On the Optimal Combination of Promotion Tournaments and Individual Performance Pay*

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This version: May 2007

Abstract

We analyze the optimal combination of promotion tournaments and individual performance pay in an agency relationship. Agents' efforts are non-observable and they possess private information about their suitability for promotion. We demonstrate that the principal refrains from providing agents with individual performance payments if it is sufficiently crucial for the organization's efficiency to promote the most suitable candidate. Thus, we provide a supplementary explanation of why individual performance payments are less often observed in practise than theory predicts. Furthermore, efficiently balancing incentive and selection issues provokes a form of the Peter Principle: The less suitable agent has an inefficiently high prospect of promotion.

JEL Classification: D82, D86, M52.

Keywords: Promotion Tournaments, Piece Rates, Peter Principle, Hidden Characteristics, Hidden Action.

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*We would like to thank Dominique Demougin, Mati Dubrovinsky, Oliver Fabel, Carsten Helm, Jenny Kragl, Dorothee Schneider, and workshop participants at the University of Connecticut and the University of Bonn for insightful comments and suggestions.

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1 Introduction

Within firms, incentive schemes often serve not only as incentive but also as selection devices. This is particularly true for promotion tournaments, which provide effort incentives and help assign employees to the jobs they are best suited for (Baker et al. 1988, Milgrom and Roberts 1992). Individual performance pay may as well lead to incentive and selection effects. For example, Lazear (2000) shows that the introduction of a simple piece rate scheme in a US autoglass company increased output and attracted more able workers. There is a large body of literature analyzing either relative incentive schemes (e.g., promotion tournaments) or individual performance pay, or shows when one contractual form dominates the other.[1] Most of this literature focuses on the provision of incentives. By contrast, we examine how relative and individual compensation schemes should be combined when incentive and selection issues arise simultaneously. Our purpose is not to characterize an optimal mechanism, but rather to look at two incentive schemes that are of high practical relevance: piece rates and promotion tournaments.

In particular, we consider a relationship between the owner of a manufacturing firm (principal) and two workers (agents). The principal first employs both agents as production workers but later wishes to promote one of them to a position in middle management. In the production stage, workers perform a manufacturing task. Effort in this task is not observable. However, due to the availability of a contractible performance measure, the principal is able to establish an individual incentive scheme. In the management job, effort is also non-observable. Moreover, since middle managers usually perform difficult-to-measure tasks as supervising subordinates or organizing the workflow in production, performance measures are not available. Therefore, a manager receives a fixed salary.[2]

Agents share the same abilities in production but may differ in their skills for the management task. They learn their management skills after signing the employment contract and

[1] See, e.g., Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), or Rosen (1986).

[2] This assumption is not crucial for our results but greatly simplifies the analysis. We discuss the extension to incentive contracts at the management stage in section 5.

entering the firm. The principal never observes these skills but benefits from selecting a high-skilled worker for promotion. She therefore has to design a contract that optimally balances incentive and selection issues. This contract is composed of a fixed wage and an individual performance scheme for the manufacturing task, and a tournament prize, which is the fixed salary for the management position.

We find that individual performance pay and the promotion tournament are substitutes in the provision of incentives. On which of these instruments the principal puts more emphasis depends on how critical the selection decision is for firm performance. Under our modeling assumptions, the interplay is as follows: The more important it is to promote the high-skilled worker, the higher the management wage and the lower-powered are individual incentives. Moreover, if the selection decision is sufficiently crucial, the optimal contract does not incorporate individual incentives. Thus, even though there is a verifiable, non-distorting performance measure available at the production stage, the principal may decide not to use it in an individual incentive scheme.

The rationale for this result is as follows. Given the fixed payment at the management stage, a manager exerts a minimum required level of effort. We assume that, given any particular effort level, a high-skilled agent makes not only a higher contribution to firm value but also has lower effort cost than a low-skilled agent.[3] Therefore, under any management salary, the more able worker has a higher net benefit from being promoted. As a consequence, whenever agents are heterogeneous, the high-skilled worker exerts more effort than the low-skilled one at the production stage. The performance measure thus serves as a signal about agents' abilities: The agent who performed better in production is more likely to be high-skilled at the management stage and should hence be promoted. However, enhancing individual performance pay dilutes the informativeness of this signal. The reason is that the harder working high-skilled agent responds less strongly to intensified individual incentives than the agent less suited for the

[3] Thus, in our model, being of a superior type means to have higher marginal productivity *and* lower marginal effort cost. Usually, only one of these assumptions is made to model different abilities of agents. We require both of them because we only allow for a fixed wage at the management stage. Refer to section 5 for a detailed explanation.

management task. As a result, the latter's promotion probability increases. If this effect is sufficiently detrimental from the principal's point of view, she refrains from applying an individual performance scheme.

This result provides a possible explanation for the fact that individual incentive schemes are less often observed than theory predicts (Parent 2002). Alternative explanations can be found in the literature. Holmström and Milgrom (1991) show that it may be optimal not to implement an incentive scheme if the agent's task has different dimensions of which some are easier to measure than others. In Bernheim and Whinston (1998), contracting parties might want to leave some verifiable aspects of performance unspecified when there are other important but non-verifiable aspects of performance, since this may allow to punish undesired behavior. Another explanation originally comes from psychology. It says that monetary incentive payments may crowd out intrinsic motivation (Deci 1971). Frey and Oberholzer-Gee (1997), Benabou and Tirole (2003), and Sliwka (2006) give economic explanations for the occurrence of crowding-out.

Furthermore, in our paper, when two homogeneous agents join the firm, selection is not an issue. Hence, since agents do not differ in their abilities for the manufacturing task, efficiency dictates that effort levels should be identical in the two possible homogeneous matches of agents. However, since the principal cannot offer contracts tailored to agents' types, she is obliged to implement inefficient effort levels. In particular, if two high-skilled agents compete for promotion, they are induced to work too hard. By contrast, two low-skilled agents exert too little effort. This is a consequence from the fact that, under any given contract, high-skilled workers are more highly motivated since they benefit more strongly from promotion than those with low skills.

