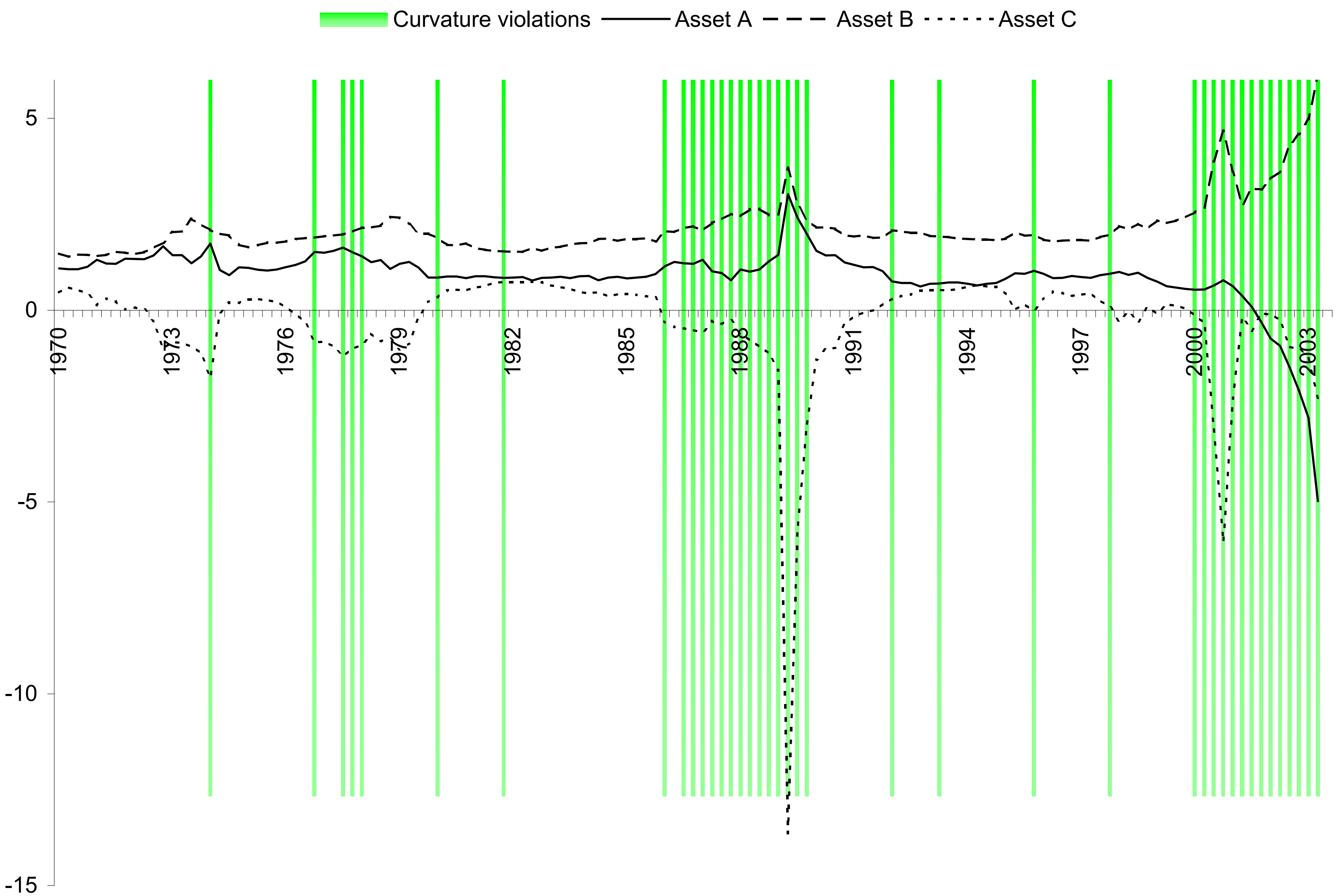

Figure 2. Morishima Elasticities of Substitution between A and B