If workers are heterogeneous, the principal gains from increasing the promotion probability of the employee who is better suited for the management position. Taking into account this benefit from differentiating effort levels, the more able agent puts in too little effort and the less able agent too much. Thus, a form of the Peter Principle (Peter and Hull 1969) arises: The promotion probability of the less-skilled agent is inefficiently high. The principal accepts this inefficiency as an outcome of optimally trading off incentive and selection issues.

Fairburn and Malcomson (2001) derive a similar result when agents are risk averse. By contrast, the agents in our model are risk-neutral. Lazear (2004) uses a quite different approach to explain the Peter Principle. In his paper, the observation that employees' performance often declines after receiving promotion is a necessary consequence of a statistical process. Agents who received promotion experienced exceptionally good random influences in the period before they got promoted. However, on average, they will be less lucky in the next period.

Selection in tournaments is also analyzed by Rosen (1986), Meyer (1991), Clark and Riis (2001), Hvide and Kristiansen (2003), and, in the context of sabotage, by Lazear (1989), Chen (2003), and Münster (2006). In contrast to these authors, we focus on the selection effect of a promotion tournament when it may be combined with a piece rate scheme. Furthermore, in our model, agents are heterogeneous in the tournament stage only because they differ in their valuations for the tournament prize. In the aforementioned papers, agents' heterogeneity is due to different abilities in the tournament stage.

The paper is organized as follows. The model is introduced in the following section. In section 3, we derive the effort chosen at the production stage given the tournament prize and individual performance pay. The optimal combination of the tournament prize and the individual incentive scheme is characterized in section 4. Section 5 discusses the impact of some our assumptions on the results. Finally, section 6 concludes.

2 The Model

We consider an employment relationship between a risk-neutral principal and two risk-neutral agents. The principal is the owner of a firm which comprises two distinct types of tasks: (*i*) pure manufacturing tasks (*production stage*); and (*ii*), management tasks (*management stage*). The principal however, lacks either the ability or the time to perform these tasks and thus, requires the service of the agents as productive parties.

There are two different types of agents in the economy, denoted A and B. They share the same abilities for performing the manufacturing tasks, but agents of type A can conduct management tasks more efficiently than agents of type B.[4] Prior to the contracting stage, neither the principal nor the agents can observe their respective types. Nevertheless, it is common knowledge that each agent is of type A with probability p, and of type B with probability $1-p$, respectively. After accepting a contract offered by the principal and entering into the employment relationship, each agent observes his own type and that of his coworker. The principal however, can never observe the agents' types. For simplicity, the agents' reservation utilities are independent of their respective types and normalized to zero.

In the *production stage*, agent i, $i = 1, 2$, is required to implement non-observable effort $e_i \in \mathbb{R}^+$, leading to the verifiable output

$$q_i = e_i + \mu_i \qquad i = 1, 2, \tag{1}$$

where μ_1 and μ_2 are identically and independently distributed random variables with zero mean. Implementing effort imposes strictly convex increasing costs $c(e_i)$, where $c'''(e_i) > 0$, i.e. marginal costs increase disproportionately.[5] To ensure the existence of a pure-strategy equilibrium in the production stage, we assume that $\inf_{e>0} c''(e) > 0$.

Since effort is non-observable, the principal cannot directly contract over e_i, $i = 1, 2$. The principal however, can exploit the verifiable information q_i in an enforceable contract to provide the agents with appropriate incentives to implement effort. In particular, the principal provides each agent with a piece rate $r \geq 0$ conditioned on his individual output q_i, in addition to a fixed payment w_1.

In the *management stage* however, the principal does not obtain any verifiable information about the conducted effort which could be utilized in an enforceable incentive contract. As a consequence, an agent assigned to perform management tasks implements a minimum required

[4]We discuss in section 5 the case where distinct types also exhibit different abilities for performing manufacturing tasks.

[5]We discuss in sections 3 and 5 how this assumption affects the subsequent results.

effort level, and receives a fixed compensation $w_2 \geq 0$ in return. As previously exposed, an agent of type A can conduct the required management tasks more efficiently than an agent of type B. This has two important implications for the subsequent analysis: First, a type A's (non-verifiable) contribution to firm value Π_A on the management stage exceeds a type B's contribution Π_B: $\Pi_A > \Pi_B$. Second, implementing the minimum required effort level imposes less disutility of effort for a type A agent than for a type B agent. To reduce the notational burden, we normalize type B's effort costs to zero, while type A's costs are $-\delta$, where $\delta > 0$. Hence, a type A's utility of being employed as a manager exceeds a type B's utility by δ.

Since $\Pi_A > \Pi_B$, it is desirable from the principal's perspective to charge an agent of type A with performing the required management tasks. As mentioned above, the principal cannot observe the agents' respective types, but she can take advantage of the fact that agents of type A have a relatively higher valuation of being promoted to the management level. Specifically, the principal can design the following promotion tournament as a screening device: In the first period, both randomly recruited agents are assigned to perform the manufacturing tasks. At the end of this period, the better performing agent, i.e. the one with the higher individual output, gets promoted to the management position. The fixed compensation w_2 for an agent employed as a manager is therefore the price for winning the tournament, where w_2 is assumed to be non-renegotiable.[6] Under this promotion rule, the accomplished performance in the manufacturing stage serves as a signal about individual skills for conducting management tasks: Whenever the employed agents are indeed heterogenous, the type A agent implements a higher effort level than the type B agent. This is because the former has a relatively higher valuation of being promoted, which translates into incentives for implementing relatively more effort.[7] Consequently, the better performing agent is more likely to be of type A.

However, utilizing a promotion tournament as screening and incentive device imposes one inefficiency: Even though both agents are equally skilled to perform the manufacturing tasks,

[6] This common assumption is necessary to ensure that agents perceive the incentive provision by means of the promotion tournament as credible.

[7] We prove this statement formally in section 3.

the application of a promotion tournament encourages heterogenous agents to implement various effort levels in the production stage. For subsequent analysis, we assume that the promotion decision is sufficiently crucial for firm performance, i.e. $\Pi_A - \Pi_B$ is adequately high. This ensures that it is in any case desirable from the principal's perspective to improve her belief about the agents' respective types by utilizing a promotion tournament.