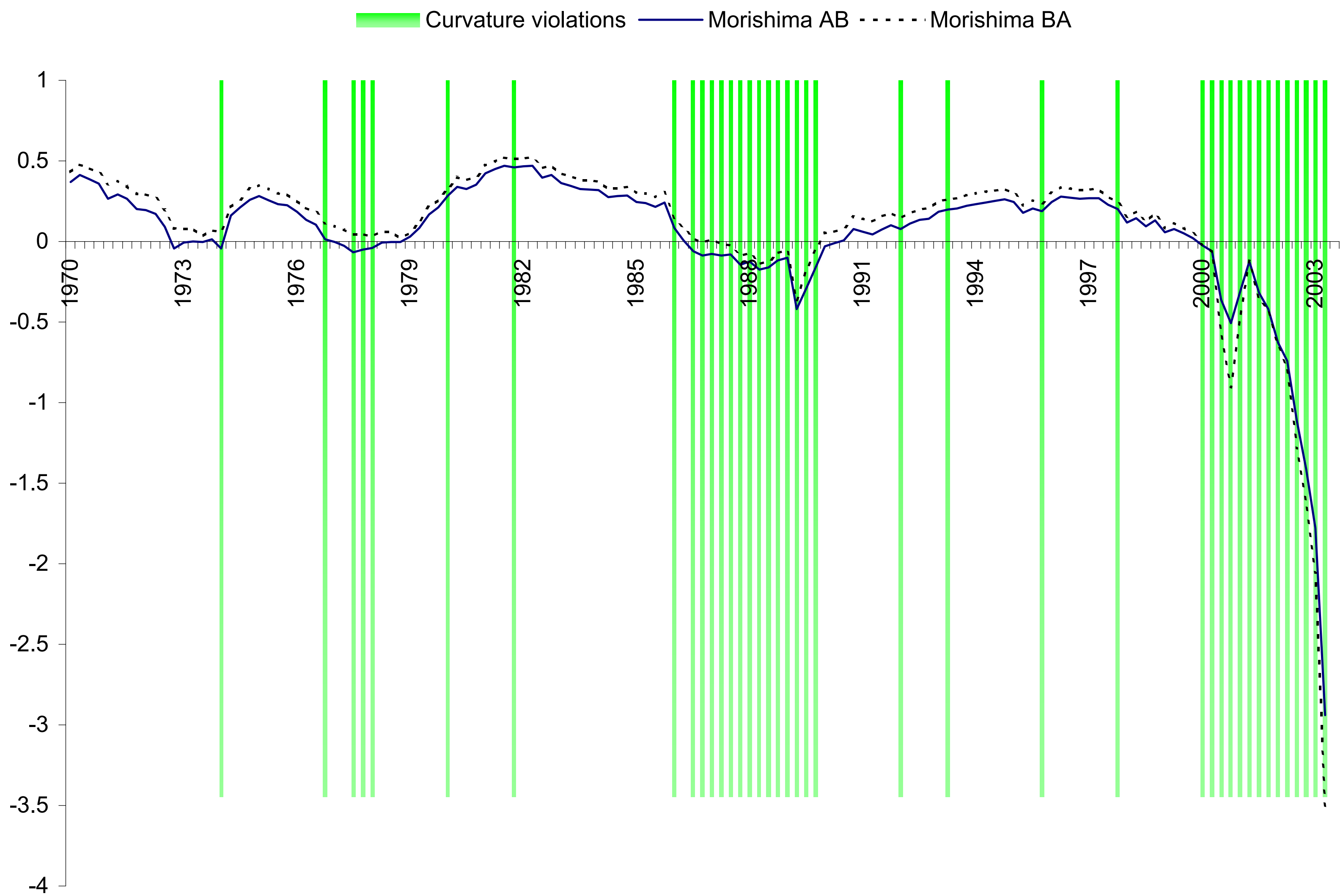

Figure 3. Morishima Elasticities of Substitution between A and C