The timing of this game is as follows. First, the principal offers each of two randomly chosen agents a contract (r, w_1, w_2). After accepting the contract, each recruited agent learns his own type and that of his coworker. Then, both agents are assigned to conduct the manufacturing tasks (*production stage*), and implement their respective effort levels e_i, $i = 1, 2$. After the individual outputs q_i, $i = 1, 2$, are realized, all payments take place. Moreover, the agent with the higher individual output gets promoted to the management level. The other agent leaves the firm and obtains his reservation utility.

3 Effort in the Production Stage

In this section, we elaborate on agents' effort choices in the production stage for a given incentive contract. To do so, we need to account for three potential matches of agents: Two homogeneous matches where both agents are either of type A or of type B; and a heterogeneous match with a type A and a type B agent.

By implementing effort in the production stage, both agents do not only affect their incentive payments conditional on their performance, but also their individual probability to get promoted to the management level. Formally, the probability of agent i to get promoted is

$$\text{Prob}[q_i > q_j] = \text{Prob}[e_i - e_j > \mu_j - \mu_i] \equiv G(e_i - e_j), \quad i, j \in \{1, 2\}, \ i \neq j, \tag{2}$$

where $G(\cdot)$ is the cumulative distribution function of the random variable $\mu_j - \mu_i$. Let $g(\cdot)$ denote the corresponding probability density function, which is assumed to be differentiable and single-peaked at zero. Since μ_i and μ_j are identically distributed, $g(\cdot)$ is symmetric around zero.

We start by investigating the case of homogenous agents. Suppose for a moment that both randomly employed agents are of type A. Agent i maximizes his expected utility by implementing effort e_i which solves

$$\max_{e_i} \; w_1 + G(e_i - e_j)(w_2 + \delta) + re_i - c(e_i). \tag{3}$$

Likewise, agent j, $j \neq i$, chooses his effort level e_j which satisfies

$$\max_{e_j} \; w_1 + [1 - G(e_i - e_j)](w_2 + \delta) + re_j - c(e_j). \tag{4}$$

Since both agents are homogenous, they choose the same effort level in a pure-strategy Nash-equilibrium. Their optimal effort level e_{AA} is thereby implicitly characterized by the first-order condition

$$g(0)(w_2 + \delta) + r = c'(e_{AA}). \tag{5}$$

Similarly, for the case where both agents are of type B, the equilibrium effort e_{BB} is characterized by

$$g(0)w_2 + r = c'(e_{BB}). \tag{6}$$

To ensure that (e_{AA}, e_{AA}) and (e_{BB}, e_{BB}) represent indeed Nash-equilibria, the agents' objective functions need to be concave. Formally, this is satisfied if

$$g'(e_i - e_j)(w_2 + \delta) - c''(e_i) \; < \; 0 \quad \forall \; e_i, e_j \geq 0, \; i \neq j, \tag{7}$$

$$\text{and} \qquad g'(e_i - e_j)w_2 - c''(e_i) \; < \; 0 \quad \forall \; e_i, e_j \geq 0, \; i \neq j. \tag{8}$$

For subsequent analysis, we assume that these conditions are satisfied for the highest w_2 the principal is willing to offer both agents.[8] Since $\inf_{e_i>0} c''(\cdot) > 0$, this is the case whenever

[8] Recall that agents' effort costs are convex, whereas the principal's expected profit is concave in effort. Since the principal has to compensate both agents for their disutility of effort aimed at ensuring their participation,

random influences on individual output are sufficiently significant, i.e. $g(\cdot)$ is 'flat' enough.[9]

We now turn to the case of heterogeneous agents. Without loss of generality, assume that agent 1 is of type A, and agent 2 is of type B. Then, type A's and type B's respective optimization problems are

$$\max_{e_1} \; w_1 + G(e_1 - e_2)(w_2 + \delta) + re_1 - c(e_1), \tag{9}$$

$$\max_{e_2} \; w_1 + [1 - G(e_1 - e_2)]w_2 + re_2 - c(e_2). \tag{10}$$

In equilibrium, type A's and type B's effort levels e_A and e_B are characterized by the following two first-order conditions:

$$g(e_A - e_B)(w_2 + \delta) + r = c'(e_A), \tag{11}$$

$$g(e_A - e_B)w_2 + r = c'(e_B). \tag{12}$$

Since the respective second-order conditions are identical to (7) and (8), (e_A, e_B) characterizes a pure-strategy Nash-equilibrium.

We can infer from (11) and (12) that $\Delta e \equiv e_A - e_B > 0$. Because the type A's prospective benefit of being promoted to the management level is relatively higher, he is motivated to work harder than the type B for a given incentive scheme. This implies further that A has a higher probability of winning the promotion tournament, i.e. $G(\Delta e) > 0.5$.

We demonstrate in the appendix (see proof of proposition 1) that e_A and e_B are increasing in r and w_2. Besides this *incentive effect*, enhancing either r or w_2 also influences the *selection effect* of utilizing a promotion tournament. The latter effect emerges from that fact that adjusting r or w_2 determines the effort difference Δe and thus, the agents' respective promotion probabilities. As the subsequent proposition emphasizes, enhancing the incentive provision deteriorates the type A's chance for promotion, and has therefore a detrimental effect on the

it cannot be efficient to induce arbitrarily high effort levels. Thus, there exists an upper bound of w_2.

[9]Compare Lazear and Rosen (1981), footnote 2.

selection effect.

Proposition 1 *In a heterogenous tournament match, the type A's probability of winning the promotion tournament is decreasing in r and w_2.*

All proofs are relegated to the appendix.

The principal can induce both agents to implement a higher effort intensity by raising either the piece rate scheme r or the tournament price w_2. However, the type A agent—who implements a higher effort level—responds less strongly to intensified incentives than the type B agent. The proof of proposition 1 in the appendix reveals that this result relies on our assumption that marginal effort costs increase disproportionately ($c'''(\cdot) > 0$). If, in contrast, $c'''(\cdot) < 0$, type A's winning probability is increasing in r and w_2. Hence, assuming $c'''(\cdot) > 0$ is equivalent to the presumption that—in a tournament with heterogeneous agents—the effort choice of the harder working agent is less sensitive to incentive adjustments.[10]

4 The Principal's Problem

In this section, we focus on the principal's problem of choosing the contract elements w_1, w_2, and r aimed at maximizing her expected profit. By combining the expected profits for each potential tournament match weighted by their respective probabilities of occurrence, the principal's problem can be stated as follows:

$$\max_{w_1,w_2,r,e_A,e_B,e_{AA},e_{BB}} \quad 2p(1-p)[(1-r)(e_A+e_B)+G(\Delta e)\Pi_A+(1-G(\Delta e))\Pi_B]$$
$$+p^2[2(1-r)e_{AA}+\Pi_A]+(1-p)^2[2(1-r)e_{BB}+\Pi_B]-2w_1-w_2 \quad (13)$$

s.t. (5), (6), (11), (12) and

$$w_1+p(1-p)[G(\Delta e)(w_2+\delta)+re_A-c(e_A)]+(1-p)p[(1-G(\Delta e))w_2+re_B-c(e_B)]$$
$$+p^2[0.5(w_2+\delta)+re_{AA}-c(e_{AA})]+(1-p)^2[0.5w_2+re_{BB}-c(e_{BB})]\geq 0. \quad (14)$$

[10]We consider this as plausible, but are not aware of any empirical evidence.

Hence, the principal maximizes her expected profit subject to the incentive compatibility constraints for each potential tournament match, and the agents' participation constraint (14).

To minimize costs, the principal sets w_1 such that (14) binds. Consequently, we can eliminate w_1 in the principal's objective function and thus, obtain the following simplified problem:

$$\max_{r,w_2,e_A,e_B,e_{AA},e_{BB}} \quad \Pi := \pi_{AB} + \pi_{AA} + \pi_{BB} \tag{15}$$

$$\text{s.t. (5), (6), (11), (12),}$$

with π_{kl}, $k, l \in \{A, B\}$, as the principal's expected profit for a tournament match where one agent is of type k and the other agent of type l, weighted by its occurrence probability. Formally,

$$\pi_{AB} := 2p(1-p)\left[e_A + e_B + \Pi_B + G(\Delta e)(\Pi_A - \Pi_B + \delta) - c(e_A) - c(e_B)\right], \tag{16}$$

$$\pi_{AA} := p^2\left[2e_{AA} + \Pi_A + \delta - 2c(e_{AA})\right], \tag{17}$$

$$\pi_{BB} := (1-p)^2\left[2e_{BB} + \Pi_B - 2c(e_{BB})\right]. \tag{18}$$

From a closer inspection of (16) – (18) it becomes clear that—under each tournament match—the principal obtains the entire surplus emerging from this employment relationship. Since agents do not possess private information prior to the contracting stage, their expected payments are tailored to just compensate them for their individual effort costs aimed at ensuring their participation. This in turn prevents both agents to extract an economic rent.

Given a specific tournament match, the corresponding surplus maximizing effort levels e_A^*, e_B^*, e_{AA}^*, and e_{BB}^* are characterized by the following first-order conditions:

$$c'(e_{AA}^*) = c'(e_{BB}^*) = 1, \tag{19}$$

$$c'(e_A^*) = 1 + g(\Delta e)(\Pi_A - \Pi_B + \delta), \tag{20}$$

$$c'(e_B^*) = 1 - g(\Delta e)(\Pi_A - \Pi_B + \delta). \tag{21}$$

We can infer from (19) that agents in AA-matches should implement the same effort levels as agents in BB-matches. This follows from the fact that both agents share the same abilities for conducting the manufacturing tasks, and there is no benefit of promoting a particular agent to the management position. In an AB-match however, it is beneficial from the principal's perspective to induce the type A agent to work relatively harder than the type B agent, i.e. $e_A^* > e_B^*$. This in turn provides the more suitable type A agent a higher chance of getting promoted to the management position.

The principal however, cannot observe the agents' respective types, and is thus not able to perfectly tailor incentives for each potential tournament match separately. This implies further that she cannot induce the surplus maximizing effort intensities and hence, is compelled to accept inefficiently chosen effort levels. Specifically, the incentive compatibility constraints (5) and (6) suggest that the chosen effort levels in an AA-match exceed the implemented effort intensities in a BB-match. Furthermore, by (11) and (12), we have

$$c'(e_A) - c'(e_B) = g(\Delta e)\delta. \tag{22}$$

In conjunction with (20) and (21), it follows

$$c'(e_A) - c'(e_B) < c'(e_A^*) - c'(e_B^*). \tag{23}$$

Accordingly, the principal is not able to motivate both agents to implement the efficient effort levels e_A^* and e_B^* for an AB-match either. To summarize the previous observations, the information asymmetry with respect to the agents' respective types prevents the principal to induce the appropriate effort levels for each potential tournament match. The subsequent proposition further clarifies the agents' relative effort choices conditional on their particular types.

Proposition 2 *Suppose the optimal contract is characterized by $r^*, w_2^* > 0$. Then, $e_{AA} > e_{AA}^* = e_{BB}^* > e_{BB}$, i.e. agents in an AA-match exert too much, and agents in a BB-match exert too little effort. In contrast, $e_A < e_A^*$ and $e_B^* < e_B$, i.e. the type A agent works too little* *while the type B agent works too hard in an AB-match. Hence, type B's promotion probability is inefficiently high.*

For a tournament with heterogenous agents, observe that condition (23) does not necessarily rule out the possibility of implementing the efficient effort difference $e_A^* - e_B^*$. To do so however, the agents' respective effort intensities would be inefficiently low. As pointed out by proposition 2, trading off incentive and selection issues in a heterogenous tournament calls for inducing an inefficiently low effort difference. As a result, type A works too little, while type B works too hard. This in turn provides a theoretical underpinning of the Peter Principle: The less suitable type B agent is 'too often' promoted to the management level. The principal deliberately accepts this supposed inefficiency as a necessary consequence of balancing selection and incentive effects appropriately.