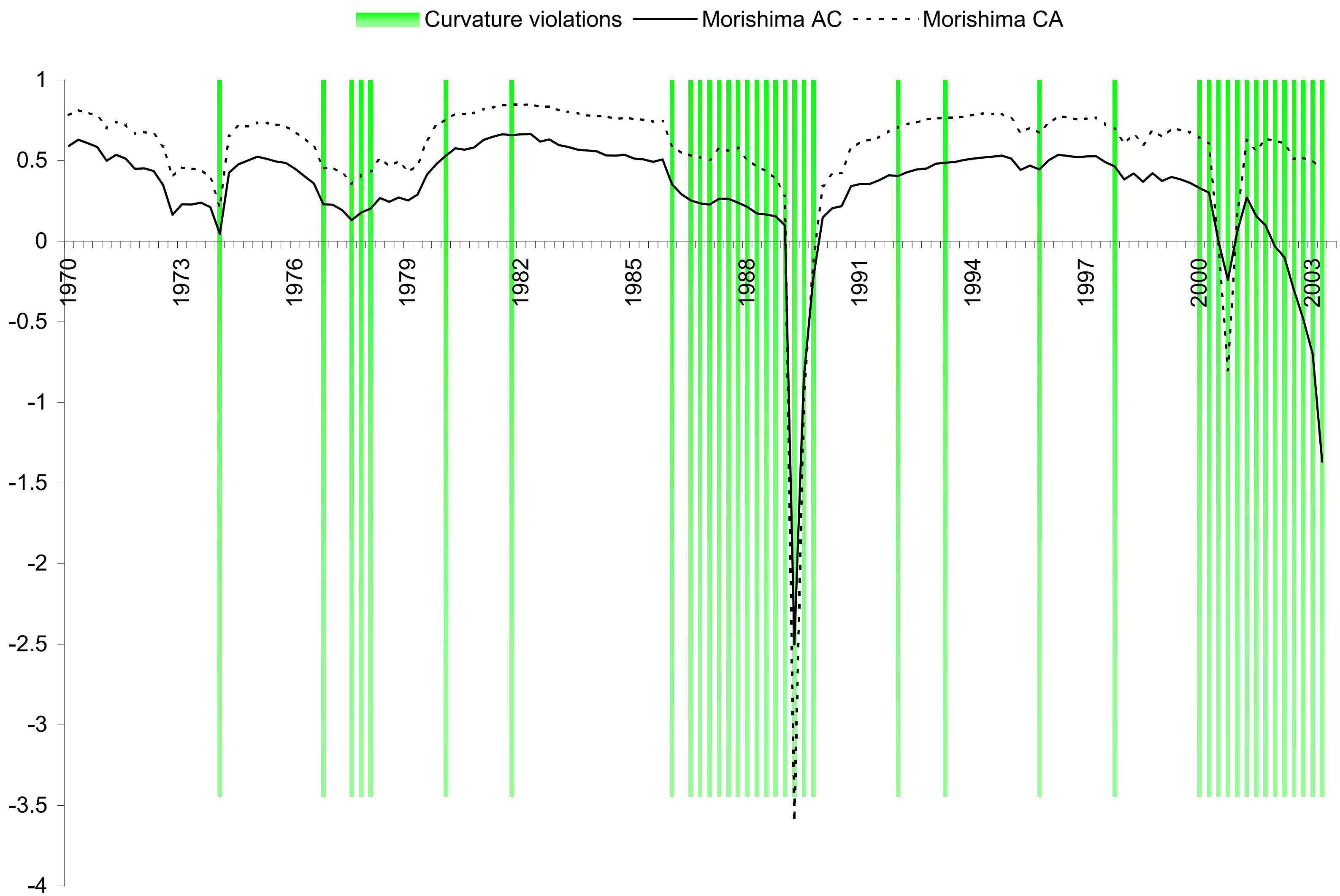

Figure 4. Morishima Elasticities of Substitution between B and C