Recall that r and w_2 are substitutes with respect to the incentive provision in the production stage. Accordingly, the principal can choose among infinite possible combinations of r and w_2 inducing the same effort levels in the AB at the same costs. To maximize her expected profit, the principal selects the combination that yields the highest expected profit in the event of homogenous tournaments. Technically, the marginal benefit of lowering incentives and thus decreasing e_{AA} should equal the marginal benefit of increasing incentives and thereby enhancing e_{BB}. However, the information asymmetry with respect to the agents' types prevents the principal to perfectly tailor the incentive provision. As a result, agents implement inefficiently high effort in AA-matches, and inefficiently low effort in BB-matches.[11]

The subsequent proposition exposes how the principal adjusts the provision of incentives when selection issues become more important.

Proposition 3 *Let* (r^*, w_2^*) *denote the contract elements that solve* (15)*. Then, the difference* $\Pi_A - \Pi_B$ *has the following effects on the optimal contract:*

[11]Formally, equations (75) and (76) represented in the appendix imply that the principal chooses r and w_2 such that $\frac{\partial \pi_{AB}}{\partial r} = \frac{\partial \pi_{AB}}{\partial w_2} = 0$ and $\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0$.

(i) If the optimal contract is characterized by $r^* > 0$ and $w_2^* > 0$,

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} = -g(0)\frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)}, \tag{24}$$

where

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} < 0. \tag{25}$$

Hence, e_A and e_B are decreasing in $\Pi_A - \Pi_B$, whereas $\Delta e = e_A - e_B$ is increasing. In contrast, the effort levels e_{AA} and e_{BB} are not affected by $\Pi_A - \Pi_B$.

(ii) If $\Pi_A - \Pi_B$ is sufficiently large, the principal sets $r^* = 0$.

If it becomes more important for the principal to promote the more suitable agent A to the management position, she decreases the piece rate r and enhances the tournament prize w_2. The intuition of this result is as follows. As exposed by proposition 1, abating r and w_2 improves the *selection effect* of the utilized promotion tournament. Intuitively, both contract elements should therefore be diminished in response to an augmented importance of the *selection effect* characterized by $\Pi_A - \Pi_B$. However, this would inevitably deteriorate the *incentive effect*, leading both agents to implement inefficiently low effort. Note however, that effort in homogenous tournaments should not be affected by a change of $\Pi_A - \Pi_B$ because selection issues are then irrelevant. From the agents' incentive compatibility constraints (5), (6), (11), and (12), and the fact that $g(\Delta e) < g(0)$, it can be inferred that adjusting the tournament price w_2 has a weaker marginal effect on agents' effort choices in the heterogenous as compared to homogenous tournament matches. By contrast, the marginal effect of adjusting the piece rate scheme r is identical across all potential tournament matches. Hence, the improvement of the *selection effect* in a tournament with heterogenous agents can be only accomplished by lowering the piece rate scheme r. This adjustment eventually motivates the type A agent to implement a relatively higher effort intensity than the type B agent, resulting in a higher probability that A wins the promotion tournament. Nonetheless, to keep the agents' effort intensities in a homogenous tournament match constant, the principal contemporaneously enhances the tournament price w_2, as emphasized by condition (24). Finally, if promoting the type A agent to the management position becomes sufficiently important, the principal utilizes only the tournament price w_2 to provide the agents with appropriate incentives. Omitting the provision of a piece rate scheme r is optimal from the principal's perspective since it would compromise the *selection effect*.

5 Discussion

In this section, we discuss some of our assumptions in light of how they affect our results exposed by propositions 1 and 3.[12]

Throughout this paper, we elaborated on an employment relationship where two randomly recruited agents share the same skills to conduct the required manufacturing tasks. In contrast to the preceding analysis, suppose for a moment that type A's costs of implementing effort in the production stage modifies to $\alpha c(e_i)$, $\alpha > 0$, whereas B's effort costs persist with $c(e_j)$. Apparently, if $\alpha < 1$, type A is not only more suitable for the management position, but also more capable of conducting management tasks. Otherwise, a type B agent would perform the manufacturing tasks relatively more efficient than an agent of type A. Whenever $\alpha > 1$, we assume that B's comparative advantage in conducting manufacturing tasks is not too significant such that—in equilibrium—agent A still implements a relatively higher effort intensity.[13] Then, for a heterogenous tournament match, agent A's winning probability is decreasing in the tournament price w_2 and piece rate r if and only if[14]

$$c''(e_B) - \alpha c''(e_A) < 0. \tag{26}$$

[12] The subsequently discussed modifications however, generally do not affect the results pointed out by proposition 2. Only if agents differ in their abilities to conduct pure manufacturing tasks, we would observe that $e^*_{AA} \neq e^*_{BB}$.

[13] Otherwise, we would obtain the counterintuitive result that the principal prefers to promote the worse performing agent to the management level, who will be on average the type A agent.

[14] This can be directly inferred from (31) and (35) represented in the appendix.

Given the underlying assumption that marginal cost of effort increases disproportionately ($c'''(\cdot) > 0$), (26) is always satisfied as long as $\alpha > 1$. In this case, improving the *incentive effect* through enhancing either r or w_2 results in an even stronger deterioration of the *selection effect* in comparison to the event of identical abilities for conducting manufacturing tasks. Hence, the results exposed by propositions 1 and 3 remain valid. If $\alpha > 1$ however, condition (26) can be violated, which would entail two important implications. First, intensifying incentives would improve the *selection effect* such that the result pointed out by proposition 1 reverses. Second, the principal would respond to an enhanced importance of the *selection effect* $(\Pi_A - \Pi_B)$ by increasing the piece rate scheme r and decreasing the tournament price w_2. The same observations can be made in the event that both agents share the same abilities for conducting the manufacturing tasks ($\alpha = 1$), but marginal effort costs are no longer disproportionately increasing (i.e. $c'''(\cdot) < 0$).

Finally, we focused on the case where the more suitable agent for the management position is characterized by a higher productivity *and* faces relatively lower costs of conducting the required management tasks. Typically, only one of these characteristics is assumed to differentiate among various types of agents. The necessity of this assumption is founded on the fact that—in our framework—enforceable incentive contracts for the management stage are not feasible. Hence, the agent implements a minimum required effort level in the management stage and obtains a fixed compensation in return. Incorporating both agent-specific characteristics in our framework thus ensures that a type A agent benefits relatively more from getting promoted to the management position than a type B agent. In an extended framework where incentive contracts are feasible at the management stage, it would suffice when type A is either more productive or more cost efficient in performing the management tasks. With such an extension, the promoted agent could choose from a menu of incentive contracts as considered in standard adverse selection models. Independent of being either more productive or more cost efficient, a high-skilled type extracts a higher economic rent than a low-skilled type under their respectively preferred incentive contracts. Accordingly, type A still benefits relatively more from being promoted to the management position as compared to type B. Furthermore, despite extracting a higher rent, type A contributes relatively more to firm value. Hence, the principal still prefers to utilize a promotion tournament aimed at enhancing the likelihood of assigning the management tasks to the more suitable type A agent.

6 Conclusion

The provision of individual incentive payments and the use of promotion tournaments in agency relationships are thoroughly analyzed by extent literature. As a next logical and imperative step, this paper investigates the optimal combination of these incentive devices aimed at motivating efficiently high effort (*incentive effect*) and matching heterogenous agents with jobs their are most suitable for (*selection effect*).

The analysis in this paper brings to light that—in a context with heterogenous agents—promotion tournaments and individual performance pay are substitutes in the provision of incentives. Specifically, the principal puts more emphasis on the tournament scheme relative to individual performance payments when selecting the more suitable candidate for promotion becomes more important. If selection is sufficiently crucial for the efficiency of an organization, the principal is found to even refrain from providing agents with individual performance payments. The rationale for this observation is that individual rewards dilute the *selection effect* of promotion tournaments. We thus provide a theoretical underpinning of the phenomenon that individual incentive payments are less often applied in practice than theory predicts.

This paper further demonstrates that—in a tournament match with heterogenous agents—the agent less suited for promotion implements too much effort, whereas the more capable agent does not work hard enough. This leads to the conclusive observation that the less suited agent has an inefficiently high prospect of promotion. Nevertheless, the emergence of this form of the Peter Principle is deliberately accepted by the principal as a natural consequence of efficiently balancing incentive and selection issues in an employment relationship with hidden action and hidden characteristics of agents.

7 Appendix

Proof of Proposition 1.

From (11) and (12) we obtain

$$H \begin{pmatrix} \frac{\partial e_A}{\partial r} \\ \frac{\partial e_B}{\partial r} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \tag{27}$$

where

$$H := \begin{pmatrix} g'(\Delta e)(W_2+\delta) - c''(e_A) & -g'(\Delta e)(W_2+\delta) \\ g'(\Delta e)W_2 & -g'(\Delta e)W_2 - c''(e_B) \end{pmatrix}. \tag{28}$$

Since $\Delta e > 0$, we have $g'(\Delta e) < 0$. With (7) and (8) it follows that $\det(H) > 0$. Applying Cramer's Rule to (27) yields

$$\frac{\partial e_A}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_B)}{\det(H)} > 0, \tag{29}$$

$$\frac{\partial e_B}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_A)}{\det(H)} > 0. \tag{30}$$

Consequently,

$$\frac{\partial \Delta e}{\partial r} = \frac{c''(e_B) - c''(e_A)}{\det(H)}. \tag{31}$$

Recall that $c'''(\cdot) > 0$. Thus, $c''(e_B) < c''(e_A)$, which further implies that Δe is decreasing in r. Next, (11) and (12) lead to

$$H \begin{pmatrix} \frac{\partial e_A}{\partial w_2} \\ \frac{\partial e_B}{\partial w_2} \end{pmatrix} = \begin{pmatrix} -g(\Delta e) \\ -g(\Delta e) \end{pmatrix}. \tag{32}$$

By utilizing (29) and (30) we obtain

$$\frac{\partial e_A}{\partial w_2} = g(\Delta e)\frac{\partial e_A}{\partial r}, \tag{33}$$

$$\frac{\partial e_B}{\partial w_2} = g(\Delta e)\frac{\partial e_B}{\partial r}. \tag{34}$$

Consequently,

$$\frac{\partial \Delta e}{\partial w_2} = g(\Delta e)\frac{\partial \Delta e}{\partial r} < 0. \tag{35}$$

□

Proof of Proposition 2.

Let $\mathcal{L}$ denote the Lagrangian of problem (15), and $\lambda_1, \ldots, \lambda_4$ the Lagrange multipliers for the constraints (5), (6), (11), and (12), respectively. The corresponding first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial r} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \tag{36}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = [\lambda_1 + \lambda_2]\, g(\Delta e) + [\lambda_3 + \lambda_4]\, g(0) = 0, \tag{37}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_A} &= 2p(1-p)\left[1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A)\right] \\ &\qquad + \lambda_1\left[g'(\Delta e)(w_2 + \delta) - c''(e_A)\right] + \lambda_2 g(\Delta e) w_2 = 0,\end{aligned} \tag{38}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_B} &= 2p(1-p)\left[1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B)\right] \\ &\qquad - \lambda_1 g'(\Delta e)(w_2 + \delta) - \lambda_2\left[g'(\Delta e) w_2 + c''(e_B)\right] = 0,\end{aligned} \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial e_{AA}} = 2p^2\left[1 - c'(e_{AA})\right] - \lambda_3 c''(e_{AA}) = 0, \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial e_{BB}} = 2(1-p)^2\left[1 - c'(e_{BB})\right] - \lambda_4 c''(e_{BB}) = 0. \tag{41}$$

Since $g(\Delta e) < g(0)$, we can infer from (36) and (37) that $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0$. Suppose for a moment that $\lambda_3 = \lambda_4 = 0$. In this case, (40) and (41) would imply that $c'(e_{AA}) = c'(e_{BB})$, which is a contradiction to (5) and (6). Thus, $\lambda_3 = -\lambda_4 \neq 0$. Consequently, one of the first terms in (40) and (41) must be negative while the other is positive. Moreover, we can infer from (5) and (6) that $c'(e_{AA}) > c'(e_{BB})$. Therefore, $1 - c'(e_{AA}) < 0$ and $1 - c'(e_{BB}) > 0$, implying that $e_{AA} > e^*_{AA}$ and $e_{BB} < e^*_{BB}$.

Next, suppose for a moment that $\lambda_1 = \lambda_2 = 0$. Then, (38) and (39) in conjunction with (5) and (6) imply

$$1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) = g(\Delta e)(w_2 + \delta) + r, \quad (42)$$

$$1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) = g(\Delta e)w_2 + r. \quad (43)$$

By subtracting the second from the first equation, we obtain $2(\Pi_A - \Pi_B) + \delta = 0$, which is a contradiction since $\Pi_A > \Pi_B$ and $\delta > 0$. Thus, $\lambda_1 = -\lambda_2 \neq 0$. By using this observation, (38) and (39) can be transformed to

$$2p(1-p)\underbrace{[1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A)]}_{:=F1} + \lambda_1 \underbrace{[g'(\Delta e)\delta - c''(e_A)]}_{:=F2} = 0, \quad (44)$$

$$2p(1-p)\underbrace{[1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B)]}_{:=G1} - \lambda_1 \underbrace{[g'(\Delta e)\delta - c''(e_B)]}_{:=G2} = 0. \quad (45)$$

Observe that either $F2$ or $G2$ is negative, while to other is positive. Then, $F1$ and $G1$ must have the respective opposite signs. Transforming these terms by utilizing (11) and (12) yields

$$1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A) = 1 + g(\Delta e)(\Pi_A - \Pi_B - w_2) - r \equiv \phi_1, \quad (46)$$

$$1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B) = 1 - g(\Delta e)(\Pi_A - \Pi_B + w_2 + \delta) - r \equiv \phi_2. \quad (47)$$

Since $\phi_1 > \phi_2$ it follows

$$1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A) > 0, \quad (48)$$

$$1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B) < 0. \quad (49)$$

As a result, $e_A < e_A^*$ and $e_B > e_B^*$. □

Proof of Proposition 3.

This proof proceeds in two steps: (*i*) we identify the change of r^* and w_2^* in $\Pi_A - \Pi_B$; and (*ii*), we demonstrate that $r^* = 0$ if $\Pi_A - \Pi_B$ is sufficiently large.

(i) The principal's maximization problem can be further simplified to

$$\max_{r,w_2} \Pi(r, w_2) = \pi_{AB}(r, w_2) + \pi_{AA}(r, w_2) + \pi_{BB}(r, w_2), \tag{50}$$

where $\pi_{AB}(r, w_2)$, $\pi_{AA}(r, w_2)$, and $\pi_{BB}(r, w_2)$ are defined as in (16) – (18). The only difference is that e_A, e_B, e_{AA}, and e_{BB} are now expressed as functions of r and w_2 implicitly given by (5), (6), (11), and (12). We assume that the functional forms of $\pi_{AB}(r, w_2)$, $\pi_{AA}(r, w_2)$, and $\pi_{BB}(r, w_2)$ are such that $\Pi(r, w_2)$ is concave for all $p \in (0, 1)$. The optimal contract elements $r^*, w_2^* > 0$ are characterized by the following first-order conditions:

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0, \tag{51}$$

$$\frac{\partial \Pi}{\partial w_2} = \frac{\partial \pi_{AB}}{\partial w_2} + \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0. \tag{52}$$

With $y \in \{r, w_2\}$, we obtain

$$\frac{\partial \pi_{AB}}{\partial y} = 2p(1-p)\left[(1 - c'(e_A))\frac{\partial e_A}{\partial y} + (1 - c'(e_B))\frac{\partial e_B}{\partial y} + g(\Delta e)\frac{\partial(\Delta e)}{\partial y}(\Pi_A - \Pi_B + \delta)\right], \tag{53}$$

$$\frac{\partial \pi_{AA}}{\partial y} = 2p^2[1 - c'(e_{AA})]\frac{\partial e_{AA}}{\partial y}, \tag{54}$$

$$\frac{\partial \pi_{BB}}{\partial y} = 2(1-p)^2[1 - c'(e_{BB})]\frac{\partial e_{BB}}{\partial y}. \tag{55}$$

Then, (51) and (52) imply

$$K \begin{pmatrix} \frac{\partial r}{\partial(\Pi_A - \Pi_B)} \\ \frac{\partial w_2}{\partial(\Pi_A - \Pi_B)} \end{pmatrix} = \begin{pmatrix} -2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r} \\ -2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial w_2} \end{pmatrix}, \tag{56}$$

where

$$K := \begin{pmatrix} \frac{\partial^2 \Pi}{\partial r^2} & \frac{\partial^2 \Pi}{\partial r \partial w_2} \\ \frac{\partial^2 \Pi}{\partial r \partial w_2} & \frac{\partial^2 \Pi}{\partial w_2^2} \end{pmatrix}. \tag{57}$$

Since $\Pi(\cdot)$ is concave, K must be negative definite. Thus, $\det(K) > 0$. Moreover, we can infer from (33) and (34) that $\frac{\partial(\Delta e)}{\partial w_2} = g(\Delta e)\frac{\partial(\Delta e)}{\partial r}$. Hence, applying Cramer's Rule to (56) yields

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} \det(K) \;=\; 2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r} \underbrace{\left[g(\Delta e)\frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2}\right]}_{:=F}, \tag{58}$$

$$\frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \det(K) \;=\; 2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r} \left[\frac{\partial^2 \Pi}{\partial r \partial w_2} - g(\Delta e)\frac{\partial^2 \Pi}{\partial r^2}\right]. \tag{59}$$

As a next step, we transform F. To do so, we utilize following relationships, where $i = A, B$:

$$\frac{\partial e_i}{\partial w_2} \;=\; g(\Delta e)\frac{\partial e_i}{\partial r}, \tag{60}$$

$$\frac{\partial e_{ii}}{\partial w_2} \;=\; g(0)\frac{\partial e_{ii}}{\partial r}, \tag{61}$$

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} \;=\; g(0)\frac{\partial^2 e_{ii}}{\partial r^2} = \frac{1}{g(0)}\frac{\partial^2 e_{ii}}{\partial w_2^2}. \tag{62}$$

Equation (60) is identical to (33) and (34), respectively. Equation (61) can be obtained by applying the implicit function theorem to (5) and (6), respectively. The last equation can be derived as subsequently exposed. From (5) and (6) we obtain $\frac{\partial e_{ii}}{\partial r} = 1/c''(e_{ii})$ and thus,

$$\frac{\partial^2 e_{ii}}{\partial r^2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial r}. \tag{63}$$

Therefore,

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2} g(0)\frac{\partial e_{ii}}{\partial r} = g(0)\frac{\partial^2 e_{ii}}{\partial r^2}. \tag{64}$$

Analogously, $\frac{\partial e_{ii}}{\partial w_2} = g(0)/c''(e_{ii})$ and hence,

$$\frac{\partial^2 e_{ii}}{\partial w_2^2} = -g(0)\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2}. \tag{65}$$

Consequently,

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2} = \frac{1}{g(0)}\frac{\partial^2 e_{ii}}{\partial w_2^2}. \tag{66}$$

Now we can make the following transformation:

$$\begin{aligned} g(\Delta e)\frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2} &= g(\Delta e)\left(\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{AA}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{BB}}{\partial r \partial w_2}\right) \\ &\quad - \left(\frac{\partial^2 \pi_{AB}}{\partial w_2^2} + \frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right), & (67) \\ &= g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} + \left(\frac{g(\Delta e)}{g(0)} - 1\right)\left(\frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right), & (68) \\ &= g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} \\ &\quad - g(0)\left(g(0) - g(\Delta e)\right)\left(\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right). & (69) \end{aligned}$$

To obtain (68) and (69) we used that

$$\frac{\partial^2 \pi_{AA}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{BB}}{\partial r \partial w_2} = \frac{1}{g(0)}\left[\frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right] = g(0)\left[\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right], \tag{70}$$

which follows directly from (61) and (62). Furthermore, by utilizing (60), it can be shown that

$$\begin{aligned} g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} &= 2p(1-p)\Bigg\{[1 - c'(e_A) + g(\Delta e)(\Pi_A - \Pi_B + \delta)]\left[g(\Delta e)\frac{\partial^2 e_A}{\partial r \partial w_2} - \frac{\partial^2 e_A}{\partial w_2^2}\right] \\ &\quad + [1 - c'(e_B) - g(\Delta e)(\Pi_A - \Pi_B + \delta)]\left[g(\Delta e)\frac{\partial^2 e_B}{\partial r \partial w_2} - \frac{\partial^2 e_B}{\partial w_2^2}\right]\Bigg\}. & (71) \end{aligned}$$

We can infer from (60) that

$$\frac{\partial^2 e_i}{\partial w_2^2} = g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\frac{\partial e_i}{\partial r} + g(\Delta e)\frac{\partial^2 e_i}{\partial r \partial w_2}. \tag{72}$$

Thus, (71) can be rewritten as

$$\begin{aligned} g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} &= 2p(1-p)g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\left\{[1 - c'(e_A) + g(\Delta e)(\Pi_A - \Pi_B + \delta)]\frac{\partial e_A}{\partial r}\right. \\ &\quad \left. + [1 - c'(e_B) - g(\Delta e)(\Pi_A - \Pi_B + \delta)]\frac{\partial e_B}{\partial r}\right\}, \qquad (73) \\ &= 2p(1-p)g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\frac{\partial \pi_{AB}}{\partial r}. \qquad (74) \end{aligned}$$

Next, (60) and (61) imply that the first-order conditions (51) and (52) are equivalent to

$$\begin{aligned} \frac{\partial \Pi}{\partial r} &= \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0, \qquad (75) \\ \frac{\partial \Pi}{\partial w_2} &= g(\Delta e)\frac{\partial \pi_{AB}}{\partial r} + g(0)\left(\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r}\right) = 0. \qquad (76) \end{aligned}$$

Since $g(\Delta e) < g(0)$, we can infer from (76) that $\frac{\partial \pi_{AB}}{\partial r} = 0$ and $\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0$ at any interior solution. Using this observation with (74) yields

$$g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} = 0. \tag{77}$$

By utilizing (69), we finally obtain

$$g(\Delta e)\frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2} = -g(0)\left(g(0) - g(\Delta e)\right)\left(\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right). \tag{78}$$

Similarly, we can compute

$$\frac{\partial^2 \Pi}{\partial r \partial w_2} - g(\Delta e)\frac{\partial^2 \Pi}{\partial r^2} = \frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r^2} + (g(0) - g(\Delta e))\left\{\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right\} \quad (79)$$

$$= (g(0) - g(\Delta e))\left\{\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right\}, \quad (80)$$

where (80) follows from the fact that $\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r^2} = 0$. Combining (58), (59), (78), and (80) eventually yields (24).

Since $\Pi = \pi_{AA}$ for $p = 1$ and $\Pi = \pi_{BB}$ for $p = 0$, concavity of Π for all $p \in [0, 1]$ implies concavity of π_{AA} and π_{BB}. Thus, $\frac{\partial^2 \pi_{ii}}{\partial r^2} < 0$. By using this observation together with $\frac{\partial \Delta e}{\partial r} < 0$, we can infer that $\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} < 0$.

Finally, by using (24) in a comparative statics analysis of (5), (6), (11), and (12), one can verify that

$$\frac{\partial e_A}{\partial(\Pi_A - \Pi_B)}, \ \frac{\partial e_B}{\partial(\Pi_A - \Pi_B)} < 0, \quad \frac{\partial(e_A - e_B)}{\partial(\Pi_A - \Pi_B)} > 0, \quad \frac{\partial e_{AA}}{\partial(\Pi_A - \Pi_B)} = \frac{\partial e_{BB}}{\partial(\Pi_A - \Pi_B)} = 0.$$

(ii) Recall first that $\frac{\partial \Delta e}{\partial r} < 0$. Then, we can infer from (51) and (53) – (55) that one can find a pair $(\hat{\Pi}_A, \hat{\Pi}_B)$ such that $\max_{w_2} \frac{\partial \Pi}{\partial r}\big|_{r=0} < 0$ for all $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. Since Π is concave, $\frac{\partial \Pi}{\partial r} < 0$ for all $r > 0$ and $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. Hence, $r^* = 0$ for all $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. □

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